HANDWRITTEN NOTES

OF

(FLUID MECHANICS)

BY

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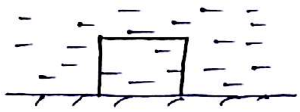
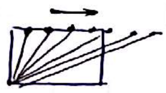
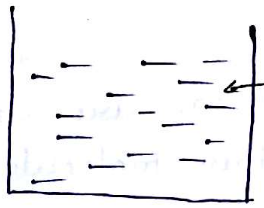
- Solid → ∀ fixed and shape fixed
- liquid → ∀ fixed & shape not fixed
- gas → ∀ not fixed & shape not fixed
- Plasma

Fluid → liquid
→ gas

which deforms continuously (flow) under the action of shear force/ tangential force.







not in a position to flow
⇒ Shear force = zero.

→ If a fluid is static ⇒ shear force = zero.

" liquid and gases both are having the same property by virtue of which they deforms continuously under the action of shear or tangential force.
that means - they are having the property to flow under the action of these forces.

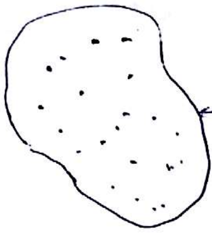
just bcz of having flow property, which is not present in the case of solid, liquid & gases are kept in a different category known as fluid.

Note - In a static fluid the development of the shear stress b/w the layer will always be equal to zero.

Fluid as continuum :-

⇓

fluid as continuous matter.



1) Macroscopic

⇓

Classical mechanics

whatever the effect of input of one ~~system~~ molecule. will be the same to all the molecule.

bcz intermolecular forces is close to zero.

eg. solid

2) Microscopic

⇓

Statistical mechanics - deals with every particular molecule in details.

" To analyse the system in one stroke we are assuming that adjaunt to one molecule the other molecule is there. there is no interspace b/w them.

∴ the fluid system as a whole can be treated as a continuous matter & this cont. matter known as continuum

Basic fluid properties :-

1) **Density (ρ)** :- It is defined as mass of substance per unit volume.

$$\boxed{\rho = \frac{m}{\forall}}$$ (Intensive property)

unit → MKS - kg/m^3

CGS - gm/cm^3 (gm/cc)

$$\boxed{1\frac{kg}{m^3} = \frac{1000 gm}{10^6 cm^3} = 10^{-3} gm/cc}$$

Dimension - ML^{-3}

2) **Specific volume (ϑ)** :- The reciprocal of density is known as specific volume.

$$\boxed{\vartheta = \frac{\forall}{m}} \quad \frac{m^3}{kg}$$

→ Volume of given fluid per unit mass is known as specific volume. for a given mass, the volume of gas will be more than liquid. this is the reason why after turbine the hot gases are condensed into liquid.

bcz. for compressing gas (whose volume is very large) very large compressor is required.

3) **Specific wt. (w)** :- It is the wt. of substance per unit volume

$$\boxed{w = \frac{mg}{\forall} = \rho g} \quad \frac{N}{m^3}$$

dimension - $[ML^{-2}T^{-2}]$

4) **Specific gravity : (SG)** :- It is the ratio of density of fluid to the density of std. fluid.

$$\boxed{S.G = \frac{\text{density of fluid}}{\text{density of std fluid}}}$$

Std. fluid → liquid (H_2O) ← $\rho_{water} = 1000 \frac{kg}{m^3}$
→ gas (Air | H_2)

5) Relative density (R.D) :- It is density of fluid with respect to other fluid

$$\boxed{R.D = \frac{\rho_1}{\rho_2}}$$

6) Compressibility (β) :-

On an increase in pressure, if there is decrease in volume then it is called compressibility.

-ve sign is due to dec. in volume.

$$\beta = \frac{\left(-\frac{d\forall}{\forall}\right)}{dP}$$ ← if there is change in volume
← on increase in pressure

$$\Rightarrow \boxed{\beta = -\frac{1}{\forall}\left(\frac{d\forall}{dP}\right)} \quad \text{—— 1)}$$

→ mass $= \forall \times \rho$ = constant

differentiating wrt $\forall$

$$\Rightarrow \forall \frac{d\rho}{d\forall} + \rho = 0$$

$$\forall \frac{d\rho}{d\forall} = -\rho$$

$$\frac{d\rho}{\rho} = -\frac{d\forall}{\forall} \quad \text{—— 2)}$$

putting ② in ①

$$\Rightarrow \beta = \frac{d\rho}{\rho} \times \frac{1}{dP}$$

$$\boxed{\beta = \frac{1}{\rho}\frac{d\rho}{dP}} \longrightarrow$$ on change in pressure if there is change in density then it is called compressibility.

→ The reciprocal of compressibility is bulk modulus of elasticity (K)

Incompressible :-

$\boxed{\beta = 0}$ ⇒ ρ should be constant wrt pressure.

→ liquid

→ In general liquid are compressible.

→ 1 atm $\rho_{H_2O} = 998\ kg/m^3$

↓

100 atm $\rho_{H_2O} = 1003\ kg/m^3$

⇒ 100 times increase in pressure results 0.5% change in ρ_{H_2O}.

∴ Assumed - Incompressible

gas

→ Highly compressible

7) Isothermal compressibility of gas :-

Isothermal → $\boxed{T = \text{constant}}$

$P = \rho R T$

$\rho = \dfrac{P}{RT}$ —— 1)

But $\beta = \dfrac{1}{\rho}\left(\dfrac{d\rho}{dP}\right)$

diff. eq^n 1) w.r.t P

$\Rightarrow \dfrac{d\rho}{dP} = \dfrac{1}{RT}$

$\Rightarrow \beta = \dfrac{1}{\rho} \times \dfrac{1}{RT} = \dfrac{1}{\rho RT} = \dfrac{1}{P}$

$\Rightarrow \boxed{\beta = \dfrac{1}{P}}$

Note :-

$P\forall = nR_oT$ (universal const.)

$n = \dfrac{m}{M}$ — no. of mole; m ← mass; M ← molecular wt.

$\Rightarrow P\forall = \dfrac{m}{M} R_o T$

$P\forall = m\left(\dfrac{R_o}{M}\right)T$ — gas const (R)

$\boxed{P\forall = mRT}$

→ Isothermal bulk modulus of elasticity $= \dfrac{1}{\beta} = P$

8) Adiabatic compressibility of gas :-

For adiabatic, $\boxed{P\forall^{\gamma} = \text{const}}$

$\Rightarrow P\left(\dfrac{m}{\rho}\right)^{\gamma} = \text{const}$ (m ← const.)

$\Rightarrow P\dfrac{m^{\gamma}}{\rho^{\gamma}} = \text{const}$ (m^{γ} ← const.)

$\Rightarrow \dfrac{P}{\rho^{\gamma}} = \text{const}$

$\Rightarrow \dfrac{\rho^{\gamma}}{P} = \text{const}$

$\Rightarrow \rho^{\gamma} P^{-1} = \text{const}$

Note -

γ = adiabatic exponent

represent atomicity of gas (whether monoatomic or dia) diff. above eqⁿ w.r.t P

$$\rho^{\gamma}\left(-\frac{1}{P^2}\right) + \frac{1}{P}\gamma(\rho)^{\gamma-1}\cdot\frac{d\rho}{dP} = 0$$

$$\frac{\gamma}{\cancel{P}}(\rho)^{\gamma-1}\cdot\frac{d\rho}{dP} = \frac{\rho^{\gamma}}{P^{\cancel{2}}}$$

$$\frac{\rho^{\cancel{\gamma}}}{\rho}\cdot\frac{d\rho}{dP} = \frac{\rho^{\gamma}}{\gamma P}$$

$$\frac{d\rho}{dP} = \frac{\rho}{\gamma P} \quad \text{—— 2)}$$

$$\Rightarrow \beta = \frac{1}{\cancel{\rho}} \times \frac{\cancel{\rho}}{\gamma P}$$

$$\Rightarrow \boxed{\beta = \frac{1}{\gamma P}}$$

$$\longrightarrow \boxed{\frac{\beta_{adia}}{\beta_{iso}} = \frac{1/\gamma P}{1/P} = \frac{1}{\gamma}}$$

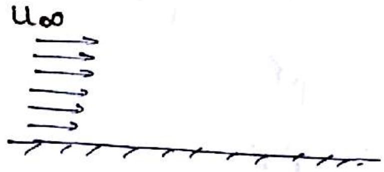
$$\longrightarrow \boxed{\frac{K_{adia}}{K_{iso}} = \gamma}$$ { where k = bulk modulus }

NOTE :- $\rho = f(P,T)$

$$\left[\begin{array}{l} P\uparrow \Rightarrow \forall\downarrow \Rightarrow \rho\uparrow \\ T\uparrow \Rightarrow \forall\uparrow \Rightarrow \rho\downarrow \end{array}\right.$$

Viscosity :-

The two adjacent layer of fluid resist the motion of each other. such a fundamental property of fluid is known as viscosity.

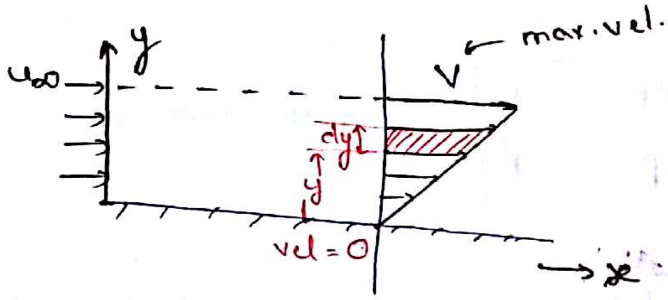


The layer of fluid which are in contact with surface has max. ~~fluctuation~~ friction force.

$\therefore$ the vel. of fluid which are in contact with surface is zero.

(i.e. velocity of layer wrt boundary will be same (that is zero relative velocity).

given by maxwell



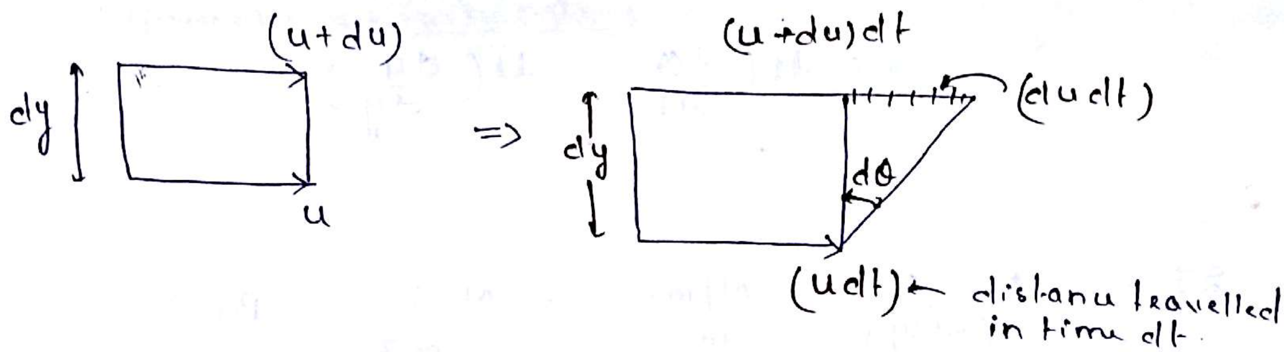
Direction y = transverse to flow

$\Downarrow$

In the transverse direction there is development of Velocity gradient

$\left(\frac{du}{dy}\right)$ ← change in vel. wrt. change in distance.

→ let us take a section of adjacent layer at a distance y from surface.



In dt time

$$\tan d\theta = \frac{du\,dt}{dy}.$$

$$d\theta = \frac{du\,dt}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

$\frac{d\theta}{dt}$ = Rate of shear / angular deformat^n

$\frac{du}{dy}$ = Vel. gradient in transverse direct^n of flow

Newton's law of viscosity :-

The shear stress b/w the layer of fluid,

$$\tau \propto \left(\frac{d\theta}{dt}\right)$$

$$\boxed{\tau = \mu\left(\frac{d\theta}{dt}\right)}$$

where μ = coefficient of viscosity
Absolute viscosity
Dynamic viscosity

μ → constant for fluid (depend on)

- Property of fluid — for same fluid μ is same but for diff. fluid μ is diff.
- funct^n of temp

$$\Rightarrow \boxed{\tau = \mu\left(\frac{d\theta}{dt}\right) = \mu\left(\frac{du}{dy}\right)}$$

μ units :-

SI $\rightarrow$ $\mu = \dfrac{\tau}{(du/dy)} = \dfrac{N/m^2}{\frac{m}{s\text{-}m}} = \dfrac{N\text{-}s}{m^2} = Pa\text{-}s$

MKS $\rightarrow$ $\dfrac{kg\text{-}m}{s^2} \cdot \dfrac{s}{m^2} = \dfrac{kg}{m\text{-}s}$

CGS $\rightarrow$ 1 Poise $= \dfrac{1\,gm}{cm\text{-}s} = \dfrac{10^{-3}\,kg}{10^{-2}\,m\text{-}s} = \cdot 1\,\dfrac{kg}{m\text{-}s} = \cdot 1\,pa\text{-}s$

$\rightarrow$

	fluid 1	fluid 2	
thin fluid	if $\left(\frac{d\theta}{dt}\right)$ = more	$\frac{d\theta}{dt} \rightarrow$ less	thick fluid
	$\Rightarrow$ $\tau \rightarrow$ ~~less~~ more	$\tau \rightarrow$ less	
	$\Rightarrow$ $\mu \rightarrow$ less	$\mu \rightarrow$ ~~more~~ high.	
	$\rightarrow$ flow is easy.	$\rightarrow$ difficult to flow	

Kinematic viscosity :- (ν)

It is also known as momentum diffusibility.

$$\boxed{\nu = \frac{\mu}{\rho}}$$

unit $\dfrac{m^2}{sec}$ (M.K.S) (SI)

CGS 1 stroke $= \dfrac{1\,cm^2}{sec} = 10^{-4}\,\dfrac{m^2}{s}$

Note :- kinematic viscosity (ν) is more imp than μ bcz.

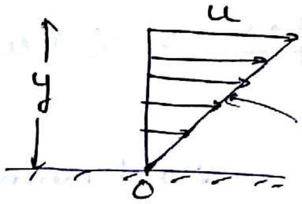
linearisation of newton law of viscosity :-

→ valid only if gap (y) is very very small

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{(u-0)}{y}$$

$$\boxed{\tau = \mu \frac{u}{y}}$$ ← y is very very small



linear only when y is very small.

→ $\tau = \mu \frac{u}{y}$

$$\frac{F_D}{A} = \mu \frac{u}{y}$$

{ F_D = drag force → the force acting b/w layer & surface

$$\boxed{F_D = \mu \frac{A u}{y}}$$

Variation of viscosity with temperature :-

Viscosity
↓
basic reason
↓
cohesion

liquid

- cohesion is dominant bcz intermolecular attract^n is more

→ when T↑ intermolecular attract^n is weak bcz intermolecular bond breaks ⇒ μ↓

gas

- cohesion → absent (almost)
- T↑ ⇒ randomness ↑ ⇒ addition resistance in flow ⇒ μ↑

$\Rightarrow$ $\boxed{\begin{array}{l} T\uparrow \ \mu\downarrow \ \text{(liquid)} \\ T\uparrow \ \mu\uparrow \ \text{(gas)} \end{array}}$

Effect of pressure on viscosity :-

→ liquid ⇒ There is no effect of pressure on viscosity bcz liquid is incompressible.

gas ⇒ (keeping temp. constant)

root mean square (collision) $\boxed{(\bar{C}) = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}}$

const.

$\Rightarrow \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$ ← on increasing pressure ⇓ volume decreases.

⇒ $\uparrow P \Rightarrow V\downarrow \Rightarrow \bar{C}$ (collision remain constant)

⇒ no effect of pressure on viscosity, keeping temp. constant.

→ $\bar{C} = \sqrt{\frac{3RT}{M}}$ ⇒ when $T\uparrow \Rightarrow C\uparrow \Rightarrow \mu\uparrow$

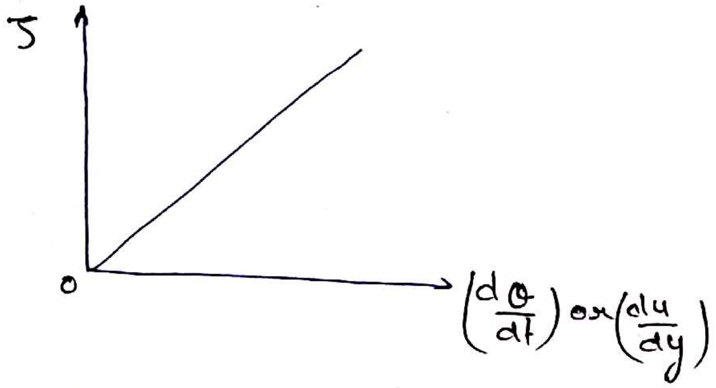
Different types of fluid :-

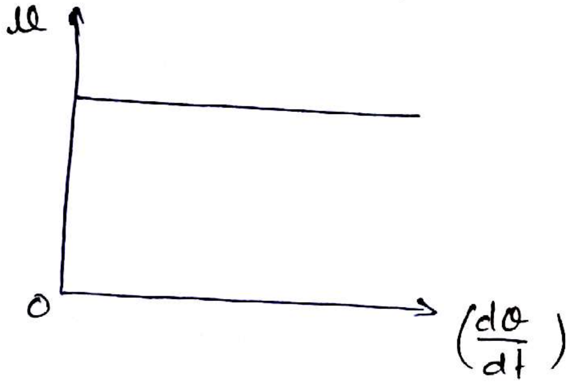
1) Newtonian fluid :-

the fluid which obeys newton law of viscosity is newtonian fluid.

$$\boxed{\tau = \mu \frac{du}{dy}}$$

eg. Air, water, oil, petrol, mercury.





2) Non-newtonian fluid :-

the fluid which donot obeys newton law of viscosity is non-newtonian fluid.

In general :-

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$

A, B const. & n const.

i) Dilatent fluid :-

If $\boxed{\begin{array}{l}\beta = 0 \\ n > 1\end{array}}$
&

$$\Rightarrow \tau = A\left(\frac{du}{dy}\right)^n$$

$$= A\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy}$$

$$\boxed{\tau = \mu_{app}\frac{du}{dy}}$$ where $\mu_{apparent} = A\left(\frac{du}{dy}\right)^{n-1}$

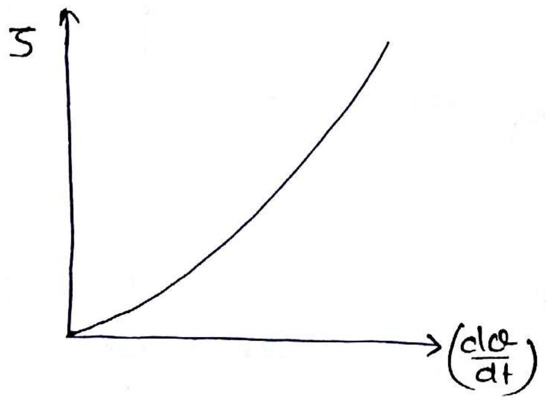
$\Downarrow$

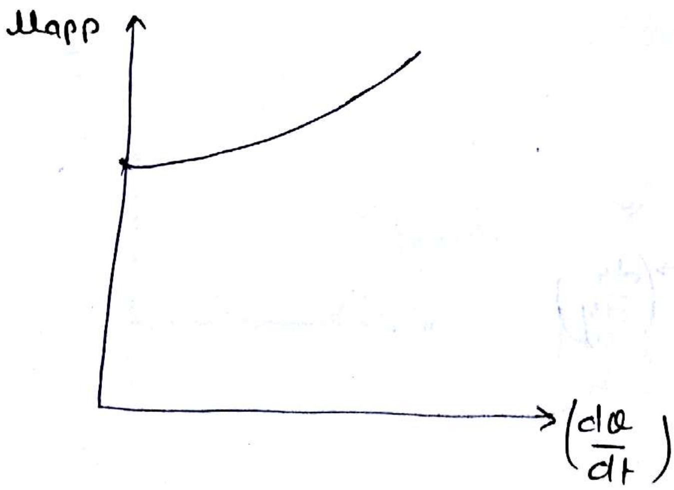
as $n > 1 \Rightarrow$ on $\uparrow\left(\frac{du}{dy}\right) \Rightarrow \mu_{app}\uparrow$

$\Rightarrow$ flow become hard

$\therefore$ also known as shear thickening fluid

eg. Sugar-water mixture, Rich-starch.





ii) Pseudo-plastic fluid :-

If $\boxed{\begin{matrix}\beta = 0 \\ n < 1\end{matrix}}$

$$\tau = A\left(\frac{du}{dy}\right)^n$$

$$= A\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy}$$

$$\boxed{\tau = \mu_{ap}\frac{du}{dy}}$$

as $n < 1$

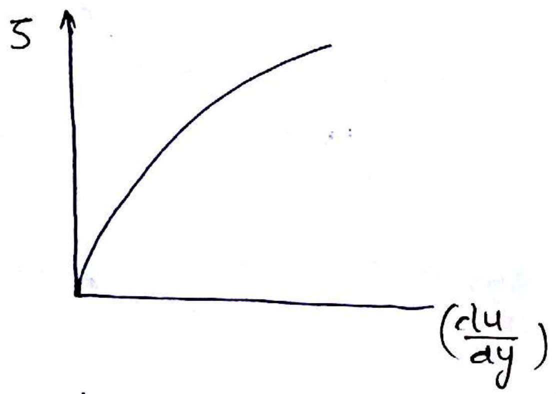
⇒ on ↑ $\left(\frac{du}{dy}\right)$

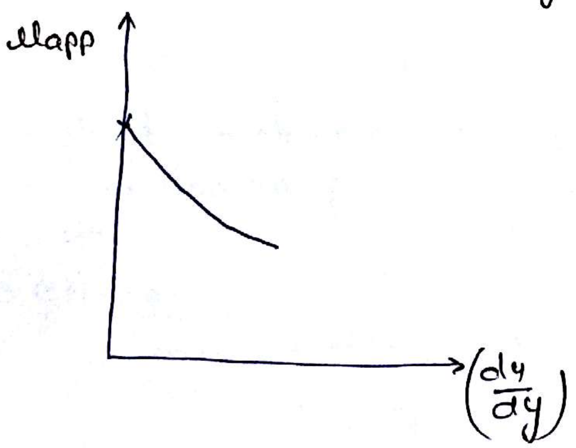
⇒ μ ↓

⇒ ∴ also known as <u>shear-thinning fluid</u>

eg. milk, blood.

imp



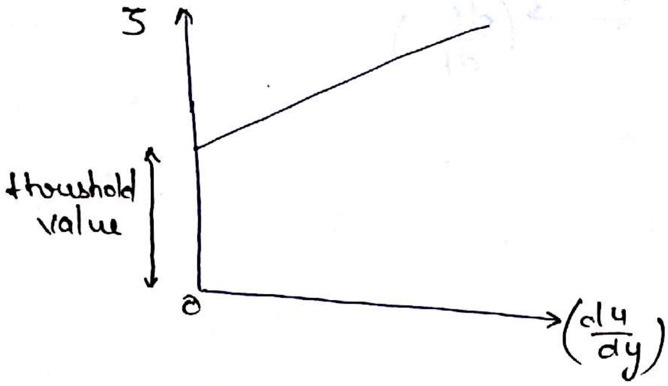


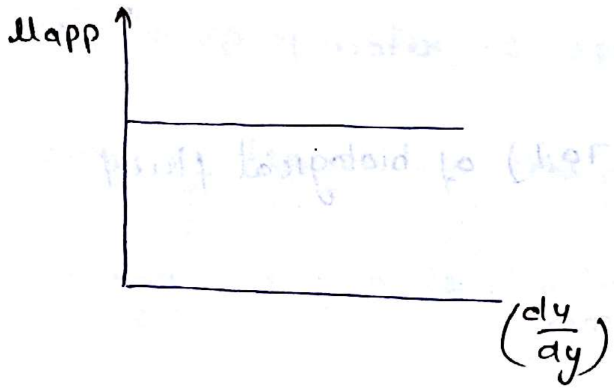
(iii) Bingham - plastic fluid :-

if $\boxed{\begin{matrix} B > 0 \\ n = 1 \end{matrix}}$

$$\boxed{\tau = A\left(\frac{du}{dy}\right) + B}$$ due to $B > 0$, here some min value (threshold) is required to start

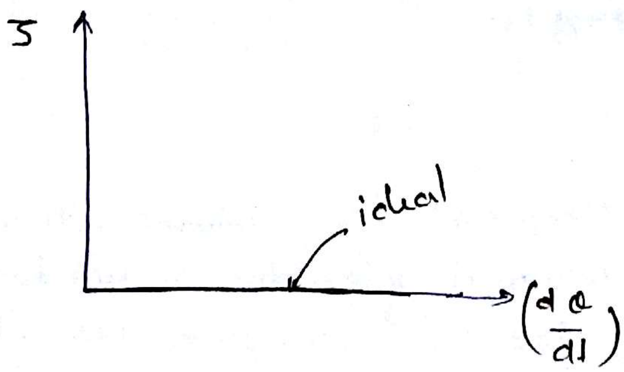
eg: tooth paste, gel.





③ Ideal fluid :-

those fluid



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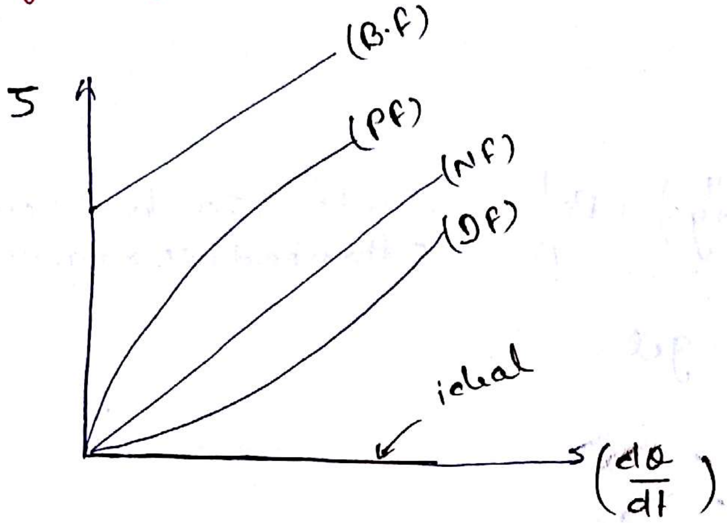
combining all graphs :-



Two other types of fluid are :-

4) Thixotropic fluid :-

when time ↑ μ ↓

eg) Paints

5> Rheopectic fluid :- (60-70%) of biological fluid

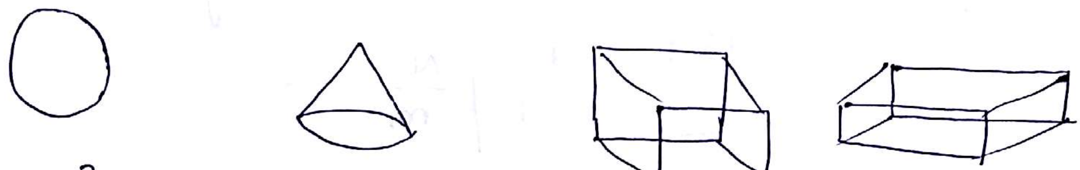
μ ↑ when time ↑

eg.> fevicol, cement-water mix.

Surface tension :-

"every fluid is having very imp. property by virtue of which it tries to minimize its surface area upto a max. limit. this fundamental property of fluid known as surface tensⁿ".

experimently



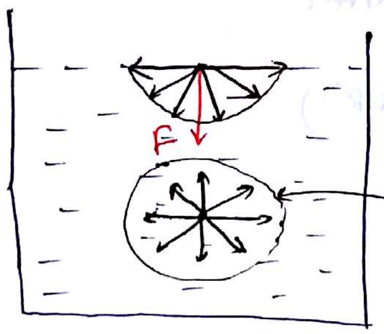
Volume -	$1 m^3$	$1 m^3$	$1 m^3$	$1 m^2$
find -	r	r, h	a	a, b, h
find area -	A = ?	A = ?	A = ?	A = ?

(A = ? of sphere → min. area)

∴ each drop of water is spherical.

→ Surface tension → tension due to surface

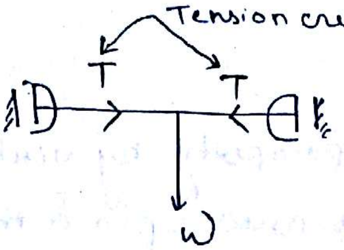
Surface tension is due to cohesive force



→ on the surface, the horizontal component of forces is cancelled. but due to vertically downward component of force, there will be net force (F) will act downward.

This force (F) Pulls the horizontal component in downward directⁿ (or) Pulls surface force in downward directⁿ which creates tension.

eg.



→ Surface Tension = force per unit length

$$\boxed{S.T = \frac{F}{l}} \ \frac{N}{m}$$

→ surface energy = T·A

ω = change in surface energy

$$\Rightarrow \boxed{\omega = T(\Delta A)}$$

Pb)

find change in energy

Sol^n

Initial volume = final volume

$$\frac{4}{3}\pi R^3 = 1000\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \boxed{r = \frac{R}{10}}$$

$$(Area)_{initial} = 4\pi R^2$$

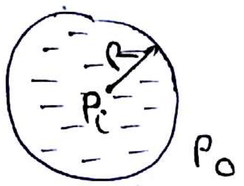
$$(A)_{final} = 1000 \times 4\pi\left(\frac{R}{10}\right)^2 \Rightarrow A_f = 40\pi R^2$$

$$\omega = E_f - E_i = T(A_f - A_i)$$

$$= T(40\pi R^2 - 4\pi R^2)$$

$$\boxed{\omega = T \times 36\pi R^2} \text{ Joule.}$$

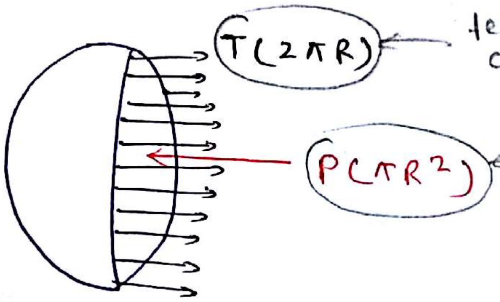
Formation of a drop :-



$\Rightarrow$ $P_i - P_o$ = excess pressure inside the drop

$\Rightarrow$ $\boxed{P_i - P_o = P}$

$\rightarrow$ let us divide the drop in section.



tensile force due to surface tension acting around the circumference of portion.

pressure force on area πR^2

At equilibrium,

$$T(2\pi R) = P(\pi R^2)$$

$$\Rightarrow \boxed{P = \frac{2T}{R}} = \text{excess pressure inside the drop.}$$

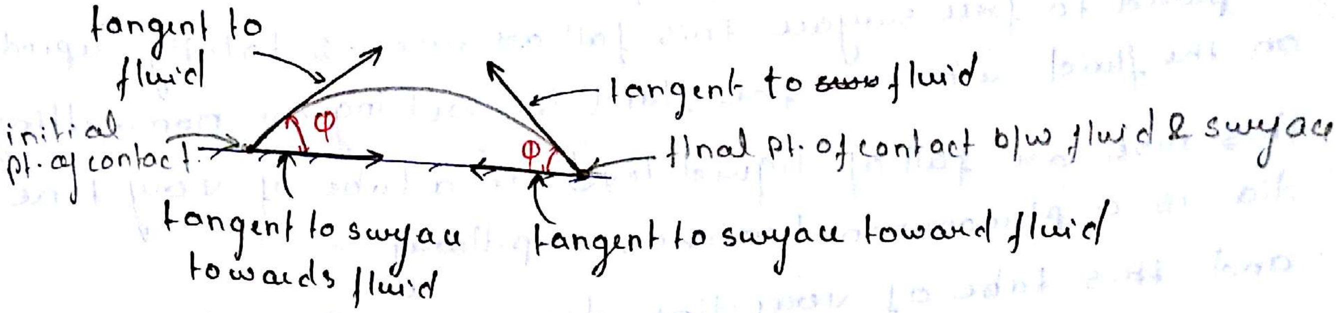
$$P \propto 1/R$$

Formation of bubble :-

1) wetting fluid :-

↓

Adhusion > cohesion



φ = angle b/w tangent to fluid and tangent to surface called angle of contact.

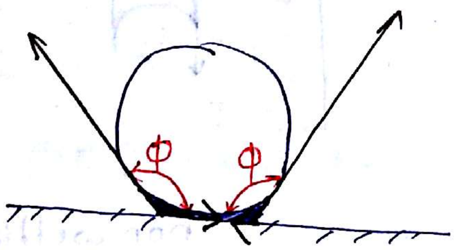
$\boxed{\phi < \frac{\pi}{2}}$ ≈ 29°

eg. contact b/w glass & water.

2) Non wetting fluid :-

↓

Cohesion > Adhusion.



$\boxed{\phi \geq \frac{\pi}{2}}$ ~ 132°

eg. Hg / glass.

Capillarity :-

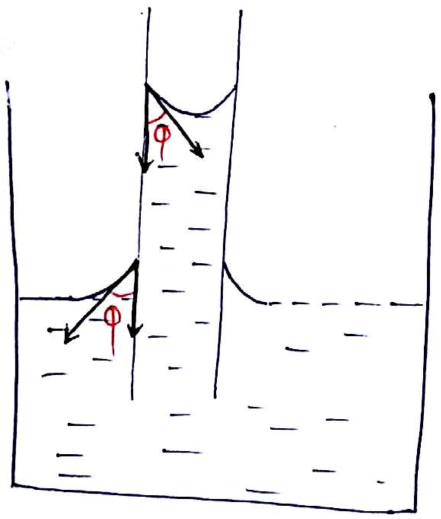
when a tube of very fine dia is immersed in a fluid (liquid) then there is fall or rise of liquid level. inside the tube as compared to free surface this fall or rise is totally dependent on the fluid whether the fluid is wetting or non wetting.

this rise or fall of liquid level in a tube of very fine dia is a phenomenon known as capillary.

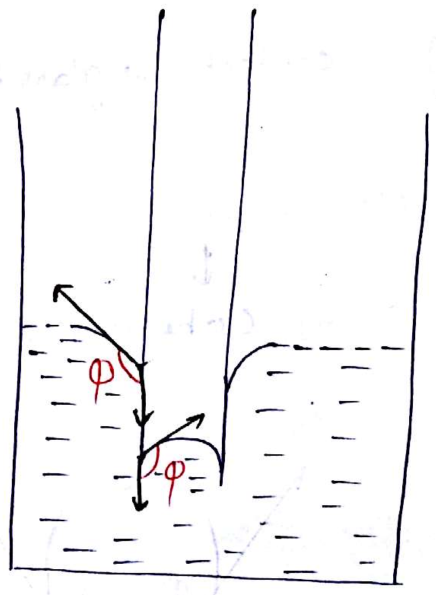
and this tube of very fine dia is capillary tube.

→ Basic cause of capillary action is due to cohesive and adhesive both.

whereas basic cause of surface tension is due to cohesive only.



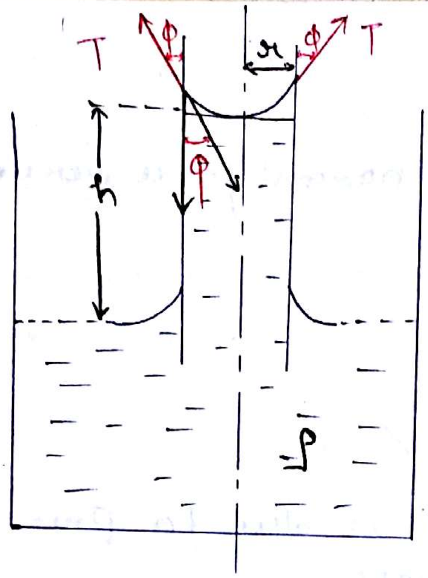
wetting



non wetting

→ shape of miniscus is concave
→ rise of liquid
→ $\phi < 90°$

→ shap → convex
→ fall of liquid
→ $\phi > 90°$



→ rise of liquid is due to verticle component of tension. horizontal component is cancelled.

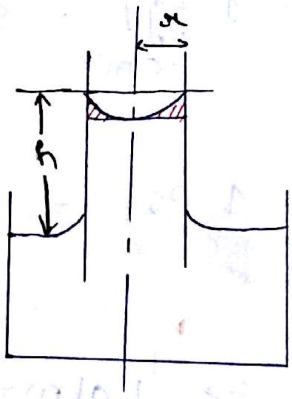
rise is till, wt. of liquid of height h is balanced by force at the ~~due~~ to surface of the liquid in the tube (due to surface tension)

⇒ wt of liquid of ht. h = verticle component of tension force

$$(\pi r^2 h)\rho \cdot g = (T\cos\theta)\, 2\pi r$$

$$\Rightarrow \boxed{h = \frac{2T\cos\theta}{r\rho g}}$$ approx.

exact →



$$T\cos\theta\,(2\pi r) = \left[\pi r^2 (h+r) - \frac{2}{3}\pi r^3\right]\rho \cdot g$$

$$T\cos\theta \cdot 2\pi r = \pi r^2 \left[h + r - \frac{2r}{3}\right]\rho g$$

$$T\cos\theta \cdot 2\pi r = \pi r^2 \left[h + \frac{r}{3}\right]\rho g$$

$$\boxed{h + \frac{r}{3} = \frac{2T\cos\theta}{r\rho g}} \Rightarrow \boxed{h = \frac{2T\cos\theta}{r\rho g} - \frac{r}{3}}$$

Pressure :-

Pressure at a pt. is defined as normal force per unit area.

- It is scalar quantity

$$\boxed{P = \frac{F}{A}} \quad \frac{N}{m^2}$$

- The basic reason of pressure is due to presence of molecule or presence of mass.

units

1) mks - $\frac{N}{m^2} = \frac{kg\text{-}m}{s^2 - m^2} = \frac{kg}{m\text{-}s^2} \Rightarrow 1\frac{N}{m^2} = 1\frac{kg}{m\text{-}s^2}$

2) S.I - Pascals $\Rightarrow 1\,Pa = 1\frac{N}{m^2}$

3) $1\,bar = (1\times10^5)\,Pa$

4) * $\boxed{1\,atm = 1,01,325\,Pa} = 1,01,325\,\frac{N}{m^2} = 1.01325\times10^5\,Pa$

$= 1.01325\,bar.$

5) $1\,\frac{kgf}{cm^2} = \frac{9.807}{10^{-4}}\,\frac{N}{m^2} = (9.807\times10^4)\,Pa$

6) $1\,Psi = \frac{1\,\text{Pound force}}{inch^2} = \frac{1\,lb}{inch^2}$

7) * $\Rightarrow \boxed{1\,atm = 1,01,325\,Pa = 14.696\,psi}$

Different types of pressure :

1) atmospheric pressure (P_{atm}) :-

Pressure exerted by environmental mass is known as atm. pressure.

2) Absolute pressure (P_{abs}) :-

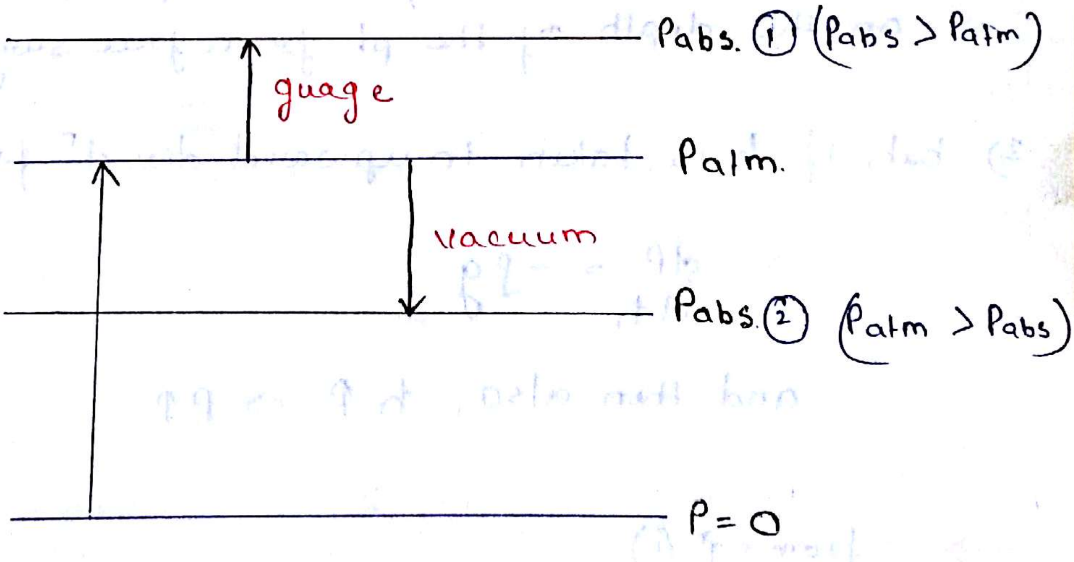
It is a real pressure of the system measured from zero level.

3) guage pressure :

$$\boxed{P_{guage} = P_{abs} - P_{atm}}$$

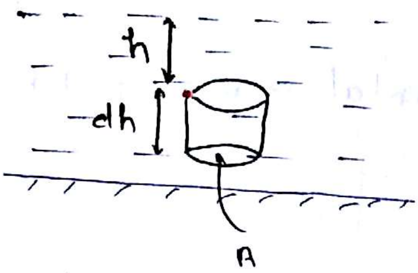
3) Vacuum pressure :

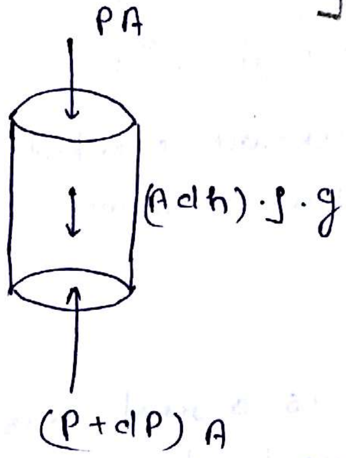
$$\boxed{P_{vacuum} = P_{atm} - P_{abs}}$$



Pressure at a pt. in a static fluid :-

$w = mg$
$= \rho V g$





$$\Rightarrow \quad PA + A\,dh \cdot \rho\, g = (P + dP)\,A$$

$$\Rightarrow \quad \boxed{\frac{dP}{dh} = \rho g} \quad \text{— 1)}$$

1) h is taken from free surface in downward direction.

$$\frac{dP}{dh} = +ve = \rho g$$

$$\Rightarrow \quad h \uparrow \Rightarrow P \uparrow$$

$\Rightarrow$ Pressure in a static fluid at a pt. is only dependent on the depth of the pt. from free surface.

2) but, if h is taken in upward direct^n from free surface.

$$\Rightarrow \quad \frac{dP}{dh} = -\rho g$$

and then also, $h \uparrow \Rightarrow P \uparrow$

$\rightarrow$ from eq^n 1

$$\frac{dP}{dh} = \rho g$$

$$\int_0^P dP = \rho g \int_0^h dh \quad \Rightarrow \quad \boxed{P = \rho g h}$$

(7) Pressure in a column of liquid (fluid) height :

a) $P_{atm} = 1,01,325\ P_a = \rho_{Hg} \times g \times h$

$= 13,600 \times 9.81 \times h$

$\Rightarrow h = 760\ mm\ (Hg)$

$\Rightarrow \boxed{1\ atm = 760\ mm\ (Hg)}$

b) $P_{atm} = 1,01,325\ P_a = \rho_{H_2O} \times g \times h$

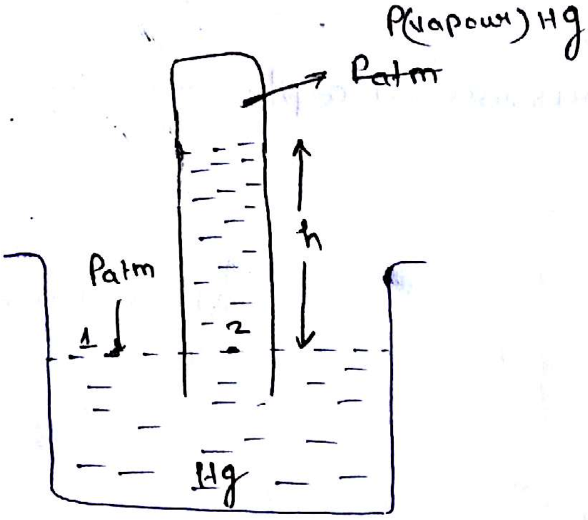
$= 1000 \times 9.81 \times h$

$\Rightarrow h = 10.3\ m\ (H_2O)$

$\Rightarrow \boxed{1\ atm = 10.3\ m\ (H_2O)}$

$\Rightarrow \boxed{760\ mm\ (Hg) = 10.3\ meter\ (H_2O)}$

Pressure measurement devices :-



when fluid is static, pressure at same level is same.

$P_1 = P_2$

$P_{atm} = P_2$

$= P(\text{vapour})_{Hg} + \rho_{Hg}\, g \cdot h$

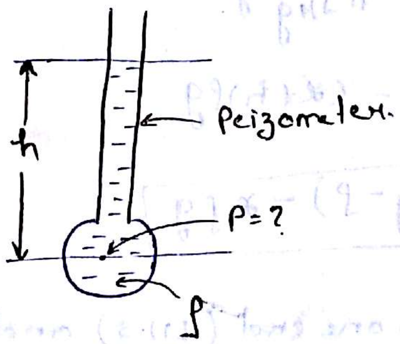
$\boxed{P_{atm} = \rho_{Hg}\, g\, h}$

2) Piezometer -

It is a device which is basically used to measure the moderate pressure.

+ This device is going to measure the guage pressure

→ no measuring fluid is used



$\boxed{P_{guage} = \rho g H}$

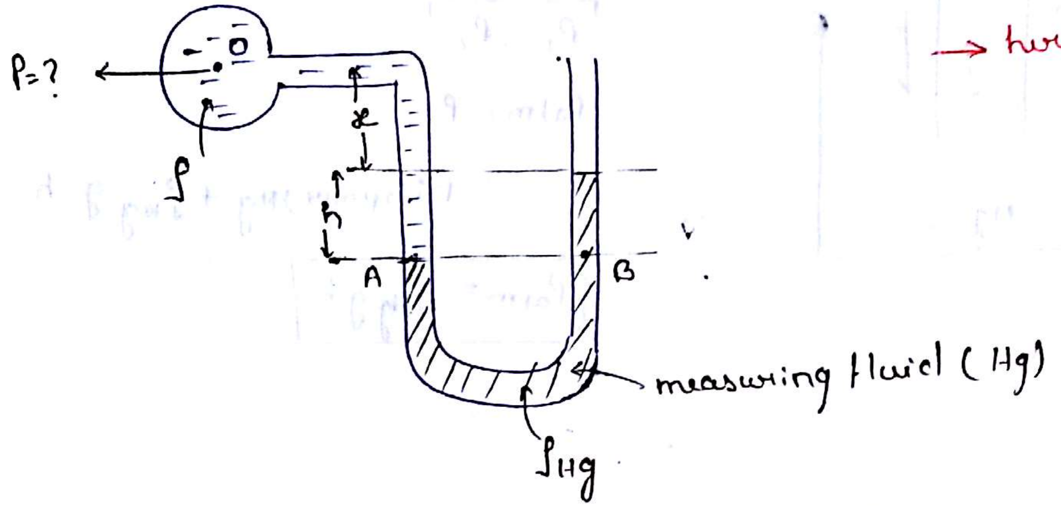
3) Manometer :-

- It is a device used to measure high pressure.
- In this device the pressure is determined in the terms of ht. of Hg column.

∴ the manometric fluid is Hg in this also.

i) simple U-tube manometer:

– used to measure guage pressure at a pt.



→ here Patm is zero.

Method 1.

Pressure is same at same level provided fluid is same

$\therefore \quad P_A = P_B$

$$P + (x+h)\rho g = h \rho_{Hg} \cdot g$$

$$P = h \rho_{Hg} \cdot g - (x+h)\rho g$$

$$\boxed{P = gh(\rho_{Hg} - \rho) - x\rho g}$$

Method .2 → start from one end (LHS) and move till last end.

take [+ve when top to bottom
–ve bottom to top.

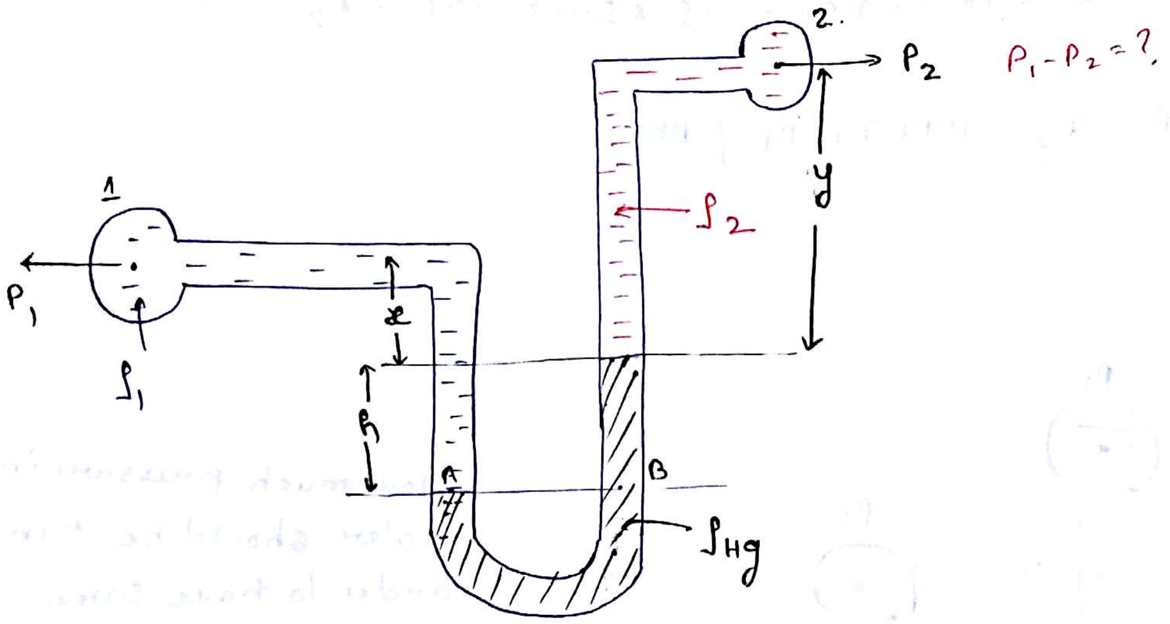
$$P + (x+h)\rho g - h\rho_{Hg} \cdot g = 0 \leftarrow \text{Patm}$$

& equate to last pt. Pressure i.e. $(\text{Pressure})_{atm} = \text{zero}$.

$$\Rightarrow \boxed{P = gh(\rho_{Hg} - \rho) - x\rho g}$$

Differential U-tube manometer :-

It is used to measure the pressure difference b/w 2 pts.

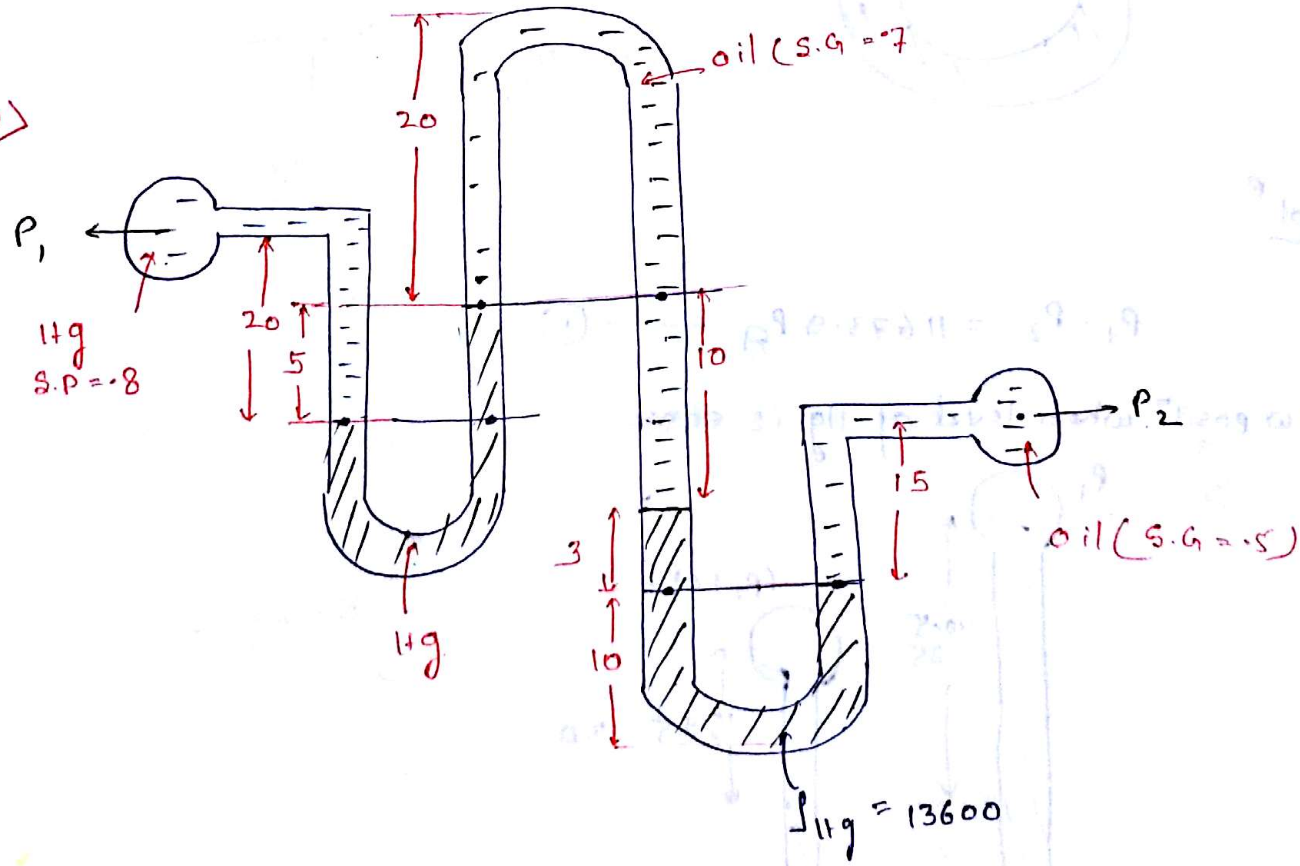


Method 2 -

$$P_1 + (x+h)\rho_1 g - h\rho_m g - y\rho_2 g = P_2$$

$$\Rightarrow \boxed{P_1 - P_2 = h\rho_m g + y\rho_2 g - (x+h)\rho_1 g}$$

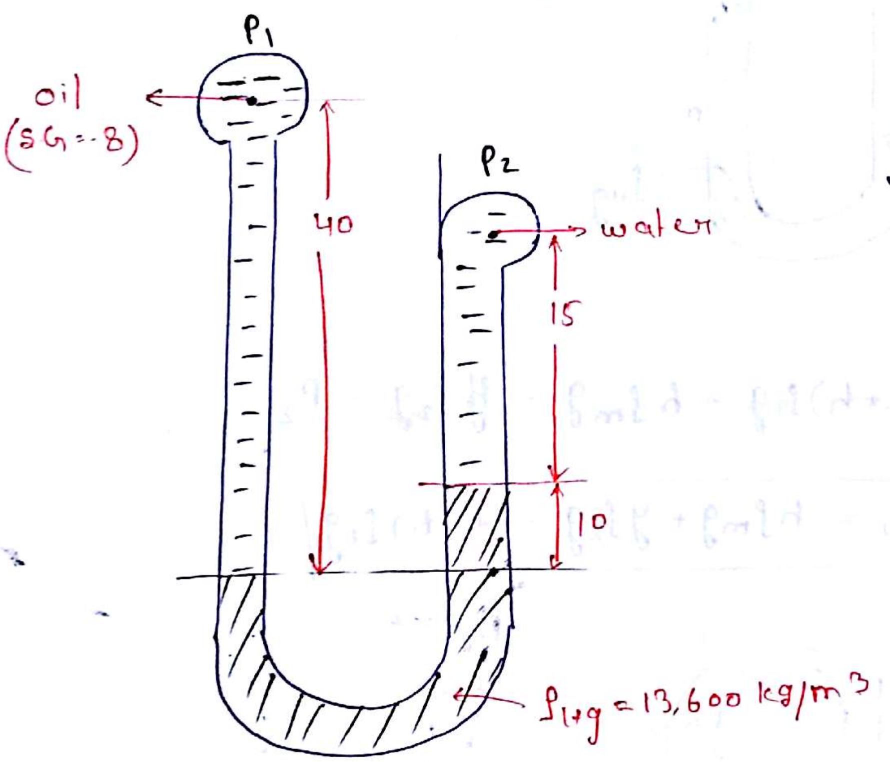
Pb)



$P_1 + (.2)\times 800\times 9.81 - .05\times 13600\times 9.81 + .1\times 700\times 9.81$
$+ .03\times 13600\times 9.81 - .15\times 500\times 9.81 = P_2$

$\Rightarrow \boxed{P_1 - P_2 = 1147.77 \text{ Pa.}}$ Ans.

Pb)

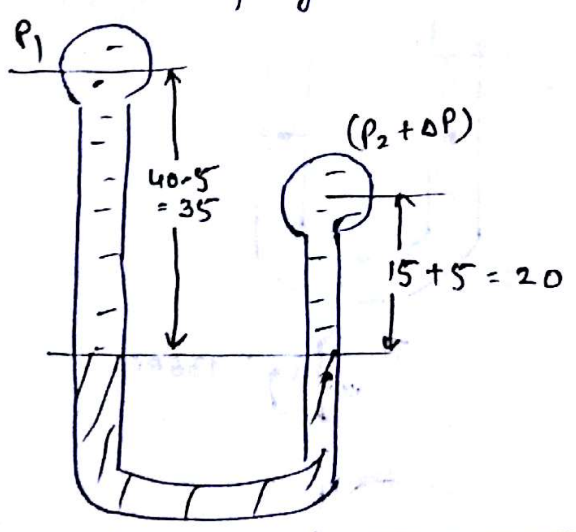


How much pressure in water should be ↑ in order to have same Hg level in both limbs.

sol^n

$P_1 - P_2 = 11673.9$ Pa ——— (1)

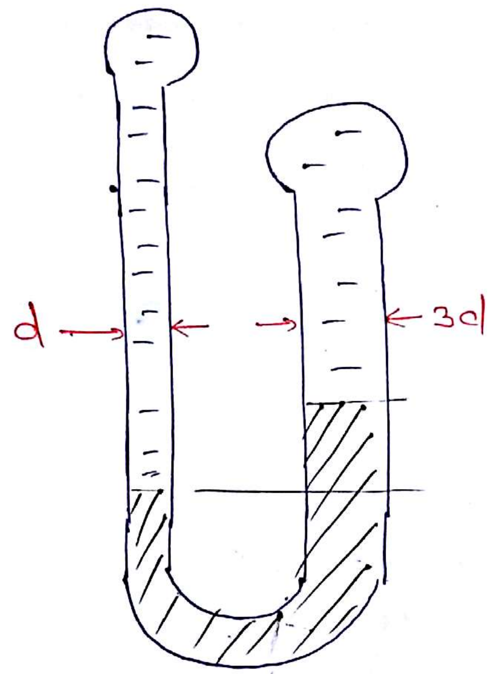
New posit^n when level of Hg is same.



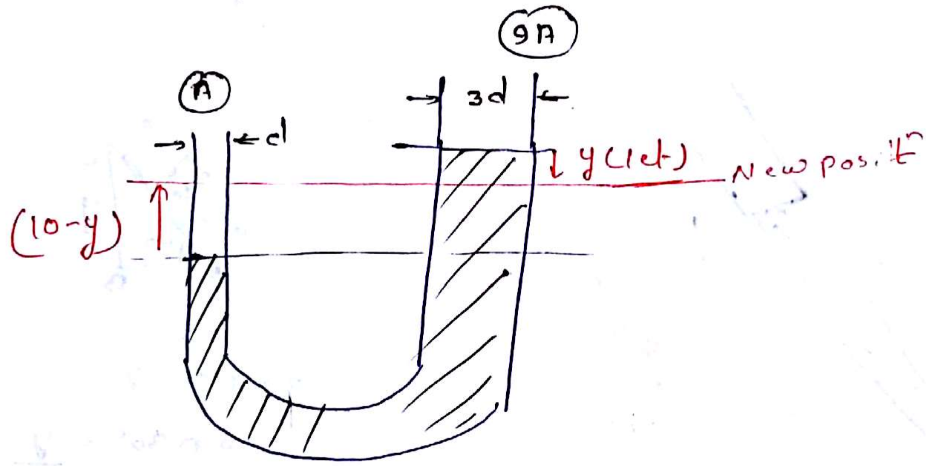
$$P_1 + .35 \times 800 \times 9.81 - .20 \times 1000 \times 9.81 = (P_2 + \Delta P)$$

$$\Rightarrow \boxed{\Delta P = 12458.7 \text{ Pa}} \quad \text{Ans.}$$

Pb) cont.



Sol^n

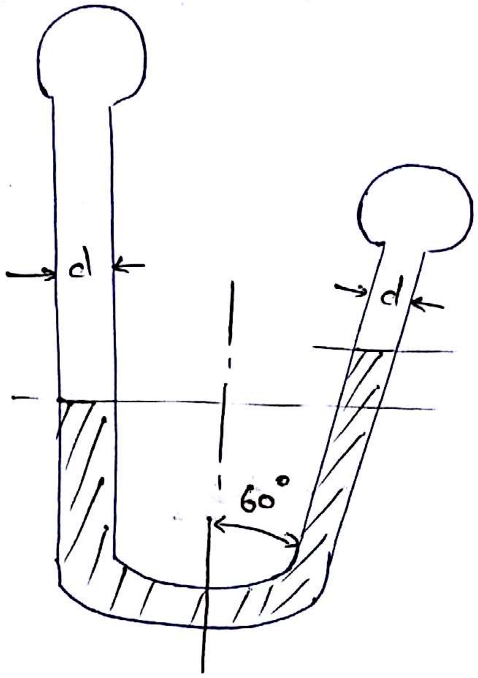


since volume is conserved.

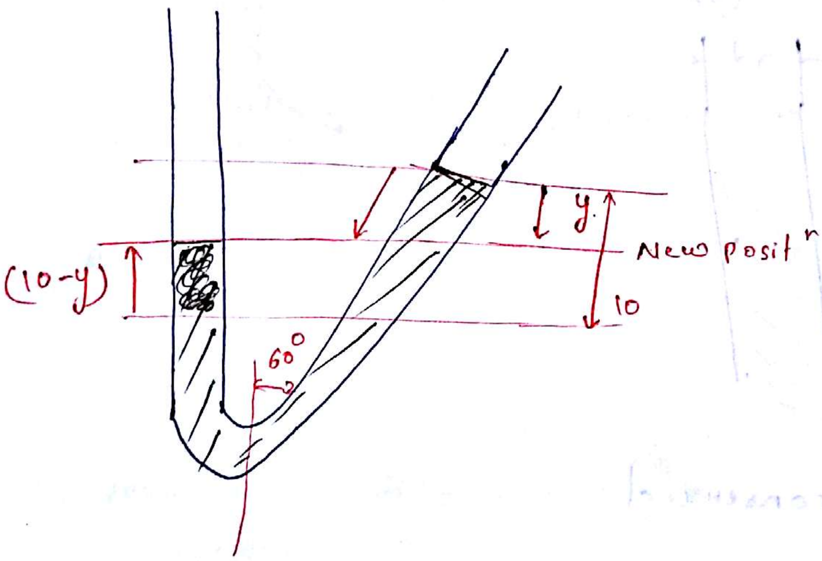
$$(9A)y = (10-y)A$$

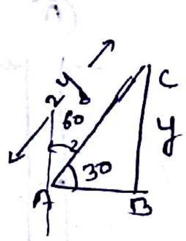
$$\Rightarrow \boxed{y = 1\text{cm}}$$

Pb>
Cont.



Sol^n





$\Rightarrow AC = ?$

$$\sin 30° = \frac{y}{AC}$$

$$\Rightarrow AC = \frac{y}{\sin 30°}$$

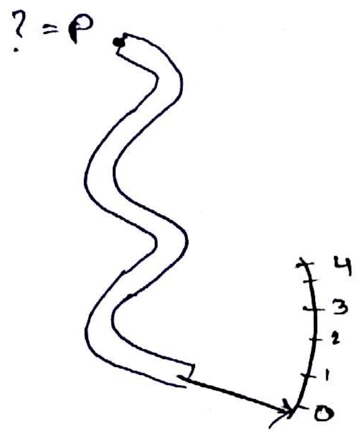
$$(2y)A = (10-y)A$$

$$\Rightarrow \boxed{y = \frac{10}{3}}$$

④ Bourdon guage :-

- this is used to measure very-very high pressure.
- tube (circular, spiral or helical) is used. which is flexible.
- one end of tube is connected to pt. where Pressure is to be measured & other end is connected to pointer.
- Pointer will be at zero when 1st end is open to atmosphere.

→ ∴ this guage measure Pressure above Patm.



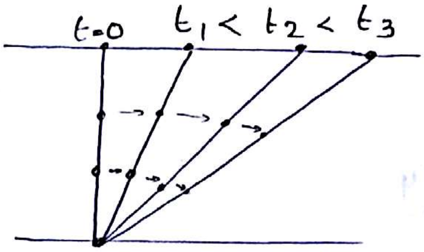
fluid :-

A fluid is a substance which is capable of flowing under the action of shear force (however the small force may be).

eg: liquids, gases, vapours etc.

Note →

If there is no shear force, the fluid will be at rest.



In solid, if the force is within elastic lt it will come back to original position after removal of the force.
but in fluid, it will not come back to the original position after the removal of the force.

In solid, deformatⁿ does not vary with time. but in fluid deformatⁿ varies with time & hence
Rate of deformatⁿ is imp. in fluid than deformatⁿ

fluid properties :-

1) Density @ mass density (ρ) :-

unit → $\boxed{\rho = \frac{m}{vol} = \frac{kg}{m^3} = \frac{M}{L^3} = ML^{-3}}$

$\boxed{\rho_{H_2O} = 1000 kg/m^3}$

- It is the ratio of mass to its volume.
- Density depends on temp. and pressure.

$$\rho \begin{cases} T\uparrow \rightarrow \rho\downarrow \\ P\uparrow \rightarrow \rho\uparrow \end{cases}$$

- It is an absolute quantity.

2) specific wt. @ wt. density (w) :-

It is the ratio of wt. of the fluid to its volume.

$\boxed{w = \frac{\text{wt. of fluid}}{\text{volume}}}$ $\frac{N}{m^3} = \frac{MLT^{-2}}{L^3} = ML^{-2}T^{-2}$

$w = \left(\frac{mg}{vol}\right) = \rho g \Rightarrow \boxed{w = \rho g}$

$w_{H_2O} = 10^3 \times 9.81 = 9810 \frac{N}{m^3}$.

specific wt. depends on temp., pressure and locatn.
As it varies from place to place, it is not absolute quantity.

3) specific gravity (S) :

- It is defined as the ratio of density of fluid to the density of std. fluid.

- As it is the ratio of same quantity ∴ it is dimensionless.

  $M^0 L^0 T^0$.

- It can also be defined as sp. wt of fluid to the specific wt. of std. fluid.

- In case of liquid std. fluid is water. and in case of gases std fluid is either H_2 @ air at a given temp. & pr.

$$\boxed{\begin{aligned} S_{H_2O} &= 1 \\ S_{Hg} &= 13.6. \end{aligned}}$$

4) compressibility (β) :

It is defined as reciprocal of bulk modulus (K)

$$\boxed{\beta = \frac{1}{K}}$$

$$K = \frac{dP}{-\frac{dV}{V}}$$

$$\rho = \frac{\text{mass}}{\text{vol}} \Rightarrow \rho V = \overset{\text{const}}{m}$$

$$\rho V = C$$

$$\rho\, dV + V\, d\rho = 0$$

$$\rho\, dV = -V\, d\rho$$

$$-\frac{dV}{V} = \frac{d\rho}{\rho}$$

$$K = \frac{dP}{-\frac{dV}{V}} = \frac{dP}{\frac{d\rho}{\rho}} \Rightarrow \boxed{K = \frac{\rho\, dP}{d\rho}}$$

$$\beta = \frac{1}{\frac{\rho\, dP}{d\rho}} \Rightarrow \boxed{\beta = \frac{d\rho}{\rho\, dP}}$$

$\beta = 0$, if $d\rho = 0$

=> density remain const.

A fluid is said to be incompressible when density remains const. w.r.t. pressure.

Otherwise the fluid is compressible.

- A fluid flow can be treated as incompressible if the $\boxed{\text{Mach} < .3}$

Note :- Bulk modulus increases with pressure bcz. at higher pressure there is greater resistance for further compressⁿ.

Bulk modulus of ideal gas under Isothermal condit^n.

$$PV = mRT$$

$$P = \frac{m}{V} RT$$

$$P = \rho RT$$

(As $T = const.$

$$\Rightarrow \frac{dP}{d\rho} = RT$$

$$K = \rho \frac{dP}{d\rho}$$

$$\therefore \quad K_T = \rho \times RT \quad \Rightarrow \boxed{K_T = P}$$

Note :- similarly Adiabatic bulk modulus (K_a)

$$PV^{\gamma} = C$$

$$\rho = \frac{m}{V} \quad \Rightarrow V = \frac{m}{\rho}$$

$$P\left[\frac{m}{\rho}\right]^{\gamma} = C \quad \Rightarrow \frac{P}{\rho^{\gamma}} = \frac{C}{m^{\gamma}} = const.$$

$$\frac{P}{\rho^{\gamma}} = C$$

$$\boxed{K_a = \gamma P}$$

\# 8) The eqn of a state for a liquid is.

$P = (3500 \rho^{\frac{1}{2}} + 2500)\ N/m^2$

Then find the bulk modulus at a pr. of $10^5\ N/m^2$.

Soln

$$P = (3500 \rho^{\frac{1}{2}} + 2500) \frac{N}{m^2}$$

$$K = \rho \frac{dP}{d\rho}$$

$$\frac{dP}{d\rho} = 3500\left(\frac{1}{2}\rho^{-\frac{1}{2}}\right) + 0$$

$$\Rightarrow \frac{dP}{d\rho} = \frac{1}{2}(3500)\rho^{-\frac{1}{2}}$$

$$K = \rho \times \frac{1}{2}\ 3500\ \rho^{-\frac{1}{2}} \Rightarrow K = \frac{1}{2}\ 3500\ \rho^{\frac{1}{2}}$$

$$K = \frac{1}{2}(P - 2500)$$

$$K = \frac{1}{2}(10^5 - 2500) = 48750 \frac{N}{m^2}.$$

\# An increase in pr. of 2 bar, decreases the volume of a liquid by 0.01 %. Then find the bulk modulus

Soln

$$K = \frac{dP}{-\frac{dV}{V}}$$

$$-\frac{dV}{V} = \frac{.01}{100}$$ (given) → change is always wrt. original.

$$dP = 2\times10^5 \, N/m^2$$

$$\Rightarrow k = \frac{2\times10^5}{\frac{.01}{100}} = 2\times10^9 \, N/m^2$$

20) when the pr. in the given mass of liquid is from 3 MPa to 3.5 MPa, the density of liquid ρ from 500 kg/m³ to 501 kg/m³. The what is bulk modulus.

<u>solⁿ</u>

$$k = \rho\frac{dP}{d\rho}$$

change is always wrt. original.

$$\Rightarrow \rho = 500 \text{ (initial)}$$

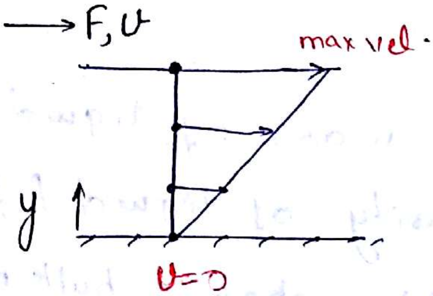
$$d\rho = 501 - 500 = 1$$

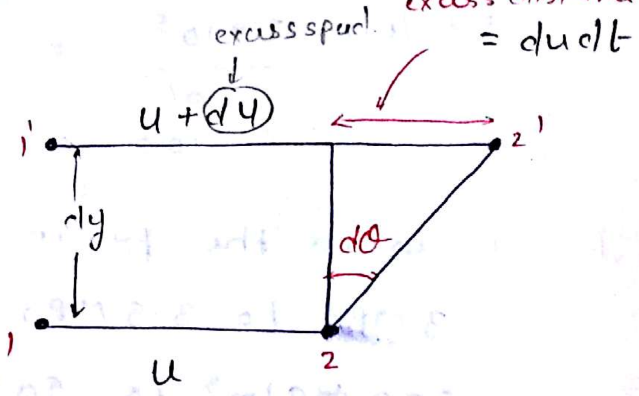
$$d\rho = 1$$

$$k = \frac{500\times.5}{1} = 250 \, MPa.$$

Viscosity :-

It is the internal resistance offered by 1 layer of fluid to the other layer.





$$\tan d\theta = \frac{du\,dt}{dy}$$

angular deformation => $d\theta = \frac{du\,dt}{dy}$.

angular deformation rate $\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$

For a given plate Area, if shear force F is large,

shear stress

τ is large,

$\tau = \frac{F}{A \to const}$

& If τ is large $\frac{d\theta}{dt}$ is large

$\Rightarrow \tau \propto F$

$\therefore \quad \tau \propto \frac{d\theta}{dt}$

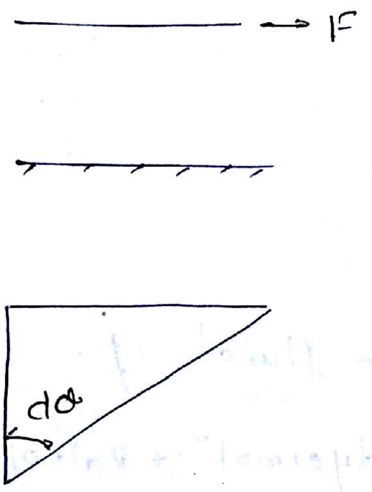
Proportionality const.

$\Rightarrow \quad \tau = \mu \frac{d\theta}{dt}$

$\Rightarrow \quad \boxed{\mu = \frac{\tau}{\frac{d\theta}{dt}}}$

Let us consider two fluid A & B

A



μ is less

$\Downarrow$

$\frac{d\theta}{dt}$ is large.

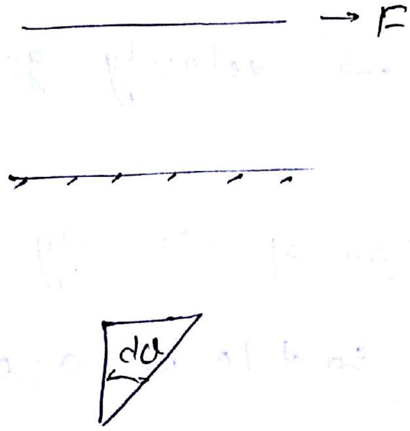
$\Downarrow$

Resistance is less.

$\Downarrow$

flow is easy.

B



μ is more

$\Downarrow$

$\frac{d\theta}{dt}$ is less

$\Downarrow$

Resistance is more

$\Downarrow$

flow is not easy.

μ represent internal resistance to the flow and this μ is known as const. of viscosity @ absolute viscosity @ dynamic viscosity.

$$\tau = \mu \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

$$\Rightarrow \boxed{\tau = \mu \frac{du}{dy}}$$

$\frac{d\theta}{dt}$ → Rate of angular deformatⁿ
@
Rate of shear strain

$\frac{du}{dy}$ → velocity gradient

Newton's law of viscosity..

A fluid is said to be a Newtonian fluid if.

Shear stress $\propto$ rate of angular deformatⁿ @ Rate of shear strain

$$\tau \propto \frac{d\theta}{dt}$$

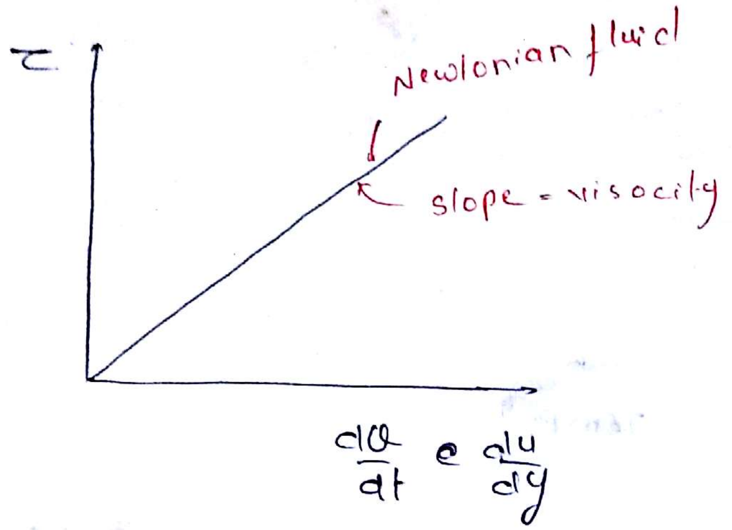
@ $\tau \propto \frac{du}{dy}$ (velocity gradient)

$$\Rightarrow \boxed{\tau = \mu \frac{du}{dy}}$$

→ for a Newtonian fluid.

For a newtonian fluid viscosity μ is const.

eg: Air, water, diesel, petrol, kerosene, Hg, Oil.



$$\frac{\tau}{d\theta/dy} = \text{slope} = \mu = \text{const.}$$

Variatⁿ of viscosity with temp.:

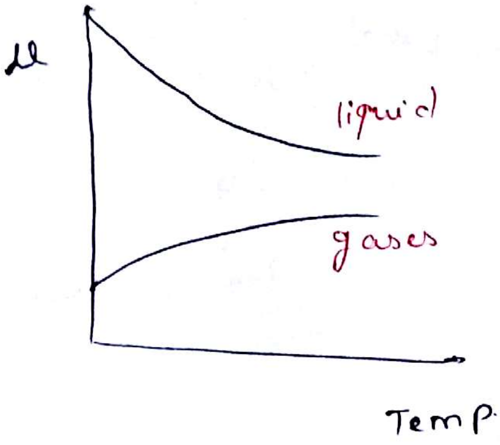
In case of liquid intermolecular distance is small and hence cohesive forces are large. with increase in temp. cohesive force will decrease and hence resistance to the force also decreases

∴ In case of liquid with rise in temp. there is reductⁿ in viscosity.

$$T\uparrow \Rightarrow CF\downarrow \Rightarrow$$

In case of gases, Intermolecular distance is large and hence cohesive forces are negligible.
with inc. in temp molecular disturbance ↑ and hence resistance to the flow also ↑.

∴ viscosity of a gas ↑ with Temp ↑



Note :- Viscosity of water at 20°C is $10^{-3} \frac{N\text{-}s}{m^2}$.

dynamic viscosity

* water is 50 to 55 times more viscous than air.

whereas → kinematic viscosity of air is greater than water

unit of viscosity (μ) :-

SI :-

$$\tau = \mu \frac{du}{dy}$$

$$\frac{N}{m^2} = \mu \times \frac{m}{s} \times \frac{1}{m}$$

$$\mu = \frac{N\text{-}s}{m^2} = Pa\text{-}s \qquad \left\{ Pa = \frac{N}{m^2} \right\}$$

$$\frac{N\text{-}s}{m^2} = \frac{kg\ m}{s^2} \times \frac{s}{m^2} = \frac{kg}{m\text{-}s}$$

$$\boxed{\mu = \frac{N\text{-}s}{m^2} = Pa\text{-}s = \frac{kg}{m\text{-}s}} \rightarrow \text{SI unit.}$$

CGS :-

$$\frac{kg}{m\text{-}s} \rightarrow \frac{gm}{cm\text{-}s} = \text{Poise}$$

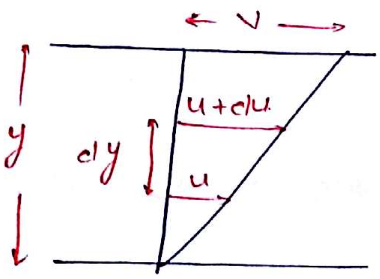
$$\frac{1\,kg}{m\text{-}s} = \frac{10^3\,gm}{10^3\,cm\text{-}s}$$

$$\frac{1\,kg}{m\text{-}s} = \frac{10\,gm}{cm\text{-}s}$$

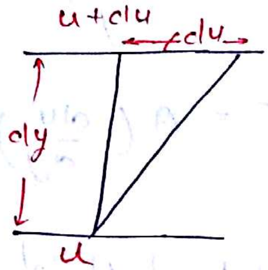
$$\boxed{\frac{1\,kg}{m\text{-}s} = 10 \text{ poise.}} \longrightarrow \text{CGS unit}$$

$$\boxed{1 \text{ Poise} = \cdot 1 \frac{kg}{m\text{-}s}}$$

Linear vel. profile :-



$\frac{V}{y}$



$\frac{du}{dy}$

$$\frac{du}{dy} = \frac{V}{y}.$$

$$\tau = \mu \frac{du}{dy} \quad \text{(for linear vel. profile)}$$

$$\tau = \mu \frac{V}{y}.$$

$$\frac{F}{A} = \mu \frac{V}{y}$$

$$** \boxed{F = \mu \frac{AV}{y}}$$

Note: viscosity varies very little with pressure. But at very high pressure there is an ↑ viscosity.

Non-Newtonian fluid:

Fluid which do not obey Newton law of viscosity are known as Non-Newtonian fluids.

The study of Non-Newtonian fluid is also known as Rheology.

The general relationship b/w shear stress (τ) and vel. gradient ($\frac{du}{dy}$) is given by

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$

Case-i Dilatant fluid.

$$B = 0 \; ; \; n > 1$$

$$\Rightarrow \tau = A\left(\frac{du}{dy}\right)^n$$

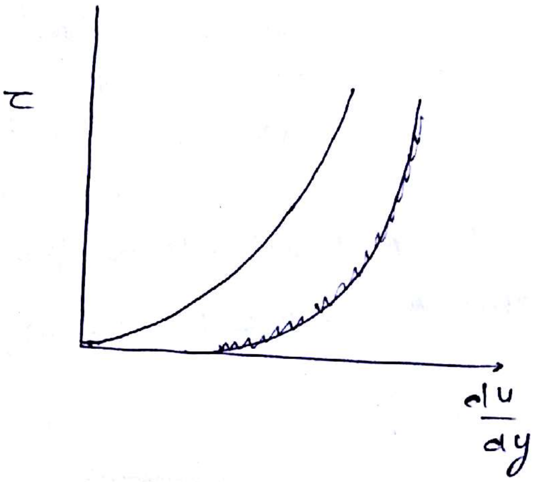
$$\Rightarrow \tau = A\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy}.$$

comparing this with $\tau = \mu\frac{du}{dy}$

$$\Rightarrow \boxed{\tau = \mu_{app}\frac{du}{dy}} \quad \text{where } \mu_{app} = A\left(\frac{du}{dy}\right)^{n-1}.$$

For a dilatant fluid apparent viscosity ↑ with rate of deformatⁿ. and hence these fluids are also known as shear thickening fluids.

eg → Rice-starch, sugar-H_2O solⁿ

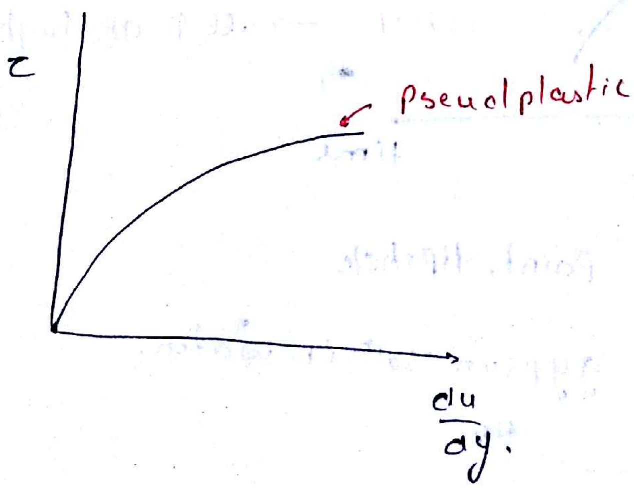


case-ii) Pseudo plastic fluid :-

$$B = 0 \;;\; n < 1$$

For these fluid, Apparent viscosity ↓ with ↑ rate of deformatⁿ and these fluids are also known as shear thinning fluids

eg. Blood, milk, coloidal solⁿ



$$\boxed{\tau = A\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy}} \quad n<1$$

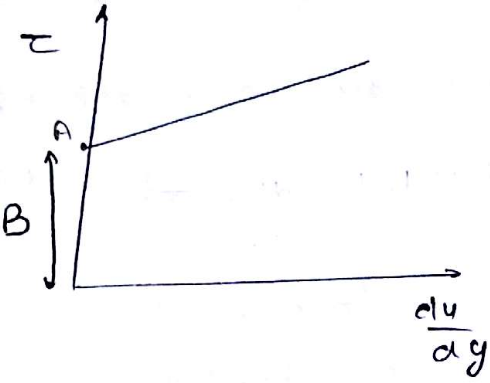
$$\Rightarrow \tau = \mu_{app}\frac{du}{dy}$$

case - iii) Bingham plastic (Ideal plastic) :

$B \neq 0$; $n = 1$

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

$$\Rightarrow \tau = A\left(\frac{du}{dy}\right) + B$$



till A, τ is there but $\frac{du}{dy}$ is not there

$\Downarrow$

flow is not there

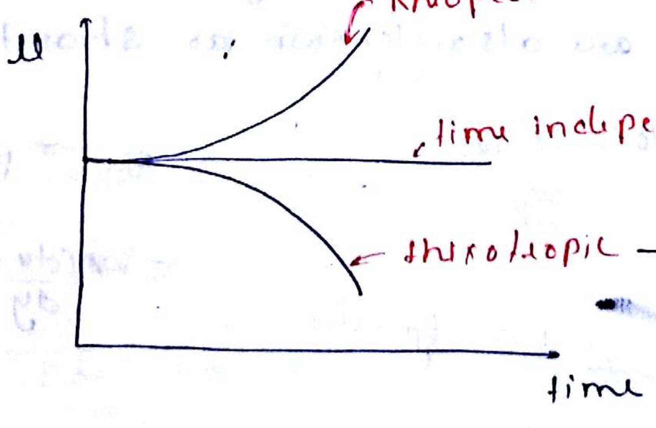
$\Rightarrow$ till A fluid acts as solid.

after A, when $\tau \uparrow$ then $\frac{du}{dy} \uparrow$

eg. Toothpaste.

Two other type of fluid :-

certain fluid show variat^n of viscosity with time like thixotropic and Rheopectic fluid.



Rheopectic → $\mu \downarrow$ at higher shear stress

thixotropic → $\mu \uparrow$ at higher shear stress

eg. thixotropic :- Paint, lipstick

Rheopectic :- gypsum sol^n in water.

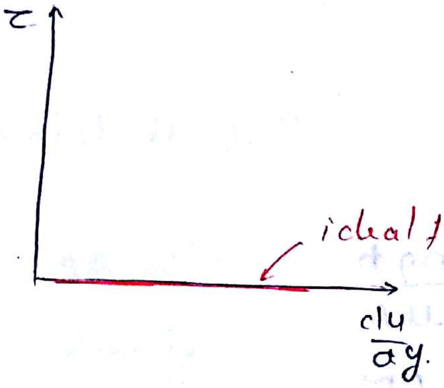
Ideal fluid :

A fluid is said to be an ideal fluid if it is non-viscous and incompressible.

$$\tau = \mu \frac{du}{dy}$$

when $\mu = 0$

$\Rightarrow \tau = 0$



A flat plate 0.1 m^2 Area is pulled at 30 cm/sec. relative to another plate located at a distance of 0.01 cm from it. If viscosity of the fluid is .001 $\frac{N\text{-}s}{m^2}$ then find the power req. to maintain the vel.

Sol^n Power = $F \times V$

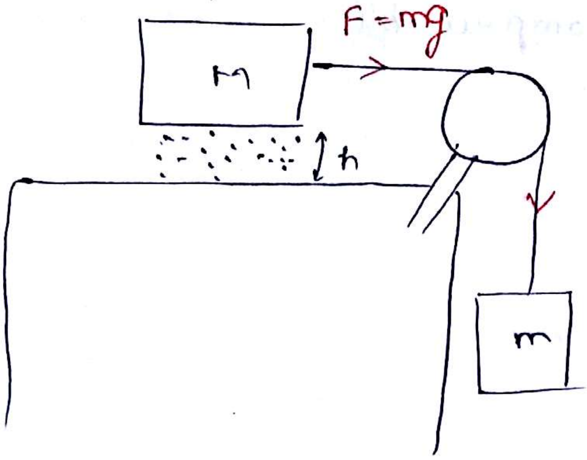
Since nothing is given $\Rightarrow$ linear vel. Profile.

$$\Rightarrow F = \frac{\mu A V}{y}.$$

$$= \frac{.001 \times .1 \times .3}{.01 \times 10^{-2}} = .3$$

$\therefore$ $P = .3 \times .3 = .09$ watt

A block of mass M slides on a horizontal table on oil film of thickness h. The mass m causes the movement and find the express^n for vel. (A = area of block)



Tension = mg.

Sol^n.

$$F = \frac{\mu A V}{y}$$

$$mg = \frac{\mu A u}{h} \Rightarrow u = \frac{mgh}{\mu A}$$

Ans.

Ans.

The foll. shear stress rate was ob. when the fluid under considerat^n is.

du/dy	0	2	3	5
τ	0	1.4	18	30

a) bingham plastic

b) Newtonian

c) Dilant

d) Pseudoplastic.

Sol^n

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

$$0 = A(0)^n + B \Rightarrow \boxed{B = 0}$$

$$\tau = A\left(\frac{du}{dy}\right)^n$$

$$\left.\begin{aligned} 1.4 &= A(2)^n \\ 18 &= A(3)^n \end{aligned}\right\} \Rightarrow \frac{18}{1.4} = \frac{A(3)^n}{A(2)^n}$$

$$\Rightarrow 12.85 = (1.5)^n$$

$$\Rightarrow n > 1$$

$\therefore$ $B = 0$ & $n > 1$ $\Rightarrow$ Dilatant fluid.

#9) match the foll.

A) sp. wt. — 1) L/T^2

B) density — 2) F/L^3

C) shear stress — 3) F/L^2

D) viscosity. — 4) FT/L^2

5) FT^2/L^4.

Sol^n

A) specific wt. $(w) = \frac{N}{m^3} \rightarrow \frac{F}{L^3}$

density

B) $w = \rho g. \Rightarrow \rho = \frac{w}{g} \rightarrow \frac{F/L^3}{L/T^2} = \frac{F}{L^3}\cdot\frac{T^2}{L} = \frac{FT^2}{L^4}$

D) $\mu = \frac{N\text{-}s}{m^2} - \frac{FT}{L^2}.$

c) Shear stress $= \mu \frac{du}{dy} \rightarrow \frac{FTL^{-2}(LT^{-1})}{L}$

or

$\tau = \frac{F}{A} \frac{\rightarrow F}{\rightarrow L^2}$ $= FL^{-2}$

A-2, B-5, C-3, D-4

A piston of 60 mm dia moves inside a cylinder of

19) 60.1 mm dia. Determin % dec. in force necessary to move the piston when the lubricant is heated from 0°C to 120°C.

$\mu_{0°C} = .0182 \ N\text{-}s/m^2$

$\mu_{120°C} = .00206 \ N\text{-}s/m^2$

sol^n

$F = \frac{\mu A v}{y}$ const.

$\Rightarrow F \propto \mu$ bcz. viscosity change with tem.

$\therefore$ % dec. in force $= \frac{F_2 - F_1}{F_1} \times 100$

$= \frac{\mu_2 - \mu_1}{\mu_1} \times 100$

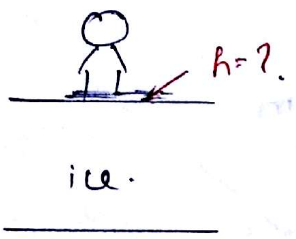
$= \frac{.06206 - .0182}{.0182} \times 100$

$= -88.68\%$

* A skater weighing 800 N skates at a rate of 15 m/sec on ice at 0°c. The avg. skating area supporting him is 10cm² and coeff. of fricⁿ b/w skates and ice is 0.02. If there is actually a thin film of water b/w skates & ice then find its thickness.

$\mu_{water} = 10^{-3} \frac{N \cdot S}{m^2}$

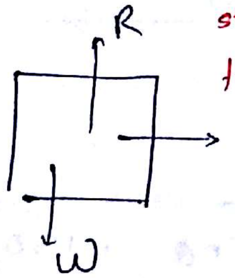
Solⁿ.



$F = \frac{\mu A V}{y} \Rightarrow y = \frac{\mu A V}{F}$

$\mu = 10^{-3}$, $A = 10 \times (10^{-2})^2 = 10^{-3}$

$V = 15 \text{ m/s}$,



shear force = fricⁿ force, coeff. of fricⁿ

$F = fR$

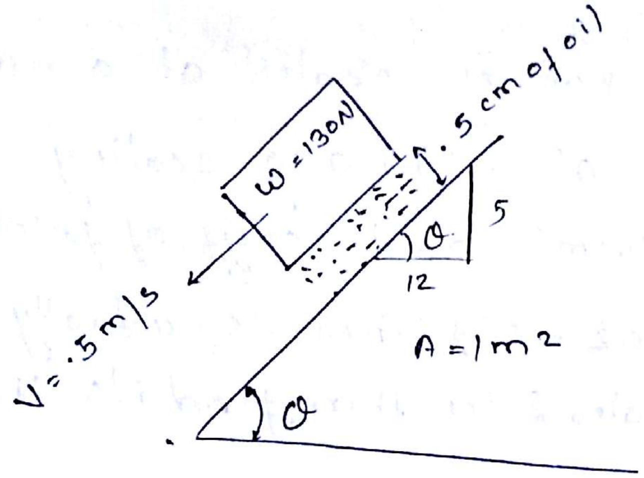
$= .02 \times 80 = 16.$

$y = \frac{10^{-3} \times 10^{-3} \times 15}{16} = \underline{9.37 \times 10^{-7}} \text{ m}$ Ans.

Find μ_{oil}.

29)

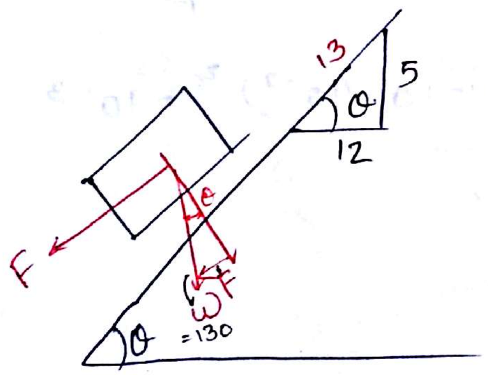
**



Soln

$$F = \frac{\mu A V}{y} \Rightarrow \mu = \frac{F y}{A V}$$

$A = 1m^2$, $V = .5 m/s$, $y = .5 \times 10^{-2} m$

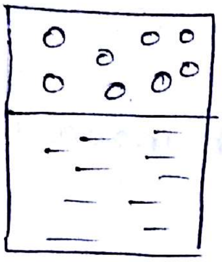


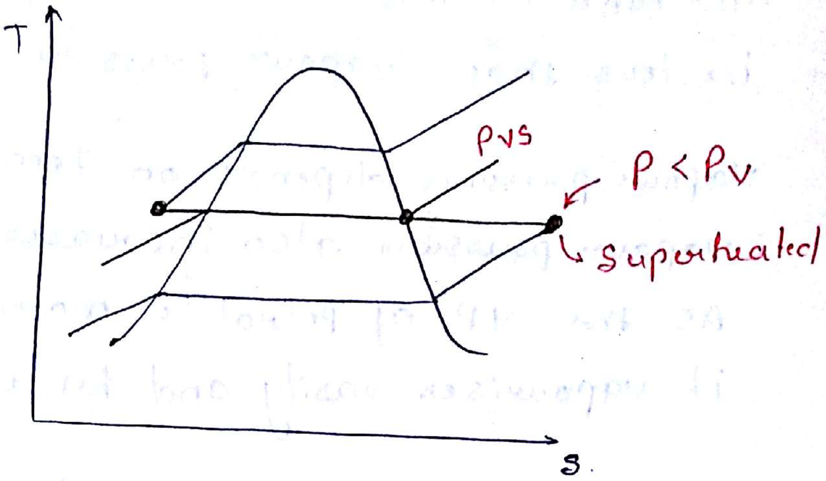
$$\sin\theta = \frac{5}{13}$$

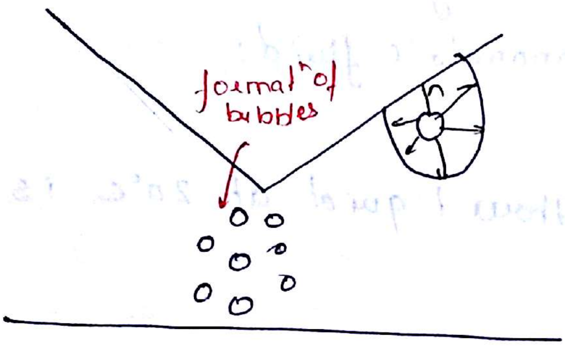
$$\sin\theta = \frac{F}{130} \Rightarrow F = 130 \sin\theta = 130 \times \frac{5}{13} = 50N$$

$$\mu = \frac{50 \times .5 \times 10^{-2}}{1 \times .5} = .5 \frac{N-s}{m^2}.$$

Vapour pressure.







The liquid molecules at the surface escapes due to more translational energy. If the container is closed the no. of molecule escaping from liquid is equal to no. of molecule rejoining the surface due to condensatn. once the equilibrium is reached.

under these conditions the pressure exerted by vapour on the surface of liquid is known as saturated vapour pressure. If the pressure of liquid at any pt. falls below vapour pressure (P_v) bubbles are formed and if these bubbles are carried to high pressure region these bubbles burst resulting in

Severe forces on the parts. This is known as Cavitat^n.

and hence to avoid cavitat^n liquid pressure must not be less than vapour pressure.

Vapour pressure depends on temp.. with increase in temp. vapour pressure also increases.

As the VP of Petrol is more than that of water it vapourises easily and hence it is more volatile.

The VP of mercury is very low. and bcz of this region it is used as a manometric fluid.

The saturat^n VP of three liquid at 20°C is given below.

Methyl alcohol → 12,500 Pa

Ethyl alcohol → 5900 Pa

Benzene → 10,000 Pa.

} which vaporises faster.

sol^n

As the VP of methyl alcohol is more ∴ it vapourises faster than Ethyl alcohol & benzene.

#22) The vel. distribut^n over a plane is given by $u = .5y - y^2$. where y is the distance above the plate. If the viscosity of fluid is $.9\ N\text{-}s/m^2$. Find the shear stress at 0.2 m above the plane.

Sol^n

$$u = .5y - y^2 \Rightarrow$$ vel. distribut^n is not linear.

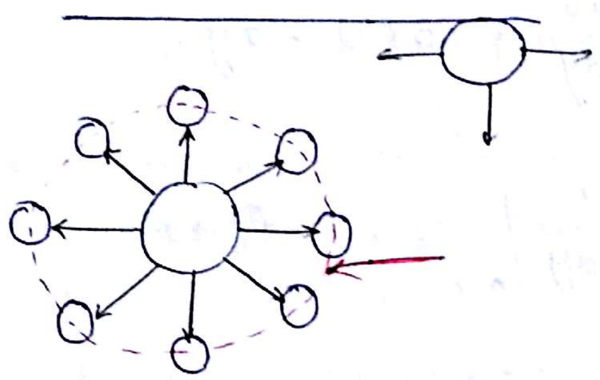
$$\tau = \mu \frac{du}{dy}, \Rightarrow \frac{du}{dy} = .5(1) - 2y.$$

$$\Rightarrow \left.\frac{du}{dy}\right|_{0.2} = .5 - 2(.2)$$

$$= \left.\frac{du}{dy}\right|_{.2} = 0.1$$

$$\Rightarrow \tau = .9 \times .1 = .09\ N/m^2$$

Surface Tension :



Consider the molecule surface A which is below free surface. This molecule is under the influence of various cohesive forces and bcz of this it will be in equilibrium.

Now consider molecule B which is on the free surface which is under the the influence of liquid. This molecule is under the of net downward force and due to this there seems to be a membrane formed at the surface which can resist small tensile load.

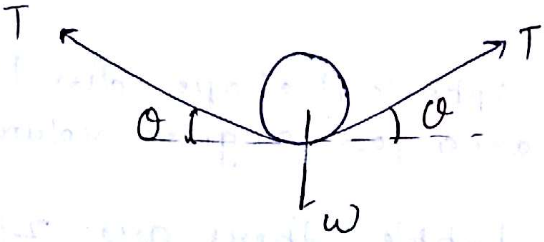
This phenomenon is known as surface tension.

- ST is a line force.
- It acts normal to the line drawn on the surface and it acts in the plane of surface.

unit → $\boxed{\sigma = \frac{F}{l}}$ $F \rightarrow N$, $l \rightarrow m$; $\frac{N}{m}$ $\quad \frac{MLT^{-2}}{L} = MT^{-2}$

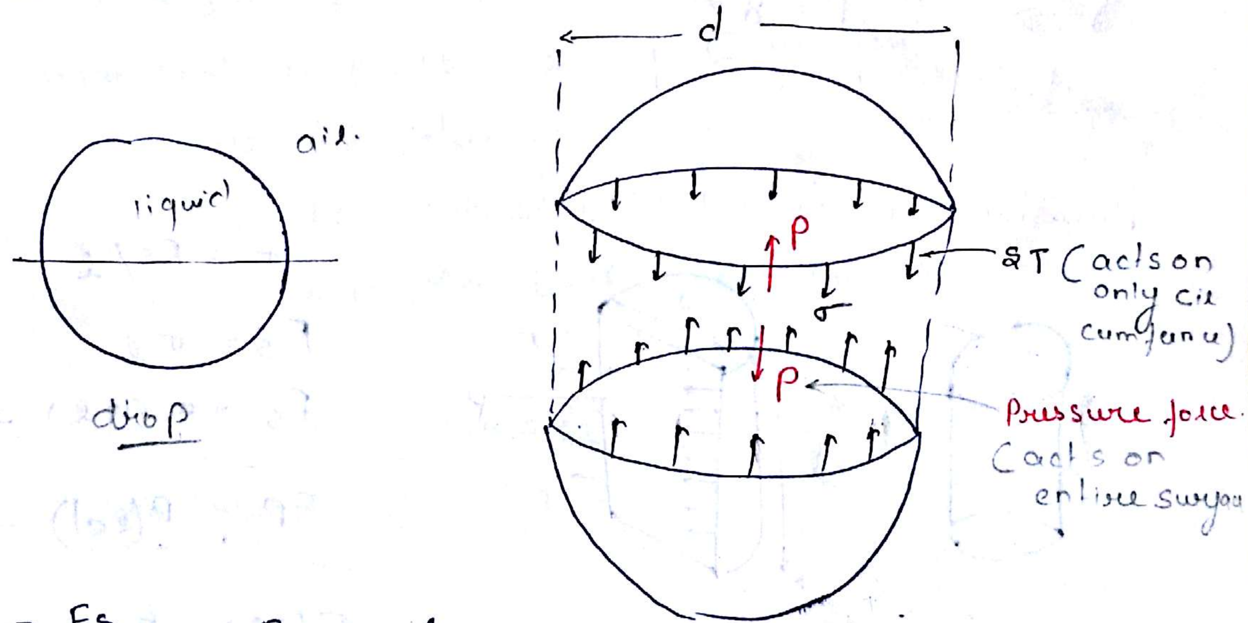
ST is basically due to ** unbalanced cohesive forces and with inc. in temp cohesive forces decreases and hence ST also decreases.

- ST for Air-water interface at 20° $(\sigma) = \cdot 0736\ N/m$
- As this force is very small force ∴ it is neglected in further analysis.
- Surfactants and detergent are used to reduce ST so that water can penetrate easily and can remove dust.



Application of ST :-

1) Pressure Inside a liquid drop in excess of atmospheric pressure.



$\sigma = \frac{F_s}{L} \Rightarrow F_s = \sigma l$

$F_s = \sigma(\pi d)$ —(1)

$p = \frac{F_p}{A} \Rightarrow F_p = PA$

$F_p = P\left(\frac{\pi}{4}\right)d^2$ —(2)

For equilibrium,

$$F_s = F_p$$

$$\sigma \pi d = P \frac{\pi}{4} d^2$$

$$\Rightarrow \boxed{P = \frac{4\sigma}{d}}$$

In a liquid drop, surface tension resist pressure force.

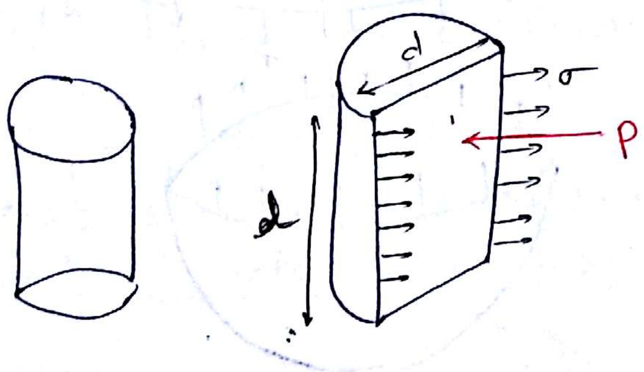
* [liquid drops assume spherical shape due to ST. bcz sphere has min. surface area for a given volume.

Note! - In case of bubble there are 2 surface. & hence

$$P = 2\left(\frac{4\sigma}{d}\right)$$

$$\Rightarrow \boxed{P = 8\frac{\sigma}{d}}$$

Pressure inside liquid jet :-



$$\sigma = F_s/\ell$$

$$F_s = \sigma\ell$$

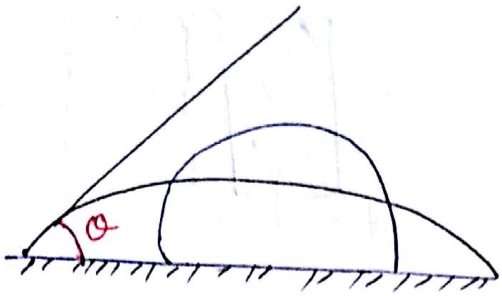
$$F_s = \sigma(2\ell) \quad \text{--- (1)}$$

$$F_P = P(\ell d) \quad \text{--- (2)}$$

$$\sigma(2\ell) = P\ell d$$

$$\Rightarrow \boxed{P = \frac{2\sigma}{d}}$$

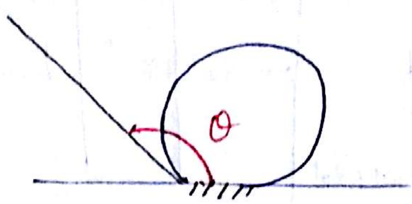
Capillarity



Adhesion is more.

wetting liquid

$\theta < 90°$



cohesion is more

non-wetting liquid

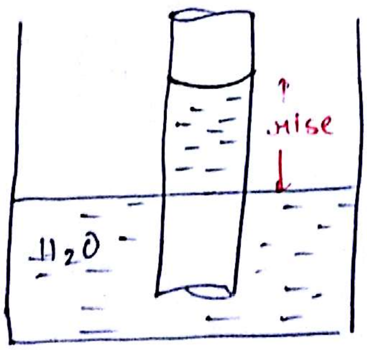
$\theta > 90°$

Cappilarity :-

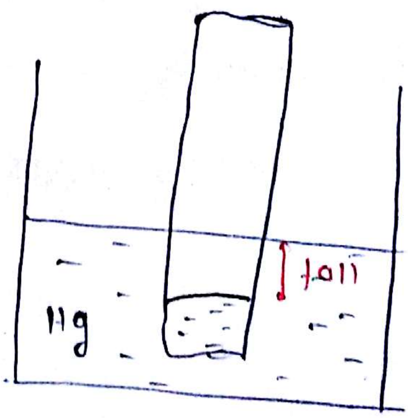
The rise @ fall of a liquid when a small dia of tube is immersed in it is known as cappilarity.
Capillary rise is due to adhesion and capillary fall is due to cohesion.

∴ capilarity is due to both adhesion and cohesion.

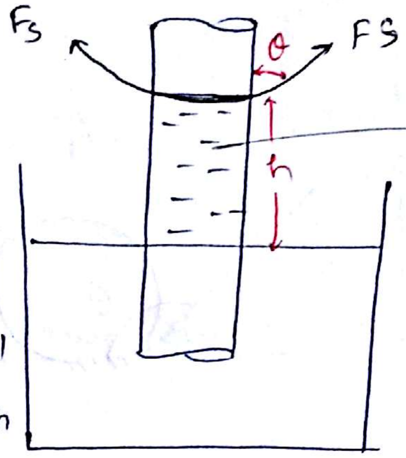
e.g. → capilary rise → water
capilary fall → Hg.

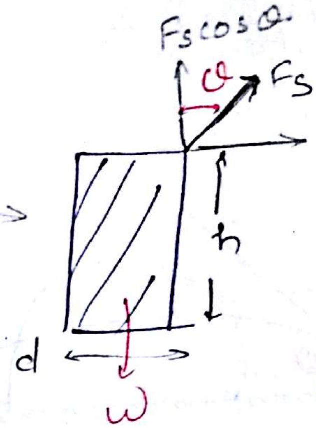


Adhesion is more



cohesion is more





$sp.\ wt = \frac{wt}{vol}$

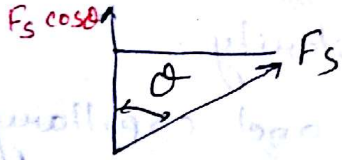
$\Rightarrow wt = sp\ wt \times vol$

$= w \times \frac{\pi}{4} d^2 h$

$$\sigma = \frac{F_s}{l} \Rightarrow F_s = \sigma l$$

$$F_s = \sigma \pi d$$

For equi, $\boxed{\text{Verticle component of } F_s = wt}$

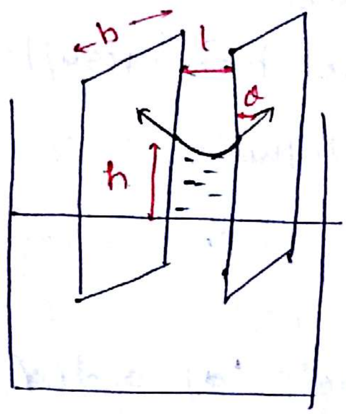


$$F_s \cos\theta = wt$$

$$\sigma \pi d \cos\theta = \rho g \frac{\pi}{4} d^2 h$$

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d} = \frac{4\sigma\cos\theta}{w d}}$$

Capillary rise b/w 2 parallel plates :



$$sp\ wt = \frac{wt}{vol}$$

$$\Rightarrow wt = w \times vol$$

$$wt = w \times hbt$$

$$wt = whbt \quad \text{—— (1)}$$

$$F_s = \sigma l$$

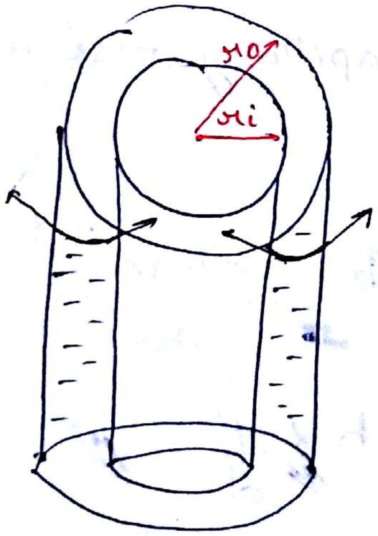
$$= \sigma \times 2b$$

verticle comp $= \sigma 2b \times \cos\theta$

$$2\sigma b\cos\theta = whbt$$

$$\boxed{h = \frac{2\sigma\cos\theta}{wt}}$$

Capillary rise b/w 2 coaxial tube :



$$sw = \frac{wt}{vol} \Rightarrow wt = sp\,wt \times vol$$

$$\Rightarrow wt = w \times \pi(r_o^2 - r_i^2)h$$

$$F_s = \sigma[2\pi r_o + 2\pi r_i)$$

verticle comp $= 2\pi\sigma(r_o + r_i)\cos\theta$

$$w \times \pi(r_o^2 - r_i^2)h = 2\pi\sigma(r_o + r_i)\cos\theta$$

$$w(r_o - r_i)h = 2\sigma\cos\theta$$

$$\Rightarrow \boxed{h = \frac{2\sigma\cos\theta}{\cancel{\cos}\, w(r_o - r_i)}}$$

Que) How does the rise of liquid get affected when the tube has insufficient length to the possible rise of the liquid.

gate

A small droplet of water at 20°C of a dia of 0.05 mm. If the pressure within the droplet is ·6 kPa higher than atm. pressure. Then find the ST.

Sol^n.

for liquid drop → $P = \frac{4\sigma}{d}$ => $\sigma = \frac{Pd}{4}$

$$\Rightarrow \sigma = \frac{\cdot 6 \times 10^3 \times \cdot 05 \times 10^{-3}}{4} = 7.5 \times 10^{-3} \frac{N}{m}$$

If the dia of tube is 1 mm then the capillary rise is 3 cm. what will be the capillary rise when the dia changes to ·2 mm

Sol^n for tube, capillary rise, $h = \frac{4\cancel{\sigma\cos\theta}}{\cancel{w}d}$ bcz of same liquid

$$\Rightarrow h \propto \frac{1}{d}$$

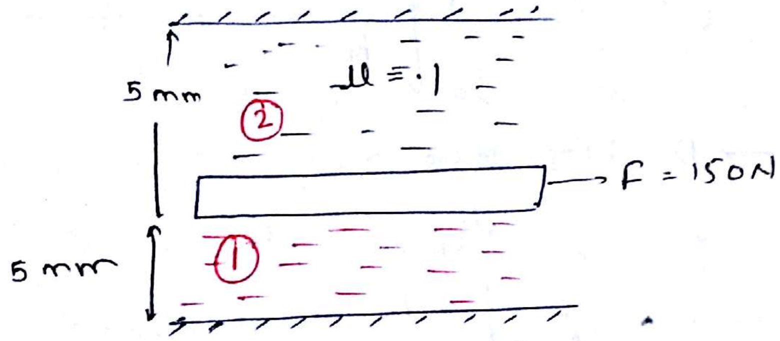
$$\Rightarrow \frac{h_2}{h_1} = \frac{d_1}{d_2}$$

$$\Rightarrow \frac{h_2}{3cm} = \frac{1\cancel{mm}}{\cdot 2\cancel{mm}}$$

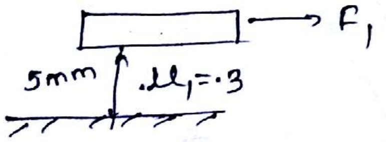
$$\Rightarrow h_2 = 15 cm.$$

Conventional que

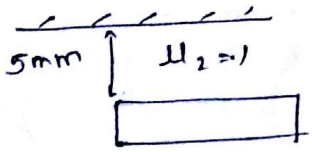
In the flow condit^n given in fig. determine the vel. at which the central plate of area 5 m² will move if a force of 150 N is applied to it. The viscosity of 2 oil are in the ratio is 1 : 3 and viscosity of top oil is 0.1 $\frac{N\text{-}s}{m^2}$



sol^n



$$F_1 = \frac{\mu_1 A V}{y_1} = \frac{.3 \times 5 \times V}{5 \times 10^{-3}}$$



$$F_2 = \frac{\mu_2 A V}{y_2} = \frac{.1 \times 5 \times V}{5 \times 10^{-3}}$$

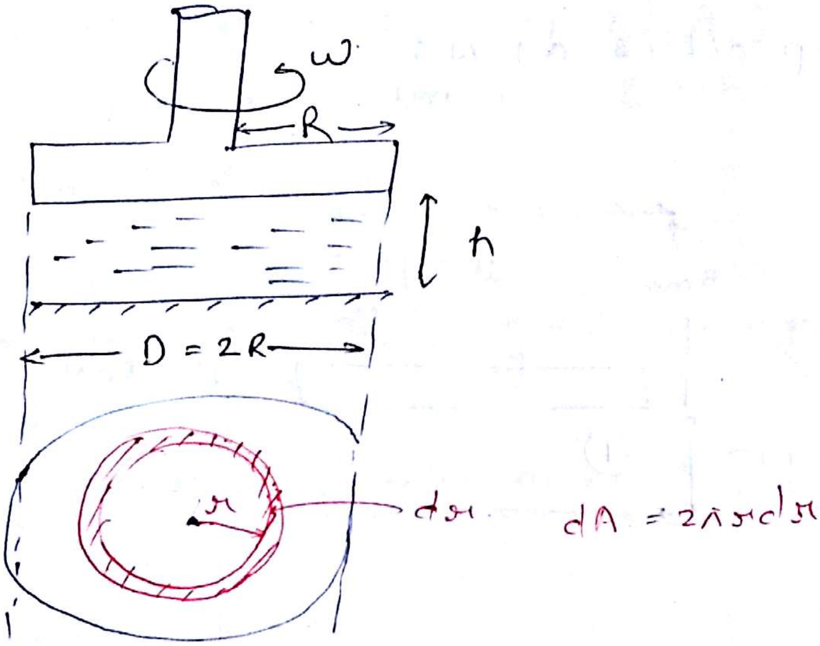
$$F = F_1 + F_2$$

$$150 = \frac{.3 \times 5V}{5 \times 10^{-3}} + \frac{.1 \times 5V}{5 \times 10^{-3}} \Rightarrow V = .375 \text{ m/s.}$$

imp
#

Same as (25)

A circular disc of dia d is kept a small ht h above a fixed surface by a layer of oil having a viscosity μ. Determine the expression for torque on the disc.



Sol^n.

$$\Rightarrow T = F \times r \qquad \text{(i)}$$

$$F = \frac{\mu A V}{y}$$

$$dF = \frac{\mu\,(2\pi r\,dr)(r\omega)}{h}$$

$$\Rightarrow dT = dF \times r$$

$$dT = \frac{2\pi\mu r^2\,\omega\,dr}{h} \times r.$$

$$\int dT = \int_0^R \frac{2\pi\mu\omega r^3\,dr}{h}$$

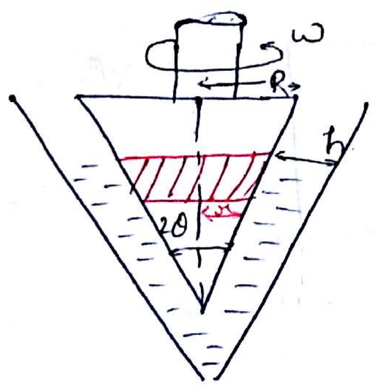
$$D = 2R$$
$$R = \frac{D}{2}$$
$$R^4 = \frac{D^4}{16}$$

$$T = \frac{2\pi\mu\omega}{h}\left[\frac{r^4}{4}\right]_0^R$$

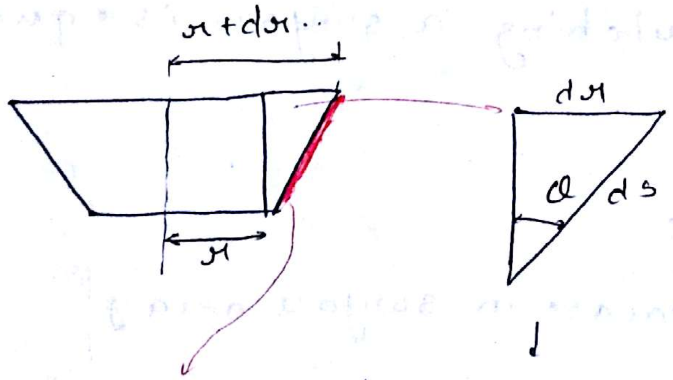
$$= \frac{\pi\mu\omega R^4}{2h} \Rightarrow \boxed{T = \frac{\pi\mu\omega D^4}{32h}}$$

imp.

A solid cone of radius R and vertex angle 2θ is made to rotate at an angular vel. ω in a conical cavity containing oil with a viscosity μ. If h is the gap b/w cone and cavity then find the torque to rotate the cone.



Sol^n



$$dA = 2\pi r\,ds \longrightarrow dA = \frac{2\pi r\,dr}{\sin\theta}$$

$$\Rightarrow \sin\theta = \frac{dr}{ds} \Rightarrow ds = \frac{dr}{\sin\theta}.$$

$$dF = \frac{\mu\, dA\, V}{y}$$

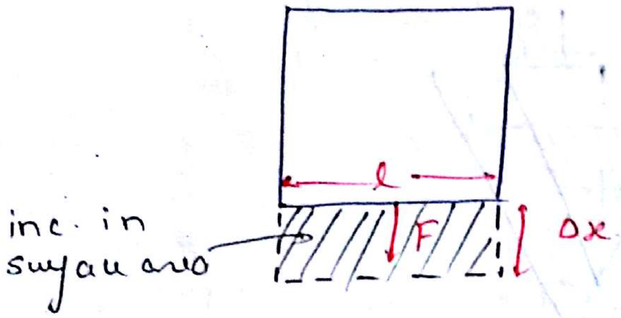
$$dF = \frac{\mu\, dA\,(\omega r)}{h} = \frac{\mu\left(\frac{2\pi r\,dr}{\sin\theta}\right)\omega r}{h}$$

$$\Rightarrow \int dF = \frac{\mu 2\pi \omega}{h \sin\theta}\int_0^R r^2\,dr.$$

$$\Rightarrow dT = dF \times r$$

$$T = \frac{2\pi\mu\omega}{h\sin\theta}\int_0^R r^3\,dr.$$

$$T = \frac{2\pi\mu\omega}{h\sin\theta}\left(\frac{R^4}{4_2}\right) = \frac{\mu\omega\pi}{2h\sin\theta} R^4$$



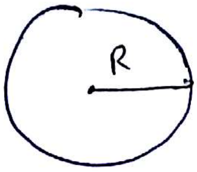
work done in stretching a surface is equal to.

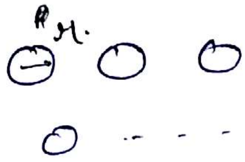
$$WD = F \times \Delta x$$

$$= \sigma l \times \Delta x$$

$$\boxed{WD = \sigma\,[\text{increase in surface area}]}$$

#28)





$V = \frac{4}{3}\pi R^3$ $\qquad$ $(V)_{each\ droplet} = \frac{4}{3}\pi r^3$

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$\Rightarrow R^3 = n r^3$$

$$r = \frac{R}{n^{1/3}}$$

$$W = \sigma\,[\text{inc. in surface area}]$$

$$W = \sigma\,[A_f - A_i]$$

$$W = \sigma\,[4\pi r^2 n - 4\pi R^2]$$

$$W = 4\pi\sigma\,[r^2 n - R^2]$$

$$W = 4\pi\sigma\left[\frac{R^2}{n^{2/3}}\cdot n - R^2\right]$$

$$\boxed{W = 4\pi\sigma R^2\,[n^{1/3} - 1]}$$

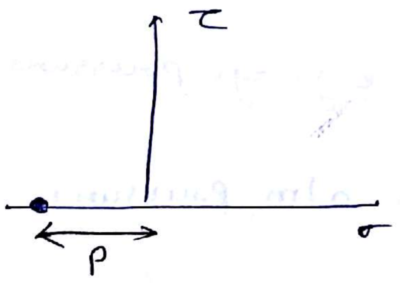
Chap-2

Pressure measurement (manometry)

Pressure is defined as normal force exerted by the fluid per unit area.

unit → $\boxed{\frac{N}{m^2} \text{ @ Pa}}$

- Pressure also sometime expressed in bar in order w.r.to compare with atmosphere which is around 1 bar.
- It is compressive in nature.
- In a static fluid as there is no shear force. There will be only normal forces (Pressure)
- ∴ For a static fluid mohr circle is a pt. as shown in fig.



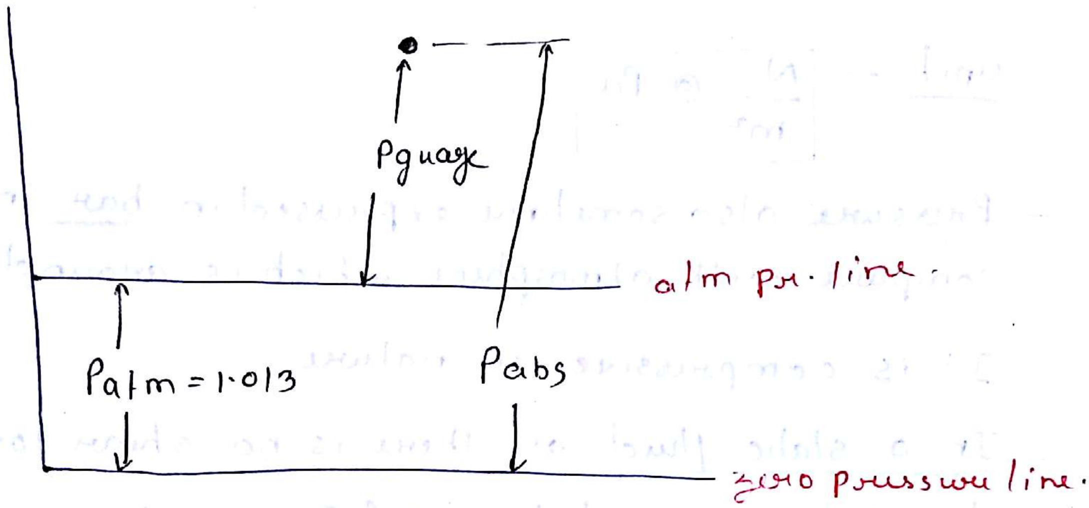
Atmospheric pressure :-

The pressure exerted by environmental mass is known as atm. pressure.

- Atm pressure is around 1.013 bar.
- Atm. pressure is measured by barometer.

guage pressure :-

The pressure measured wrt. atmospheric pressure is known as guage pressure.



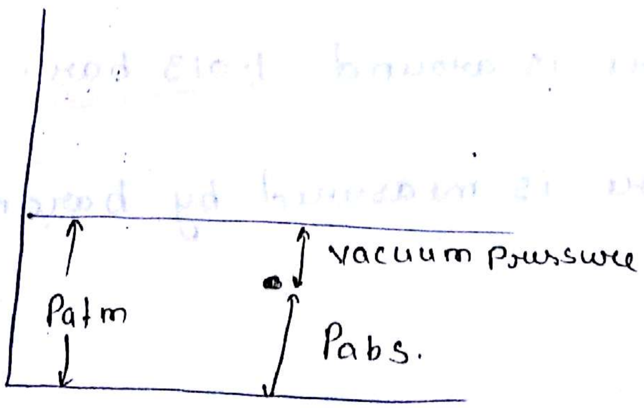
The pressure measured wrt zero pressure is known as absolute pressure.

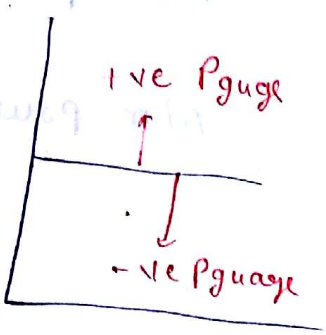
Vacuum pressure : (-ve guage pressure)

The pressure less than atm. pressure is known as vacuum pressure

Note :-

There can be +ve guage pressure @ -ve guage Pr. but there cannot be -ve absolute pressure





Pascal's law :

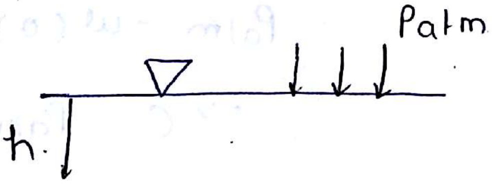
The pressure at a pt. is equal in all directⁿ in a static fluid @ If a pressure is applied at a pt in a static fluid it is transmitted equally in all directⁿ.

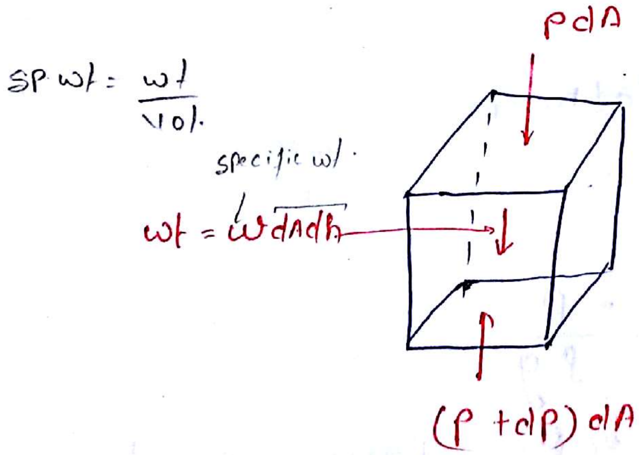
Applicatⁿ of Pascal's law

1) Hydraulic lift
2) Hydraulic brake.

Note :- Pascal's law can also be applied for a flowing fluid if the fluid is Ideal bcz in Ideal fluid there are no viscous @ shear forces.

Hydrostatic law :-





$$PdA + w\,dA\,dh = (P + dP)dA$$

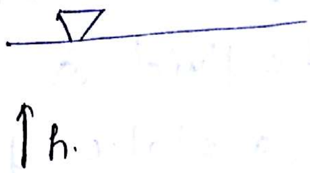
$$P + w\,dh = P + dP$$

$$\boxed{\frac{dP}{dh} = w}$$

Hydrostatic law give variatⁿ in pressure in verticle directⁿ.

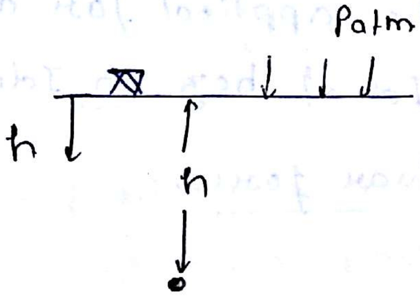
Note: If h is taken in vertically upward directn then,

$$\boxed{\frac{dP}{dh} = -w}$$



→ For guage pressure as the reference pressure is atm. pressure ∴ it is treated as zero.

Pressure at any depth h :-



at $h = 0$, $P = P_{atm}$.

$$\frac{dP}{dh} = w \Rightarrow \int dP = \int w\, dh$$

$$P = wh + C$$

$$P_{atm} = w(0) + C$$

$$\Rightarrow C = P_{atm}$$

$$\Rightarrow P = wh + P_{atm}$$

if P is guage press, $P_{atm} = 0$

$$\Rightarrow P = wh$$

$$\Rightarrow \boxed{P = \rho g h}$$

$$h = \frac{P}{\rho g}$$

$h = \frac{P}{w}$ ← Pressure head.

* Pressure is sometime expressed in ht. of liquid column this is bcz ρ ~~is~~ ~~at~~

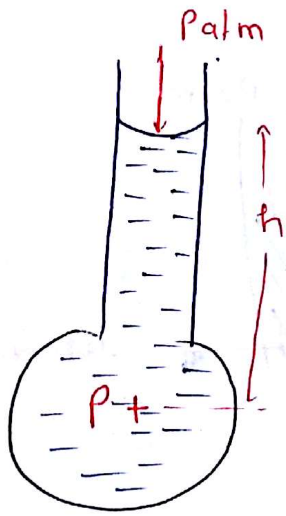
$$P = \underset{\text{const.}}{\rho}\ \underset{\text{const.}}{g}\ h. \quad \Rightarrow P \text{ depends on } h.$$

Manometry :-

The principle of finding the pressure by using hydrostatic law is known as manometry.

1) Piezometer.

Piezometer is a tube open at both ends, one end is connected to a pt. where pressure is to be found and other end is opened to atmosphere.



$$0 + \rho g h = P$$

$$\Rightarrow \boxed{P = \rho g h.}$$

Note:- Piezometers are used for finding out moderate liquid pressure. (not used for finding gas Pr.)

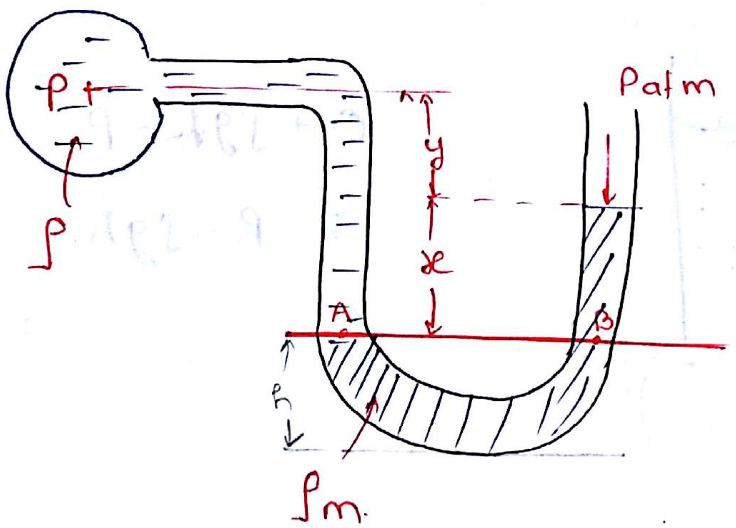
Note :- In order to nullify the effect of capilarity the dia of tube must be large (greater than 1 cm).

Manometer :-

1) Simple
(Pressure at a pt.)

2) Differential
(Pressure difference b/w 2 pts)

Simple U-tube manometer :-



$$\rho_m g h - \rho_m g h$$
$$= 0$$

that's why pressure at same level of same fluid is equal.

1) Jumping of fluid technique :-

$$P + \rho g(x+y) - \rho_m g x = 0$$ ← $P_{atm} = 0$

$$\boxed{\begin{array}{l}\text{Downward movement : +ve} \\ \text{upward movement : -ve}\end{array}}$$

$$\Rightarrow P = \rho_m g x - \rho g(x+y)$$

2) Datum technique :-

$$P_A = P_B$$

$$P + \rho g(x+y) = P_A$$

$$0 + \rho_m g x = P_B$$

$$\Rightarrow P + \rho g(x+y) = \rho_m g x$$

$$\Rightarrow P = \rho_m g x - \rho g x + y.$$

Reason for using mercury as manometric fluid :-

1) low vapour pressure
2) High density → $P = \rho g h$ ← for small h, ρ should be high.
3) Immisible with other fluid.

Pr. have be observed at 4 diffrent Pt. in diffrent units as follows.

a) 150 kPa , b) 1800 mill bar. c) 20 m of water

d) 1240 mm of Hg.

then arrange in ~~order~~ in decending order of mag. of Pressure.

Soln

$$150\,kPa = 150\times10^3\,Pa = 150\times10^3\,\frac{N}{m^2}$$

$$1800\text{ mill bar} = 1800\times10^{-3}\text{ bar} = 1800\times10^{-3}\times10^5\,\frac{N}{m^2}$$

$$20\text{ m of water} \longrightarrow P = \rho g h$$

$$= 10^3\times9.8\times20$$

$$= 196.62\times10^3\,N/m^2$$

$$1240\text{ mm of Hg} \longrightarrow P = \rho g h$$

$$= 13.6\times10^3\times9.81\times\frac{1240}{1000}$$

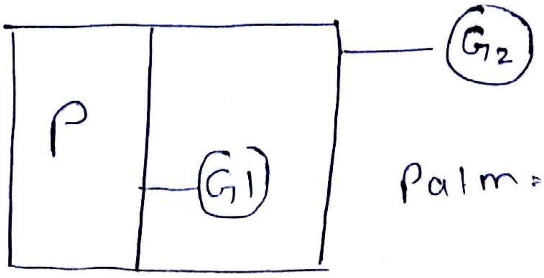
$$= 165.43\times10^3\,N/m^2$$

$\therefore\ C > B > D > A$

The pressure guages G_1 & G_2 installed on a

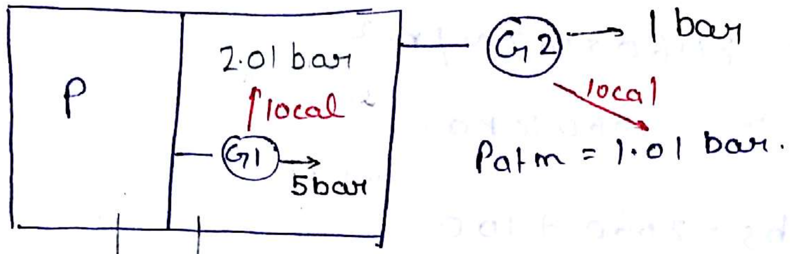
(21) system shows pressure of $P_{G1} = 5$ bar, $P_{G2} = 1$ bar

gate then find the unknown pressure P in absolute scale.



Patm = 1.01 bar.

Sol^n



$$\boxed{P_{abs} = P_g + P_{atm}}$$ local

$P_{abs} = P_{G2} + P_{atm}$

$= 1\text{ bar} + 1.01\text{ bar}$

$$\boxed{P_{abs} = 2.01\text{ bar}}$$

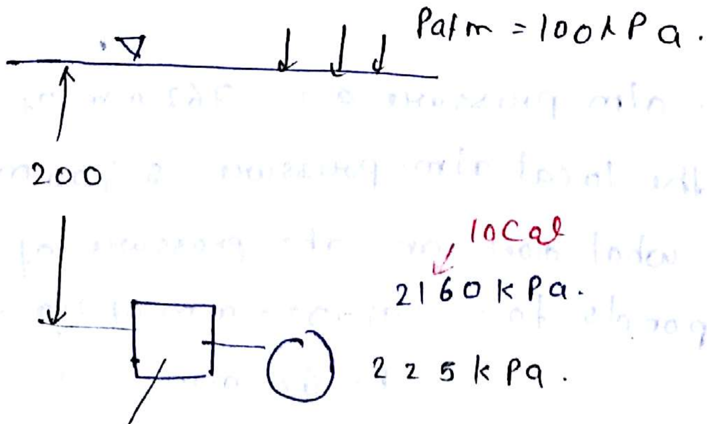
$P_{abs} = P_{G1} + P_{atm}$

$= 5\text{ bar} + 2.01\text{ bar}$

$$\boxed{P_{abs} = 7.01\text{ bar}}$$

\# A diver decents 200 m in a sea ($\rho = 1050\ kg/m^3$) to a sunken ship where in a container is formed with a pressure guage reading of 225 kPa. Taking pressure at sea surface to be atmospheric (100 kPa) then find the absolute pressure in a container.

Sol^n



$P_{abs} = P_g + P_{atm}$

$P_{abs} = 225 + 2160 = 2385$

gauge pr.

$$\rightarrow p = \rho g h$$

$$= 1050 \times 9.81 \times 200$$

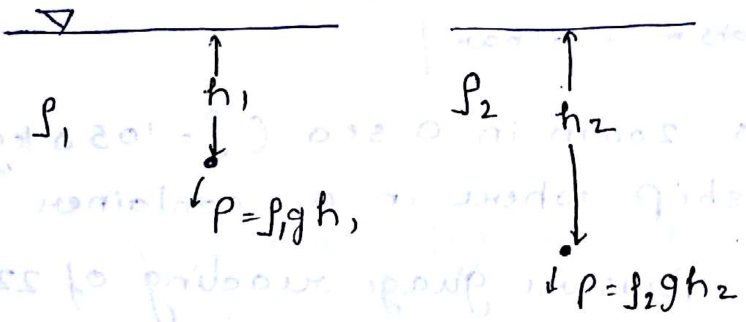
$$= 2060 \times 10^3 \ N/m^2$$

$$= 2060 \ kPa.$$

$$P_{abs} = 2060 + 100$$

$$= 2160 \ kPa.$$

conversion of one fluid column into the other fluid column



$$\rho_1 g h_1 = \rho_2 g h_2$$

$$\boxed{\rho_1 h_1 = \rho_2 h_2}$$

$$\frac{\rho_1 h_1}{\rho_{H_2O}} = \frac{\rho_2 h_2}{\rho_{H_2O}}$$

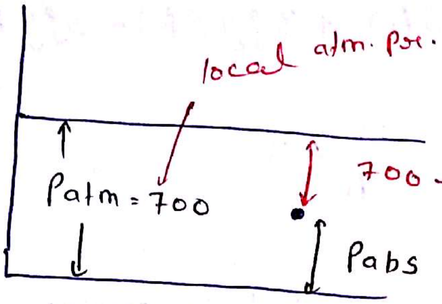
$$\boxed{S_1 h_1 = S_2 h_2}$$

③ the std. atm. pressure is 762 mm of Hg (now use of std. pr.). At a specific locat^n the local atm. pressure is 700 mm of Hg. At this place what does an abs. pressure of 380 mm of Hg corresponds to — a) 320 mm of Hg vacuum

b) 62 mm " "

c) 382 mm " "

d) 300 mm " "

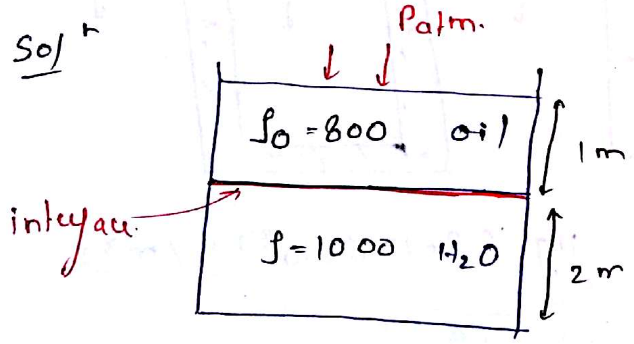


700 - 380 = 320 mm of Hg vacuum. Ans

Pabs = 380 mm

An open tank contain water to a depth of 2m & oil over it to a depth of 1m. The density of oil is 800 kg/m³. Then find the pr at the interface of 2 fluid layer.

Solⁿ

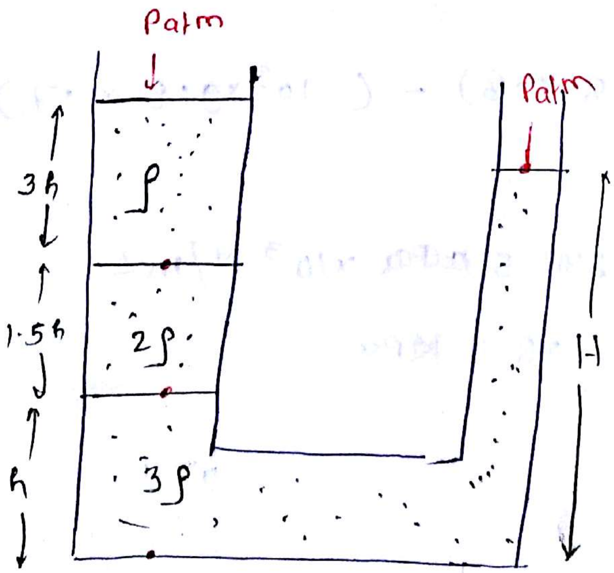


In guage pr → Patm = 0

$0 + 800 \times 9.8 \times 1 = P_{interface}$

$P_{int} = 7848\ N/m^2$

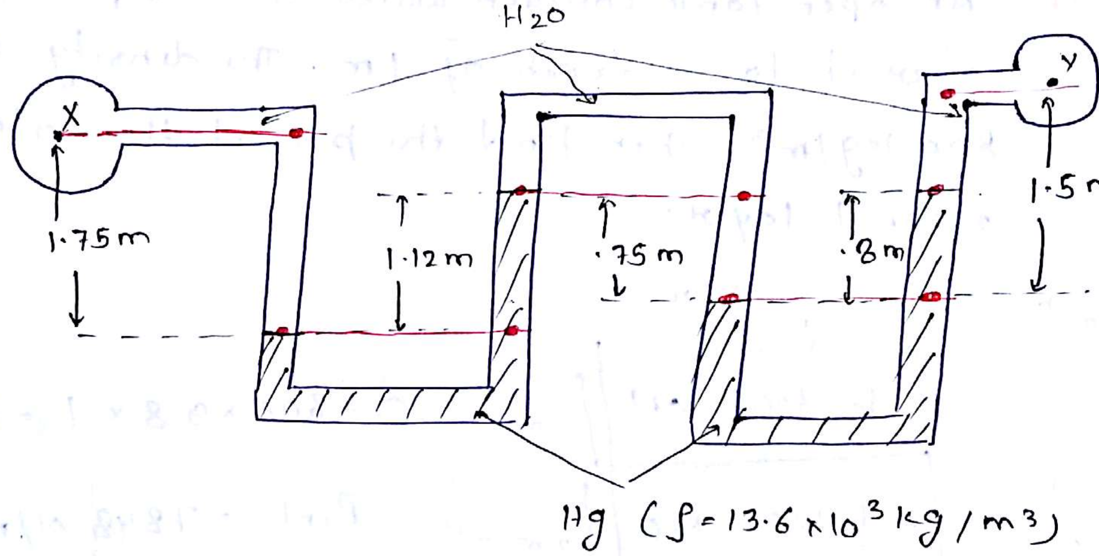
15) 3 immisible liquid of density $\rho, 2\rho, 3\rho$ are kept in a jar and a piezometer fitted to the bottom of the jar is shown in fig. Then find → H/h.



$$0 + \rho g(3h) + 2\rho g(1.5h) + 3\rho g(h) - 3\rho g H = 0$$

$$\Rightarrow \quad 9h = 3H \quad \Rightarrow \quad \frac{H}{h} = 3$$

\# 2 U-tube manometer are connected in series as shown in fig. Then find the pr. difference b/w x & y in kPa.



multi U-tube

Note: This manometer is known as multi U-tube manometer & these are used for finding out high pressure.

Sol^n

$$P_x + (10^3 \times 9.8 \times 1.75) - (13.6 \times 10^3 \times 9.8 \times 1.12) + (10^3 \times 9.8 \times .75)$$

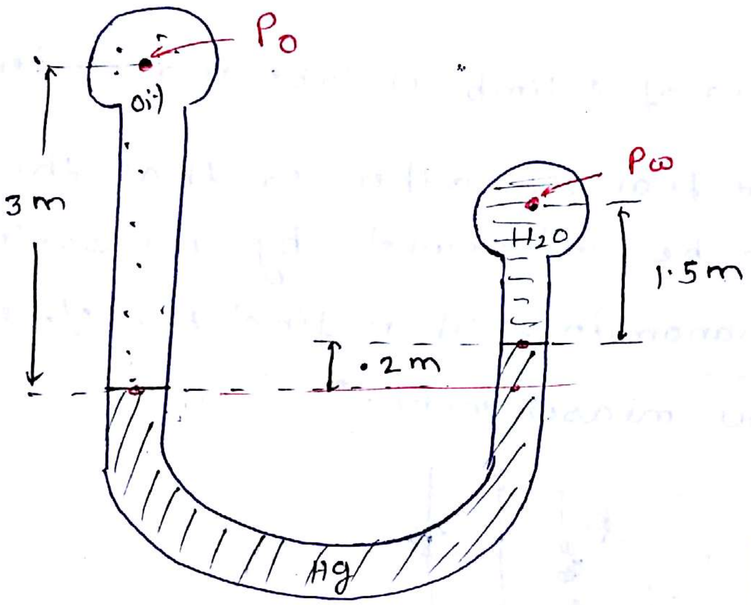
$$- (13.6 \times 10^3 \times 9.8 \times .8) - (10^3 \times 9.8 \times .7) = P_y$$

$$\Rightarrow P_x - P_y = 238.5 \ \cancel{kPa} \times 10^3 \ N/m^2$$

$$= 238.5 \ kPa.$$

imp

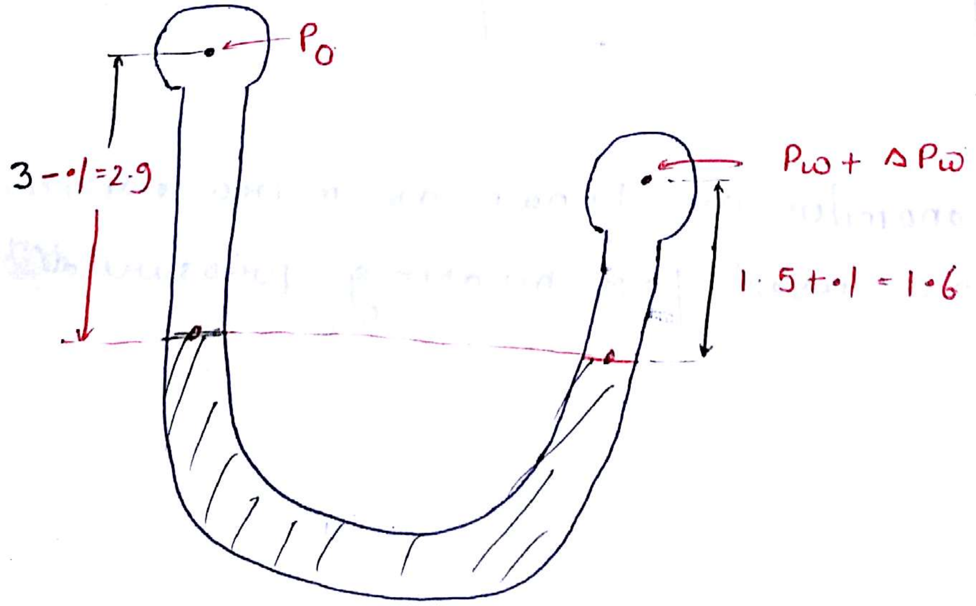
2 pipe lines one oil with oil of $\rho = 900 \, kg/m^3$ & other with water are connected to a manometer as shown in fig. By what amount the pressure in water pipe should be increased without changing oil pressure so that mercury level in both limbs become equal. Take $\rho_{Hg} = 13550 \, kg/m^3$



Soln

$$P_o + (900 \times 9.81 \times 3) - (13550 \times 9.8 \times .2) - (10^3 \times 9.8 \times 1.5) = P_w$$

$$\Rightarrow P_o - 14813.1 = P_w \quad \text{—— (1)}$$



$P_0 + (900 \times 9.81 \times 2.9) - (10^3 \times 9.81 \times 1.6) = P_w + \Delta P_w$

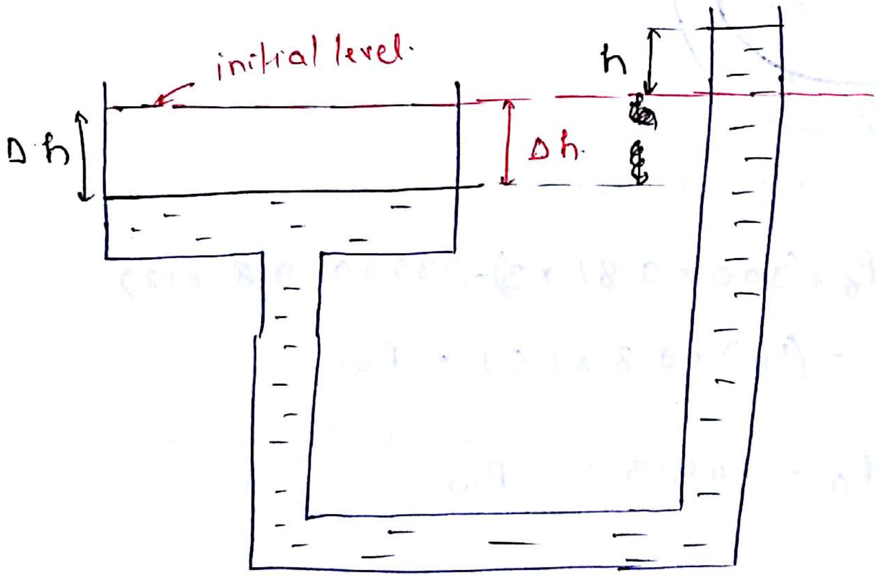
$\Rightarrow P_0 + 9908.1 = P_w + \Delta P_w$ —— (2)

from (1) & (2)

$\Delta P_w = 24712 \, N/m^2$

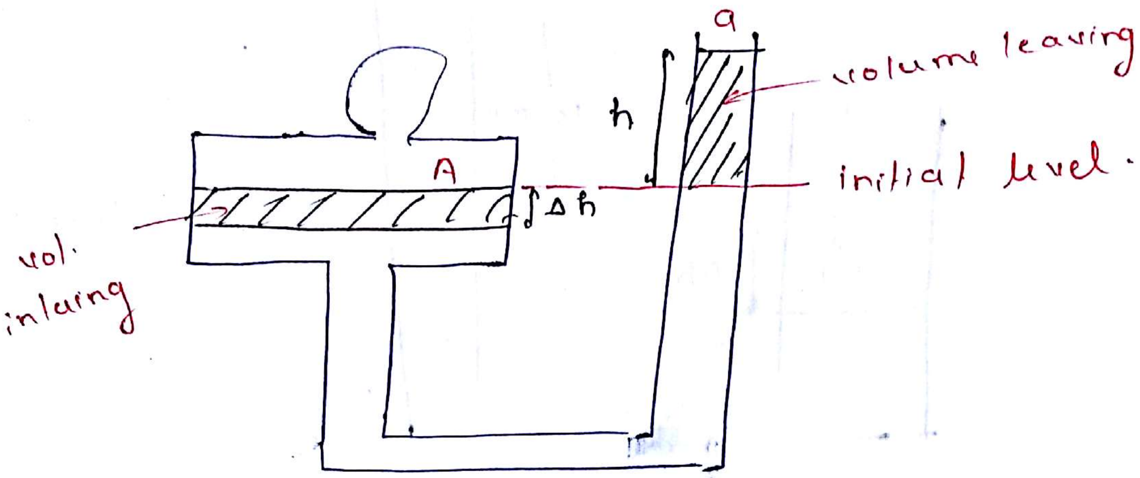
$= 24.7 \, kN/m^2$

\# (14) A x-sectional area of 1 limb U-tube manometer is made 500 times larger than the other so that the pr. diff. b/w 2 limbs can be determined by measuring 'h' on 1 limb of manometer. Then find the % error w.r.t. this pressure measurement.



Sol^n

Note [These manometer are known as micro-manometer & these are used for measing pressure diff. accurately



Volume inlaving = vol leaving

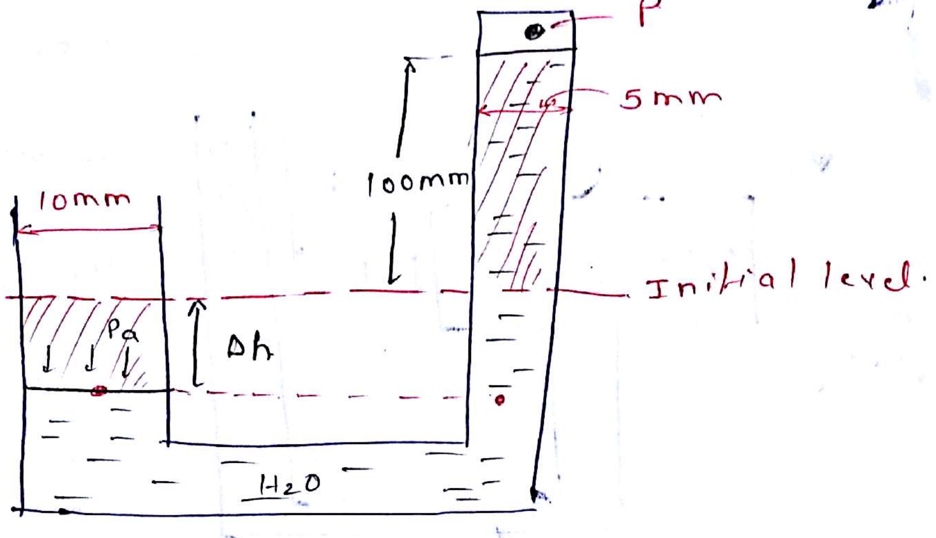
$$A \Delta h = a h.$$

$$\Rightarrow \frac{\Delta h}{h} = \frac{a}{A} = \frac{\cancel{\alpha}}{500 \cancel{\alpha}} = \frac{1}{500}$$

$$\Rightarrow \frac{\Delta h}{h} \times 100 = \frac{1}{500} \times 100 = \cdot 2 \%.$$

(12) A u-tube manometer as shown in fig. as water as a manometric fluid. when one unknown pr. 'P' acts

at 5mm dia limb the water rises in a limb by 100mm from initial level. If the other end is opened to atm. (Pa) then find $(P - Pa)$ in N/m^2



Sol^n

vol. leaving = vol. inleting.

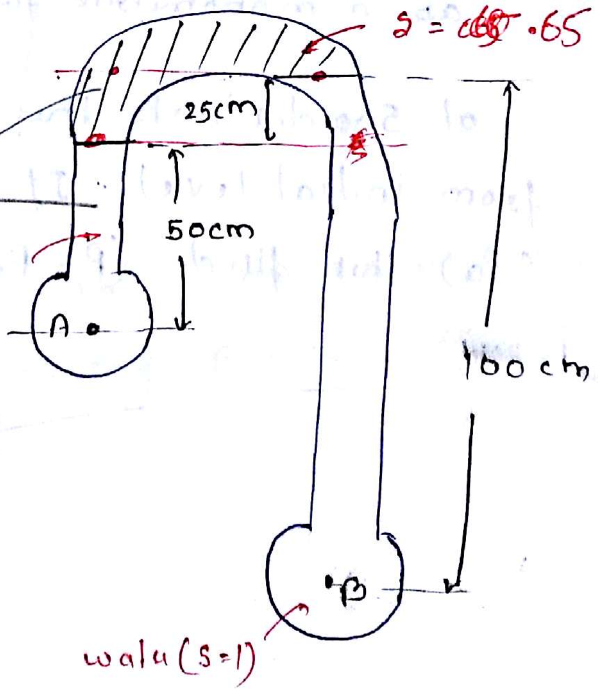
$$\frac{\pi}{4}(10)^2 \times \Delta h = \frac{\pi}{4}(5)^2 \times 100$$

$$\Rightarrow \Delta h = 25\,mm.$$

$$P_a - 10^3 \times 9.81 \times \frac{125}{1000} = P$$

$$\Rightarrow P - P_a = -1226\ N/m^2$$

Find the Pr. diff. b/w pt. B & A as shown in fig. in cm of H_2O



Sol^n

$$h_1 s_1 = h_2 s_2$$

$$50 \times .8 = h_w \times 1$$

$$\Rightarrow h_w = 40$$

$$h_1 s_1 = h_2 s_2$$

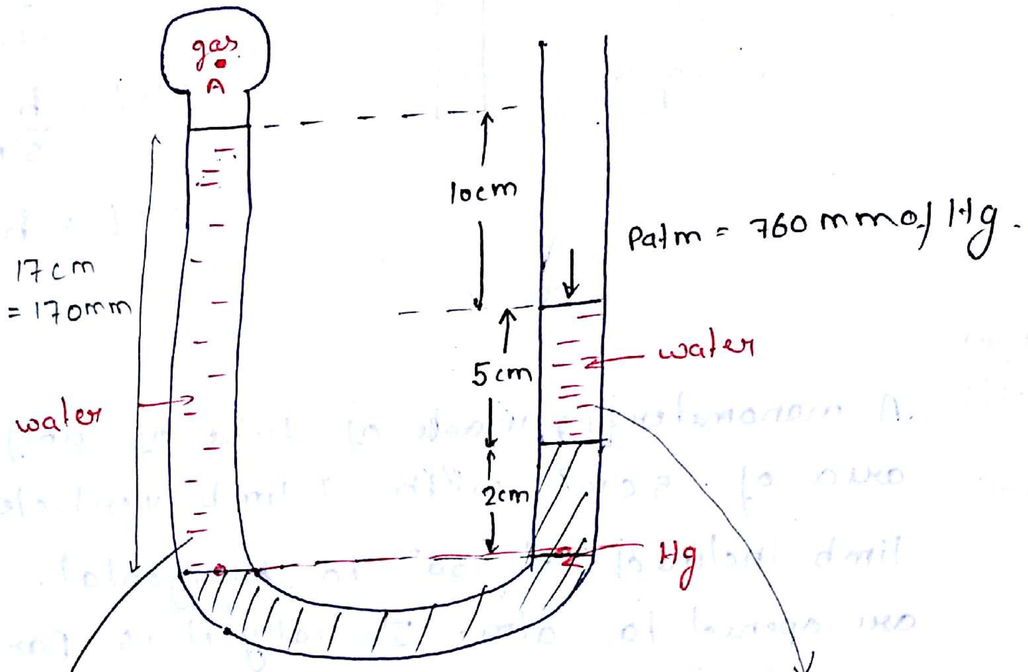
$$25 \times .65 = h_w \times 1$$

$$h_w = 16.25$$

$$P_A - \underset{\cancel{\text{[illegible]}}}{40} - \underset{\cancel{\text{[illegible]}}}{16.25} + \underset{\cancel{\text{[illegible]}}}{100} = P_B$$

$$\Rightarrow P_B - P_A = 43.75 \text{ cm of water.}$$

\# ** Refer the fig. and find absolute pr. of gas A.



Sol^n

$$h_w S_w = h_m S_m$$

$$170 \times 1 = h_m \times 13.6$$

$$\Rightarrow h_m = \frac{170}{13.6} = 12.5.$$

$$50 \times 1 = h_m \times 13.6$$

$$\Rightarrow h_m = \frac{50}{13.6} = 3.68$$

2cm = 20mm

$$P_A + 12.5 - 20 - 3.68 = 0$$

$$\Rightarrow P_A = 11.17 \text{ mm of Hg (guage)}$$

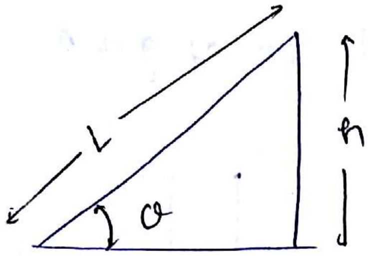
$$\therefore P_{abs} = P_g + P_{atm}$$

$$= 11.17 + 760$$

$$= 771.17$$

Note:

The sensitivity of inclined manometer is $\frac{1}{\sin\theta}$



$\sin\theta = \frac{h}{L}$

$\Rightarrow L = \frac{h}{\sin\theta}$

$\Rightarrow L = h \times \frac{1}{\sin\theta}$ ← sensitivity.

v.v.imp.

good Que.

#19 A manometer is made of tube of uniform x-sectional area of .5 cm² with 1 limb verticle and other limb inclined at 30° to horizontal. Both the limbs are opened to atm. Initially it is partly filled with a liquid of specific gravity 1.25. If now an additional vol. of 7.5 cm³ of water is added to inclined tube cal. the rise of liquid in verticle tube.

Solⁿ.

Vol. of addition water added = Vol of limb occupied by water. ← Area x length.

$\Rightarrow 7.5 = .5 \times (y_1 + y_2)$

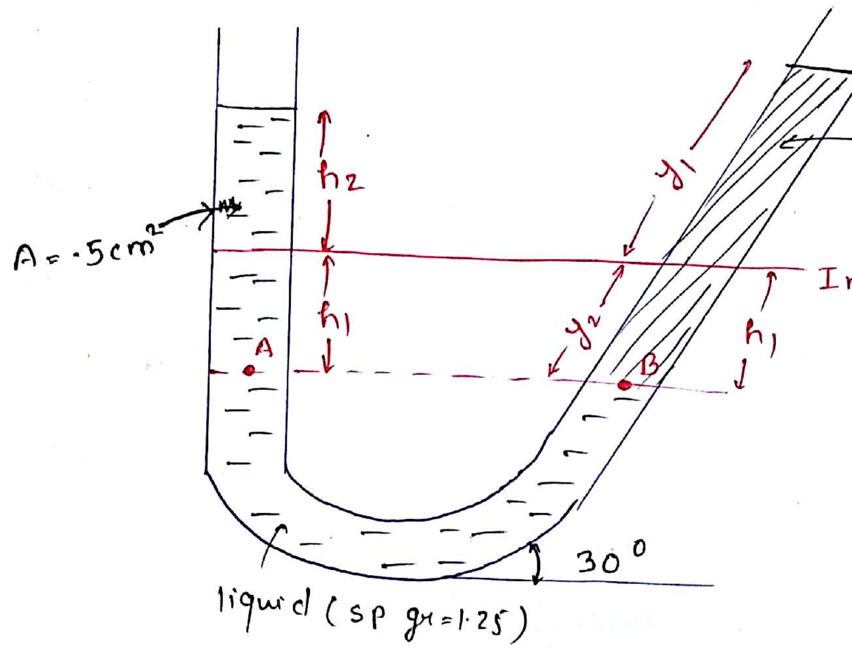
$\Rightarrow y_1 + y_2 = 15$ —— (1)

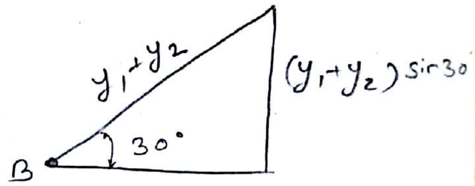
Vol. of additional water added = Volume above initial level

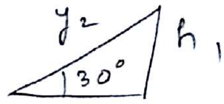
$7.5 = (.5 \times h_2) + (.5 \times y_1)$

$\Rightarrow h_2 + y_1 = 15$ —— (2)

Soln







$$P_A = P_B$$

$$\Rightarrow h_1 S_1 = h_2 S_2$$

$$(h_1 + h_2)(1.25) = \underbrace{(y_1 + y_2)}_{\text{from (1)}} \sin 30 \times 1$$

$$\Rightarrow (h_1 + h_2)\, 1.25 = 15 \times \frac{1}{2}$$

$$\Rightarrow h_1 + h_2 = 6 \quad \text{——} \quad (3)$$

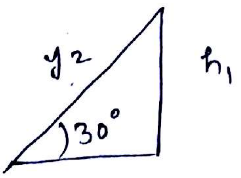
same.

$P_A = P_B$

$\rho_l g(h_1 + h_2) = \rho_w g(y_1+y_2)\sin 30$

$(1.25 \times 10^3)(h_1 + h_2) = 10^3 (15) \times \frac{1}{2}$

$h_1 + h_2 = 6$



$$\sin 30 = \frac{h_1}{y_2}$$

$$\Rightarrow \frac{1}{2} = \frac{h_1}{y_2} \Rightarrow y_2 = 2h_1 \quad \text{——} \quad (4)$$

solving all 4 eqⁿ →

we get, $h_1 = 2$

$\boxed{h_2 = 4}$ → Rise of liquid

Ans.

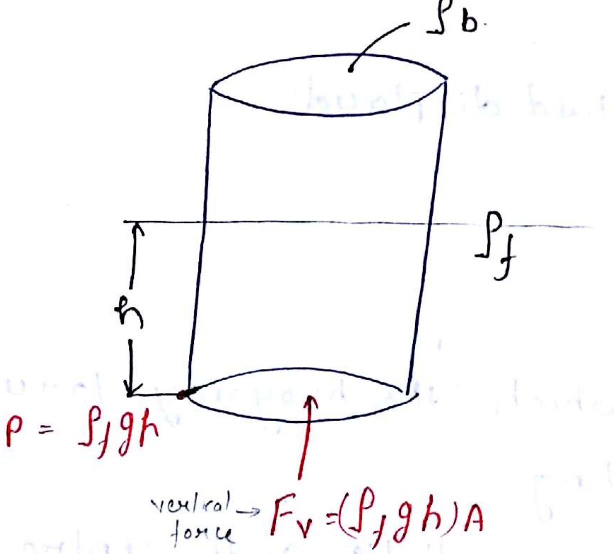
Buoyancy and flotation

Archimedis principle :

when a body is immersed either partially @ completely the net verticle upward force exerted by the fluid on the body is known as buoyancy force and this buoyancy force is equal to wt. of fluid displaced.

In a constant density fluid buoyancy force is basically due to pressure difference.

Partially immersed



$w = \frac{wt}{vol.}$

$\Rightarrow wt = w \times vol.$

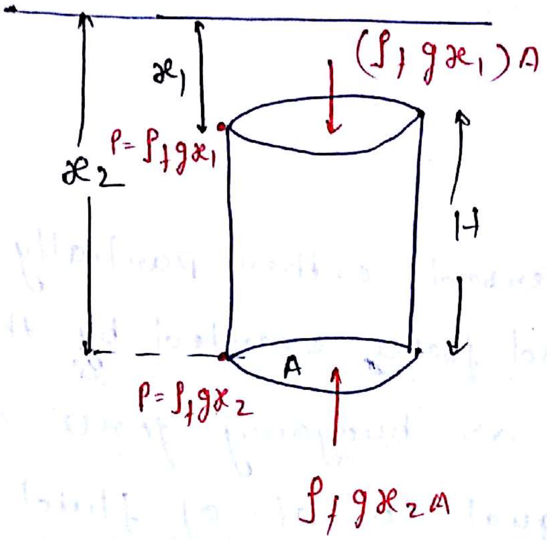
$\boxed{wt = \rho g \, vol.}$

$F_V = \rho_f g h A \Rightarrow F_V = \rho_f g V_{fd} \Rightarrow \boxed{\uparrow F_V = wt. \text{ of fluid displaced}}$

hA = vol. of body immersed in fluid

hA = vol. of fluid displaced.

fully immersed.



$$(F_v)_{net} = \rho_f g (x_2 - x_1) A$$
$$= \rho_f g H A$$

$$(F_v)_{net} = \rho_f g V_{fd}$$

$\uparrow$ $\boxed{(F_v)_{net} = \text{wt. of fluid displaced}}$

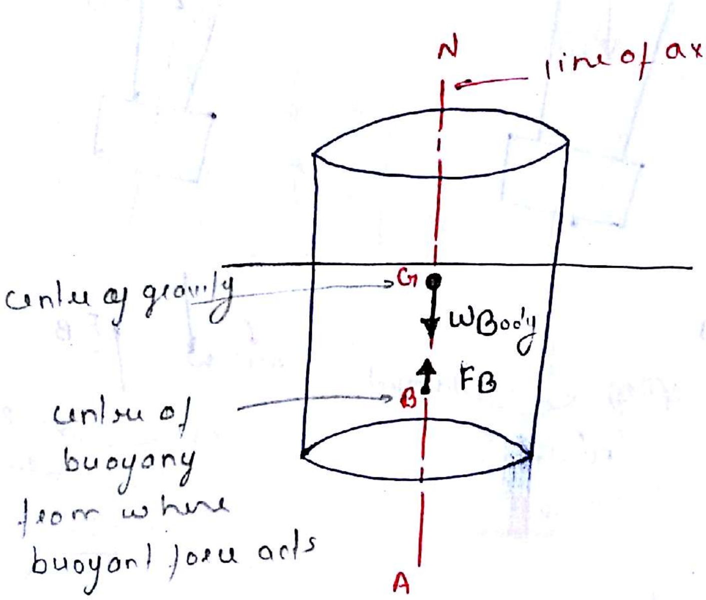
Centre of buoyancy :-

It is the pt. from which the buoyancy force is supposed to be acting.
and centre of buoyancy will lie at the centroid of the displaced volume.

Ravi

Principle of floatat^n :-

For a floating body in equilibrium wt. of the body is equal to the buoyancy force and the line of act^n of this two force must be same.



wt of body = buoyancy force

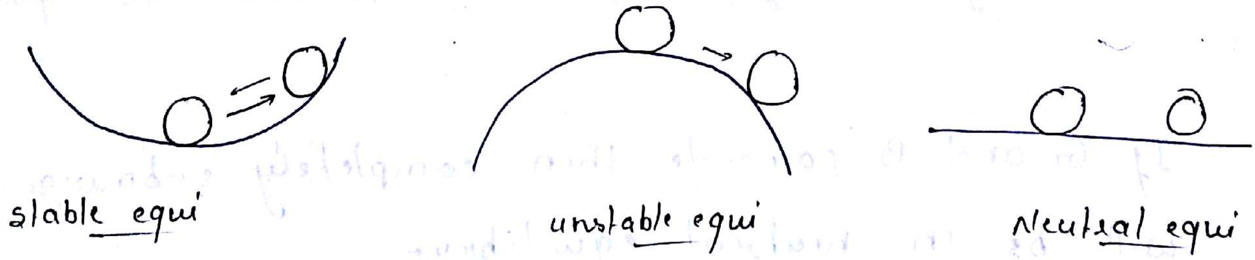
$$W_{Body} = F_B \quad \text{——} \; (1)$$

& we know,

$$F_B = W_{fd} \quad \text{——} \; (2)$$

$$\Rightarrow \boxed{W_{Body} = W_{fd}} \; ***$$

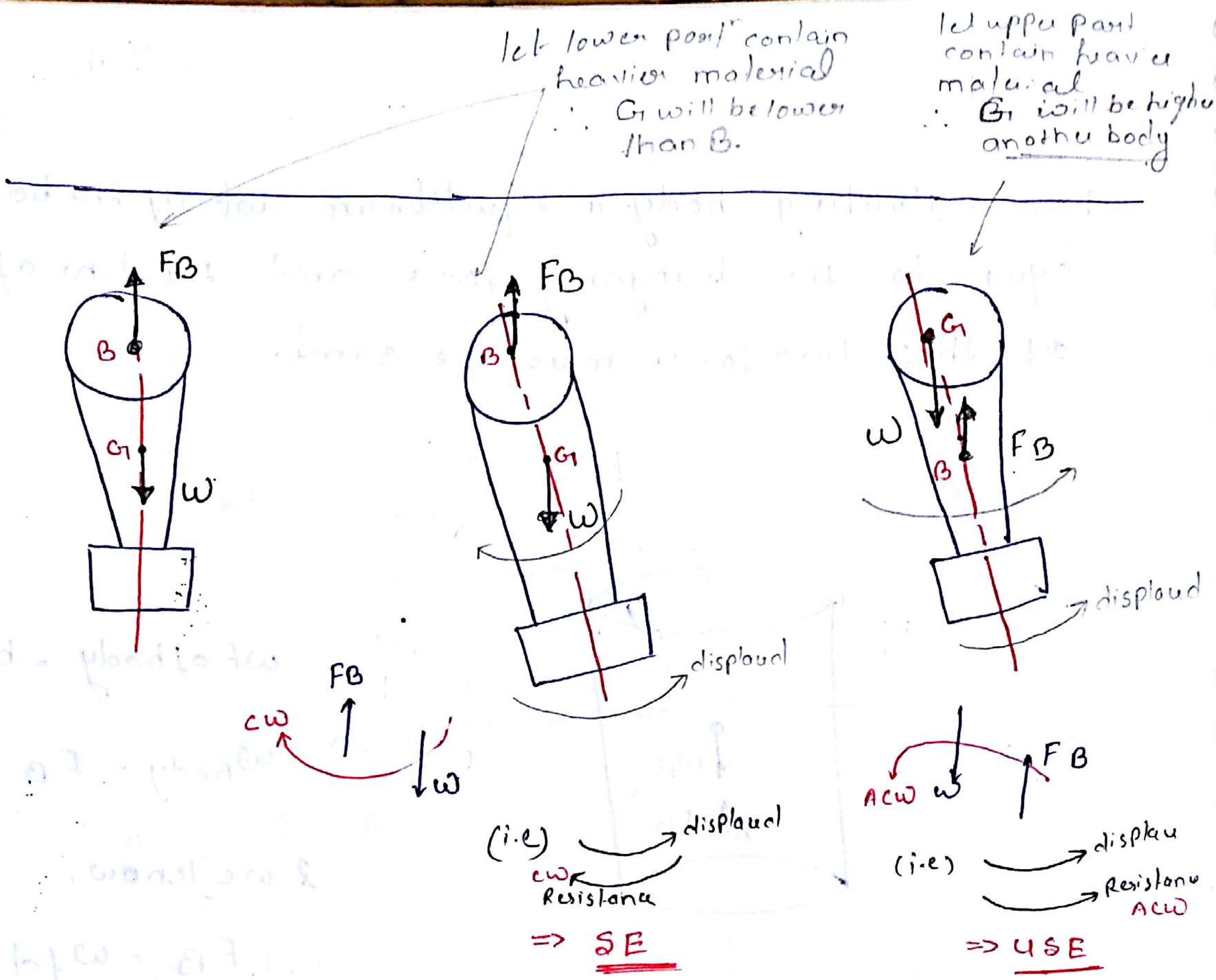
Types of equilibrium :-



Stability condition for completely submeged body :

what ever be the posit^n of body ⟶ Posit^n of B & G will be same

$$B = \rho_f \, g \, \underbrace{V_{fd}}_{const} \Rightarrow B = const$$



A completely submerged body will be in stable equiligum when centre of buoyancy(B) is above the centre of gravity (G)

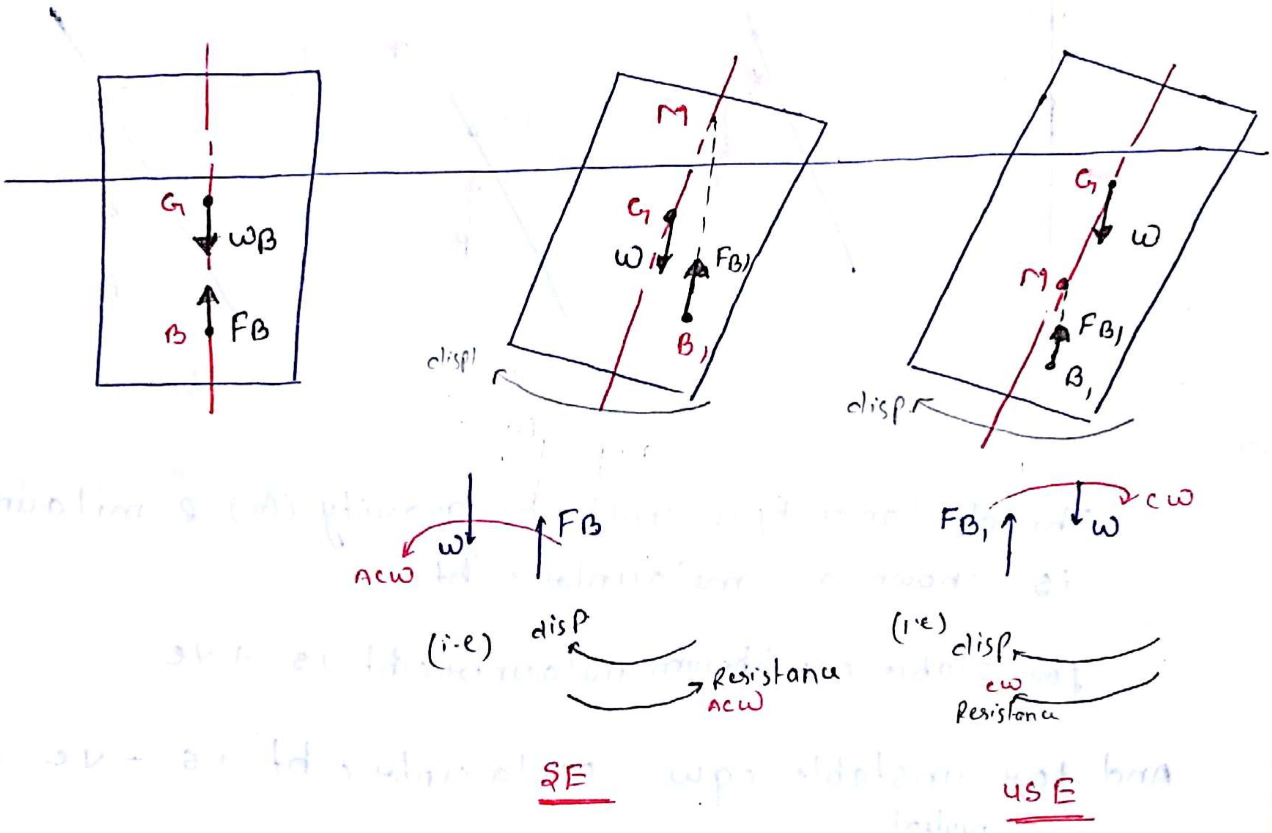
If the centre of buoyancy(B) is below the centre of gravity(G) completely submerged body will be in unstable equi.

If G and B coincide then completely submerged body will be in neutral equilibrum

Stability condit^n for partially submerged @ floating body.

G will be in same posi^n but B will not be in same posi^n.

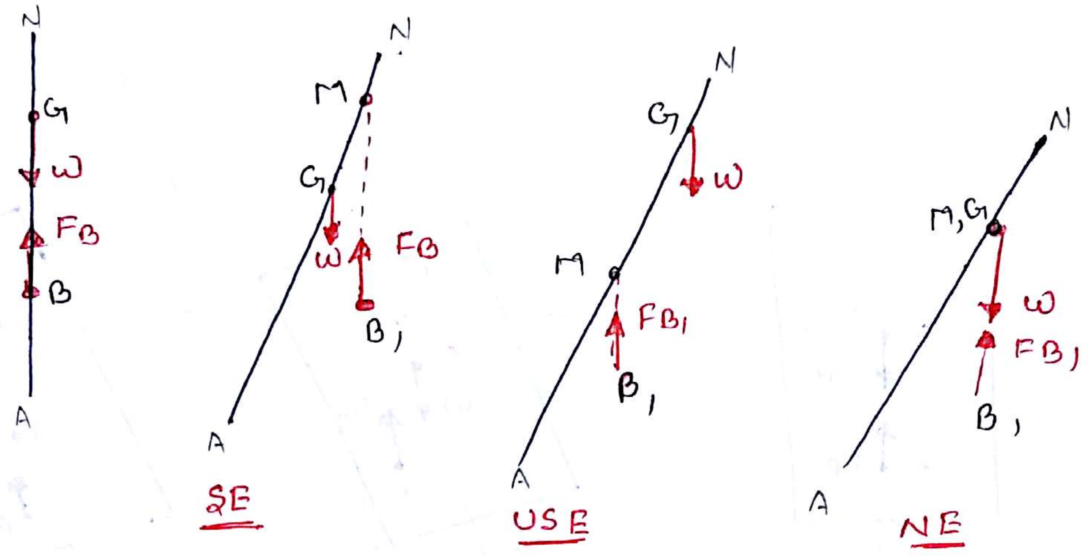
bcz: centre of buoyancy (B) move where more fluid is displaced



A floating body is in stable equi. when metacentre (M) is above centre of gravity (G)

A floating body will be in unstable equilibrium when metacentre (M) is below the centre of gravity (G)

If G & M coincide then floating body will be in neutral equilibrium



The distance b/w centre of gravity (G) & metacentre (M) is known as metacentric ht.

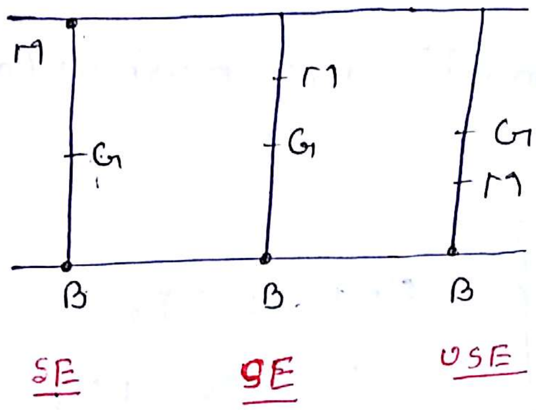
for stable equilibrium metacentric ht. is +ve.

And for unstable equi. metacentric ht is -ve and for neutral equi. metacentric ht is zero.

Metacentre :-

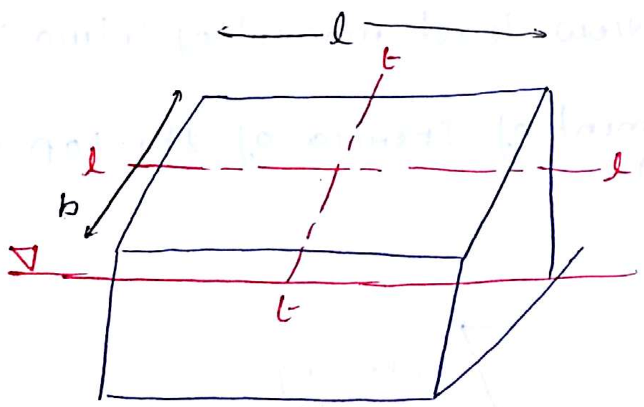
The pt. of intersect^n of normal axis with the new line of act^n of buoyancy force when the body is given small angular displacement is known as metacentre

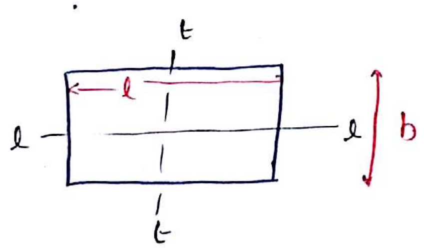
@ It is the pt. about which the body is supposed to be oscillating when tilted.



The condit^n of stable equilibrium is $\boxed{BM > BG}$

If BM @ BGM is large, the floating body will be more stable





$$I_t = \frac{bl^3}{12}$$

$$I_l = \frac{lb^3}{12}$$

since $l > b$

$\Rightarrow I_l < I_t$

$BM = \frac{I}{V}$ ← displaced volume of fluid.

$\frac{I}{V}$ = metacentric radius.

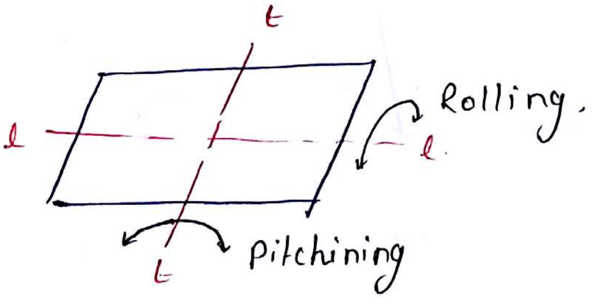
For more SE condit^n, BM must be large.

$$\left.\begin{aligned} BM_t &= \frac{I_t}{V} \\ BM_l &= \frac{I_l}{V} \end{aligned}\right\} \quad \text{since } I_t > I_l \quad \therefore BM_t > BM_l$$

whi

From design pt. of view least moment of inertia is taken. which is the moment of Inertia of the top view at the surface of liquid.

The o

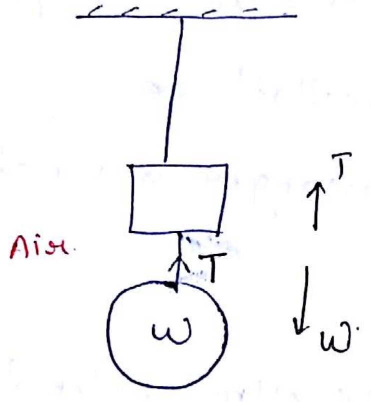


$$\boxed{\begin{aligned} & BM_l < BM_t \\ & BM_{rolling} < BM_{pitching} \end{aligned}}$$

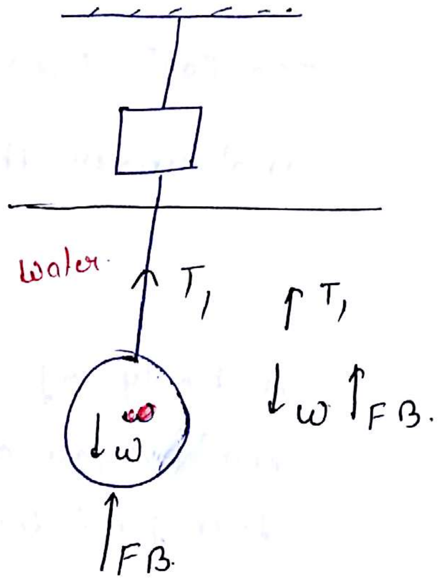
The oscillat^n about longitudinal axis is known as rolling

" " transverse axis " " pitching

wt. loss of body when immersed in fluid.



$\boxed{W = T}$



$$W = F_B + T_1$$

$$T_1 = W - F_B$$

$$\underbrace{W - T_1}_{\text{wt. loss}} = F_B$$

$\therefore$ $\boxed{\text{wt. loss} = \text{Buoyancy force}}$

The correct wt. of body is obtained when it is immersed in air bcz buoyancy effect are negligible in air.

Time period of oscillatⁿ :

$$\boxed{T = 2\pi\sqrt{\frac{k_g^2}{g\,(GM)}}}$$

k_g = radius of gyratⁿ

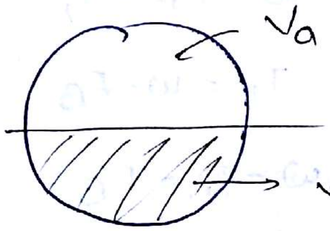
$$I = A k_g^2$$

For more stable equ. condit^n GM must be large but large GM results in smaller time period of oscillat^n (i.e) frequent oscillat^n

and under this condit^n Passanger are not comfortable.

\# A body of sp. wt = 8976 N/m^3 extends above the surface of sea water of density = 10104 N/m^3 then find the % of total volume of the body visible to an observer.

<u>sol</u>



$\Rightarrow V_{fb} = V - V_a$.

wt. of body = wt. of fluid displaced

$$W_B = W_{fd}$$

$$\rho_B g V_B = \rho_f g V_{fd}$$

$$w_B V_B = w_f V_{fd}$$

$$8976 \times V = 10104 (V - V_a)$$

$$8976 V = 10104 V - 10104 V_a$$

$$\Rightarrow \frac{V_a}{V} = \frac{1128}{10104} \times 100 = 11.10\%.$$

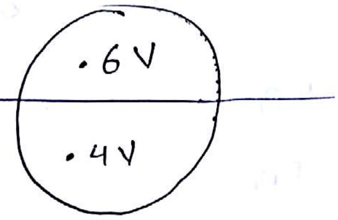
imp.

A metallic body floats at the interface of Hg & water in such a way that 40% of its volume is submerged in Hg and 60% in water. then find density of body.

Sol^n

$\rho_{water} = 10^3$

$\rho_{Hg} = 13.6 \times 10^3$



wt. of body = wt. of fluid displaced

$$w_B = w_{fd}$$

$$w_B = (w_{fd})_{H_2O} + (w_{fd})_{Hg}$$

$$\rho_B g \cancel{V} = (\rho_{H_2O} \times g \times .6 \cancel{V}) + (\rho_{Hg} \times g \times .4 \cancel{V})$$

$$\Rightarrow \rho_B = .6 \rho_{H_2O} + .4 \rho_{Hg}$$

$$= .6 \times 10^3 + .4 \times 13.6 \times 10^3$$

$$\rho_B = 6040 \, kg/m^3$$

Ans.

A body weighs 100 N in air and 80 N in water. then find the density of body.

Sol^n

$W_{air} = 100\,N$

$W = mg \Rightarrow m = \frac{W}{g} = \frac{100}{g} \Rightarrow \boxed{m = \frac{100}{g}}$

wt. loss = F_B

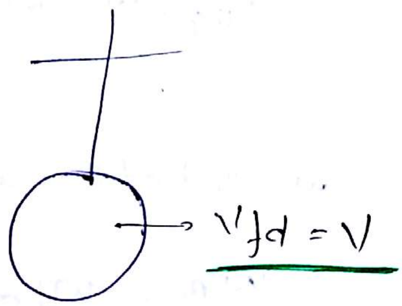
$100 - 80 = F_B$

$20 = F_B = W_{fd}$

$20 = \rho_f\, g\, V_{fd}$

$20 = 10^3 \times g \times V$

$V = \frac{20}{10^3 g}$



$\therefore \rho = \frac{m}{V} = \frac{100/g}{20/10^3 g} = \qquad = 5000\,kg/m^3$

** for completely submerged $\rightarrow$ $\boxed{V_{fd} = \text{Vol. of body}}$.

A body weighs 30 N in a liquid of density 800 kg/m^3 and 15 N in a liquid of density = 1200 kg/m^3 resp. Then find the volume of the body.

**

Sol^n V = vol. of body.

~~wt.~~ wt. loss = F_B

~~30 = F_B = W_{fd}~~

* ✓ Let W is actual body wt.

30 N	15 N
$\downarrow$	$\downarrow$
$\rho = 800$	$\rho = 1200$

wt. loss = $W - 30$

wt loss = F_B

$W - 30 = \rho_f g V_{fd}$

$W - 30 = 800 g V$ —— (1)

$W - 15 = 1200 g V$ —— (2)

from (1) & (2) —— $V = 3.82 \times 10^{-3} \, m^3$

imp.

Match the following →

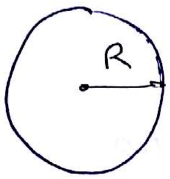
List 1	List 2
A) SE of a floating body	1) B below G
B) SE of a submerged body	2) M above G
C) USE of a floating body	3) B above G
D) USE of a submerged body	4) M below G.

It is given that a solid sphere and cube have same surface area then the ratio of buoyancy force on sphere to that of cube when they are completely submerged in a liquid is given by.

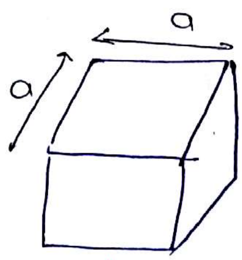
a) $\sqrt{\frac{4}{\pi}}$ b) $\sqrt{\frac{6}{\pi}}$ c) $\sqrt{\frac{8}{\pi}}$ d) $\sqrt{\frac{5}{\pi}}$

Sol^r

sphere



cube



$$\text{surface area} \rightarrow 4\pi R^2 = 6a^2$$

$$\Rightarrow a = \sqrt{\frac{2\pi}{3}}\,R \quad \text{——} \quad (1)$$

$$\frac{F_{BS}}{F_{BC}} = \frac{\rho_f\, g\, V_{fd} \leftarrow V_{sphere}}{\rho_f\, g\, V_{fd} \leftarrow V_{cube}}$$

$$\Rightarrow \frac{F_{BS}}{F_{BC}} = \frac{V_{sphere}}{V_{cube}}$$

$$= \frac{\frac{4}{3}\pi R^3}{a^3}$$

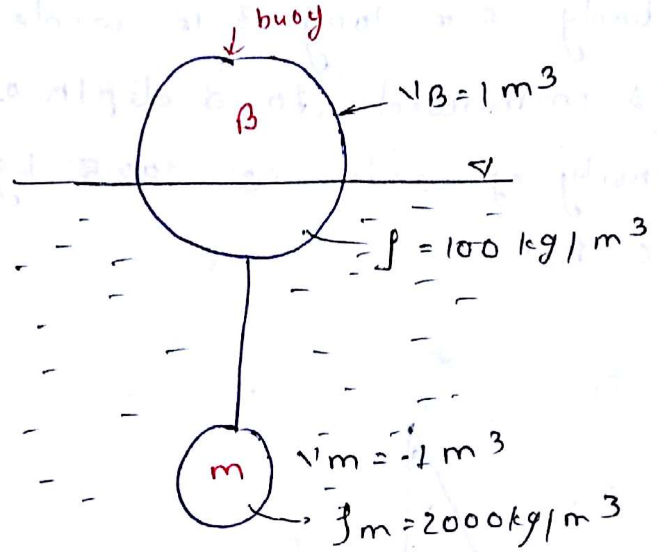
$$= \frac{\frac{4}{3}\pi R^3}{\left\{\left(\frac{2\pi}{3}\right)^{1/2} R\right\}^3} \longrightarrow \text{from (1)}$$

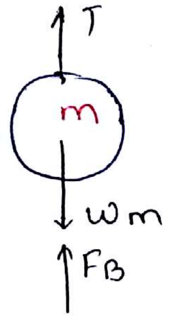
$$\frac{F_{BS}}{F_{BC}} = \left(\frac{6}{\pi}\right)^{1/2}$$

imp.

A metallic sphere of vol. 0.1 m^3 & density 2000 kg/m^3 and fully immersed in water is attached by a flexible wire to a ~~boy~~ buoy of volume 1 m^3 and its density is 100 kg/m^3. calculate the tension in wire and the vol. of buoy that is submerged.

sol^n —





$$T + F_B = W_m$$

$$T = W_m - F_B$$

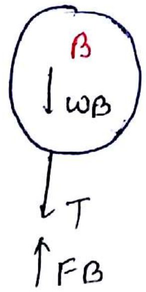
$$= \rho_m g V_m - \rho_f g V_{fd}$$

since m is completely submerged $\therefore V_{fd} = V_m$

$$= 2000 \times g \times .1 - 10^3 \times g(.1)$$

$$= g \times .1\,[200 - 10^3]$$

$$\boxed{T = 981}$$



$$W_B + T = F_B$$

$$(100 \times 9.81 \times 1) + 981 = F_B$$

$$\Rightarrow F_B = 1962\ N$$

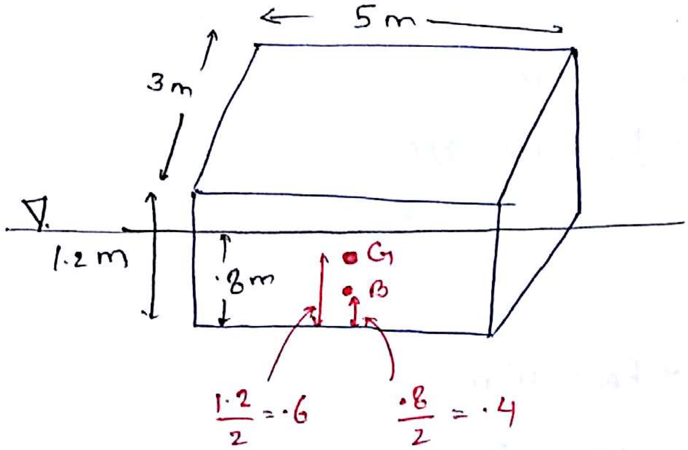
$$\Rightarrow F_B = \rho_f g V_{fd} = 1962$$

$$\Rightarrow V_{fd} = \frac{1962}{\rho_f g} = \frac{1962}{10^3 \times 9.81} = .2\,m^3$$

Ans.

for floating body $\rightarrow$ $\boxed{\text{Vol of body submerged = Vol of fluid displaced}}$

A rectangular body 5 m long 3 m wide and 1.2 m height is immersed to a depth of 0.8 m in water. The density of water is 1025 kg/m3 then find metacentric ht.

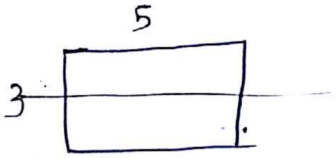


Sol^n

$BG = .6 - .4$

$BG = .2$

$BM = \frac{I}{V}$

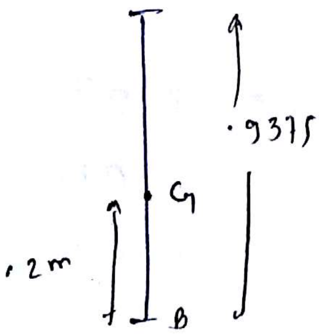


$I = \frac{5 \times 3^3}{12} =$

Vol. of fluid displace. $V = 5 \times 3 \times .8 =$

= vol. of body immersed

$\therefore BM = .9375$ m.



metacentric ht

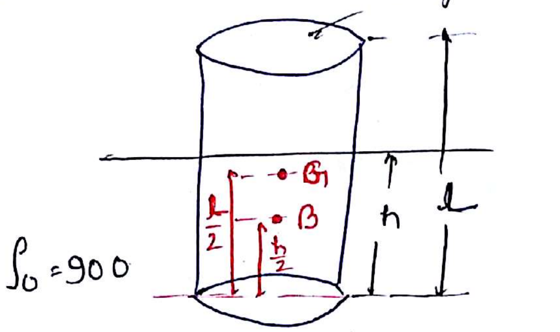
$GM = .9375 - .2$

$GM = .7375$ m

A cyl. of density 600 kg/m3 floats in oil of density 900 kg/m3 with its longitudinal axis verticle.

If l is the ht and d is the dia of the cyl. then show that for stable equi. $\frac{L}{D} < \frac{3}{4}$

Soln.



for floating body, $W_{Body} = W_{fd}$

$$\rho_B g V_B = \rho_f g V_{fd}$$

$$600 \times \frac{\pi}{4} D^2 L = 900 \times \frac{\pi}{4} D^2 \times h.$$

$$\Rightarrow 6L = 9h$$

$$\Rightarrow \boxed{h = \frac{2L}{3}} \Rightarrow \frac{h}{2} = \frac{L}{3} \quad \text{——— (1)}$$

$$BG = \frac{L}{2} - \frac{h}{2}$$

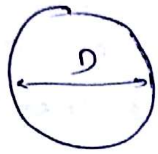
$$= \frac{L}{2} - \frac{L}{3}$$

$$\boxed{BG = \frac{L}{6}}$$

$BM = \frac{I}{V}$ V_{fd}

$$V = \frac{\pi}{4} D^2 h.$$

$$V = \frac{\pi}{4} D^2 \times \frac{2L}{3} = \frac{\pi D^2 \times 2L}{12}$$



$$I = \frac{\pi}{64} D^4$$

$$\therefore \quad BM = \frac{I}{V} = \frac{\frac{\pi}{64} D^4}{\frac{\pi D^2 \times 2L}{12}} = \frac{3D^2}{32L}$$

$\Rightarrow BM > BG \quad \longleftarrow$ for SE

$$\Rightarrow \frac{3D^2}{32L} > \frac{L}{6}$$

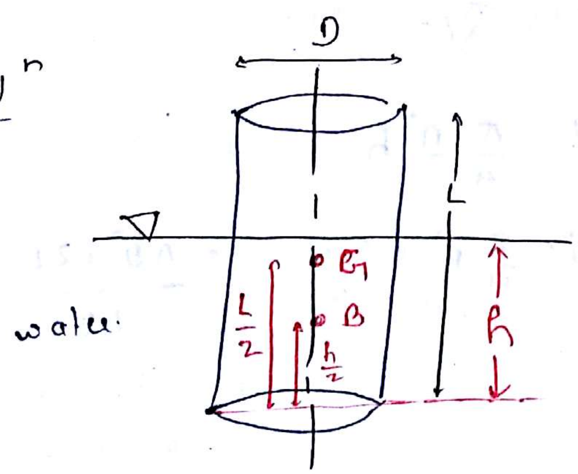
$$\Rightarrow \frac{9}{16} > \frac{L^2}{D^2}$$

$$\Rightarrow \frac{3}{4} > \frac{L}{D}$$

$$\Rightarrow \boxed{\frac{L}{D} < \frac{3}{4}} \quad \text{Ans.}$$

A solid cyl. of length 'L', dia 'D' and density 600 kg/m³ floats in Neutral equi. with its axis verticle then find $\frac{L}{D}$

Sol^n



$$W_B = W_{fd}$$

$$\rho_B g V_B = \rho_f g V_{fd}$$

$$600 \times \frac{\pi}{4} D^2 L = 10^3 \times \frac{\pi}{4} D^2 \times h$$

$$\Rightarrow \quad h = \frac{3L}{5} \quad \text{———} \; (1)$$

$$BG = \frac{L}{2} - \frac{h}{2}$$

$$= \frac{L}{2} - \frac{3L}{5}\left(\frac{1}{2}\right)$$

$$BG = \frac{L}{5}$$

$$BM = \frac{I}{V} = \frac{\pi/64 \, D^4}{\frac{\pi}{4} D^2 \times \frac{3L}{5}} = \frac{5D^2}{48L}$$

V_{fd}

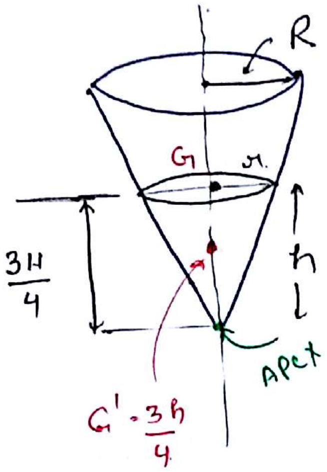
for NE $\longrightarrow$ $BM = BG$.

$$\frac{5D^2}{48L} = \frac{L}{5}$$

$$\Rightarrow \quad \frac{L^2}{D^2} = \frac{5 \times 5}{48}$$

$$\Rightarrow \quad \boxed{\frac{L}{D} = \frac{5}{4\sqrt{3}}} \quad \text{Ans.}$$

→ Cone



C.G = $\frac{3H}{4}$ from pt. → bottom @ apex.

vol. of cone = $\frac{1}{3}\pi r^2 h$

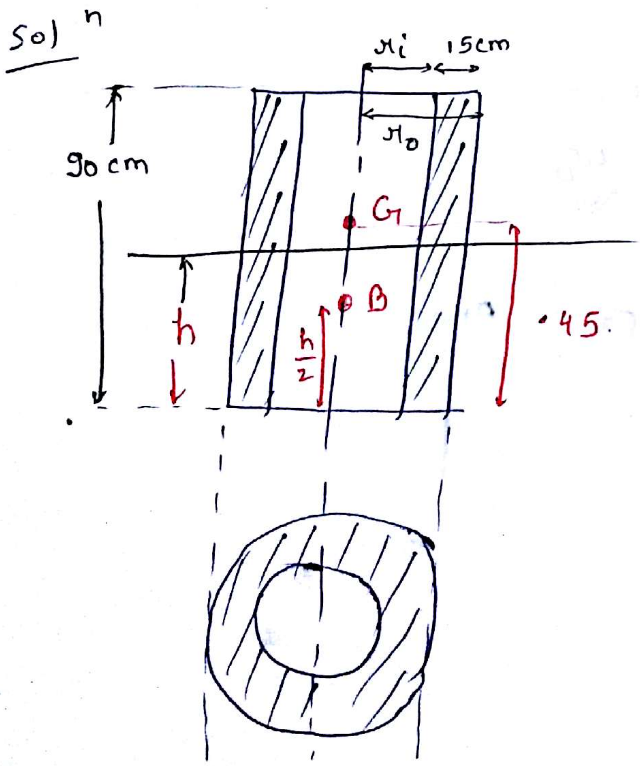
$\rho_B g V_B = \rho_f g V_f$

ht. of C.G from apex = $\frac{3}{4}H$ → where H = total ht. of cone

ht. of C.B from apex = $\frac{3}{4}h$ → h = submerged portion.

A hollow cyl. open at both ends has internal dia of 30 cm. wall thickness of 15 cm. and length of 90 cm If it weighs 625 N. Find whether the cyl. would be stable will floating in water. with its axis verticle.

Sol^n



$r_i = \frac{30}{2} = 15$cm

$r_o = 15 + 15 = 30$cm

$W_B = W_{fd}$

$625 = \rho_f g V_{fd}$

vol. of fluid disp = vol. of body under water

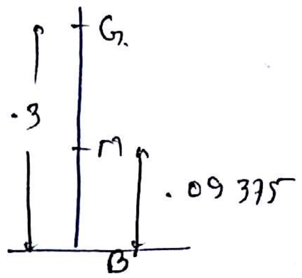
$625 = 10^3 \times 9.81 \times \pi(r_o^2 - r_i^2)\, h$

$\Rightarrow h = .3\,m.$

$BM = \dfrac{I}{V} = \dfrac{\frac{\pi}{4}(r_o^4 - r_i^4)}{\pi(r_o^2 - r_i^2) \times h}$

V_{fd} = vol. of body immersed.

$\Rightarrow BM = .09375$



since $BM < BG$

$\Rightarrow$ unstable equi.

$I = \frac{\pi}{4}(r_o^4 - r_i^4) \rightarrow$ for hollow cyl.

Que not comming in gate

Hydrostatic forces :-

1) Hydrostatic forces on plane surfaces :-

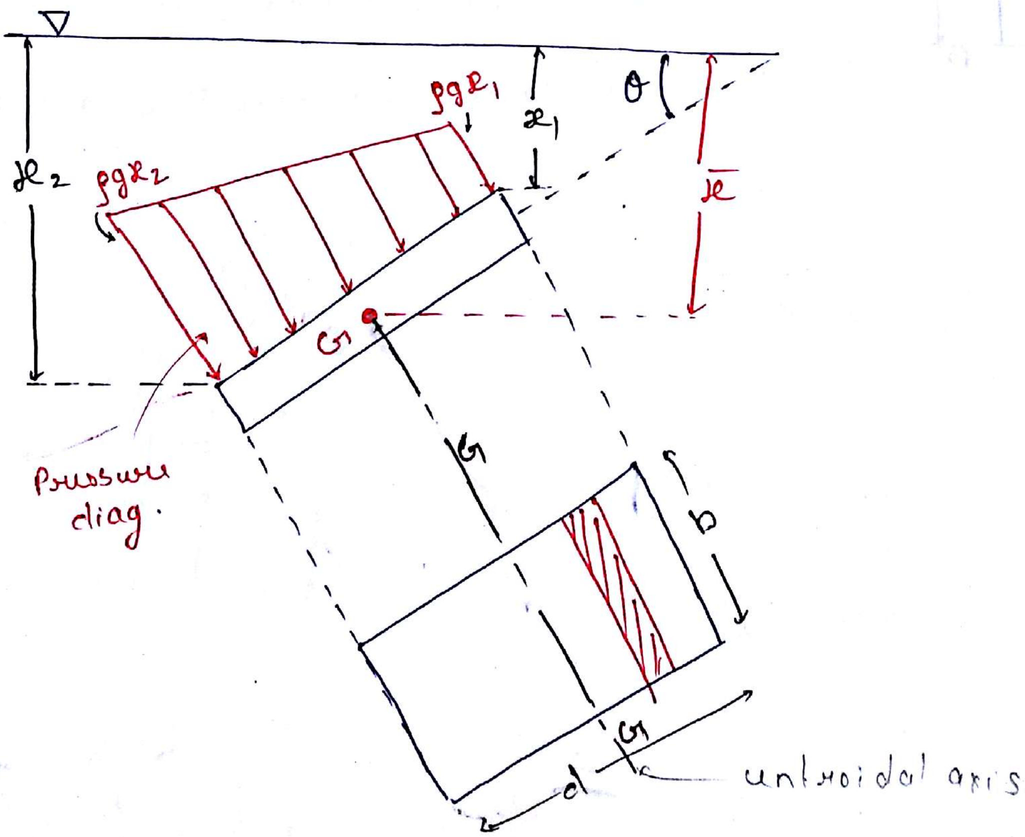
case-1 :- Inclined surface :-

Hydrostatic force :-

The force exerted by the static fluid when a body is exposed to it.

centre of Pressure :-

It is the pt. from which the total hydrostatic force is supposed to be acting.



$$\boxed{F = wA\bar{x}}$$

w = specific wt.

F = hydrostatic force.

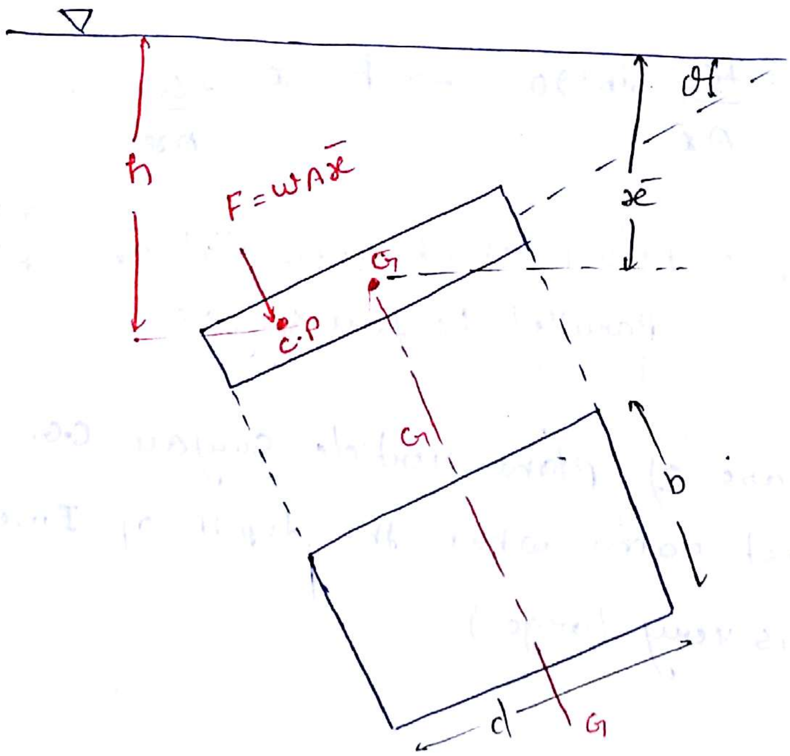
$\bar{x}$ = verticle distance of C.G from free surface.

By using principle of moment and parallel axis theorem centre of pressure can be found.

Let h be the distance of C.P from free surface.

$$\therefore \boxed{h = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2 \theta}$$

where, I_G = moment of Inertia about centroidal axis passing through G-G.

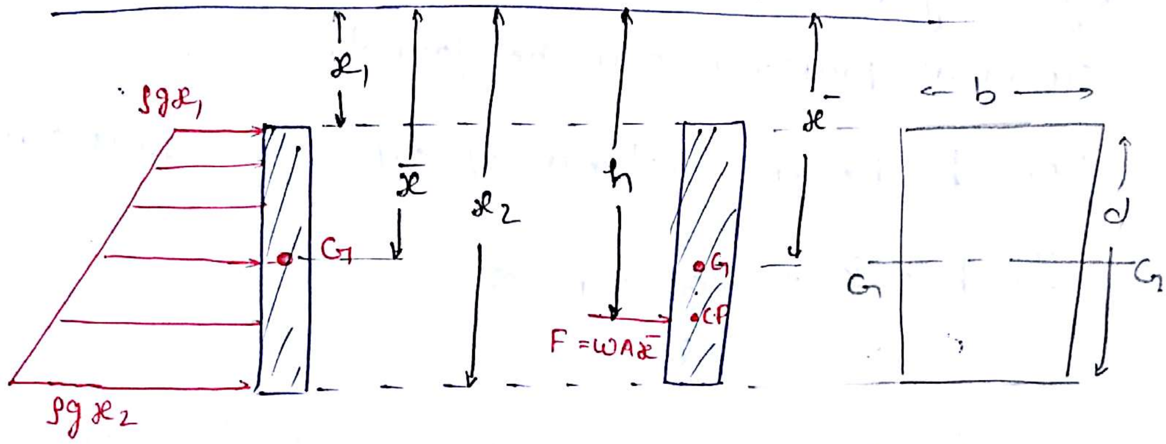


$$I = \frac{bd^3}{12}$$

$$A = bd$$

case-2 :→ verticle surface :.

Put $\theta = 90°$ in previous case.



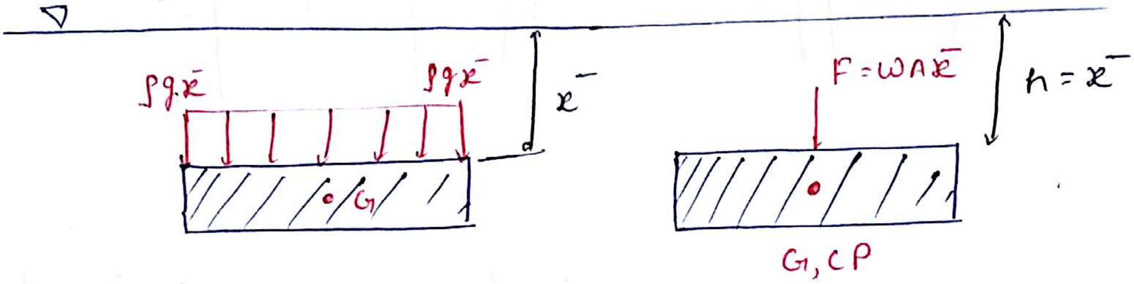
$$F = wA\bar{x}$$

$$h = \bar{x} = \frac{I_G}{A\bar{x}} \sin^2 90 \Rightarrow h = \bar{x} + \frac{I_G}{A\bar{x}}.$$

where I_G = M.O.I about centroidal axis which ~~will~~ is Parallel to free surface

Note:- In case of plane verticle surface C.G and C.P are almost same when the depth of Emersion ($\bar{x}$ is very large) case-3 :- Horizontal surface

Put $\theta = 0$ in case-1



$F = wA\bar{x}$

$h = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2 \theta \Rightarrow h = \bar{x}$

$P = \frac{F}{A}$

$\Rightarrow F = PA$

$= \rho g \bar{x} \times A$

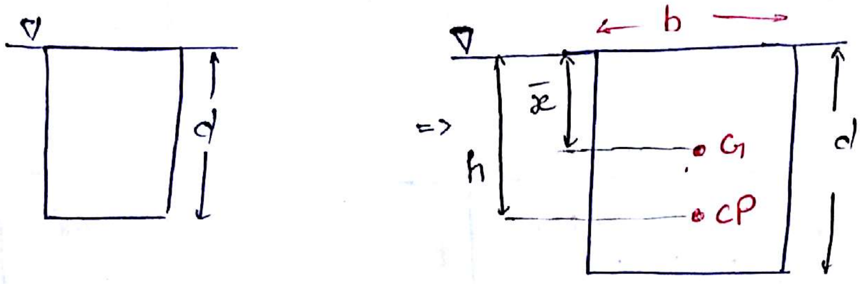
$= wA\bar{x}$

summary.

case	force	C.P.
Inclined	$wA\bar{x}$	$\bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta$
verticle	$wA\bar{x}$	$\bar{x} + \frac{I_G}{A\bar{x}}$
Horizontal	$wA\bar{x}$	$\bar{x}$

*** Certain cases

1)



$$h = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$\Rightarrow h = \frac{d}{2} + \frac{bd^3}{12} \times \frac{1}{bd \times \frac{d}{2}}$$

$$\bar{x} = \frac{d}{2}$$

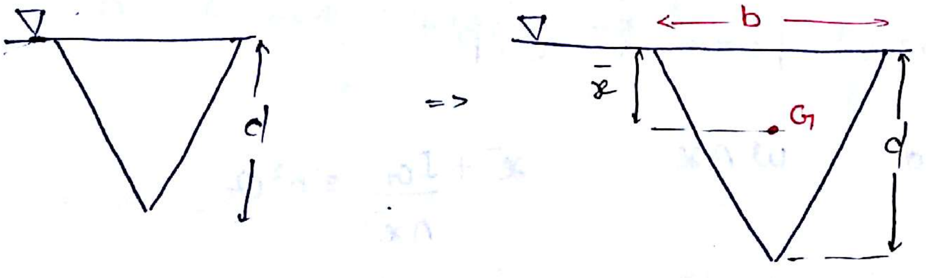
$$I_G = \frac{bd^3}{12}$$

$$A = bd$$

***

$$\Rightarrow \boxed{h = \frac{2d}{3}}$$ → Remember

2)



$$\bar{x} = \frac{d}{3}$$

$$I_G = \frac{bd^3}{36}$$

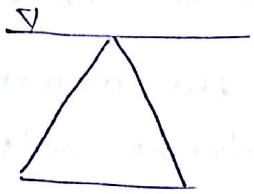
$$A = \frac{1}{2} bd$$

$$h = \bar{x} + \frac{I_G}{A\bar{x}}$$

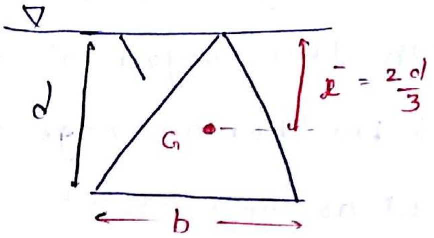
$$= \frac{d}{2} + \frac{bd^3}{36} \times \frac{1}{\frac{1}{2}bd} \times \frac{1}{d/3}$$

$$\boxed{h = \frac{d}{2}}$$

3)



$\Rightarrow$



$$A = \frac{1}{2} b d$$

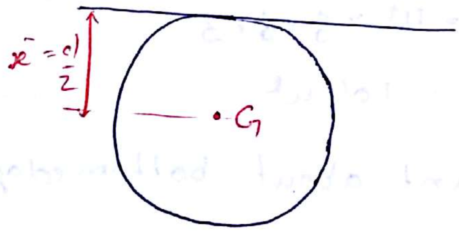
$$I_G = \frac{bd^3}{36}$$

$$h = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$= \frac{2d}{3} + \frac{bd^3}{36} \times \frac{1}{\frac{1}{2}bd} \times \frac{1}{\frac{2d}{3}}$$

$$\boxed{h = \frac{3d}{4}}$$

4)



$$\bar{x} = \frac{d}{2}$$

$$A = \frac{\pi}{4} d^2$$

$$I_G = \frac{\pi}{64} d^4$$

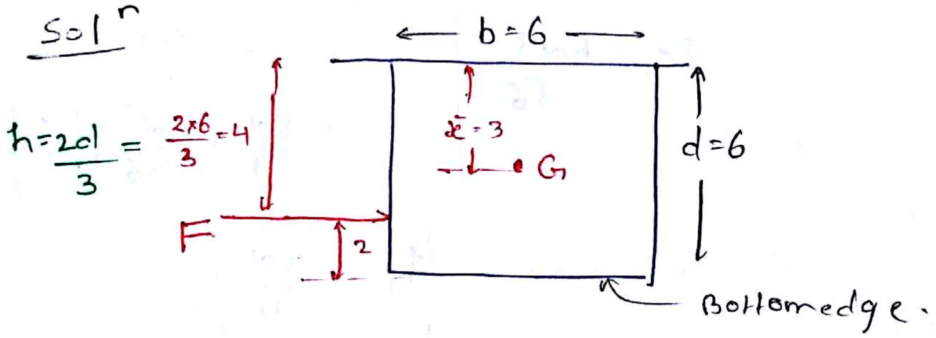
$$\Rightarrow \boxed{h = \frac{5d}{8}}$$

#

A verticle gate 6 × 6 m^2 holds water on one side with free surface at its top.. Find the moment about the bottom edge of the gate due to water force. Take w as specific wt.

a) 36 w b) 72 w c) 108 w d) 216 w.

Sol^n



$F = wA\bar{x}$

$= w \times 6 \times 6 \times 3$

$= 108 w$

moment about bottomedge $= F \times 2$

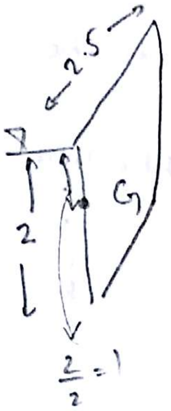
$= 108 w \times 2$

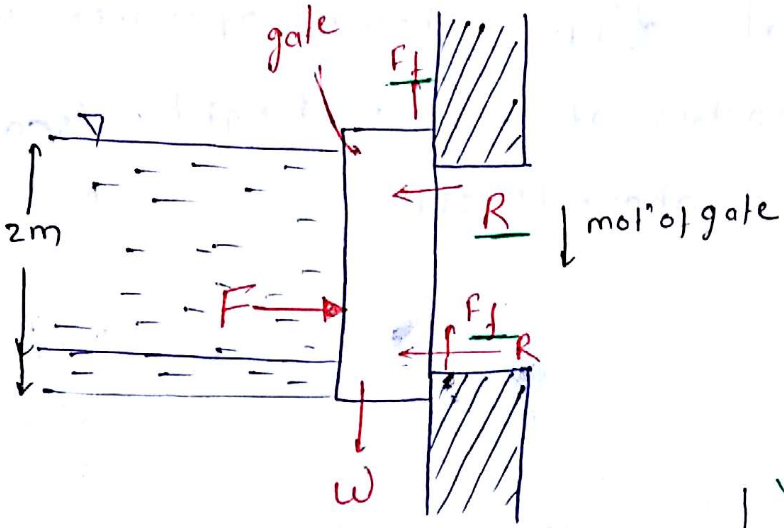
$= 216 w$.

imp.

#

A verticle gate 2.5 m wide and weighing 500 kg is held in position due to horizontal force on one side and associated frict^n as shown in fig. when the water level drops down to 2 m above the bottom of the gate the gate just starts sliding down. The find the coeff. of frict^n b/w gate and supporting str.





$F = R + R$

$F = 2R$ —— (1)

sol^n

$W_t = F_f + F_f$

$W_t = 2F_f$

$500g = 2F_f \quad \{F_f = \mu R\}$

$500g = 2\times\mu R$ —— (2)

$\Rightarrow 500g = 2\mu\times\frac{F}{2}$

$\Rightarrow 500g = \mu F$ —— (3)

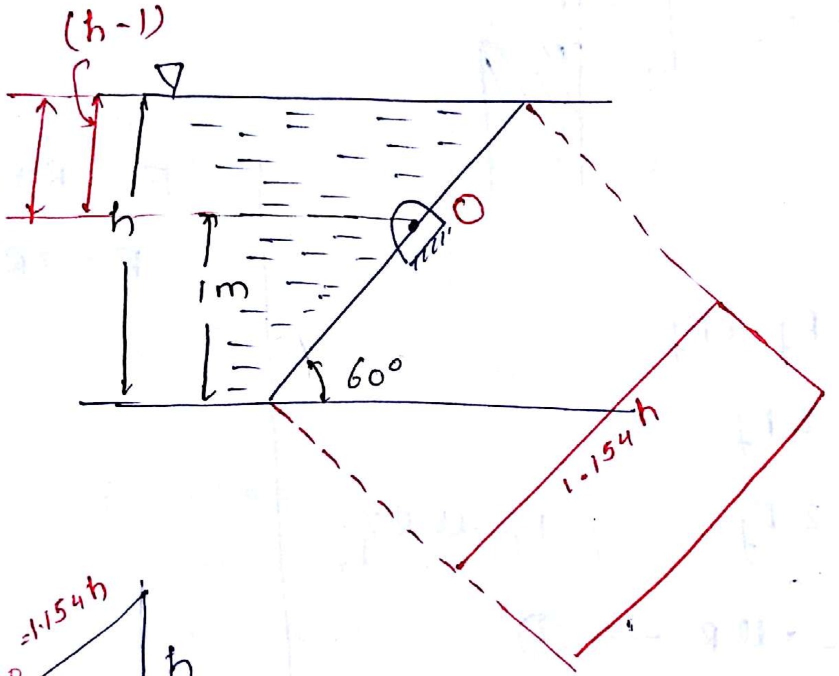
$F = w A \bar{x}$

$\Rightarrow F = \rho g A \bar{x}$

$\Rightarrow F = 10^3\times g\times(2\times 2.5)\times\frac{2}{2}$

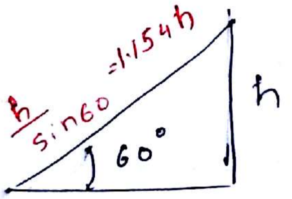
$\therefore$ from (3) $\rightarrow$ $\boxed{\mu = .1}$ Ans.

The automatic tipper shown operates when the level reaches at certain height. cal. the level when it is about to tip.



at pt. O → about to tip.

Sol^n



$\bar{x} = \frac{h}{2}$

$$Cp = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta$$

$$= \frac{h}{2} + \frac{1(1.154h)^3}{12} \times \frac{1}{1.154h} \times \frac{1}{h/2} \sin^2 60$$

$$\boxed{Cp = .666\,h}$$ ← this cp is distance from free surface

$$h - 1 = .666\,h$$

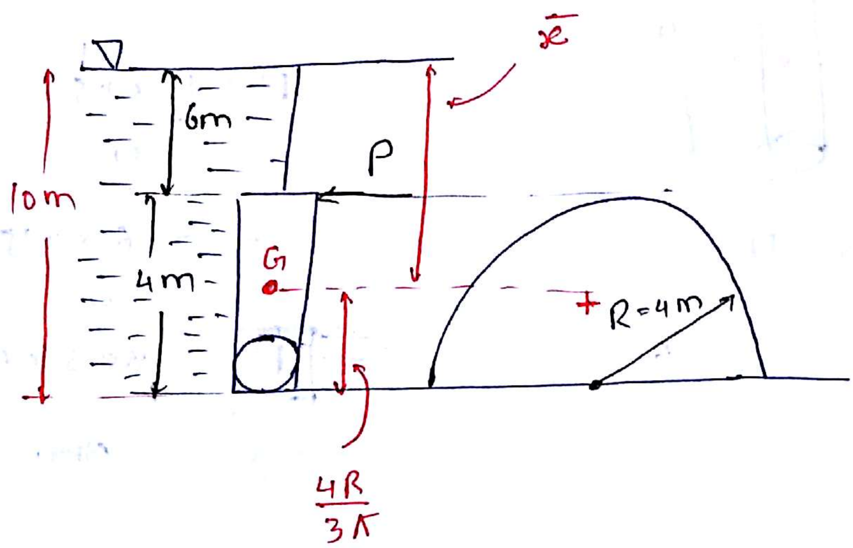
$$\Rightarrow h = 2.99\text{ m}$$

Conventional

v.v. imp.

#

A semicircular gate hinged at O is held in place by a horizontal force P acting at A. as shown in fig → cal the force P required for equilibrium. For semicircular plate the distance of centre of gravity from st. edge is $\frac{4R}{3\pi}$ and M.O.I is $.035 \pi R^4$.



Sol^n

$$\bar{x} = 10 - \frac{4R}{3\pi}$$

$$\Rightarrow \bar{x} = 10 - \frac{4\times4}{3\pi}$$

$$\Rightarrow \boxed{\bar{x} = 8.3 \text{ m}}$$

$$F = wA\bar{x} \qquad w = \rho g = 1000 \times 9.81 = 9810$$

$$= 9810 \times \frac{\pi 4^2}{2} \times 8.3$$

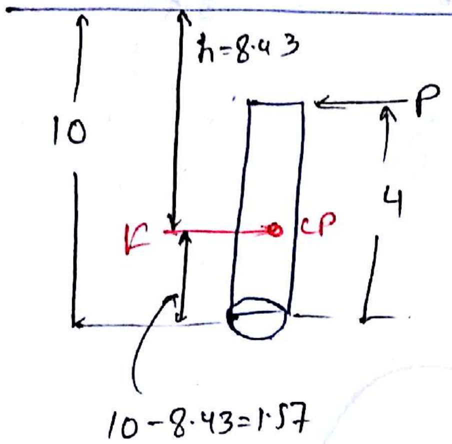
$$= 2046 \times 10^3 N$$

$$\boxed{F = 2046 \text{ kN}}$$

$$CP = h = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$h = 8.3 + \frac{.035 \times \pi \times (4)^4}{\frac{\pi 4^2}{2} \times 8.3}$$

$$\Rightarrow h = 8.43$$



$$F \times 1.57 = P \times 4$$

$$P = \frac{F \times 1.57}{4}$$

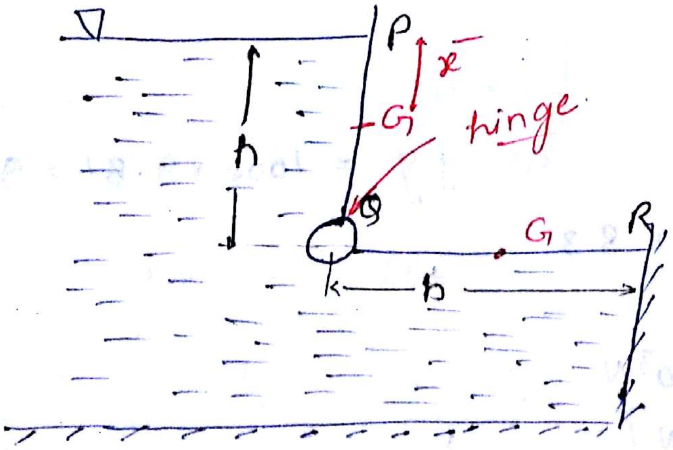
$$= \frac{2046 \times 1.15}{4}$$

$$\boxed{P = 803\ kN}$$

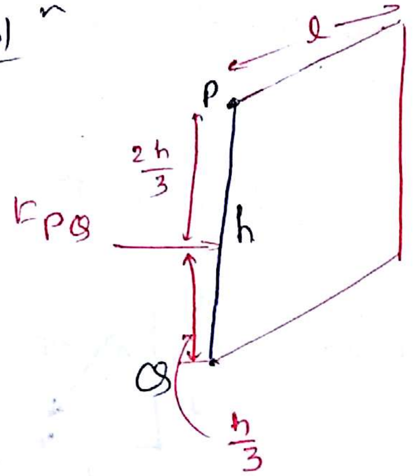
Ans.

v.v.imp.

\# The fig. shows a hydraulic gate PQR whose wt. is very small compared to hydrostatic forces find the value of h for equilibrium.



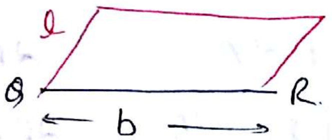
Sol^n



$$F_{PQ} = \omega A \bar{x}$$

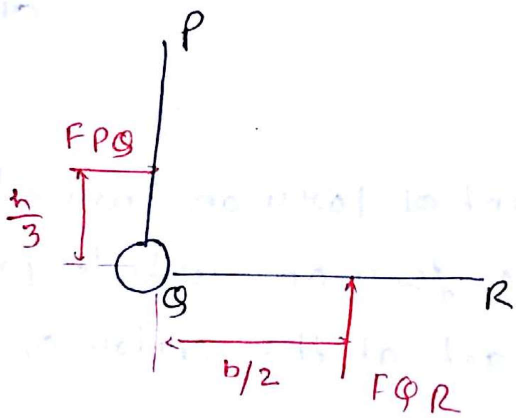
$$= \omega \times (h \times 1) \times \frac{h}{2}$$

$$F_{PQ} = \frac{\omega h^2}{2} \quad \text{—— (i)}$$



$$F_{QR} = \omega \times (b \times 1) \times h$$

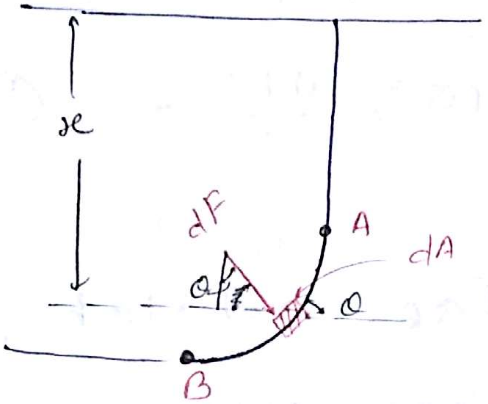
$$F_{QR} = \omega b h. \quad \text{—— (2)}$$

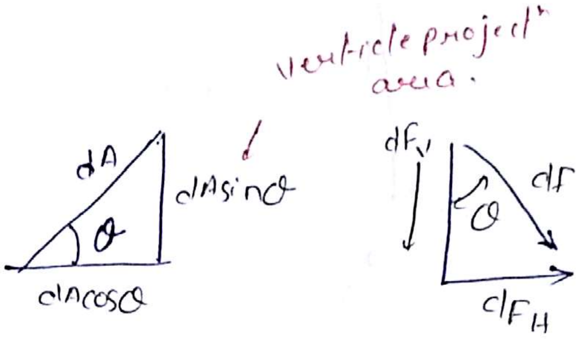


$$F_{PQ} \times \frac{h}{3} = F_{QR} \times \frac{b}{2}$$

$$\frac{\omega h^2}{2} \times \frac{h}{3} = \omega b h \times \frac{b}{2} \Rightarrow \boxed{h = \sqrt{3}\, b} \text{ Ans.}$$

Hydrostatic forces on curved surfaces :





$dF = P\,dA$

$dF = \rho g x \times dA$

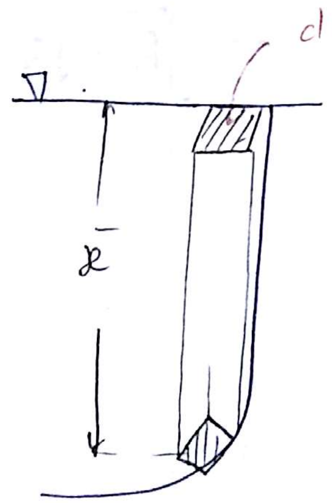
$dF_H = dF \sin\theta$

$dF_V = dF \cos\theta$

$dF_H = \rho g x\, dA \sin\theta$

*** The horizontal component of force on curved surface is equal to hydrostatic force on verticle projectⁿ area and this force will act at the centre of pressure of corresponding area.

→



$dF_V = dF \cos\theta$

$= \rho g x\, dA \cos\theta$

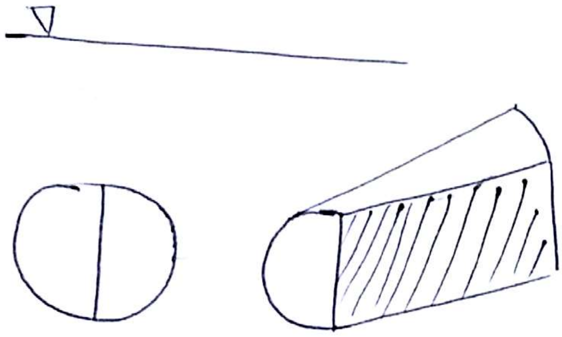
$dF_V = \rho g\, \text{vol}$

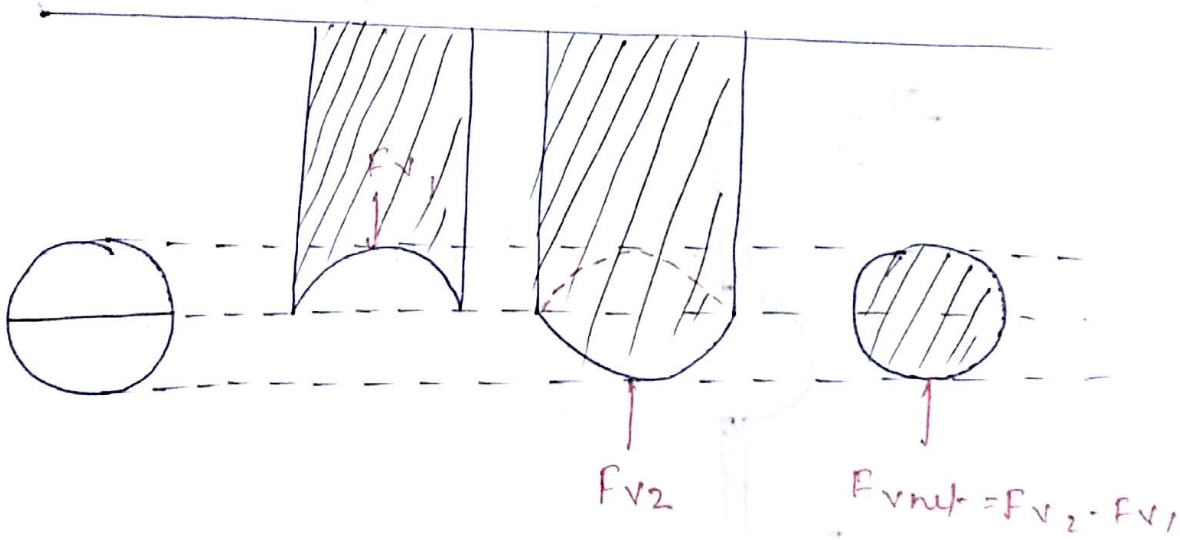
$\boxed{dF_V = \text{wt. of fluid}}$

$w = \text{wt}/\text{vol} \Rightarrow \text{wt} = w \times \text{vol}.$

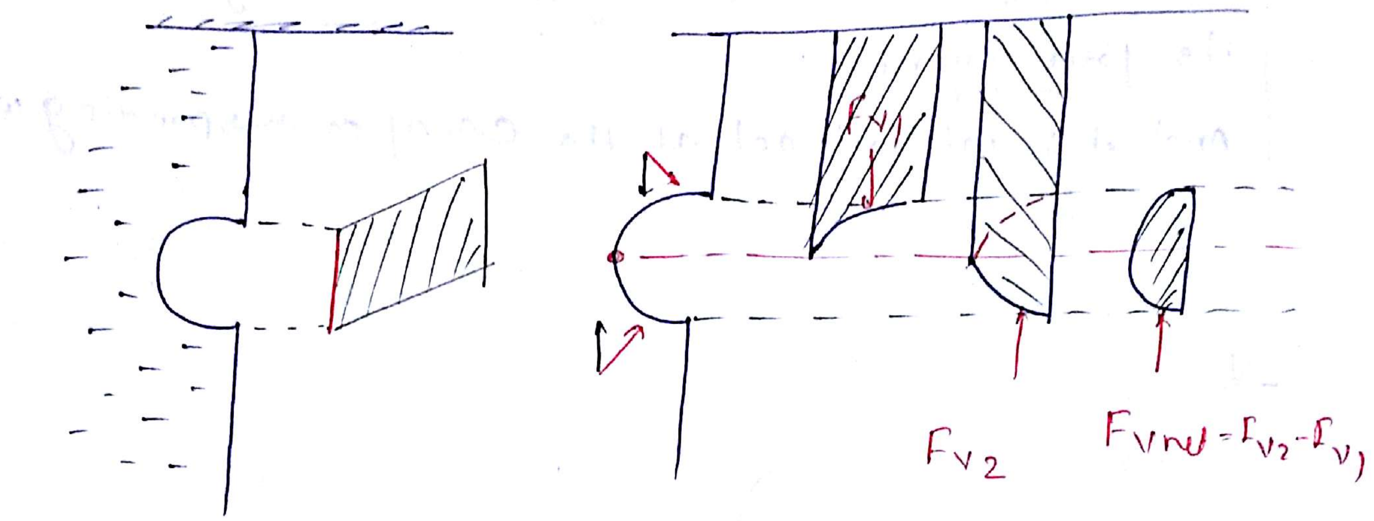
$\Rightarrow \text{wt} = \rho g\, \text{vol}.$

verticle component of the curved surface is equal to wt. of fluid contained by the curved surface up to its free surface.
And this wt. will act at the C.G of corresponding wt.

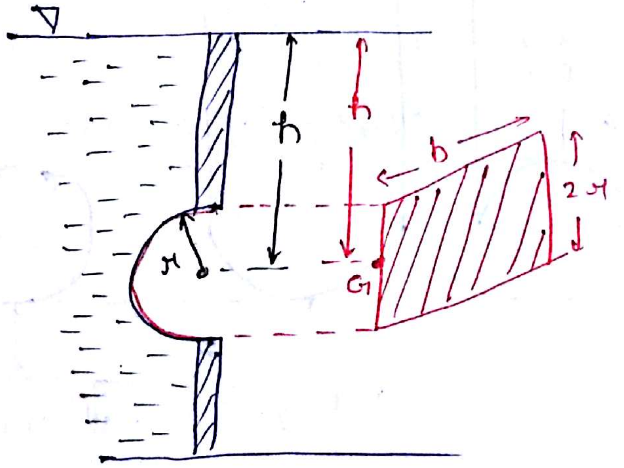




Net verticle component is equal to wt. of the fluid corresponds to cylindrical volume.



\# Find the horizontal and verticle compound of hydrostatic force on a semi-circular gate having width b.



Sol^n

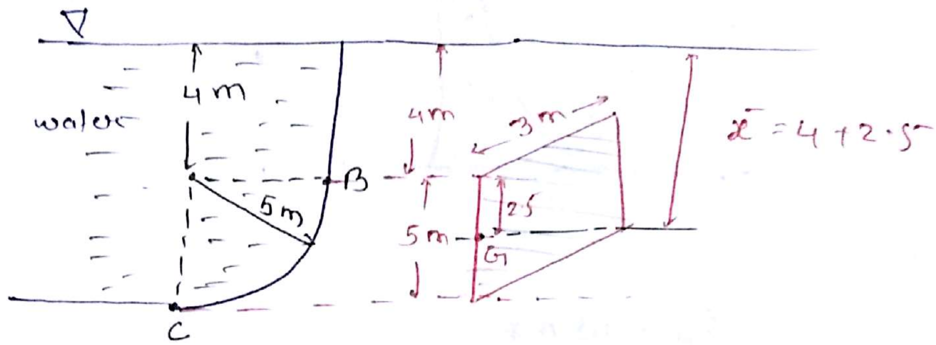
Horiz. comp = Project^n on vertic. plane of forces

$$F_H = wA\bar{x}$$
$$= w(b \times 2r) \times h$$
$$= 2wbrh$$
$$= 2\rho g b r h$$

$$F_V = wt. = \rho g V$$

$$F_V = \rho g \times \left(\underbrace{\frac{\pi r^2}{2}}_{\text{Area}} \times \underbrace{b}_{\text{width}}\right)$$

The tank in the fig. is 3 m wide cal. the hydrostatic horizontal, verticle and resultant force on $\frac{1}{4}$th of a circle BC.



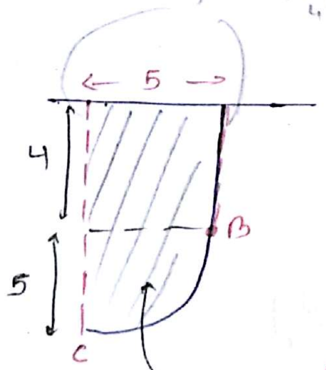
Sol^n

$$F_H = w A \bar{x}$$

$$= 9810 \times (5 \times 3) \times 6.5$$

$$= 956.41 \text{ kN}$$

Project^n of $\frac{1}{4}$th of circle on free surface



$$F_V = wt = \rho g V$$

$$F_V = 10^3 \times 9.81 \times$$

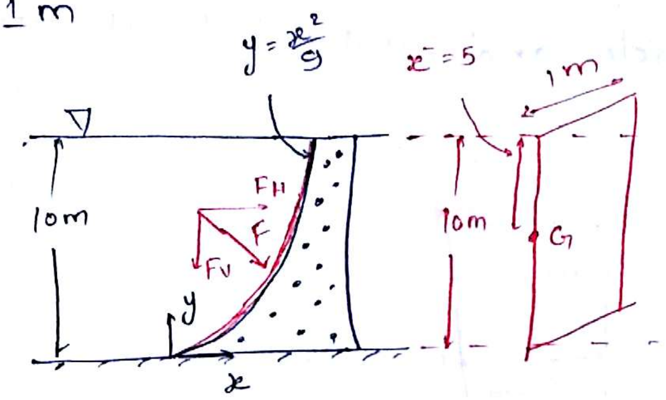
$$F_V = 1166 \text{ kN}$$

$$Vol = \left[\underbrace{5 \times 4 + \frac{\pi (5^2)}{4}}_{\text{Area}}\right] \underbrace{\times 3}_{\text{width}}$$

Find the mag. & direct^n of water force acting

curved surface of a dam which is shaped acc. to the eq^n $y = \frac{x^2}{9}$ as shown in fig --

the ht. of water retain by dam is 10 m. the width of dam is. 1 m

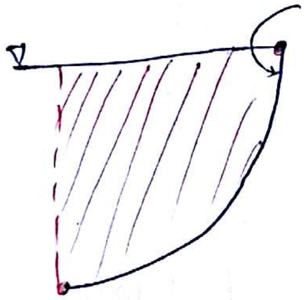


sol^n

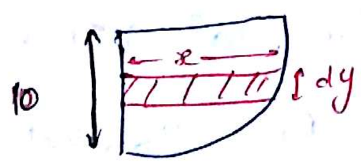
$$F_H = wA\bar{x}$$

$$= 9810 \times (10 \times 1) \times 5$$

$$= 490.5 \text{ kN}$$



$\frac{x^2}{9} = y \Rightarrow x^2 = 9y \Rightarrow x = 3\sqrt{y}$



$$dA = x\,dy$$

$$A = \int_0^{10} x\,dy.$$

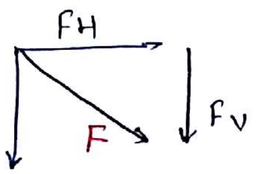
$$A = \int_0^{10} 3\sqrt{y}\,dy$$

$$\Rightarrow A = 63.24 \text{ m}^2$$

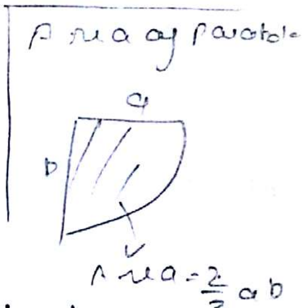
$$Vol. = A \times width$$
$$= 63.24 \times 1$$
$$= 63.24\,m^3$$

$$F_V = wt = \rho g V$$
$$= 10^3 \times 9.81 \times 63.24$$
$$= 620.38\,kN$$

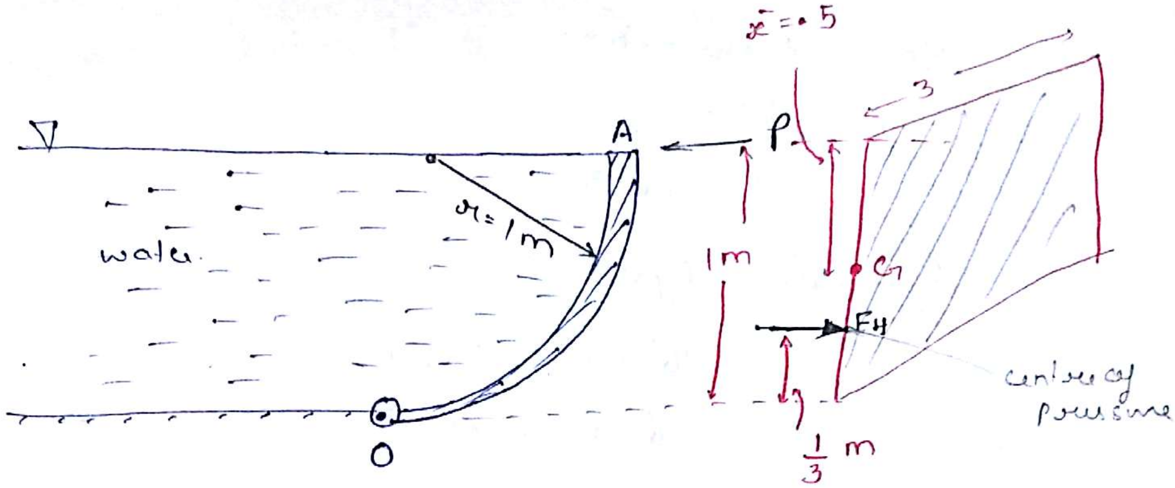
$$\therefore F = \sqrt{F_H^2 + F_V^2} \Rightarrow F = \sqrt{490.5^2 + 620.38^2} = 790.9\,m$$



$$\tan\theta = \frac{F_V}{F_H} = \frac{620.38}{490.5}$$
$$\Rightarrow \theta = 51.6^\circ$$

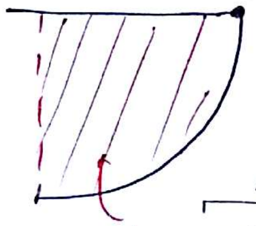


\# The gate OA as shown in fig. → is hinged at O. and is in the form of $\frac{1}{4}$th of a circle of radius 1 m. It supports water on one side as shown in fig. the width of the gate is 3 m. Find the force P req. to hold the gate in position.



sol^n

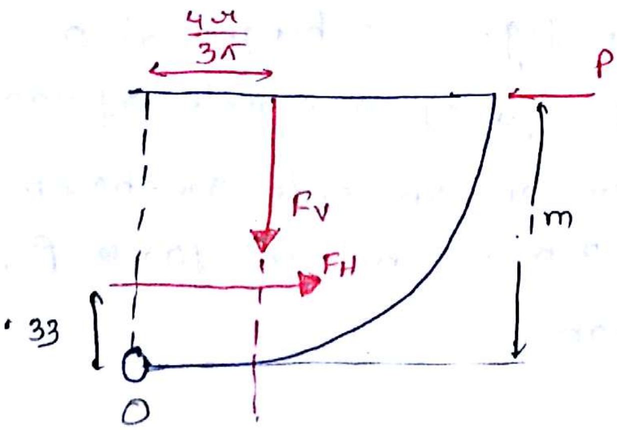
$$F_H = wA\bar{x}$$
$$= 9810 \times 3 \times 1 \times .5$$
$$F_H = 14.715\,kN$$



$$vol = \overbrace{\frac{\pi(1)^2}{4}}^{A} \times \overbrace{3}^{b}$$
$$= .$$

of fluid

$$F_V = wt = \rho g V$$
$$= 10^3 \times 9.81 \times$$
$$= 23.14\,kN$$

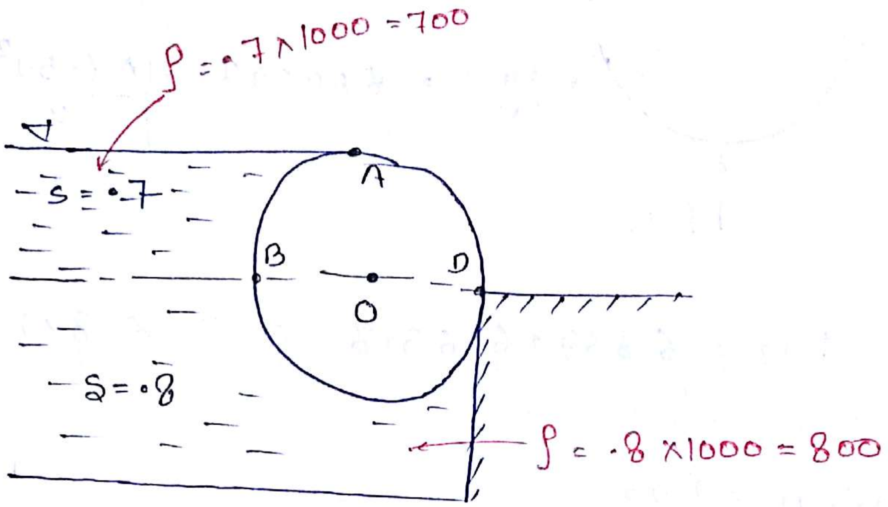


taking moment about O

$$F_H(.33) + F_V\left(\frac{4 \times 1}{3\pi}\right) = P \times 1$$
$$14.715(.33) + 23.14\left(\frac{4}{3\pi}\right) = P$$
$$P = 14.6\,kN$$

V.imp

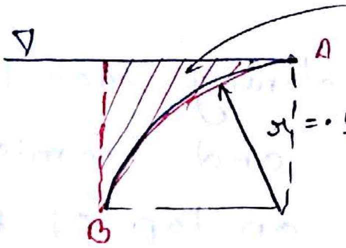
A cyl. of 1 m dia and 2 m length stays in equi. as shown in fig. cal. the density of cylinder.



Sol^n

$$W_{cyl} = \rho_{cyl}\, g\, V_{cyl}$$

$$= \rho_{cyl} \times g \times \underbrace{\frac{\pi}{4}(1)^2}_{\text{Area}} \times \underbrace{2}_{\text{width}} \quad \text{—— (i)}$$



$$\text{Area} = (.5 \times .5) - \left(\frac{\pi (.5)^2}{4}\right) = .05336 m^2$$

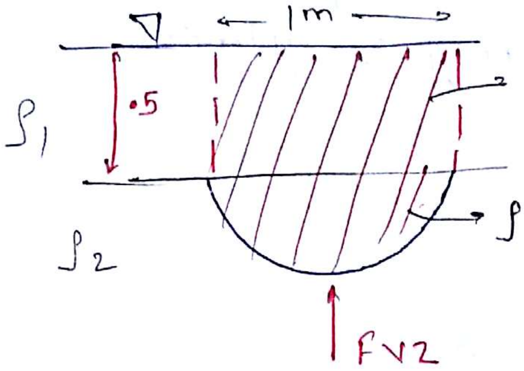
$$\therefore \text{Vol} = A \times L$$

$$= .0536 \times 2$$

$$F_{V1} = \text{wt of fluid} = \rho g V$$

$$= 700 \times 9.81 \times$$

$$F_{V1} = 736.8 \text{ N} \downarrow$$



$\rho g V = 700 \times 9.81 \times (1 \times .5 \times 2) = 6867 N$

$\rho g V = 800 \times 9.81 \times \left\{\frac{\pi (.5)^2}{2} \times 2\right\} = 6163.8 N$

$$F_{V2} = 6867 + 6163.8 = 13030.8 N$$

— for equi → net verticle force = 0.

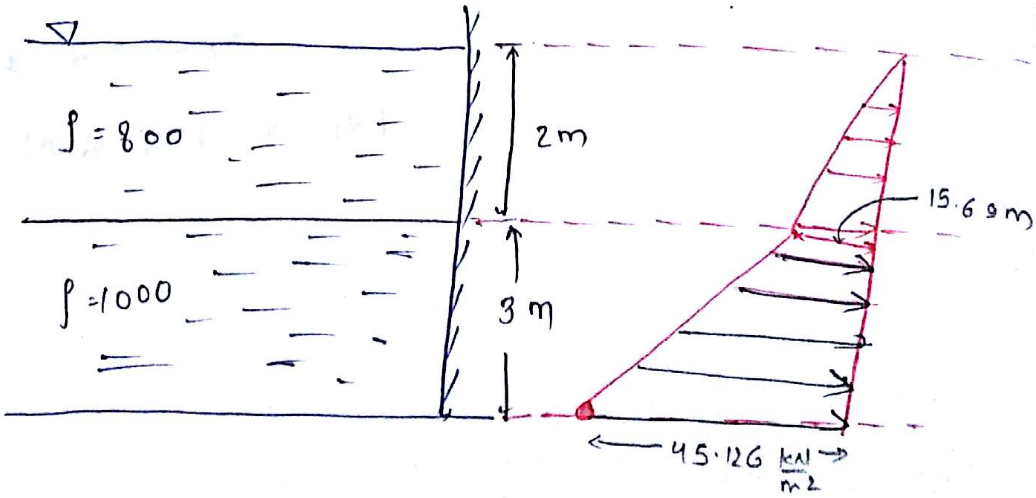
$$F_{V1} + W_{cyl} = F_{V2}$$

$$\Rightarrow W_{cyl} = F_{V2} - F_{V1}$$

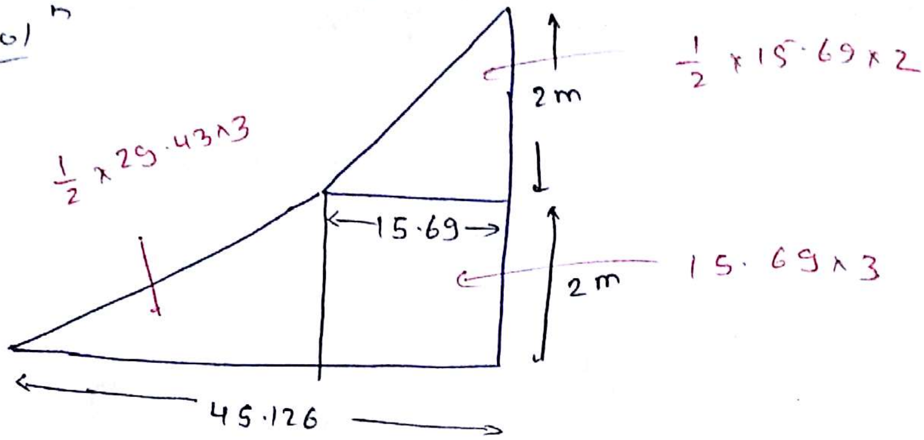
$$\Rightarrow \rho_{cyl} \times 9.81 \times \frac{\pi}{4}(1)^2 \times 2 = 13030.8 - 736.8$$

$$\Rightarrow \rho_{cyl} = 797.8 \ kg/m^2$$

A tank contains water of density 1000 kg/m3 upto a ht. of 3m about the base and immisible liquid of density 800 kg/m³ is filled on top of that over 2 m depth. cal. the force on verticle wall of width 6 m.



Sol^n



$$F = \left[\left\{\frac{1}{2} \times 15.69 \times 2\right\} + \left\{15.69 \times 3\right\} + \left\{\frac{1}{2} \times 29.43 \times 3\right\}\right] \times \underset{\text{width}}{6}$$

$$= 641.43 \text{ kN}$$

Fluid kinematics

kinematics deals with motion of fluid without any reference to force.

Fluid flow is analysed by using → lagrangian approach
→ Eulerian approach.

In lagrangian approach the behavior of single fluid particle is analysed.
where as in Eulerian approach certain sectⁿ @ pt. is taken and at this sectⁿ the fluid flow is analysed.

Due to its simplicity, Eulerian approach is mostly used in fluid mechanics.

Types of fluid flow

1) Steady and unsteady flow.

A flow is said to be steady flow if fluid property do not vary wrt time at any given sectⁿ

otherwise the flow is unsteady.

For steady flow →
$$\begin{cases} \dfrac{dV}{dt} = 0 \\ \dfrac{d\rho}{dt} = 0 \\ etc. \end{cases}$$

2) uniform and non-uniform flow :

A flow is said to be uniform if the vel. remains const. at different sectⁿ at any given instant of time.

otherwise the flow is non uniform.

For uniform flow $\rightarrow \frac{dv}{ds} = 0$

3) laminar and turbulent flow :-

→ when fluid particles moves in the form of layer with one layer sliding over the other, then that flow is known as laminar flow.

laminar flow generally occurs at low vel.

eg. flow of blood in veins.

→ when fluid flows in highly disorganised manner leading to rapid mixing of fluid particles, then that flow is known as turbulent flow.

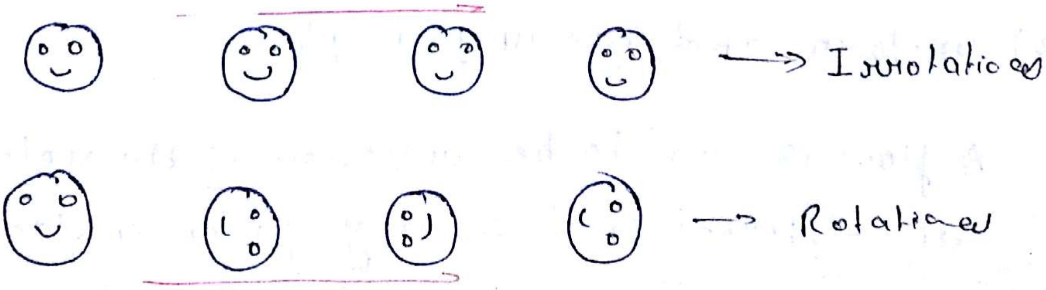
Turbulent flow generally occurs at higher vel.

eg. flow of water in river, flow of gas comming out from chimney.

4) Rotational and Irrotational flow :-

A flow is said to be rotational flow when fluid particle rotate about their mass centre during motⁿ.

otherwise the flow is Irrotatial.



In case of irrotational flow their is no rotation and hence there is no torque i.e there is no tangential force (Shear force) and this occurs generally in non viscous fluid

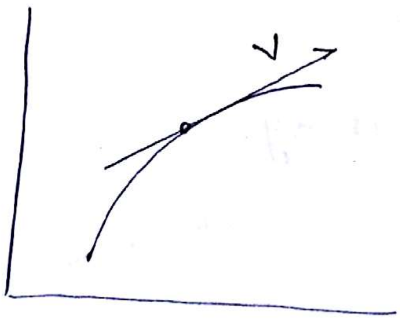
$F = \mu \frac{AV}{y}$

Stream line :-

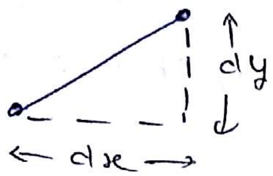
It is an imaginary line @ curve drawn on space such that a tangent drawn to it at any pt. gives velocity vector $(\vec{V})$ at that instant.

The flow is along stream line. as there is no component in $\perp$ directⁿ there is no flow across a stream line.

2 stream line can never intersect @ a single stream line can never intersect at itself bcz at any given instant at any pt. the vel. must be unique.



eqⁿ of a stream line :-



$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}$$

$$\Rightarrow dt = \frac{dx}{u}; \quad dt = \frac{dy}{v}$$

$$\Rightarrow \frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow v\,dx = u\,dy$$

$$\Rightarrow \boxed{v\,dx - u\,dy = 0}$$

↳ eqn of a stream line in 2-D.

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}} \longrightarrow 3D$$

gate

\# A flow is represented by $\vec{V} = ax\,\hat{i} + ay\,\hat{j}$. where a = const. then find the eqn of a stream line passing through (1,2)

Soln

$$\vec{V} = ax\,\hat{i} + ay\,\hat{j}$$

$$= u\hat{i} + v\hat{j}$$

where, $u = ax$, $v = ay$

we know, $\frac{dx}{u} = \frac{dy}{v} \longrightarrow$ for 2D

$$\Rightarrow \frac{dx}{ax} = \frac{dy}{ay} \Rightarrow \int \frac{dx}{x} = \int \frac{dy}{y}$$

$$\Rightarrow \ln x = \ln y + \ln c$$

$$\Rightarrow \ln x = \ln yc$$

$\Rightarrow x = y\,c$

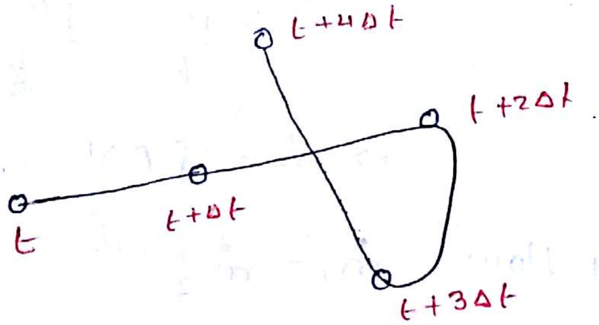
$\Rightarrow 1 = 2 \times c \Rightarrow c = \frac{1}{2}$

$x = y \times \frac{1}{2} \Rightarrow 2x = y \Rightarrow \boxed{2x - y = 0}$

Path line :

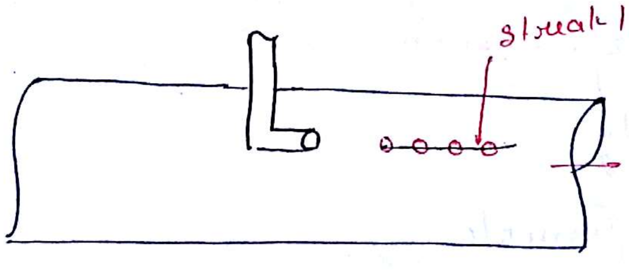
It is the path traced by a single fluid particle at different instant of time.

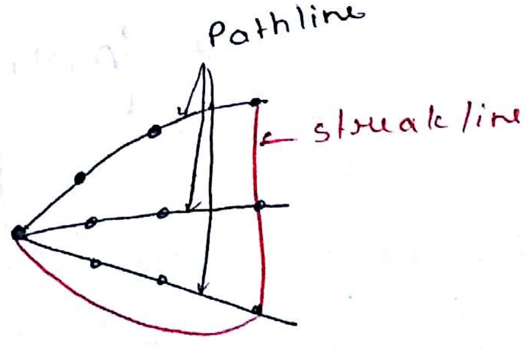
It follows lagrangian approach.



Streak line :

It is the locus of various fluid particles passing through a fixed pt.

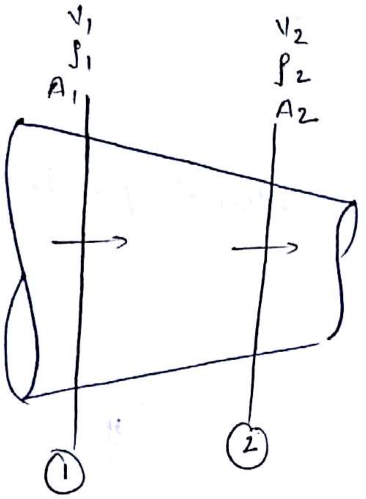




Note : In a steady flow as stream line are fixed. so, Pathline, streamline, streak line are identicle.

conservation of mass (continuity eqⁿ)

case-1 :- steady 1-D flow



$\rho = \frac{mass}{vol}$

$\Rightarrow m = \rho \times vol.$

$m = \rho \times AL$

$\Rightarrow \dot{m} = \frac{m}{t} = \frac{\rho A L}{t}$.

$\Rightarrow \dot{m} = \rho A V$

for steady flow, $\dot{m}_1 = \dot{m}_2$

** $\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$ → continuity eqⁿ

↳ for steady & 1-D flow.

Incompressible → $\boxed{\rho = const.}$ $\Rightarrow \rho_1 = \rho_2$

$\Rightarrow$ $\boxed{A_1 V_1 = A_2 V_2}$ → continuity eqⁿ

→ steady
→ 1-D
→ incompressible

** Pipe, nozzle, diffuser → 1D

case-2 :- generalised continuity eq^n.

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

u, v, w are component of vel. in x, y, z

↳ i) steady / unsteady flow
ii) uniform / non-uniform flow
iii) compressible / incompressible flow.

case-2a) steady flow :-

For steady flow → $\boxed{\frac{d\rho}{dt} = 0}$

$$\Rightarrow \boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

case-2b) Incompressible flow :-

for incompressible flow → $\boxed{\rho = \text{const.}}$ ⇒ $\boxed{d\rho = 0}$

$$\Rightarrow \cancel{\frac{d\rho}{dt}} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] = 0$$

$$\Rightarrow \boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

↳ continuity eq^n for Incompressible flow → 3-D

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$ → For flow to be Incompressible It must satisfy this eq^n.

↳ continuity eq^n for incomp 2D flow

→ the term $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$ → is called divergence of V $(\nabla \cdot \vec{V})$

Note:

*** Every fluid must satisfy continuity eq^n bcz continuity eq^n is conservat^n of mass.

1-D.

A compressible fluid is flowing steadily through a pipe whose area is reduces by 40% from sect^n 1 to sect^n 2. It is further known that the corresponding reduct^n in density is 15%. compared to the vel. of fluid at sect^n 1 the vel. of sect^n 2 is Increased by a factor of

Sol^n

continuity eq^n → $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$$\cancel{\rho_1} \times \cancel{A_1} V_1 = .85 \cancel{\rho_1} \times .6 \cancel{A_1} V_2$$

$$\Rightarrow V_1 = .85 \times .6 V_2$$

$$\Rightarrow V_2 = \frac{V_1}{.85 \times .6} = 1.96 V_1$$

Ans. → 1.96.

The vel. component in x & y direct^n are given by

(26)

$u = \lambda x y^3 - x^2 y$

$v = x y^2 - \frac{3}{4} y^4$.

then find λ for incompressible flow.

sol^n 2D- compressible continuity eq^n →

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = \lambda x y^3 - x^2 y$$

$$\Rightarrow \frac{\partial u}{\partial x} = \lambda y^3 - 2xy$$

$$v = xy^2 - \frac{3}{4} y^4$$

$$\frac{\partial v}{\partial y} = 2xy - 3y^3$$

$$\therefore \lambda y^3 - \cancel{2xy} + \cancel{2xy} - 3y^3 = 0$$

$$\Rightarrow \underline{\lambda = 3} \quad \text{Ans.}$$

The vel. for a flow is given by.

(39) * $\vec{V} = (5x + 6y + 7z)\hat{i} + (6x + 5y + 9z)\hat{j} + (3x + 2y + \lambda z)\hat{k}$

and density varies as $\rho = \rho_0 e^{-2t}$ in order that mass is conserved that value of λ ~~unchanged~~. should be

sol^n

$$\vec{V} = (5x + 6y + 7z)i + (6x + 5y + 9z)\hat{j} + (3x + 2y + \lambda z)\hat{k}$$

comparing with →

$$\vec{V} = ui + vj + wk$$

$$\Rightarrow u = 5x + 6y + 7z, \quad v = 6x + 5y + 9z, \quad w = 3x + 2y + \lambda z$$

$$\rho = \rho_0 e^{-2t}$$

$$\Rightarrow \frac{\partial \rho}{\partial t} = -2\rho_0 e^{-2t}$$

$$\Rightarrow \frac{d\rho}{dt} = -2\rho \quad \text{———} \quad (2)$$

general continuity eq^n for 3D $\rightarrow$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho u) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$-2\rho + \rho[5 + 5 + \lambda] = 0$$

$$\Rightarrow \boxed{\lambda = -8}$$

Acceleration of a fluid particle:

$$u = f(x, y, z, t)$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\underbrace{\frac{dx}{dt}}_{u} + \frac{\partial u}{\partial y}\cdot\underbrace{\frac{dy}{dt}}_{v} + \frac{\partial u}{\partial z}\cdot\underbrace{\frac{dz}{dt}}_{w} + \frac{\partial u}{\partial t}\cdot\cancel{\frac{dt}{dt}}$$

$$\Rightarrow a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

similarly $a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

The acc^n due to change of vel. wrt space is known as convective acc^n.
and acc^n due to change of vel with time is known as temporal @ local acc^n.

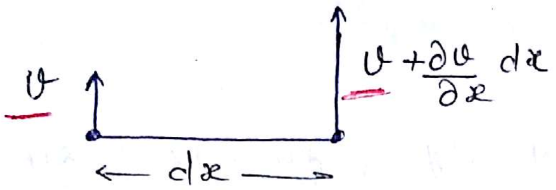
$\therefore$ total acc^n = convective + temporal.

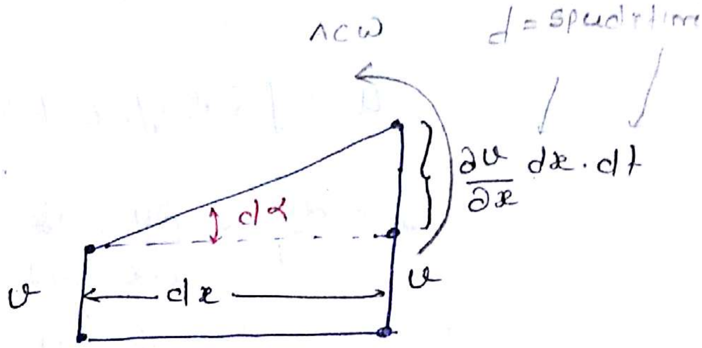
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$a_x = \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] + \frac{\partial u}{\partial t}$$

Type of flow	convective	Local @ temporal
steady & uniform	0	0
steady & non-uniform	exist	0
unsteady & uniform	0	exist
unsteady & non-uniform	exist	exist

Rotational components :-





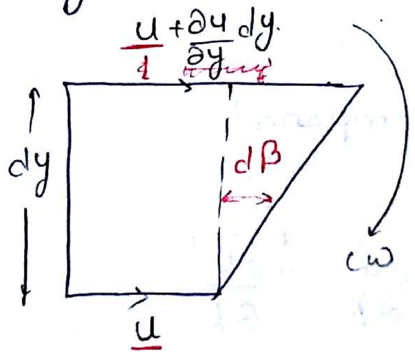
$$\tan d\alpha = \frac{\frac{\partial v}{\partial x} dx\, dt}{dx}$$

$$\Rightarrow \tan d\alpha = \frac{\partial v}{\partial x} dt$$

$$\Rightarrow d\alpha = \frac{\partial v}{\partial x} dt$$

$$\Rightarrow \frac{d\alpha}{dt} = \frac{\partial v}{\partial x} \quad \text{——— (1)}$$

similarly,



$$\frac{d\beta}{dt} = \frac{\partial u}{\partial y}. \quad \text{——— (2)}$$

considering cw rotation as $-ve$
ACW rotat^n as $+ve$

we have, $\frac{d\beta}{dt} = -\frac{\partial u}{\partial y}$.

In fluid mechanics angular vel. ω is defined as the avg. angular vel. of initially 2 ⊥ line segment.

$$\therefore \quad \omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} + \frac{d\beta}{dt}\right)$$

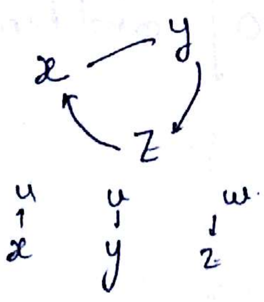
$$\Rightarrow \boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$

Rotational vector $\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$

where, $\omega = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$

$$\Rightarrow \underbrace{2\omega}_{\text{vorticity.}} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Flow is irrotational if $\boxed{\omega_x = \omega_y = \omega_z = 0}$



$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

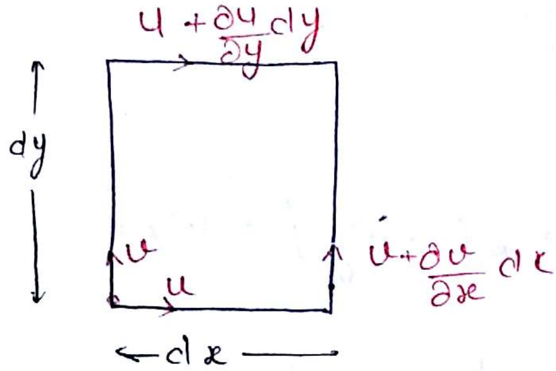
$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

v. imp.

Circulation : (Γ)

It is the line integral of tangential component of vel. taken around a closed curve.



$$\Gamma = u\,dx + \left(v + \frac{\partial v}{\partial x} dx\right) dy - \left(u + \frac{\partial u}{\partial y} dy\right) dx - v\,dy$$

$$= u\,dx + \cancel{v\,dy} + \frac{\partial v}{\partial x} dx\,dy - u\,dx - \frac{\partial u}{\partial x} dx\,dy - \cancel{v\,dy}$$

$$\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{dy}\right) dx\,dy.$$

$\underbrace{2\omega_z}_{\text{vorticity}} = \frac{\partial v}{\partial x} - \frac{\partial u}{dy}$

****

$\boxed{\text{circulat}^n = \text{vorticity} \times \text{Area.}}$

In an irrotational flow the $\boxed{\text{vorticity} = 0}$ and hence

$\boxed{\text{circulat}^n = 0}$

Vel. potential funct^n (ϕ) :

It is a funct^n of space and time defined in such a manner that its negative derivative with space will give vel. in that direct^n.

It is another way of representing vel. components.

$$\boxed{\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}}$$

→ −ve sign is taken bcz. the flow is in the direct^n of decreasing potential

vel. potential funct^n can be defined in 3-D.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Big] \rightarrow \text{2-D, incomp. cont. eq}^n$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right]$$

case-i) if ϕ satisfies laplace eq^n, then $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \text{continuity eq}^n \text{ is satisfied}$$

⇒ flow is possible.

case (ii) :. if ϕ does not satisfy laplace eqⁿ →

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \neq 0$$ ⇒ continuity eqⁿ is not satisfied
⇒ flow is not possible.

For a possible flow field, vel. potential functⁿ (ϕ) must satisfy laplace eqⁿ.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right)\right]$$

$$= \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x}\right]$$

⇒ $\omega_z = 0$ ⇒ Irrotational.

vel. potential functⁿ (ϕ) exists only for irrotational flow (i.e) the existence of vel. potential functⁿ (ϕ) implies that the flow is irrotational.

Stream funct^n (ψ) :

A funct^n of space and time defined in such a manner that it satisfies the continuity eq^n.

$$\therefore \boxed{u = -\frac{\partial \psi}{\partial y} \quad ; \quad v = \frac{\partial \psi}{\partial x}}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Note :-

Stream funct^n is defined for 2-D flow. whereas, vel. potential funct^n ~~is~~ can be defined for 3-D flow.

→

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

case-i)) If ψ satisfies laplace eq^n → $\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

$$\Rightarrow \omega_z = \frac{1}{2}(0) = 0$$

$$\Rightarrow \omega_z = 0$$ — Irrotational

case ii) If ψ does not satisfies laplace eq^n → $\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \neq 0$

$$\Rightarrow \omega_z \neq 0$$ → Rotational

Stream funct^n exists for rotational as well as irrotational flow.

If the stream funct^n satisfies laplace eqn then the flow is irrotational otherwise the flow is rotational.

Discharge (Q) :

The volume flow rate is known as discharge.

$$Q = \frac{\text{Volume}}{\text{time}} = \frac{A \times L}{\text{time}} = A \overset{\text{velocity}}{V}$$

$$\Rightarrow \boxed{Q = AV}$$

unit → $\frac{m^3}{s}$, lps, lpm

Note: In a steady, 1-D, incompressible flow, discharge remains const.

$$A_1V_1 = A_2V_2$$

$$Q_1 = Q_2$$

$$\Rightarrow \boxed{Q = \text{const.}}$$

i) 1-D
ii) incomp.
iii) steady

Cauchy - Reimann eqn :

$$u = -\frac{\partial \phi}{\partial x} \quad ; \quad u = -\frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\Rightarrow -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} \qquad \Rightarrow -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\Rightarrow \boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}} \qquad \Rightarrow \boxed{-\frac{\partial \phi}{\partial y} = +\frac{\partial \psi}{\partial x}}$$

C-R eqn

Significance of stream funct^n :

$$\frac{dx}{u} = \frac{dy}{v}$$

$v\,dx = u\,dy \Rightarrow v\,dx - u\,dy = 0 \longrightarrow$ eqn of streamline

$$v = \frac{\partial \psi}{\partial x} \qquad u = -\frac{\partial \psi}{\partial y}$$

$$\frac{\partial \psi}{\partial x}dx - \left(-\frac{\partial \psi}{\partial y}\right)dy = 0$$

$$\frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0 \quad \text{——— (1)}$$

$$\psi = f(x, y, t)$$

At any given instant $\longrightarrow \psi = f(x, y)$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy \quad \text{——— (2)}$$

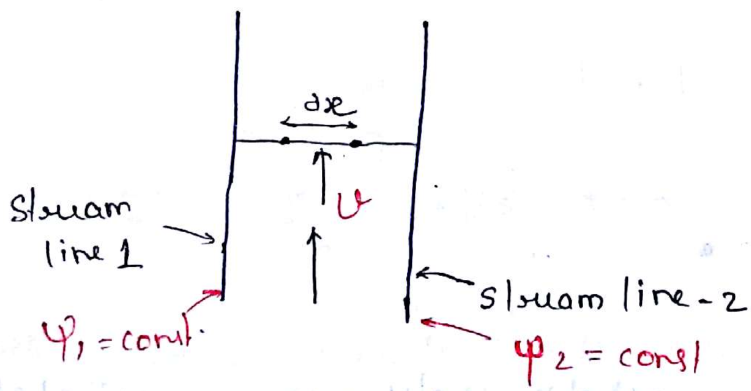
from (1) & (2)

$$d\psi = 0 \Rightarrow \boxed{\psi = \text{const.}}$$

***

For a paticular stream line, stream funct^n remains const.

→



Q = Area × vel.

$dQ = dx \times 1 \times u$

$$dQ = u\,dx \quad \text{——— (1)}$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$\Rightarrow d\psi = \frac{\partial \psi}{\partial x} dx \Rightarrow \partial\psi = u\,dx \quad \text{——— (2)}$$

$$\frac{\partial \psi}{\partial x} = u$$

from (1) & (2)

$d\phi = -d\psi$

$\Rightarrow \boxed{q = \psi_2 - \psi_1}$

the differnce b/w any 2 stream funct^n gives discharge per unit width.

Relationship b/w equi-potential lines and const. stream funct^n lines

$\psi = \text{const.}$

$\phi = \text{const.}$ (equipotential)

$\phi = f(x, y)$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$0 = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$\frac{\partial \phi}{\partial x} dx = -\frac{\partial \phi}{\partial y} dy$$

$$\frac{dy}{dx} = \frac{\partial \phi}{\partial x} \Big/ -\frac{\partial \phi}{\partial y}$$

$$\frac{dy}{dx} = \frac{-u}{v} \qquad u = -\frac{\partial \phi}{\partial x} \Rightarrow -u = \frac{\partial \phi}{\partial x}$$

slope of equipotential line $= \frac{-u}{v}$ —— (1) $\qquad v = \frac{-\partial \phi}{\partial y}$

$$\psi = \text{const.}$$

$$\psi = f(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$0 = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$\Rightarrow \frac{\partial \psi}{\partial x} dx = -\frac{\partial \psi}{\partial y} dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{\partial \psi}{\partial x} \Big/ -\frac{\partial \psi}{\partial y}.$$

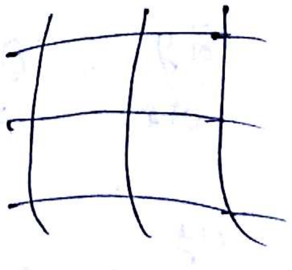
$$\Rightarrow \frac{dy}{dx} = \frac{v}{u}$$

slope of const. stream line $= \frac{v}{u}.$ —— (2)

Product of slope ⟶

$$-\frac{u}{v} \times \frac{v}{u}$$

$$= -1$$



⇒ $\boxed{\text{slope of equipot. line} \times \text{slope of const. stream funct}^n \text{ line} = -1}$

equi-potential line and const. stream funct^n line are othogonal @ $\perp$ to each other. in flow

The stream funct^n is given by $\psi = 3xy$ then find the vel. at (2,3)

Sol^n

$V = ui + vj$

$|V| = \sqrt{u^2 + v^2}$

$u = -\dfrac{\partial \psi}{\partial y}$. $\qquad v = +\dfrac{\partial \psi}{\partial x}$.

$u = -\dfrac{\partial}{\partial y}(3xy) \qquad v = +\dfrac{\partial}{\partial x}(3xy)$

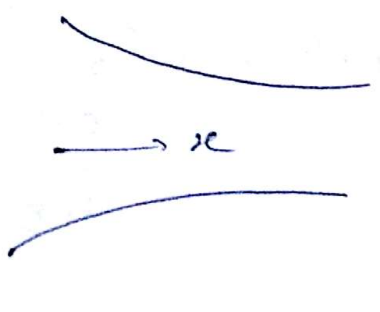
$u = -3x. \qquad v = +3y$

$u = -3(2) = -6 \qquad v = 3y = 3(3) = 9$

$\Rightarrow |V| = \sqrt{(-6^2) + 9^2} = 10.82$.

A fluid with a dischage of 5 m^3/sec enters the nozzle as shown in fig → the x-sectional area varies with $A(x) = \dfrac{1}{1+x^2}$, Assuming the flow to be steady, 1-D, then the acc^n at any pt in the nozzle is given by →

Sol



a) $50(x + x^3)$

b) $50(1 + x^2)$

c) 0

d) $50(x^2 + x^3)$

$\underline{\text{Sol}^n}$ $A(x) = \frac{1}{1+x^2} \Rightarrow A = \frac{1}{1+x^2} \Rightarrow \frac{1}{A} = 1+x^2$

$a = a_x \hat{i} + \cancel{a_y \hat{j}} + \cancel{a_z \hat{k}}$

$\Rightarrow a = a_x \hat{i}$

$\Rightarrow a_x = u\frac{\partial u}{\partial x} + v\cancel{\frac{\partial u}{\partial y}} + w\cancel{\frac{\partial u}{\partial z}} + \cancel{\frac{\partial u}{\partial t}}$

$\Rightarrow a_x = u\frac{\partial u}{\partial x}$ ——— (1)

$Q = A \times u \Rightarrow u = \frac{Q}{A} = Q \times \frac{1}{A} = 5(1+x^2)$

$\Rightarrow u = 5(1+x^2)$

$\Rightarrow a_x = u\frac{\partial u}{\partial x} = 5(1+x^2) \cdot \frac{\partial}{\partial x}[5(1+x^2)]$

$= 50(x+x^3)$

\# The x-component vel. in an incompressible, 2D flow is $u = 1.5x$. It is found that at a pt. 1, 0 the y-component vel is zero then find the y-comp. vel.

$\underline{\text{Sol}^n}$ for 2D, incompressible $\longrightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$1.5 + \frac{\partial v}{\partial y} = 0$

$$\Rightarrow \frac{\partial v}{\partial y} = -1.5 \Rightarrow \int \partial v = \int -1.5\, dy$$

$$\Rightarrow v = -1.5y + c$$

at $(\overset{x}{1}, \overset{y}{0}) \rightarrow v = 0$

$$\Rightarrow 0 = -1.5(0) + c$$

$$\Rightarrow c = 0$$

$$\Rightarrow v = -1.5y.$$

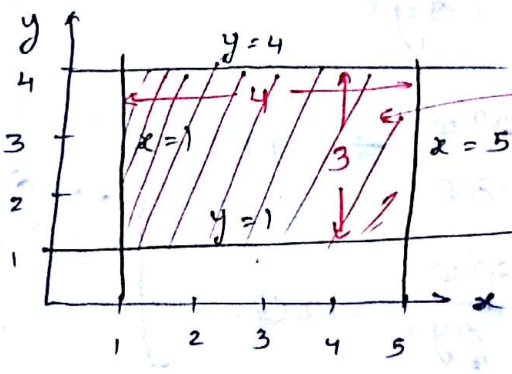
v.imp.

Determine the circulat^n around a rectangle defined by $x = 1$, $y = 1$, $x = 5$ and $y = 4$ for a vel. $u = 2x + 3y$

$v = -2y$

Sol^n

$x = 1$
$y = 1$
$x = 5$
$y = 4$



Area $= 4 \times 3$
$= 12$

circulat^n = vorticity × Area

$$\text{vorticity} = 2\omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$u = 2x + 3y \rightarrow \frac{\partial u}{\partial y} = 3$$

$$v = -2y \rightarrow \frac{\partial v}{\partial x} = 0$$

$$\text{vorticity} = (0 - 3) = -3$$

circulat^n = -3×12 = -36 units

A stream for a 2D - incomp. flow is given by eq.

** $\psi = Px^2 + Qy^2$ where P & Q are const. the vel. potential

31 funct^n for this flow exists only when

a) $P = Q$ b) $P = -Q$ c) $P = 2Q$ d) $P = -2Q$.

Sol^n

If the vel. potential funct^n is to exists ⟶ then the flow must be irrotational and for irrotational flow stream funct^n must satisfy laplace eq^n.

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

$$\psi = Px^2 + Qy^2$$

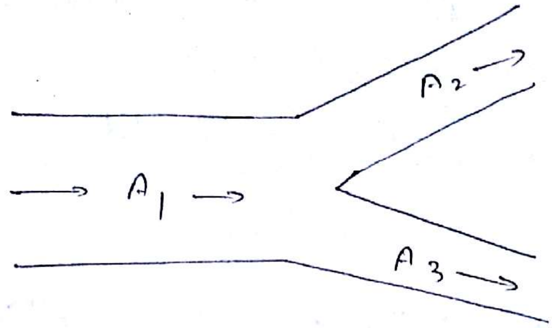
$$\left.\begin{aligned} \frac{\partial^2 \psi}{\partial x} &\neq 2P \\ \frac{\partial^2 \psi}{\partial y^2} &= 2Q. \end{aligned}\right\} \Rightarrow \frac{\partial^2 \psi}{\partial x} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

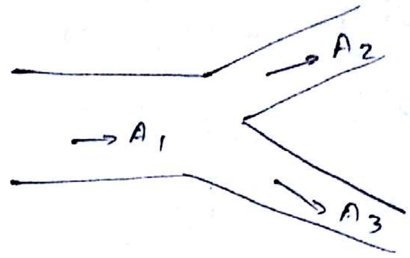
$$\Rightarrow 2P + 2Q = 0$$

$$\Rightarrow \boxed{P = -Q}$$

* water enters a pipe of x-sect^n area A, then divides into 2 sect^n of equal area A_2 & A_3. At one instant the flow vel. are $v_1 = 2$ m/s, $v_2 = 3$ m/sec. and $v_3 = 5$ m/s. At another instant $v_1 = 3$ m/sec, $v_2 = 4$ m/sec then find v_3 at this instant.

Sol^n





$V_1 = 2 m/s, V_2 = 3, V_3 = 5$

mass entering = mass leaving.

same fluid

$$\cancel{\rho_1} A_1 V_1 = \cancel{\rho_2} A_2 V_2 + \cancel{\rho_3} A_3 V_3$$

$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$A_1 (2) = A_2 (3) + A_2 (5)$$

$$\Rightarrow 2A_1 = A_2 (3+5)$$

$$\Rightarrow A_1 = 4A_2 \quad \text{--- (1)}$$

$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$A_1 (3) = A_2 (4) + A_2 (V_3)$$

$$\Rightarrow 3(4A_2) = A_2 (4 + V_3)$$

$$\Rightarrow \boxed{V_3 = 8}$$

In a steady flow nozzle the flow vel. at nozzle axis is given by $\vec{V} = u_0 (1 + \frac{3x}{L}) \hat{i}$ where x is the distance along the axis of nozzle from inlet and L is the length of the nozzle. then find the time req. for a fluid particle travelling from inlet to exit.

Sol^n

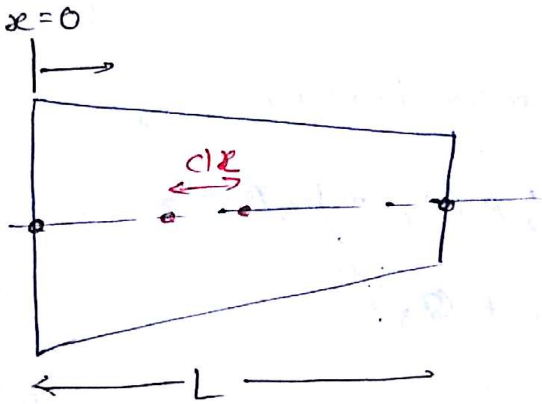
$$\vec{V} = u_0 (1 + \frac{3x}{L}) \hat{i} \Rightarrow \text{1-D flow}$$

$$\Rightarrow \vec{V} = u\hat{i}$$

where $u = u_0\left(1 + \frac{3x}{L}\right)$

$V = \sqrt{u^2 + \cancel{v^2} + \cancel{w^2}} \Rightarrow V = u$

$\Rightarrow V = u = u_0\left(1 + \frac{3x}{L}\right)$ —— (1)



$V = \frac{D}{t} \Rightarrow$ ~~dist = vel × t~~ $t = \frac{D}{V}$

$\Rightarrow \int dt = \int_0^L \frac{dx}{u_0\left(1 + \frac{3x}{L}\right)}$

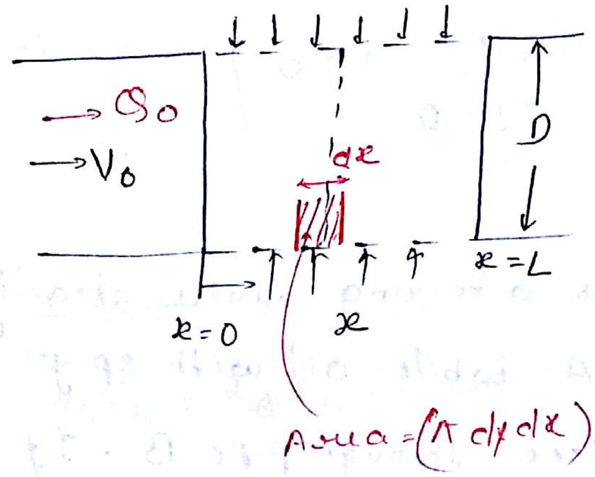
$t = \frac{1}{u_0}\int_0^L \frac{dx}{1 + \frac{3x}{L}}$

$\Rightarrow t = \frac{1}{u_0}\left[\frac{\ln\left(1 + \frac{3x}{L}\right)}{3/L}\right.$

$= \frac{L}{3u_0}\left[\ln\left(1 + \frac{3x}{L}\right)\right]_0^L = \frac{L}{3u_0}\left[\ln\left(1 + \frac{3L}{L}\right) - \ln\left(1 + \frac{3(0)}{L}\right)\right]$

$\Rightarrow t = \frac{L}{3u_0} \times \ln 4$

A pipe has a porous sect^n of length L as shown in fig. vel. at the start of this sect^n is V_0. if fluid leaks into the pipe through the porous sect^n at a volumetric rate per unit area $q\left(\frac{x}{L}\right)^2$. what will be the vel. in the pipe at any distance x. Assume incompressible, 1-D flow.



sol^n

$$\int dQ = \int \frac{q x^2}{L^2} \times \pi D\, dx$$

$$Q = \frac{q \pi D}{L^2} \frac{x^3}{3} + C$$

At $x = 0$; $Q = Q_0$

$$\Rightarrow Q_0 = 0 + C \Rightarrow C = Q_0$$

$$\Rightarrow Q = \frac{q \pi D x^3}{3L^2} + Q_0$$

$$Q = AV \Rightarrow V = \frac{Q}{A}$$

$$V = \frac{1}{A}\left[\frac{q\pi D x^3}{3L^2} + Q_0\right]$$

$$= \frac{1}{A}\frac{q\pi D x^3}{3L^2} + \frac{Q_0}{A}$$

$$= \frac{1}{\left(\frac{\pi}{4}D^2\right)}\frac{q\pi D x^3}{3L^2} + V_0$$

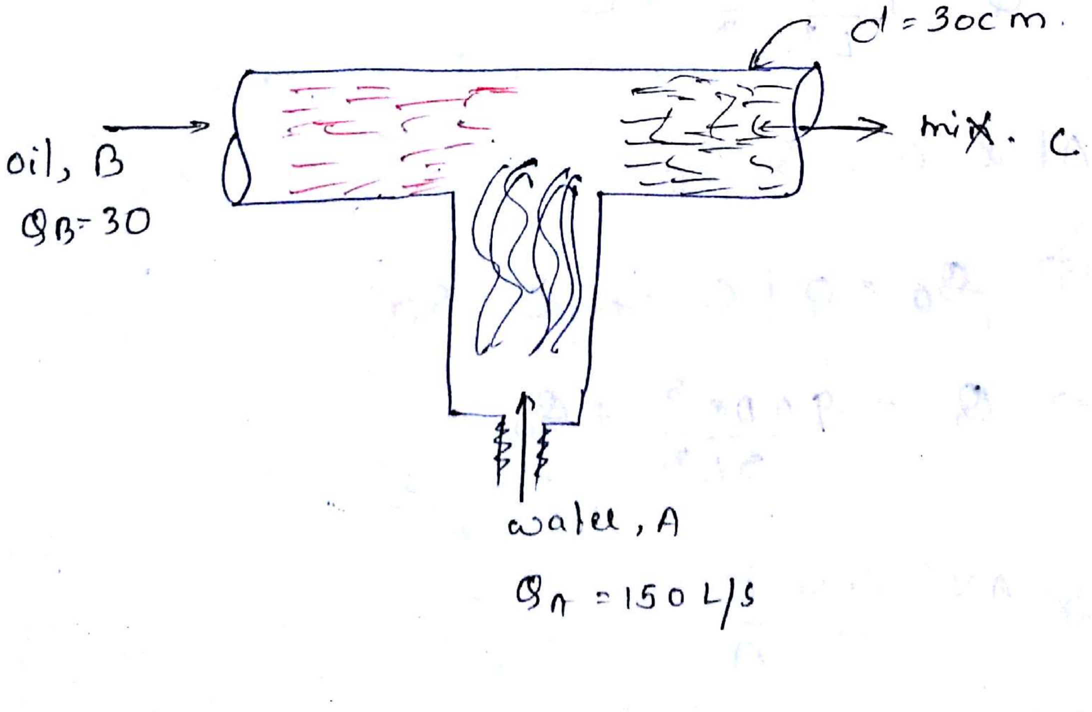
$Q_0 = AV_0$

$\Rightarrow V_0 = \frac{Q_0}{A}$

$$\boxed{V = \frac{4qx^3}{3L^2D} + V_0}$$

v.v.imp.

water enters a mixing device steadily at 150 L/sec through pipe A while oil with sp.gr = .8 is forced in at 30 L/sec through pipe B. If liquids are incompressible and form a homogeneous mix. of oil and water. Find the avg. vel. and density of mix. leaving through 30 cm dia pipe C.



Sol^n

mass flow rate of water entering = mass flow rate of water leaving

$\rho_1 A_1 V_1 + \rho_2 V_2 A_2 = \rho_3 V_3 A_3$ (for incomp. ρ = const)

$\Rightarrow Q_A + Q_B = Q_C$

$150 + 30 = Q_C \Rightarrow Q_C = 180\ L/sec$

$= 180 \times 10^{-3}\ m^3/sec$

$Q = AV \rightarrow V_C = \frac{Q_C}{A_C} = \frac{180 \times 10^{-3}}{\frac{\pi}{4}(.3)^2} = 2.546\ m/sec$

for steady flow $\rightarrow \dot{m}_A + \dot{m}_B = \dot{m}_C$

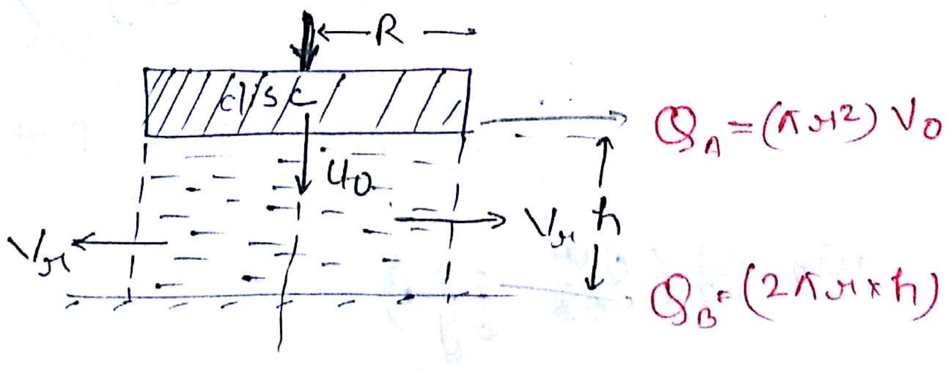
$\rho_A A_A V_A + \rho_B A_B V_B = \rho_C A_C V_C$

$\rho_A Q_A + \rho_B Q_B = \rho_C Q_C$

$10^3 \times 150 + 800 \times 30 = \rho_C \times 180$

$\Rightarrow \rho_C = 966.6\ kg/m^3.$

\# A circular disc of radius R is pushed through a vel. V_0 towards a large plate with gap h b/w the two and this is occupied by a fluid as shown in fig → then find the vel. V_r at the edge of disc.



sol^n

$$Q_A = Q_B$$

$$\pi R^2 u_0 = 2\pi R \cdot h \times V_r$$

$$\boxed{V_r = \frac{R u_0}{2h}}$$

The vel. for a 2-D flow is as follows →

linked qu.

*

$u = \frac{u_0 x}{L}$; $v = -\frac{u_0 y}{L}$ if $L = \cdot 2$ and the resultant of total acceal^n in x & y. direct^n at $x = L, y = L$ is $10\, m/s^2$ then the value of u_0 in m/s is .

a) 1.414 b) 2.38 c) 1.19 d) 11.9

this flow is → a) Rotational & compressible

b) Irrot & comp.

c) rot & incomp.

d) irrot & incomp.

sol^n

$$\left.\begin{aligned} u = \frac{u_0 x}{L} &\Rightarrow \frac{\partial u}{\partial x} = \frac{u_0}{L} \\ v = -\frac{u_0 y}{L} &\Rightarrow \frac{\partial v}{\partial y} = -\frac{u_0}{L} \end{aligned}\right\} \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

⇒ it is incompressible flow

we know $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$\Rightarrow \omega_z = \frac{1}{2}(0-0) = 0$

$\Rightarrow \omega_z = 0 \Rightarrow$ Irrotational

$\therefore$ flow is incomp & irrot

$a = a_x \hat{i} + a_y \hat{j}$

$|a| = \sqrt{a_x^2 + a_y^2}$ —— (a)

$a_x = \cancel{a_x}\; u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\cancel{\frac{\partial u}{\partial z}} + \frac{\partial u}{\partial t}$

$= \frac{u_0 x}{L}\left(\frac{u_0}{L}\right) + \left(-\frac{u_0 y}{L}\right)(0) + 0$

$\Rightarrow a_x = \frac{u_0^2 x}{L^2}$ —— (1)

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\cancel{\frac{\partial v}{\partial z}} + \frac{\partial v}{\partial t}$

$a_y = \frac{u_0 x}{L}(0) + \left(-\frac{u_0 y}{L}\right)\left(-\frac{u_0}{L}\right) + 0$

$\Rightarrow a_y = \frac{u_0^2 y}{L^2}$

At $x = L$; $y = L$

$\Rightarrow a_x = \frac{u_0 \times L}{L^2} = \frac{u_0^2}{L}$ {from (1)}

$$a_y = \frac{V_0^2 \times \cancel{K}}{L^{\cancel{2}}} = \frac{V_0^2}{L}$$

=> from (a) => $10 = \sqrt{\left(\frac{V_0^2}{L}\right)^2 + \left(\frac{V_0^2}{L}\right)^2}$

$$10 = \sqrt{2} \times \frac{V_0^2}{L}$$

$$\Rightarrow \underline{u_0 = 1.19}$$

\# For a 2D irr. flow, the vel. potential

** (22) is defined as $\phi = \log(x^2+y^2)$ then which of the foll is a possible stream funct^n.

a) $\frac{1}{2}\tan^{-1}(y/x)$ (b) $\tan^{-1}(y/x)$ (c) $2\tan^{-1}(y/x)$ d) $2\tan^{-1}(x/y)$

Sol^n

$$\phi = \log(x^2+y^2)$$

Chauchy-Rieman eq^n → $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ —— (1)

$$\frac{\partial \phi}{\partial x} = \frac{1}{x^2+y^2}(2x+0)$$

$$\frac{\partial \phi}{\partial x} = \frac{2x}{x^2+y^2} \quad \text{—— (2)}$$

from 1) & 2)

$$\frac{\partial \psi}{\partial y} = \frac{2x}{x^2+y^2}$$

$$\Rightarrow \frac{\partial \psi}{\partial y} = \frac{2x}{x^2\left[1+\frac{y^2}{x^2}\right]}$$

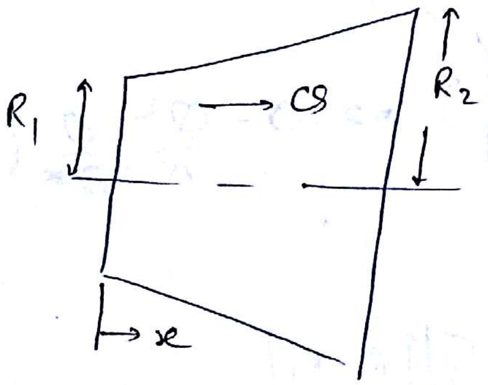
$$\Rightarrow \int \partial \psi = \int \frac{2\,\partial y}{x(1+y^2/x^2)}$$

$$\psi = 2\tan^{-1} y/x$$

conventional

For a fluid flowing through a diversion passage of length L having inlet and outlet radii r_1 & r_2 & a const. flow rate of Q. Find the acc^n at the exit of pipe. Assume the flow to be steady, 1-D, incomp.

Sol^n



$$a = a_x\hat{i} + \cancel{a_y\hat{j}} + \cancel{a_z\hat{k}}$$

1-D

$$\Rightarrow a = a_x\hat{i}$$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{a_x^2 + 0 + 0}$$

$$\Rightarrow |a| = a_x$$

$$a_x = u\frac{\partial u}{\partial x} \cancel{\frac{\partial u}{\partial t}} + \cancel{} + \cancel{} + \cancel{\frac{\partial u}{\partial t}}$$

steady

$$\Rightarrow a_x = u\frac{\partial u}{\partial x}$$

$$V = u\hat{i} + \cancel{v\hat{j}} + \cancel{w\hat{k}}$$

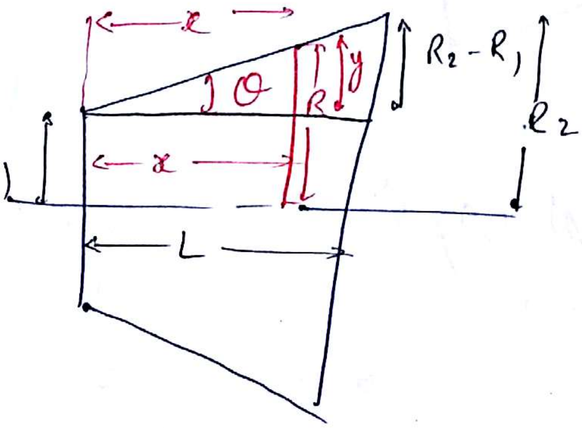
$$\Rightarrow |V| = \sqrt{u^2} = u$$

$$\Rightarrow V = u$$

$$a = a_x = V\frac{\partial V}{\partial x}$$

const.

$$a = \frac{Q}{A}\cdot\frac{\partial}{\partial x}\left(\frac{Q}{A}\right) \Rightarrow \boxed{a = \frac{Q^2}{A}\cdot\frac{\partial}{\partial x}\left(\frac{1}{A}\right)}$$



$$\tan\theta = \frac{R_2 - R_1}{L} = k$$

$$R = R_1 + y$$

$\tan\theta = \frac{y}{x} \Rightarrow k = y/x \Rightarrow y = kx.$

$R = R_1 + kx$

$A = \pi R^2$

$A = \pi(R_1 + kx)^2$

$Q = \frac{2(R_1 - R_2)\phi^2}{\pi^2 L R_2^5}$