

Differentials



Bolt it up and let it happen.
- Anonymous



Outline

- Introduction to Differentials
- Types of Differential
 - The Open Differential
 - The Spool
 - The Limited Slip Differential*
- Differential Set-Up
- Linear Differential Model
- Calculation of Locking for Frictional Contact Differentials
- Locking Percentage Rating

* = we will discuss several different limited slip differentials

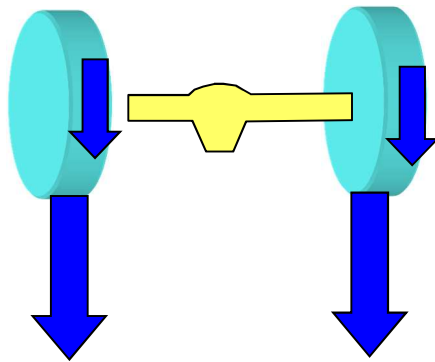


Introduction to Differentials

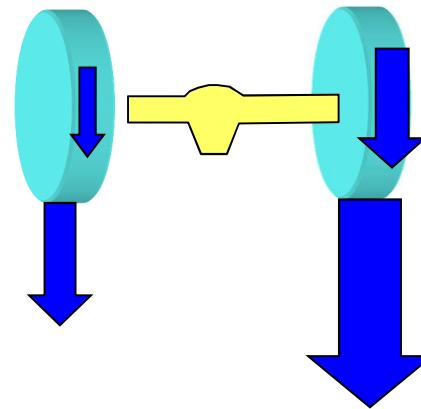
Differential - involving differences in speed or direction of motion.

Source: The American Heritage® Dictionary of the English Language, Fourth Edition

The size of the arrows is representative of the magnitude of speed.



Both of these wheels are rotating at the same rate. No differentiation.



One wheel is rotating faster than the other in this case; therefore, differentiation occurs.

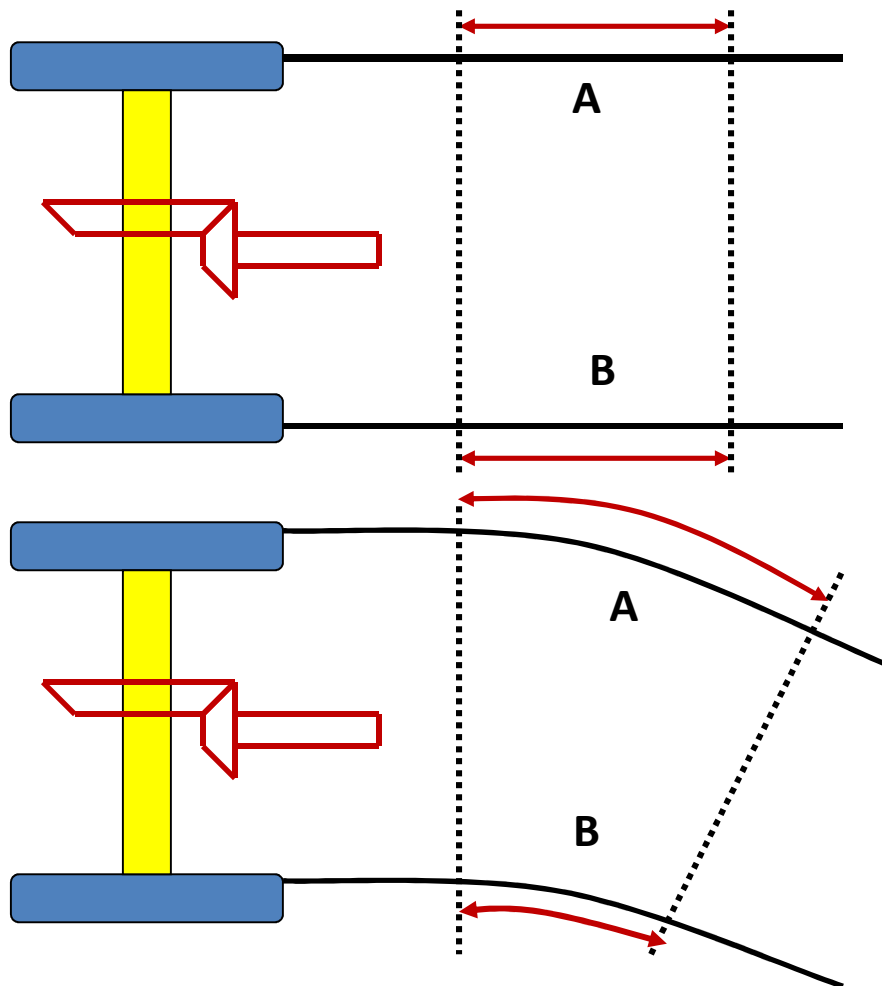
A differential, as defined, allows the two axles it connects to have a difference in speed or direction.



Introduction to Differentials

Why use a Differential

To enable the driving wheels to rotate at different speeds around a corner.



In a straight, wheels travel through the same distance:

$$A=B$$

In a corner, the outer wheel travels further than the inner wheel: $A>B$.



Introduction to Differentials

Why use a Differential

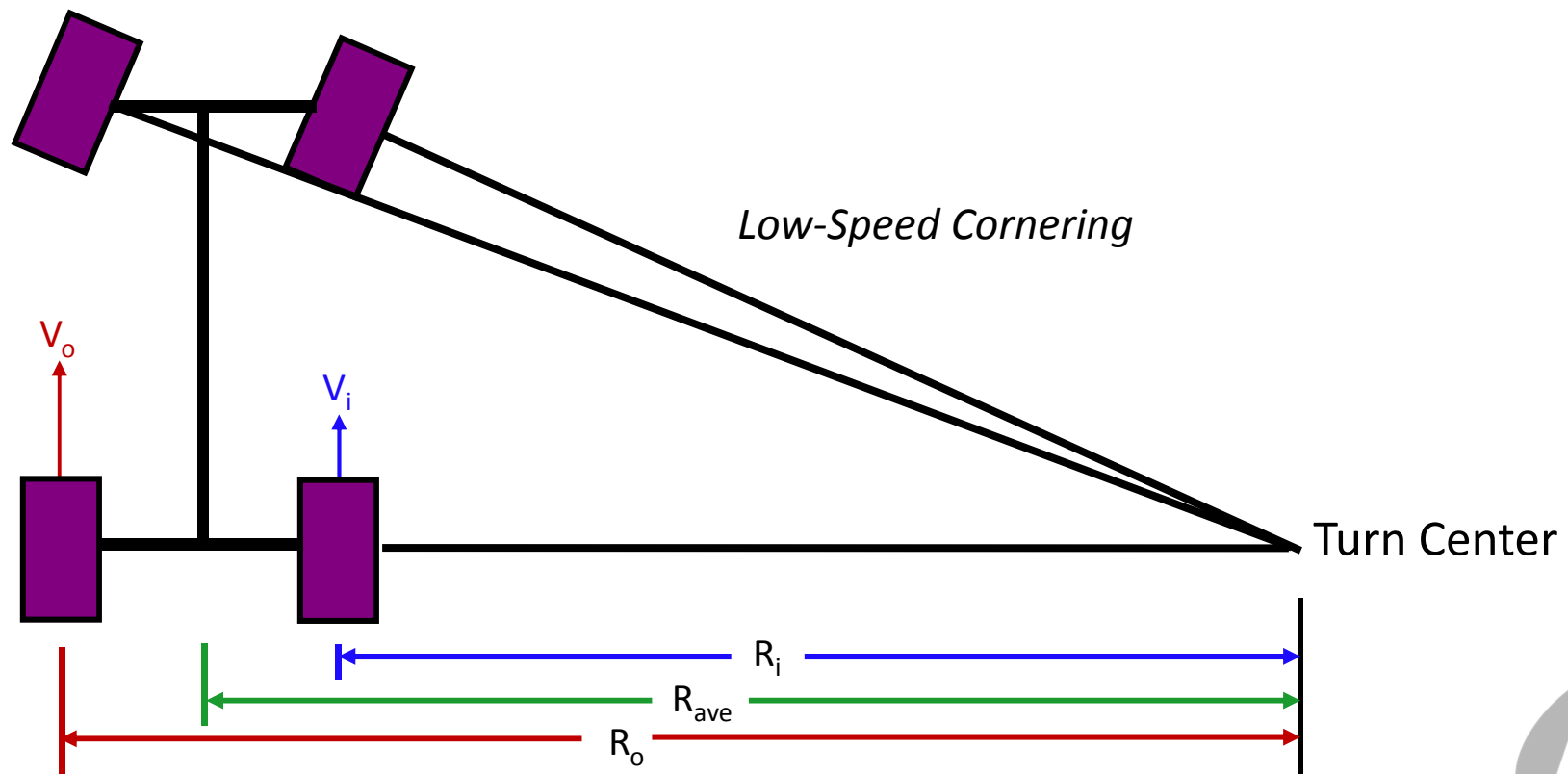
- A differential is not used only because of the different lengths of travel of the inner and outer wheels through a corner.
- During a corner the outside wheel will receive an increased load due to lateral weight transfer. Therefore, the outer wheel is now loaded more than the inner wheel, which forces the outer wheel to rotate about a speed indicative of its path.
- The inner wheel, traveling a shorter path with a smaller load, will thereby move with a combination of drag and rotation.
- We cannot allow the inside tire on a driven axle to drag in terms of comfort, tire wear, driveline consistency, general handling, etc...
- That is why most modern vehicles are equipped with a differential system which allows the driven wheels to rotate at different speeds.



Introduction to Differentials

Why use a Differential

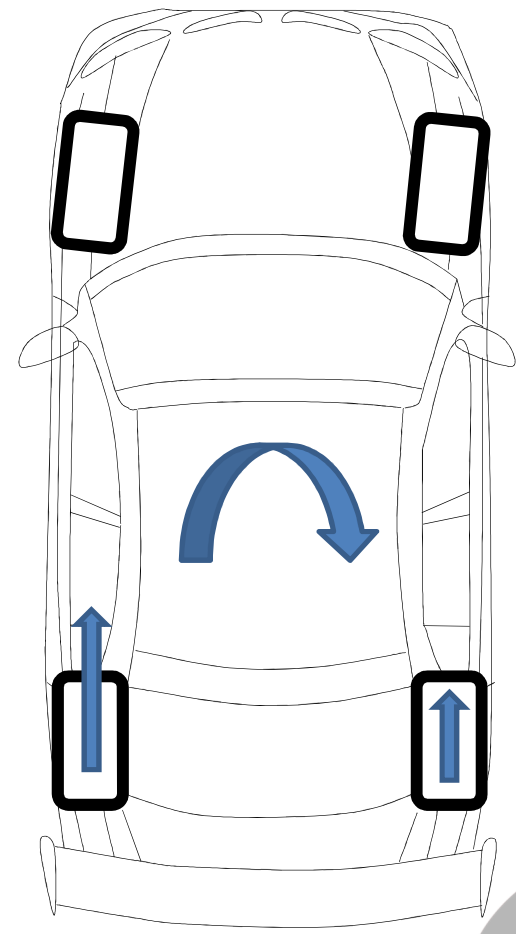
- Less power required to turn vehicle.
- Reduction of turning circle for a given steering angle.
- Accommodation of smooth low speed operation.



Introduction to Differentials

Why use a Differential

- Perhaps the most important reason to use a differential, particularly in racing, is its influence on the longitudinal acceleration and the yaw moment.
- The differential can determine the maximum longitudinal acceleration of the vehicle in the way it splits torque between the drive wheels. Drive wheels will have different levels of traction available due to surface differences and load transfer effects.
- Because of this torque difference between left and right drive wheels the differential can also cause a yaw moment that contributes to the understeer/oversteer characteristics of the vehicle.
- We will see later how these effects can be tuned and used to our advantage.



Introduction to Differentials

Calculation of Induced Slip

Amount of tire slip induced (for locked axle) can be calculated according to the following formulas:

$$V_{diff} = \frac{R_o - R_i}{R_i} = \frac{R_o}{R_i} - 1$$

V_{diff} = velocity difference

R_o = outside wheel corner radius

R_i = inside wheel corner radius

OR

$$V_{diff} = \frac{T}{R_{ave}}$$

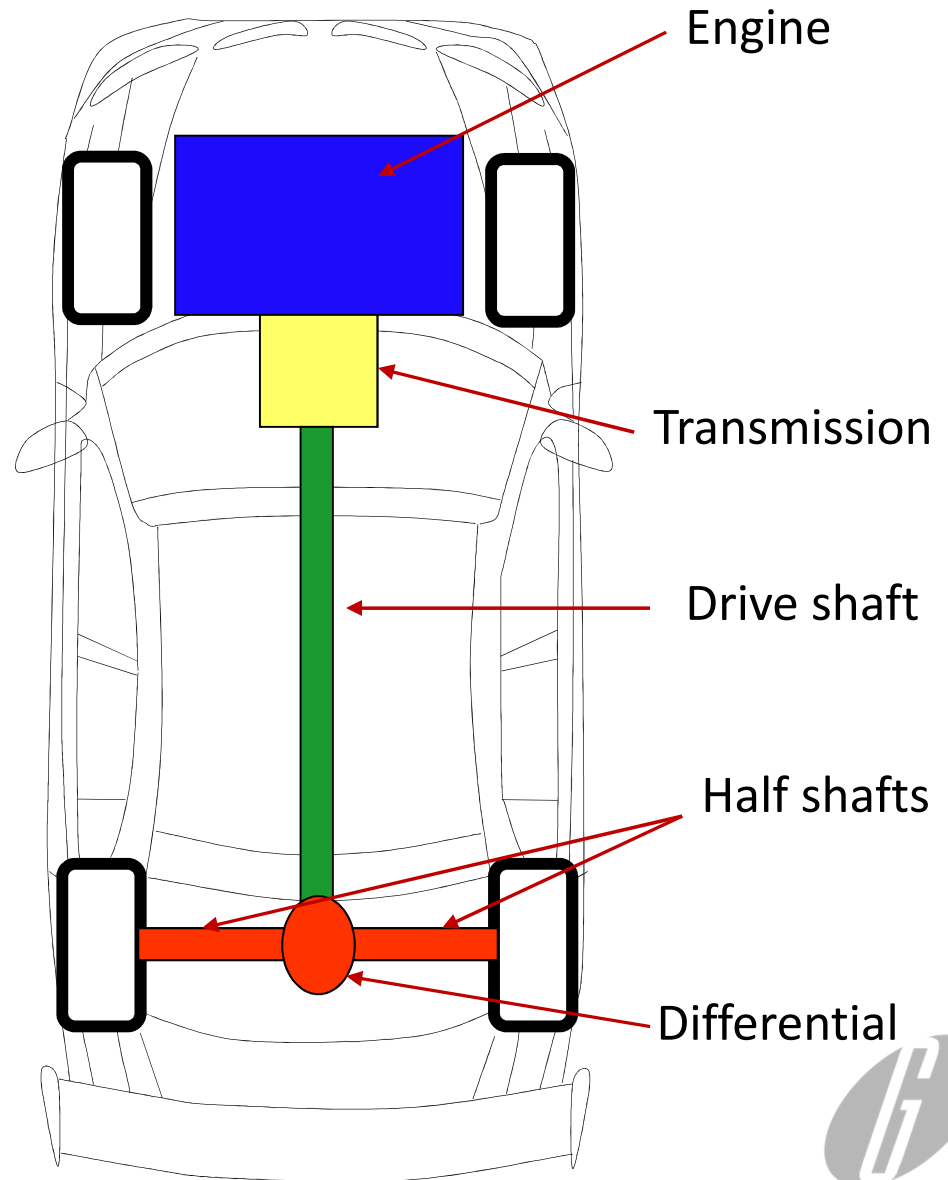
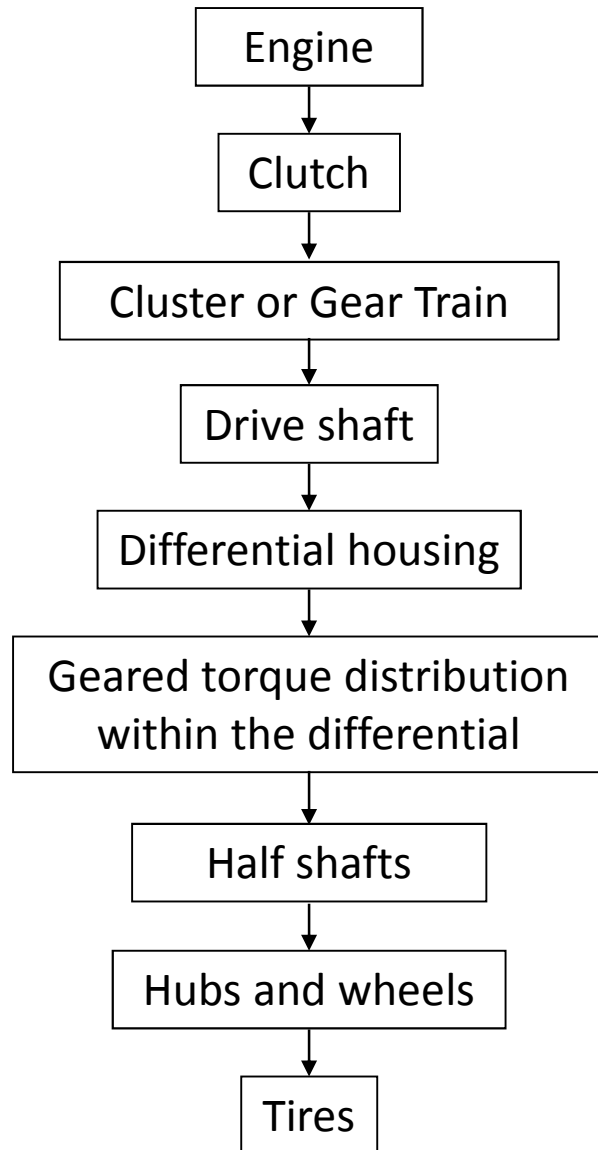
T = axle track width

R_{ave} = Radius of corner measured from the center of the axle



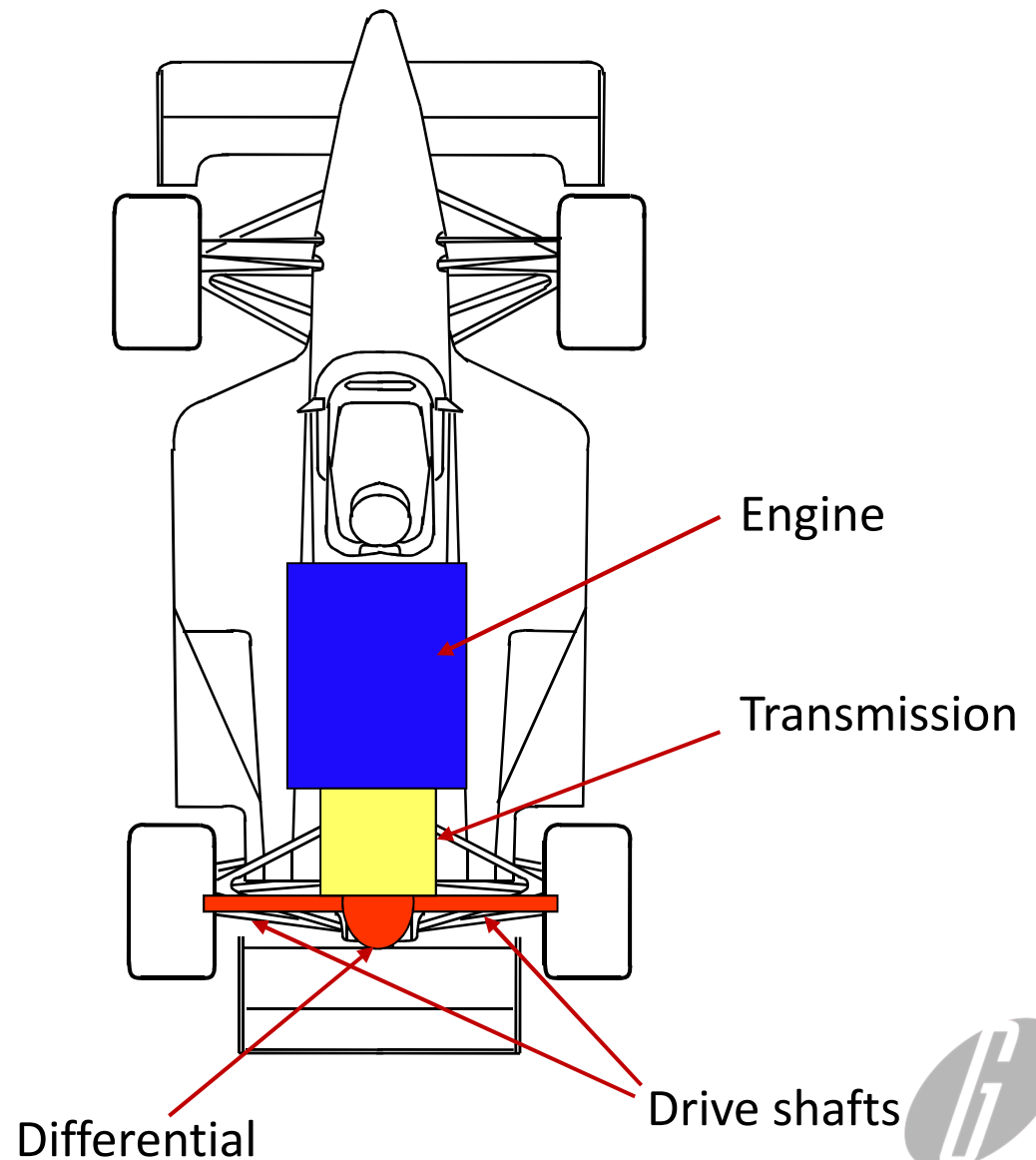
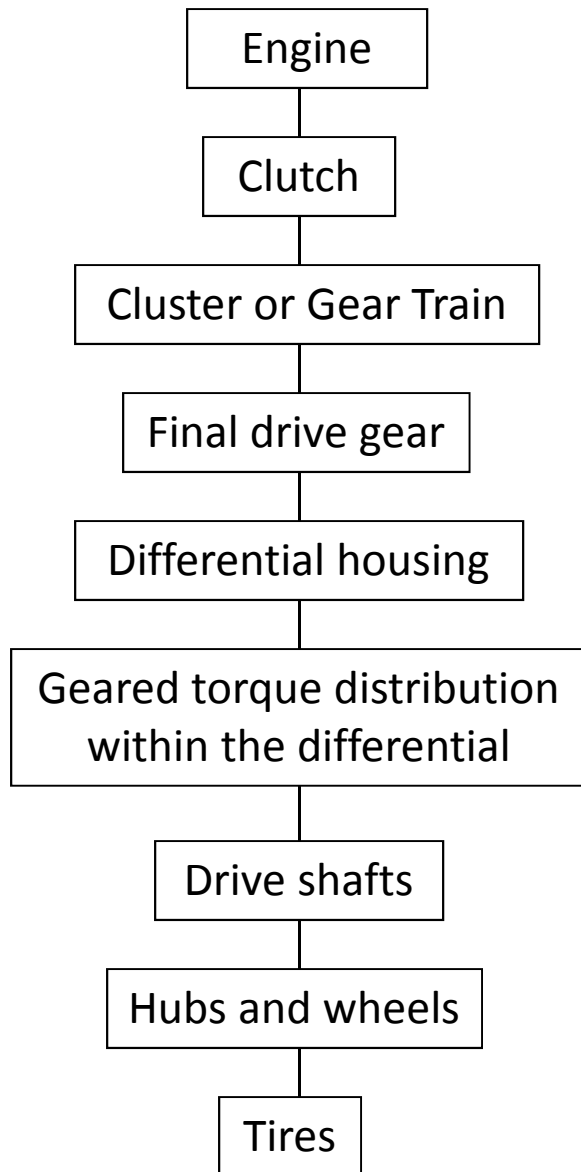
Introduction to Differentials

Simplified Torque Path for a Front Engine, Rear Wheel Drive Car



Introduction to Differentials

Simplified Torque Path for a Rear Engine, Rear Wheel Drive Car

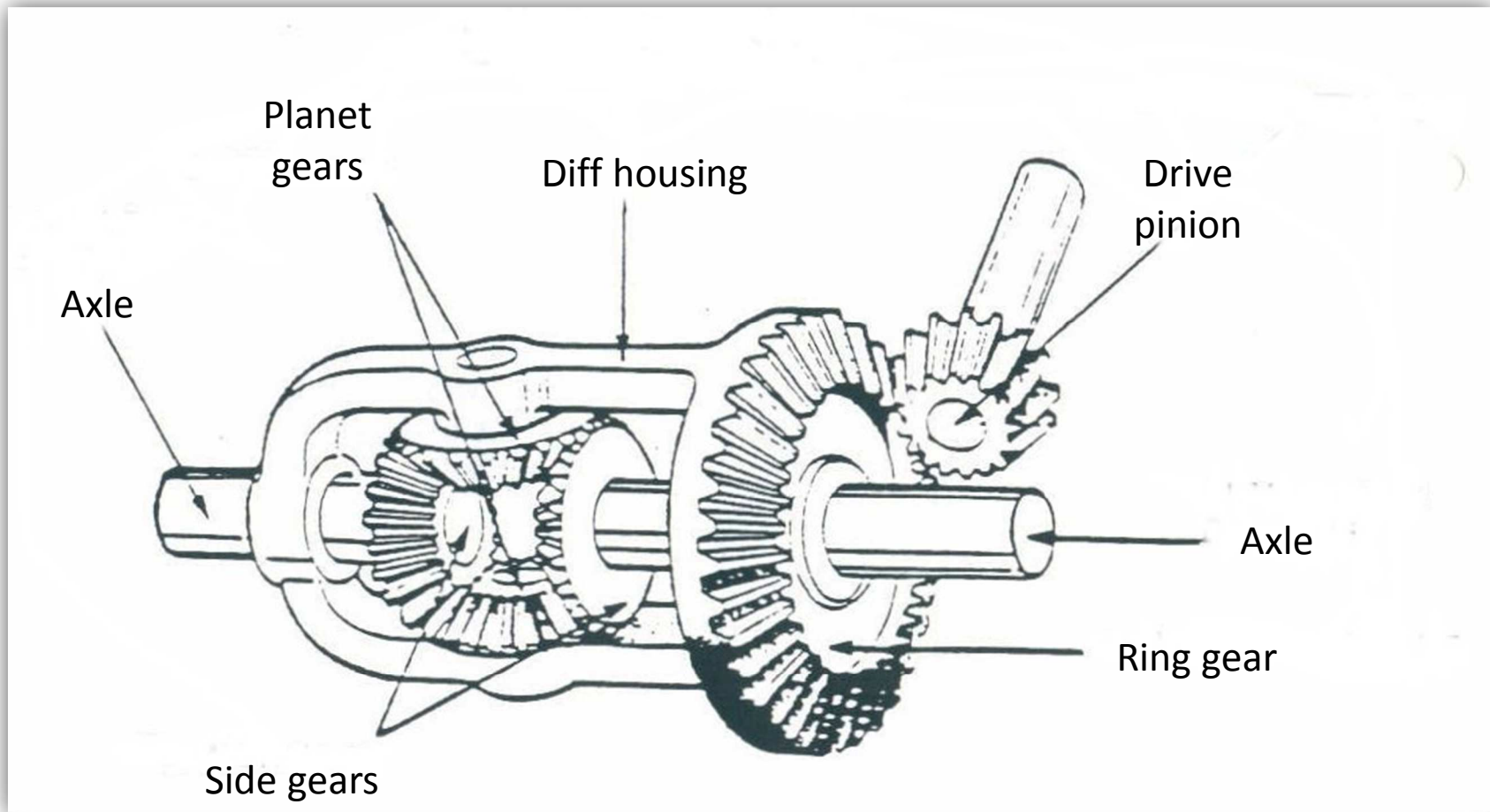


Types of Differential

- **Open Differential**
 - Completely free to differentiate at all times.
- **Spool (Locked) Differential**
 - Axles are “Locked” together. No differentiation.
- **Limited Slip Differential**
 - Axles actuate from open to locked depending on relative speed and/or torque.
 - There are several completely different types of limited slip differentials.



The Open Differential



The Open Differential

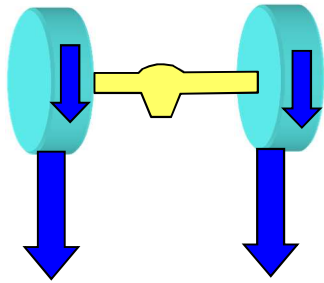
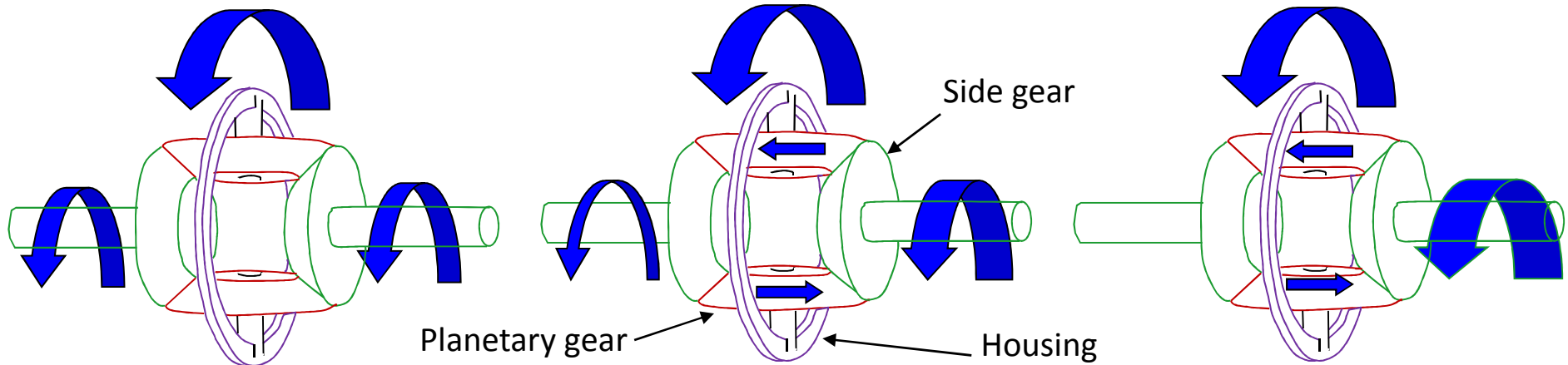
Introduction

- The open differential is the most basic of all differentials.
- Engine torque is transferred through the clutch and the gear train to the pinion gear (drive shaft then pinion gear in NASCAR).
- The pinion drives the ring gear which is fixed to the differential housing
- The planet gears are connected to the diff housing and also meshed with the side gears. Therefore, the housing transmits the torque to the planet gears, which then transfers to the side gears.
- The side gears are splined to the axles, which drive the wheels.

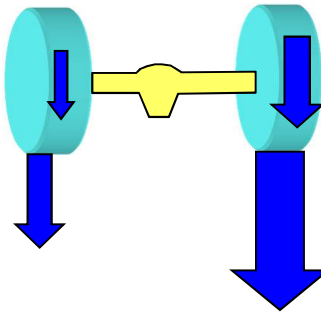


The Open Differential

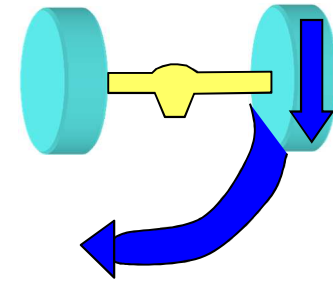
How it Works



Equal load on each axle. No relative motion within the differential gears and the rotating housing applies its energy through the bevel gears.



Unequal load. The differential gears rotate. Around a turn, the wheels are required to rotate at different speeds. The rotation of the differential gears speeds up the outer wheel and slows the inner, allowing equal torque to be applied to both wheels.



One wheel unable to spin and housing turned. The free wheel would spin in the same direction as the housing but at twice the speed.



The Open Differential

The Problem of the Open Differential

- The problem of an open diff is that both wheels are able to turn at different rate however the torque is split evenly.
- Therefore the longitudinal acceleration of the car will be limited by the wheel with the lowest traction. In an extreme case wheel spin will occur.
- This is a particular problem in race vehicles which see a large amount of lateral load transfer which unload the inside wheel and therefore decreases its traction.
- An open differential is beneficial for passenger vehicles because they corner at low speeds and seldom see a surface difference between the driving wheels.
- Therefore, the passenger vehicle will almost never experience wheel spin, which is the biggest downfall of the open diff.

Although the open diff is beneficial to most passenger cars it is NOT beneficial in racing. An open differential should only be used if it is the only differential the rules allow.



The Open Differential

Advantages and Disadvantages

Advantages:

- Light weight, inexpensive
- The open differential allows the driven wheels to move with different speeds.
- This is very beneficial for corner entry because it prevents the inside wheel from dragging, thereby reducing tire scrub.

Disadvantages:

- The open differential attempts to balance the torque applied to each axle.
- If one wheel is experiencing less load than the other the differential will increase the axle speed of the light wheel and reduce the speed of the heavy wheel until torque is equalized.
- Therefore, the open differential increases the likelihood of wheel spin and limits the maximum achievable longitudinal acceleration

One possible alternative, the complete opposite of an open differential is a Locked (Spool) differential.



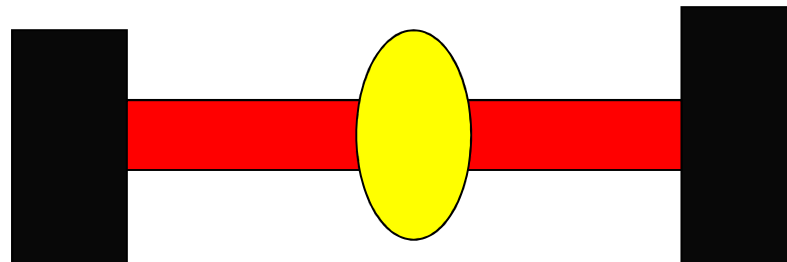
The Spool

Introduction

The spool, or locked differential directly connects the driving axles and forces them to rotate with the same velocity at all times. In this sense it is not really a differential.

The spool is often used in oval racing (Cart, IRL, Nascar,...)

Since the majority of oval tracks have corners with similar radii and degrees of banking a differential can be eliminated by using rear tires with different diameters. This is referred to as “stagger.” Stagger, if done properly, can eliminate the problems of inside tire drag in corners.



A spool (locked) differential will virtually eliminate wheel spin since the driving wheels are connected and must rotate with the same velocity. On an oval, the proper use of stagger can eliminate inside tire drag, which is the adverse effect of a spool.



The Spool Tire Stagger

Tire stagger = difference in the circumference of one tire in relation to another.

Stagger should be used if you are turning only one direction. (ie – ovals)

Stagger is used to lower the average rolling tire drag. This is accomplished by “splitting the difference” between the revolutions per minute across the driving axles while the vehicle is turning and moving in a straight line.

Interesting Note: with the presence of a load difference (inside vs. outside tire) the actual measured static tire circumference likely cannot be directly related to the true revolutions per minute that the tire experiences while at speed.

This explains why stagger numbers are often bigger than simple rolling distance calculations.



The Spool

Advantages and Disadvantages

Advantages of the Spool:

- Light weight, Inexpensive, Reliable
- The spool keeps the driving wheels locked together, which reduces wheel spin.

Disadvantages of the Spool:

- Must use stagger to compensate for corner entry understeer.
- Stagger causes straight line tire drag, which reduces both top speed and acceleration.
- If a failure occurs in the drive shaft or drive shaft joint while the car is under power it will turn in the direction away from the failure at unbelievable speed, and with a lot of violence. *No recovery!*
- Tire stagger does not always work. At road courses, for example, since stagger cannot be adjusted before every corner, there will be times when the inside wheel will drag, which will cause significant corner entry understeer.



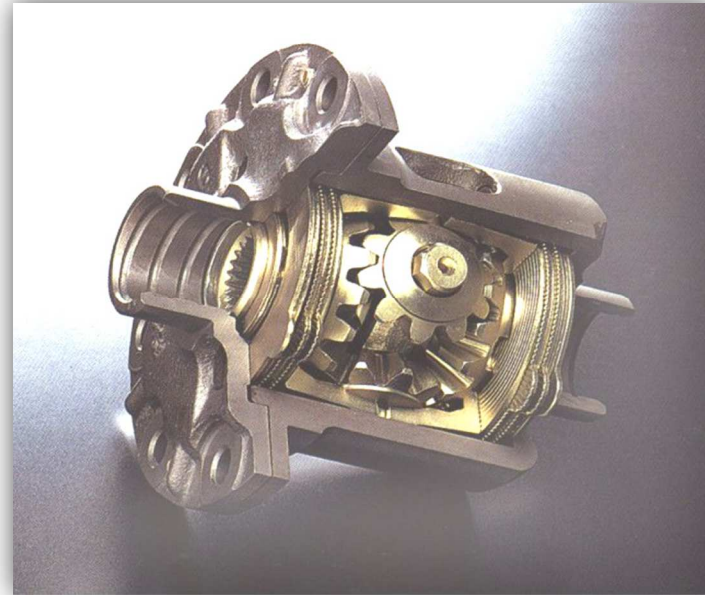
The Limited Slip Differential

Introduction

We have seen that the best scenario is to have an open differential on corner entry and a spool on corner exit. Essentially we need a differential capable of allowing different wheel speeds during corner entry, and limiting un-laden wheel spin under high torque loads to enable the car to corner without tire drag or wheel spin.

This can be achieved by using a:

Limited Slip Differential.



These differentials will limit wheel spin to varying degrees by delivering the majority of the torque to the wheel with the greater traction.



The Limited Slip Differential

Types of Limited Slip Differentials

- Detroit locker
- Salisbury differential
- Viscous differential
- Torsen T-1 differential
- Torsen T-2R differential
- Quaife differential
- Central differential
- Hydraulic and Active differential



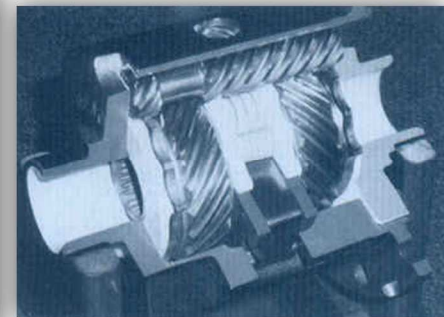
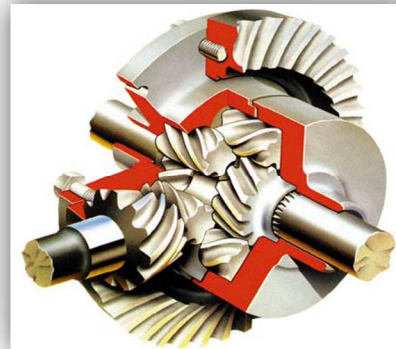
The Limited Slip Differential

Basic Governing Equation

The following equation applies to all limited slip differentials (with the exception of viscous coupling differentials):

$$T_r - T_l = \text{sgn}(\Omega_l - \Omega_r)T_c$$

It states that the difference in torque the differential can support is equal to T_c . The sign of the difference is determined by the difference in wheel speed left to right. Where the various designs differ is in the function that controls T_c



The Limited Slip Differential

Detroit Locker



The Limited Slip Differential

Detroit Locker



The Detroit locker moves from completely locked to completely open with no time in-between.

The transition from open to locked can be adjusted slightly by changing the preload inside the locker. However, this adjustment has a very small effect.

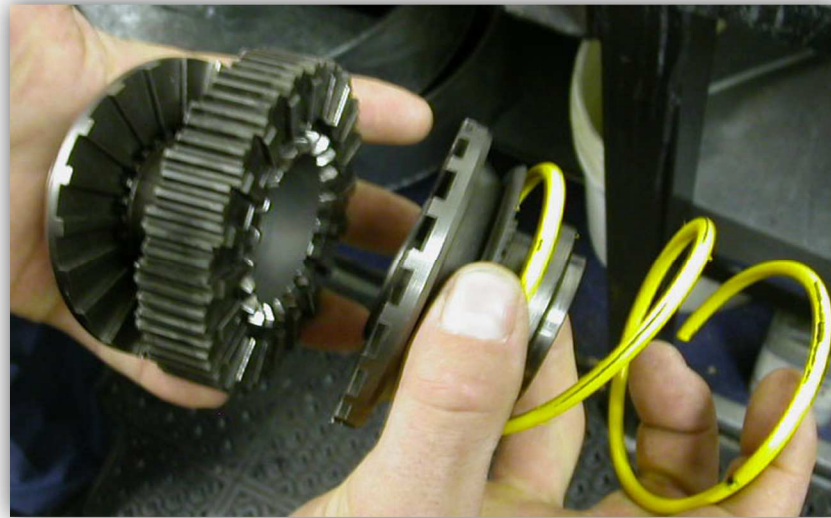
The Detroit locker is often referred to as an overrunning clutch type differential. Upon corner exit, when drive torque is applied, power is transmitted to the axle which is rotating slower, while the faster axle “overruns.” Once the rpm of the inside wheel matches that of the outside wheel the two immediately lock together causing the driving wheels to act as a spool.

Depending on the amount of power required at the wheels in the middle of a corner, the outside wheel may experience no power at all, which will allow it to generate maximum lateral force.



The Limited Slip Differential

Detroit Locker



Advantages

- The Detroit locker nearly eliminates inside tire drag during corner entry by functioning as an open differential. Then it changes to completely locked in an attempt to negate wheel spin while exiting a corner.
- While the locker is acting as an open differential in corners it effectively drives the inside (slower) wheel and disconnects from the outside (faster moving) wheel.
- It is a rugged positive locking differential. It is often used for off-road purposes since it can handle axles with 3000 to 70000 lb capacities.



The Limited Slip Differential

Detroit Locker

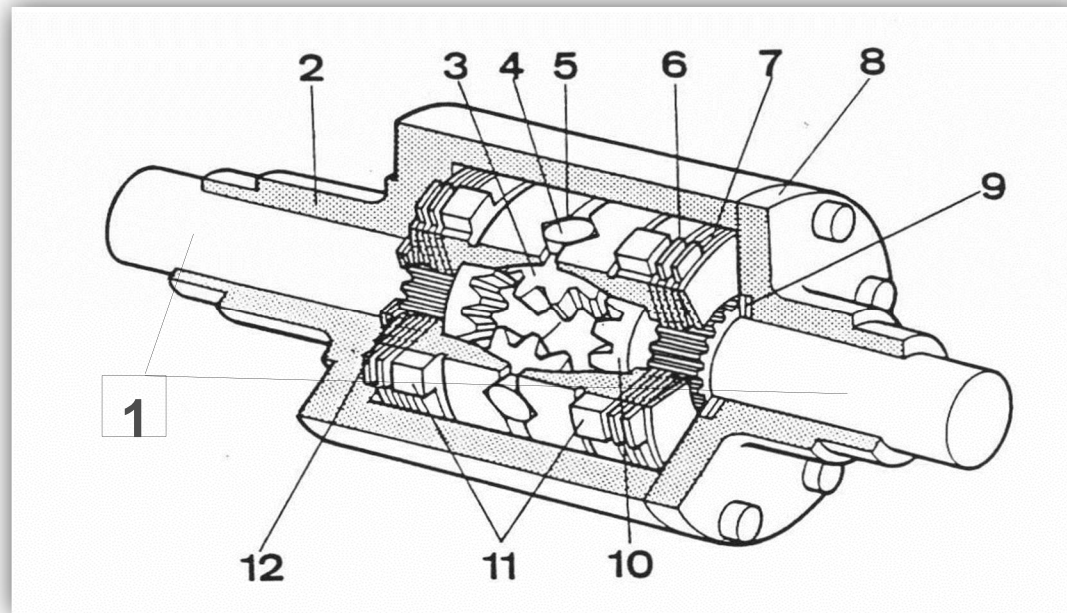
Disadvantages

- As we've seen, the Detroit locker gives desired results of an open diff on corner entry and a spool on corner exit. However, major problems reside in the transition from open to locked.
- A common complaint of the Detroit locker is that the driver can hear and feel "backlash" going from drive to coast and coast to drive. The transition from open to locked and vice versa happens very quickly with a Detroit causing sudden changes in handling characteristics.
- The Detroit locker also tends to have problems with mid-range throttle inputs (ie – anything between no throttle and full throttle), which is often the time spent in the center of a corner. The locking mechanism will continuously transition between locked and unlocked causing the car to switch between understeer and oversteer several times in one corner.



The Limited Slip Differential

Salisbury Differential



1: axles

2: differential housing

3: planet gear

4: cross-shaft

5: prismatic notch (ramp)

6: outside discs fixed in rotation with diff housing

7: inside discs fixed in rotation with side gears

8: diff case

9: stop washer

10: side gear

11: pressure ring

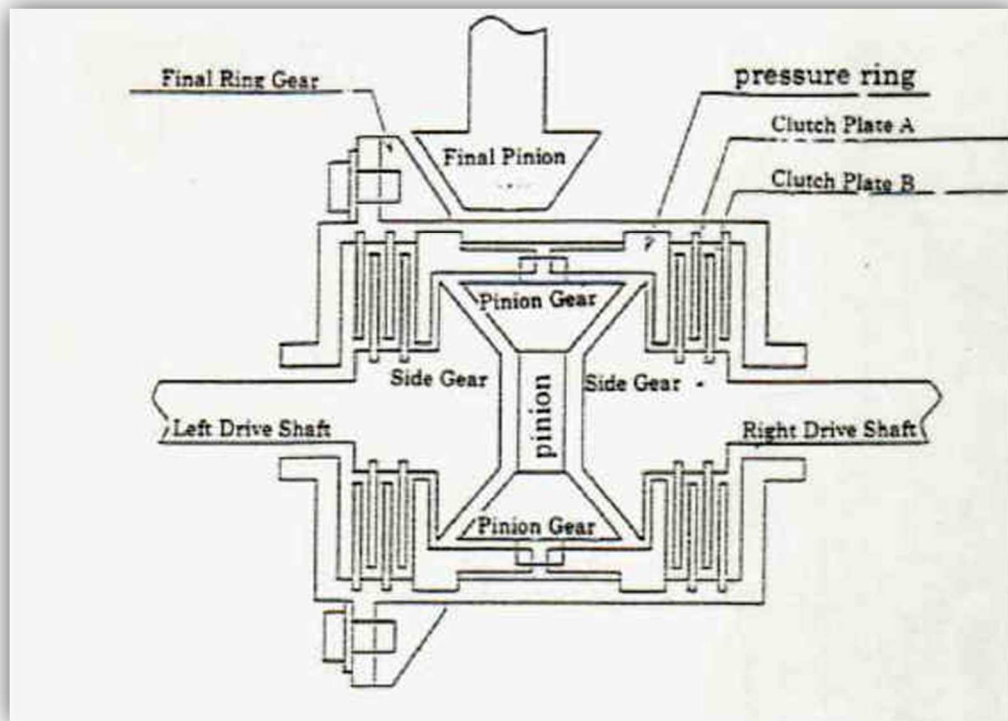
12: Belleville washer



The Limited Slip Differential

Salisbury Differential

The Salisbury differential is a limited slip differential whose characteristics are regulated by clutches, pressure rings, and ramp angles.



The clutches of the Salisbury differential lie between the pressure rings and the diff wall. Generally, each side of the differential has the same number of clutches, and they are configured in an alternating fashion where one is fixed in rotation with the diff housing, and the next is fixed to the side gear.

The pressure rings fit securely over the planetary/side gear cage, and rest against the pinion that connects the planetary gears via a prismatic notch, which is often referred to as a ramp.



The Limited Slip Differential

Salisbury Differential

Ramps: the prismatic notches in the pressure rings that envelope the pinion.

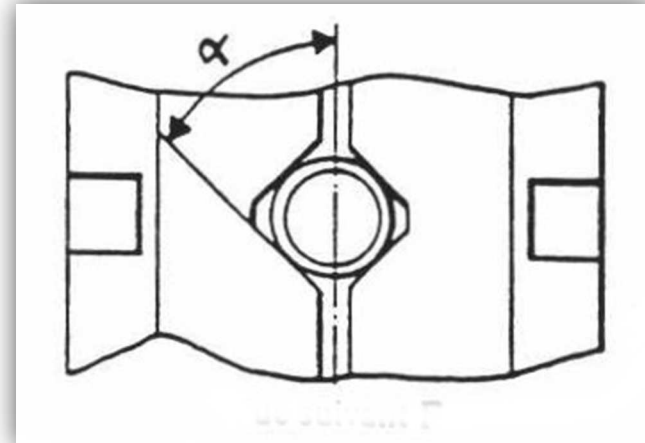
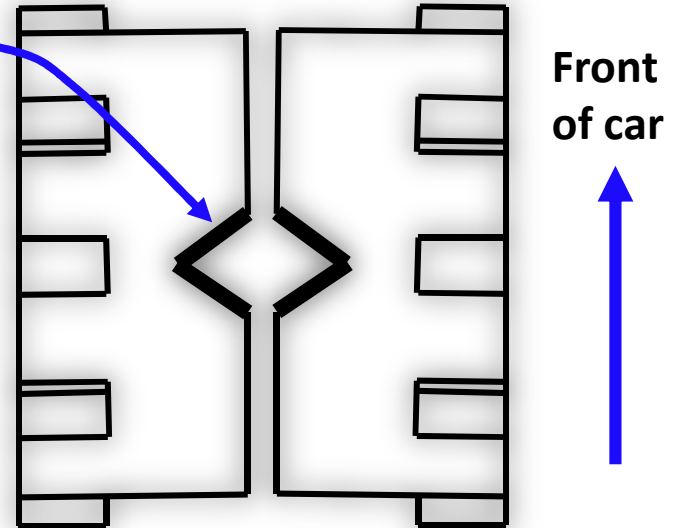
Ramps are a key factor in Salisbury type clutch differentials. There are three common types of ramps: 1-way, 1.5-way, and 2-way.

The type of ramp is dependent upon the ramp angle " α " which we measure from the longitudinal axis of the vehicle. The axis you measure from is not important, as long as you are consistent and understand that some people measure differently.

1-way ramp: one side of each pressure ring has a ramp angle = 90° , the other side is $< 90^\circ$

1.5-way ramp: both sides of a pressure ring have ramp angles $< 90^\circ$; however, angles are not equal.

2-way ramp: both sides of a pressure ring have ramp angles $< 90^\circ$, and they are equal.



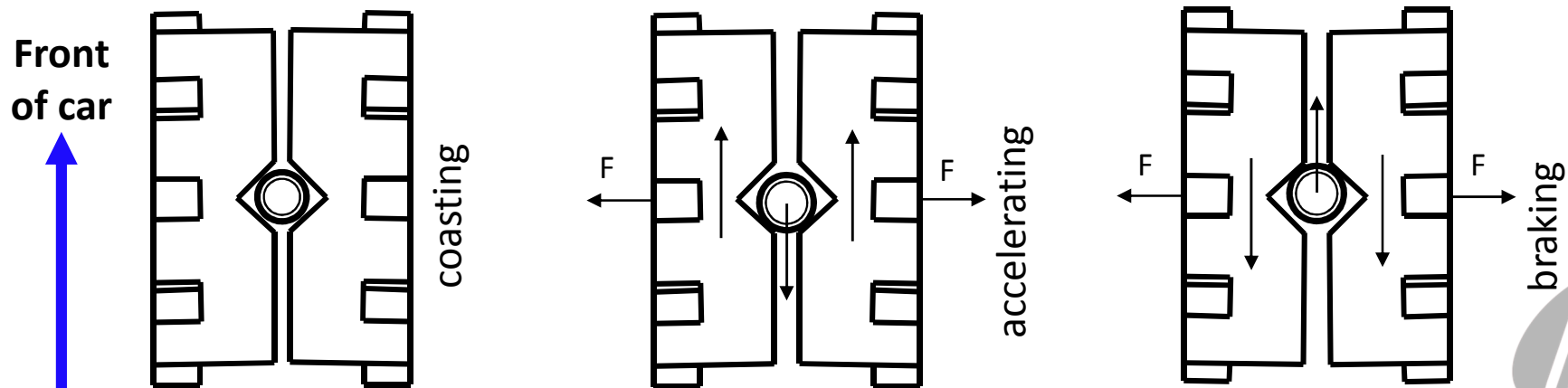
The Limited Slip Differential

Salisbury Differential

The amount of lock experienced by a Salisbury differential is dependent on the amount of friction in the clutch packs. Friction is proportional to the force applied to the surface.

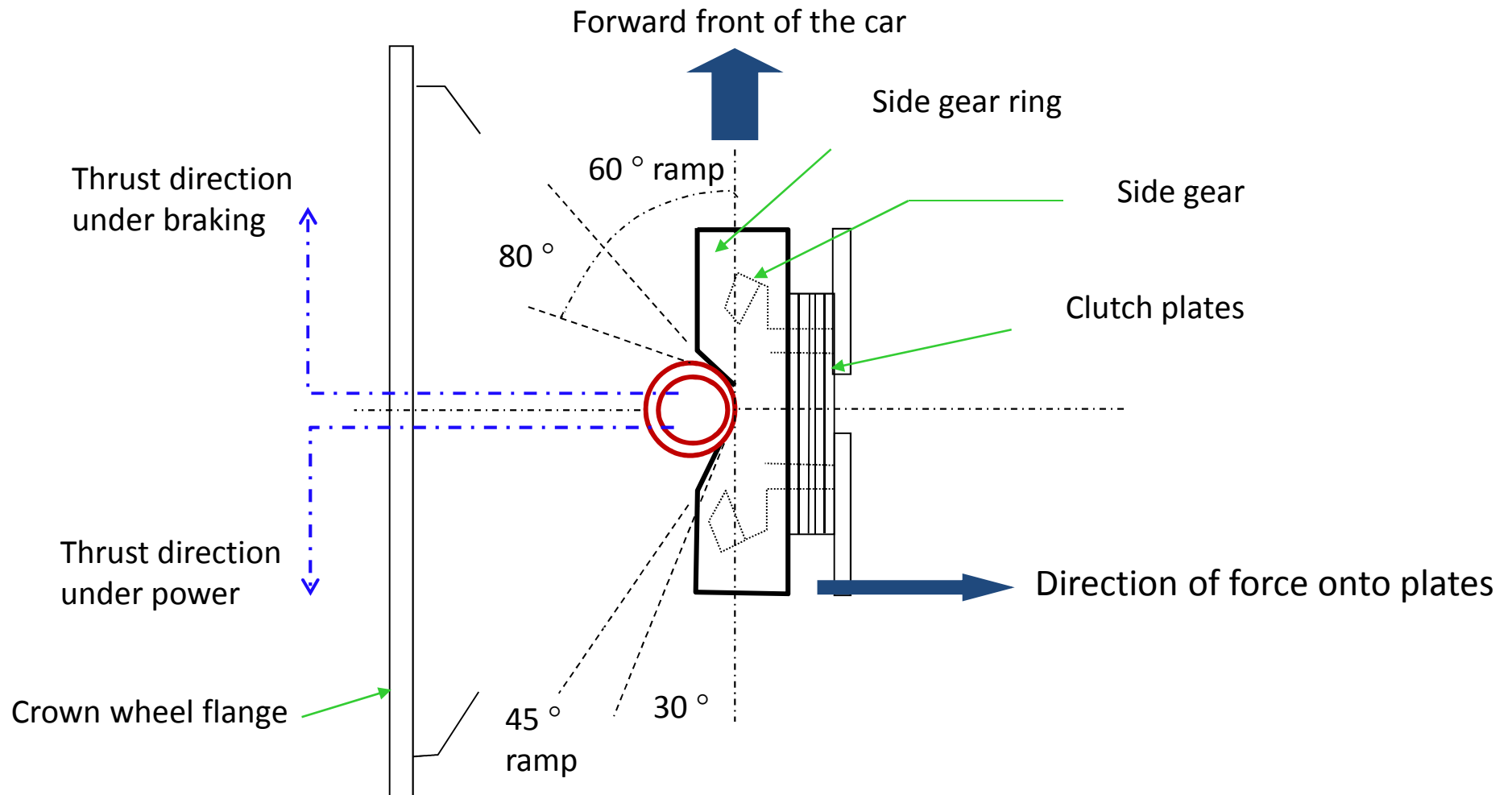
The existence of engine torque, regardless of wheel speed, creates a speed difference between the differential casing and the planetary/side gear cage. This forces the ramps to move the pinion, since the pressure rings are connected to the casing. Due to Newton's first and third laws of motion the pinion resists this motion with an equal and opposite force, which creates a spreading force (F) in the axial direction on the pressure rings. The magnitude of this spreading force is a function of the engine torque and the ramp angle.

The friction created in the clutch disks will try to lock the half-shaft to the diff housing. Since spreading forces are proportional to the torque, the tightening force of the discs will increase proportionally with increased engine torque.



The Limited Slip Differential

Salisbury Differential

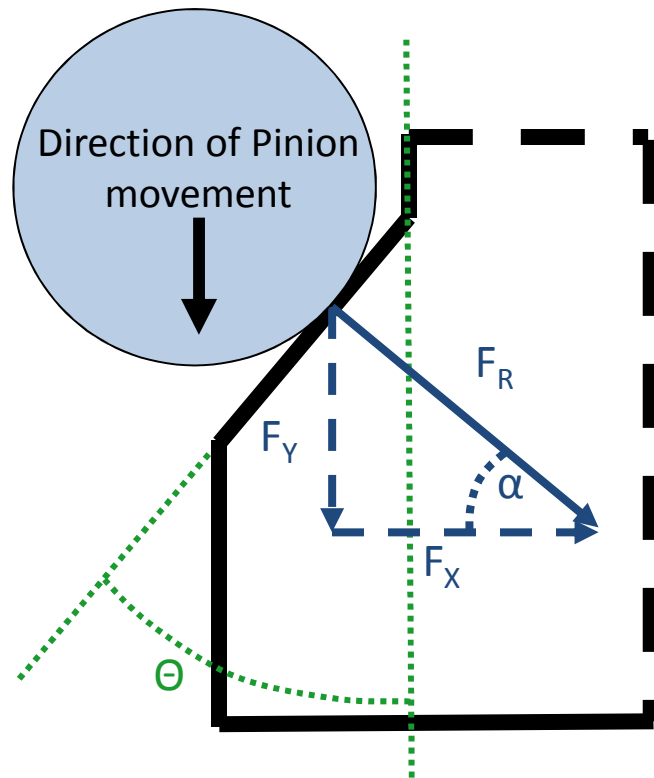


An easy method of tuning the clutch pack, (the percentage of lock), is by changing ramp angles and number of active clutches. The least dispersal of lateral load is transmitted by the 90° ramp. Generally a 30° ramp is the lowest angle used to maximize the lateral load.



The Limited Slip Differential

Salisbury Differential



How to Find the Spreading Force

F_Y = Force applied by pinion in Y (longitudinal) direction

F_X = Resultant force in X (lateral) direction (spreading force)

F_R = Resultant combined force applied to ramp by pinion

Θ = ramp angle

$\alpha = \Theta$ (by rule of triangles)

$$F_X = F_Y / \tan \alpha = F_Y / \tan \Theta$$

F_Y for each side of a pressure disc is found by dividing the engine torque at the differential by (2)*(effective ramp radius)).

$$F_Y = \frac{T @ Diff}{(2 * R_{eff})}$$



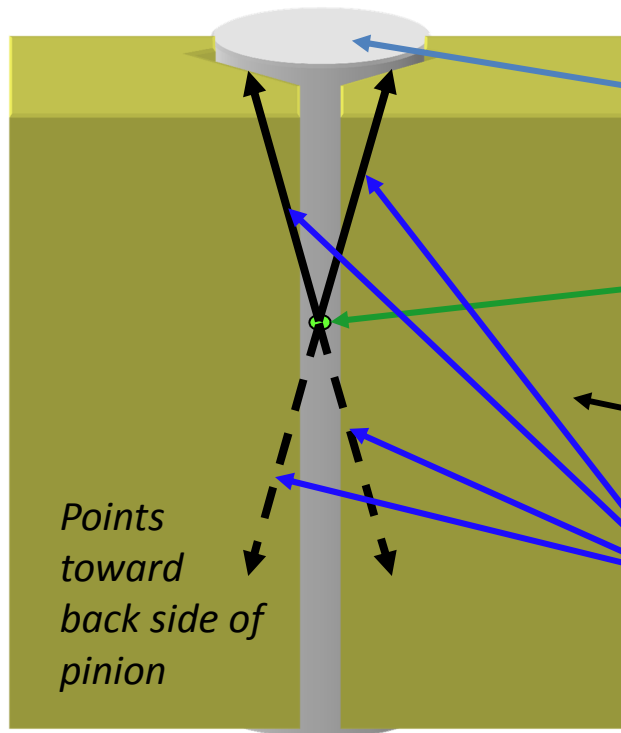
The Limited Slip Differential

Salisbury Differential

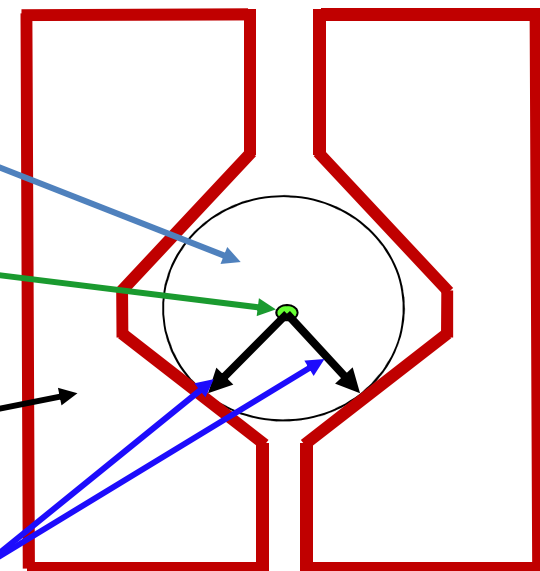
How to Find the Effective Ramp Radius

Example of Braking with 1.5-way Ramps

Side View



Top View



2-D views of effective radius

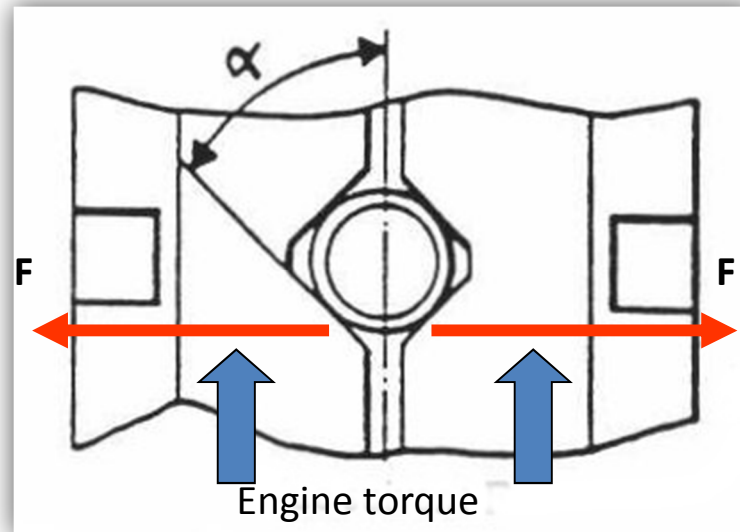
The effective ramp radius is the distance measured from the longitudinal and lateral center of the center pin (pinion) to the point where the pinion and the ramp touch. The radius will be slightly different when the car is accelerating as opposed to braking if the ramp angles are different.



The Limited Slip Differential

Salisbury Differential

Transmitted Torque During Clutch Slip



The transmitted torque during the clutch's slip is equal to:

n = number of clutch discs

μ = coefficient of friction

P = Pressure applied by spreading force = F_x/A_s (F_x = spreading force; A_s = friction surface area)

R_2 = outside radius of clutch's friction surface

R_1 = inside radius of clutch's friction surface

$$T_t = n * \mu * P * 2\pi \frac{(R_2^3 - R_1^3)}{3}$$



The Limited Slip Differential

Salisbury Differential



30° is generally the smallest angle used to transmit spreading load to the friction discs, while a 90° ramp is the largest.

We usually want minimum lock up under braking, therefore, an 80° ramp is a common choice for the coast side, and a 45° ramp is common on the power side.

Generally, a formula car, like a F3 runs with a 60/80 ramps. (power/coast)

The 90° ramps have a tendency to chip and are therefore usually replaced by 80-85° ramps. By experience, in fast corners with a lot of downforce, the 30/60 ramps can work well. There are very few people (except in UK) using 45/45 ramps (too much corner entry and mid corner understeer, especially on slow corners and bumpy tracks).

80/80 ramps with little or no preload are a good starting point for an unknown car and track. From there you can gradually decrease the ramp angle on the power side.

By adjusting ramp angles, preload and number of active clutches, the differential bias ratio can evolve from an open diff to nearly a spool.

Remember that the more lock you have, the less differentiation and the more trailing on-throttle understeer you will experience.



The Limited Slip Differential

Salisbury Differential

Effect of preload

It is often said that the amount preload on the clutch plates is the determining factor for the amount of lock up. This can be true, but it is generally better if it is not, since it would indicate very little engine and braking torque, or an excessive magnitude of preload. The spreading force of the pressure rings are generally an order of magnitude greater than any conceivable preload.

However we always use minimum preload to ensure that:

- the differential's transitional parameters from power off to power on remain consistent.
- the differential will limit slip when a wheel is totally unloaded.

One of the biggest problems of a clutch pack is when a driving tire is completely unloaded, the differential runs on preload only, and the ramps have no effect.

The preload is setup by shimming the friction discs. It is more accurate to measure preload with a depth micrometer or with a feeler gauge than with a torque wrench.

Several people are using little or even no preload since the galling of the friction plates is greatly reduced and the differential is more consistent without it.



The Limited Slip Differential

Salisbury Differential

The bias ratio or amount of lock up is dependant of 5 major variables:

1. Driving and braking torque
2. The number of active clutches and the order of stacking used.

More active clutches provide more lock.

3. Material used in the friction discs

4. Angle of ramps

This angle will determine how much of the driving torque is directed axially against the friction discs.

5. The preload



The Limited Slip Differential

Salisbury Differential



It is possible to calculate, and graph the percentage of differential lock with math channels in your data acquisition software. You can simply formulate a ratio between the left and right wheel speeds (of the powered wheels). This ratio can simulate the locking percentage of the driving wheels.

On a road course, the ratio will fluctuate from +1 to -1, depending on the direction of the corner.

$$\% \text{ DiffLocking} = \frac{\text{LeftWheelSpeed}}{\text{RightWheelSpeed}} * 100$$

You can compare this ratio to the wheel speed ratio with the engine torque, RPM, throttle position, lateral G's, brake pressure, and track map to determine the effect of preload and ramps.



The Limited Slip Differential

Salisbury Differential

The major advantages of the clutch pack or plate limited slip differential are:

- Relatively easy to service and adjust
- Variable slip between on-power, braking and coasting
- Smooth transition from open to nearly locked compared to lockers
- Several different adjustments can be made (gear friction, ramp angles, preload, clutch material, number of clutches, size of clutches, Belleville washers, etc. . .)
- Widely used for racing purposes

The Salisbury clutch-type limited slip differential's major advantages lie in the smoother transition period than previous differentials, and the amount of adjustments that can be made with little difficulty.



The Limited Slip Differential

Salisbury Differential

The major disadvantages of the clutch pack or plate limited slip differential are:

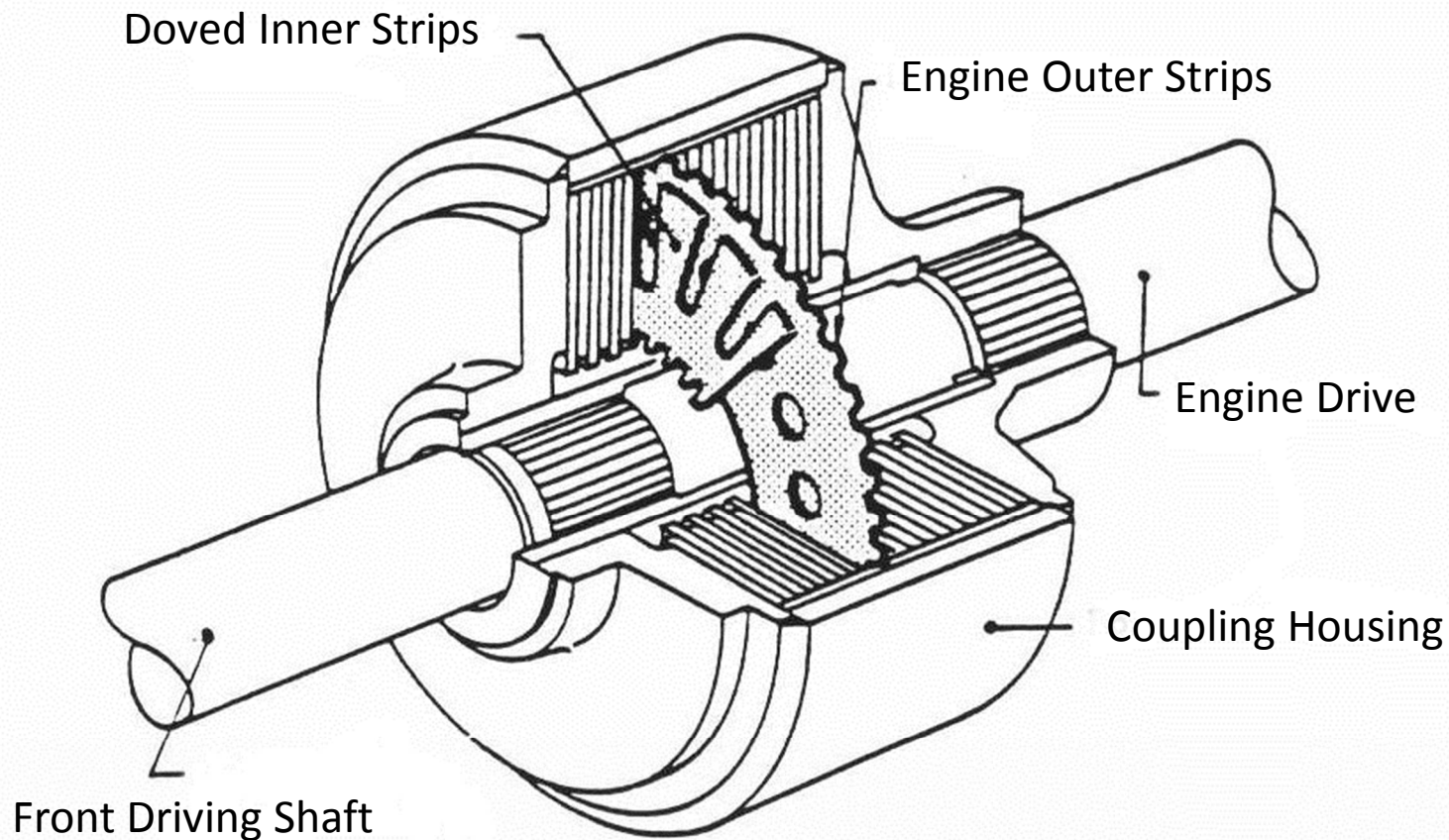
- Lack of consistency (basically in the transitional period from overrun to power on)
- Percentage of lock up and time necessary to achieve it are functions of ramp angles, friction plate material and preload
- Preload is function of temperature and plate wear
- Effective ramp angles which produce spreading force is a variable function of the point load against ramps, bevel gear wear, etc...

Wear of any component is liable to change the operating characteristics of the differential during the race or practice period. When performance changes during a race or practice period one is never quite sure whether one is chasing chassis, tires or differential.



The Limited Slip Differential

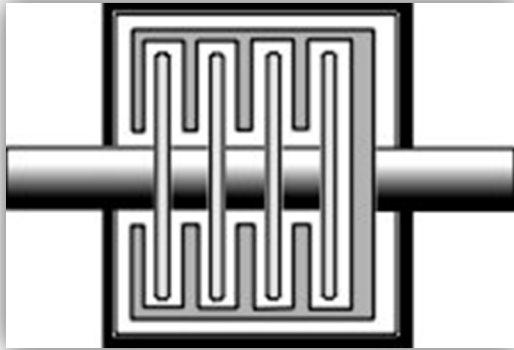
Salisbury Differential



The Limited Slip Differential

Viscous Differential

The viscous was invented in 1945 by Harry Ferguson.



The viscous coupler functions as a differential by positioning many circular plates very close to each other, and filling the space between plates with a high viscosity fluid.

The output shaft and the differential case are splined to alternating perforated discs that have an arrangement of slots and holes. The unit is filled with silicon-based high viscosity fluid, which resists shearing. Crosspiece rings keep the necessary distance between discs (about 0.15 mm or 0.006).

When tire slip occurs in one of the axles, the viscosity of the liquid resists shearing between the discs (a difference in speed). As a result, torque is transferred from the faster drive shaft through the liquid to the slower drive shaft. The greater the speed difference, the larger the torque transfer, which limits the amount of slip between drive shafts.



The Limited Slip Differential

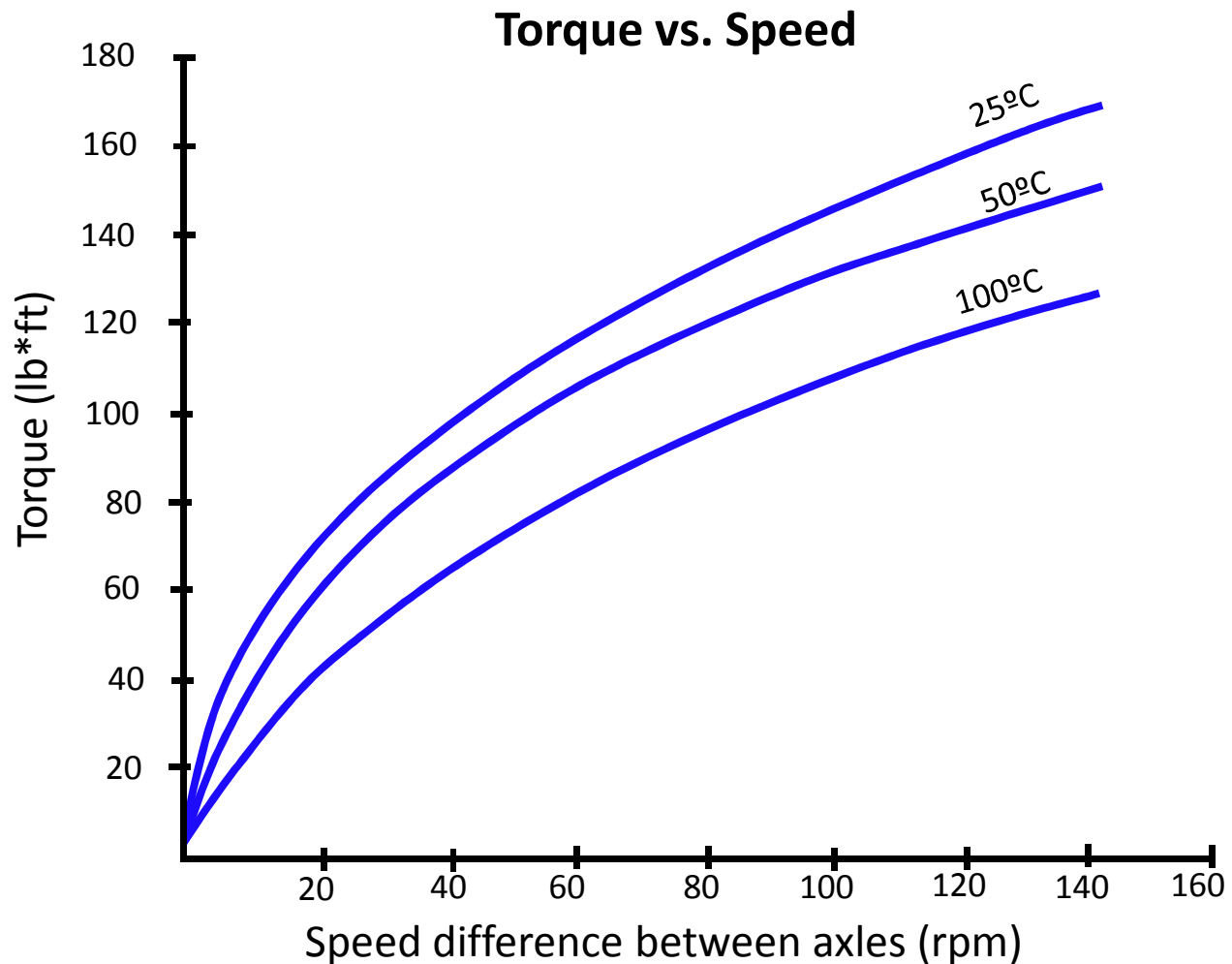
Viscous Differential

- The Viscous Differential is often used as a center differential in four wheel drive cars. However, it is also setup as the front and/or rear differential in several other cases.
- The fluid used inside the diff is a silicon fluid that can be specified in different viscosities. The diff must be assembled in a clean environment, and special care should be taken to ensure that the proper amount of fluid is added.
- The viscous differential, offers minimal resistance between axles at small speed differences. This accommodates for maneuvering the car and for tire diameter differences caused by load, inflation, or wear differences.
- As the speed difference between the axles increases, the amount of lock in the differential increases, which provides better traction in acceleration and braking.
- A benefit of the viscous differential is the smooth transition from open to nearly locked. Since there is no shock loading on the axles, drivers do not even notice the transition.
- The diff is sensitive to torque, temperature, and pressure, fluid viscosity, disc size, separation, holes, etc. However, the torque, temp, and pressure variations are minimal, and quite stable during normal operating conditions as seen in the following graph.



The Limited Slip Differential

Viscous Differential



****Graph representative of cars that produce large amounts of torque.***



The Limited Slip Differential

Viscous Differential

The Special Case: **Hump mode** (*outside of normal operating conditions*)

Like any differential, the viscous generates a large amount of heat when there is a prolonged period of speed difference between the drive wheels.

Excessive heat causes thermal expansion of the fluid, which compresses the residual air, and so, pushes the metal plates together.

The resulting metal-to-metal contact transfers more torque than the fluid does in the viscous mode, which results in improved traction under such extreme conditions.

This is called the “*hump mode*.”

The time of slip until the onset of the hump mode can be controlled in manufacture by varying the volume of the viscous fluid to air within the unit, the shape and the size of slots and holes, etc. . .

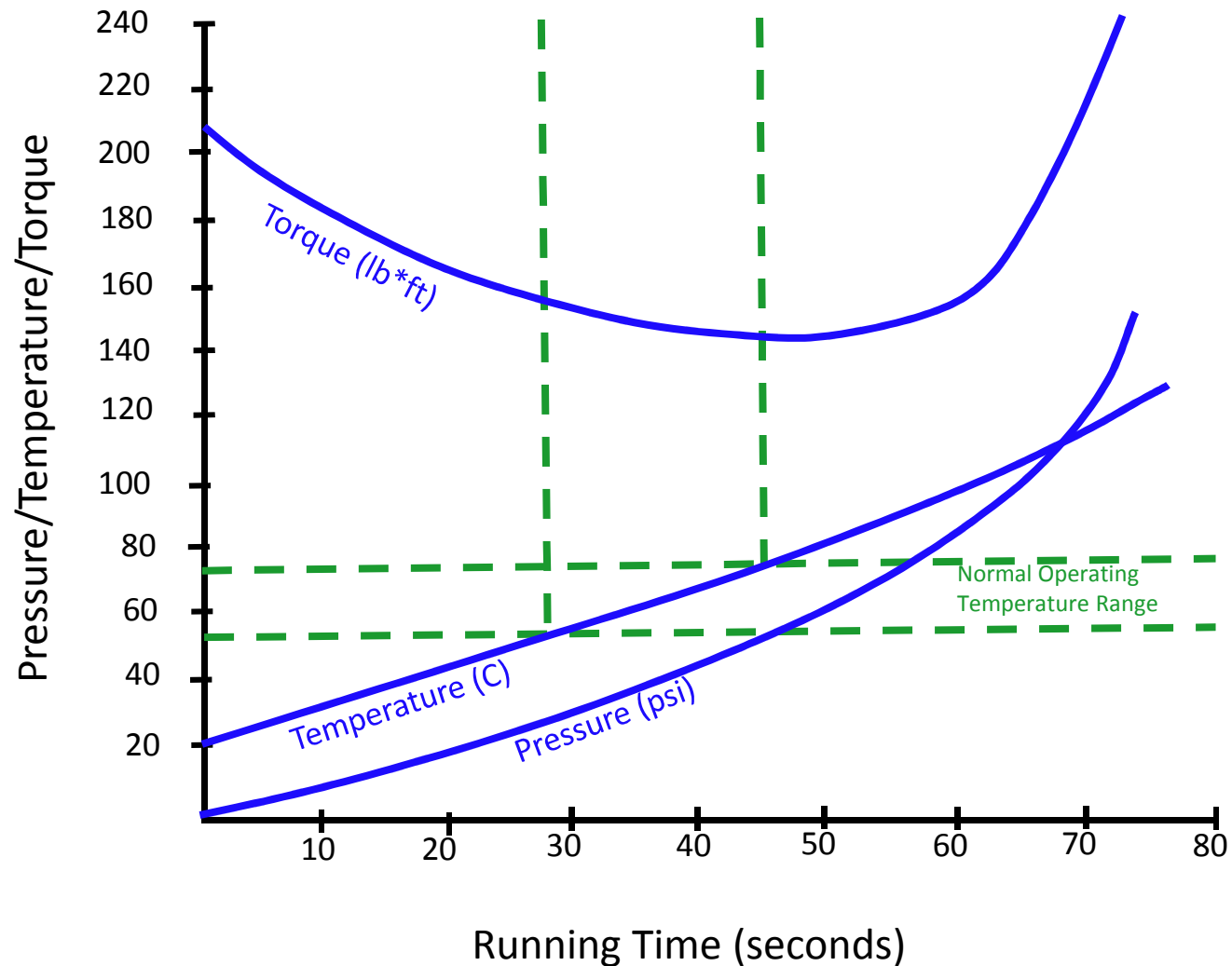
As soon as the unit reaches its “hump mode” the pressure will rapidly decompose, and the unit will almost immediately revert back to its normal, non “hump” mode.



The Limited Slip Differential

Viscous Differential

Torque vs. Temperature (Hump Curve)

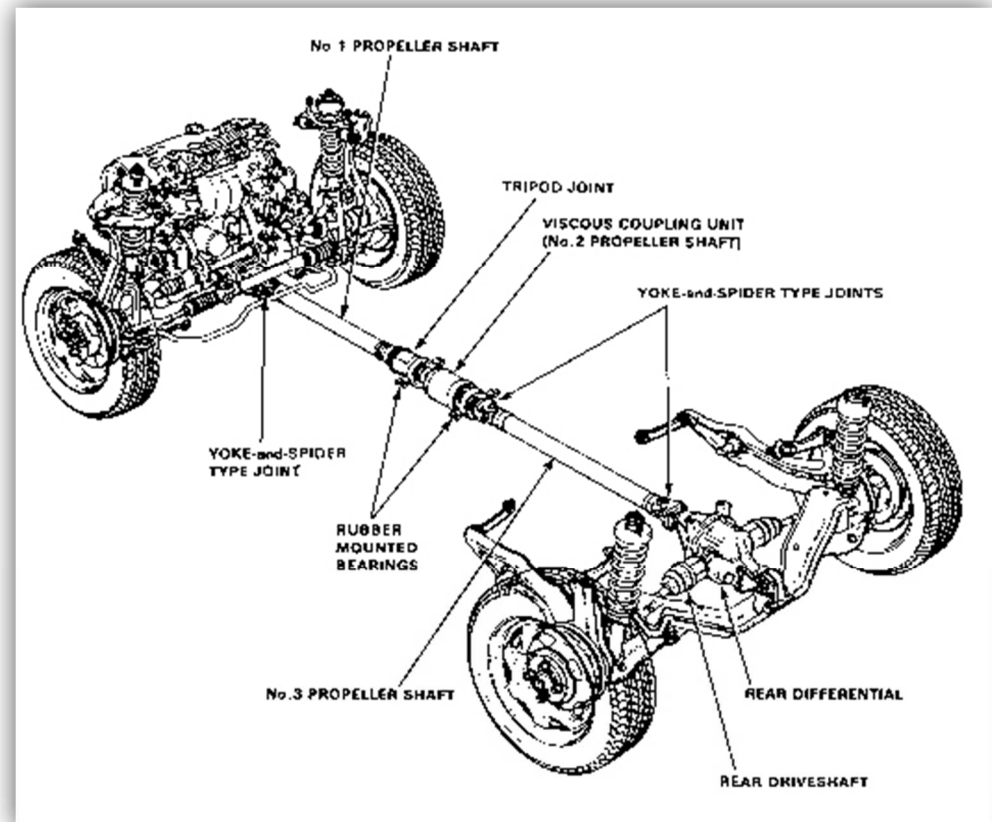


The Limited Slip Differential

Viscous Differential

The amount of lockup and the characteristic shape of the torque-biasing curve can be varied by:

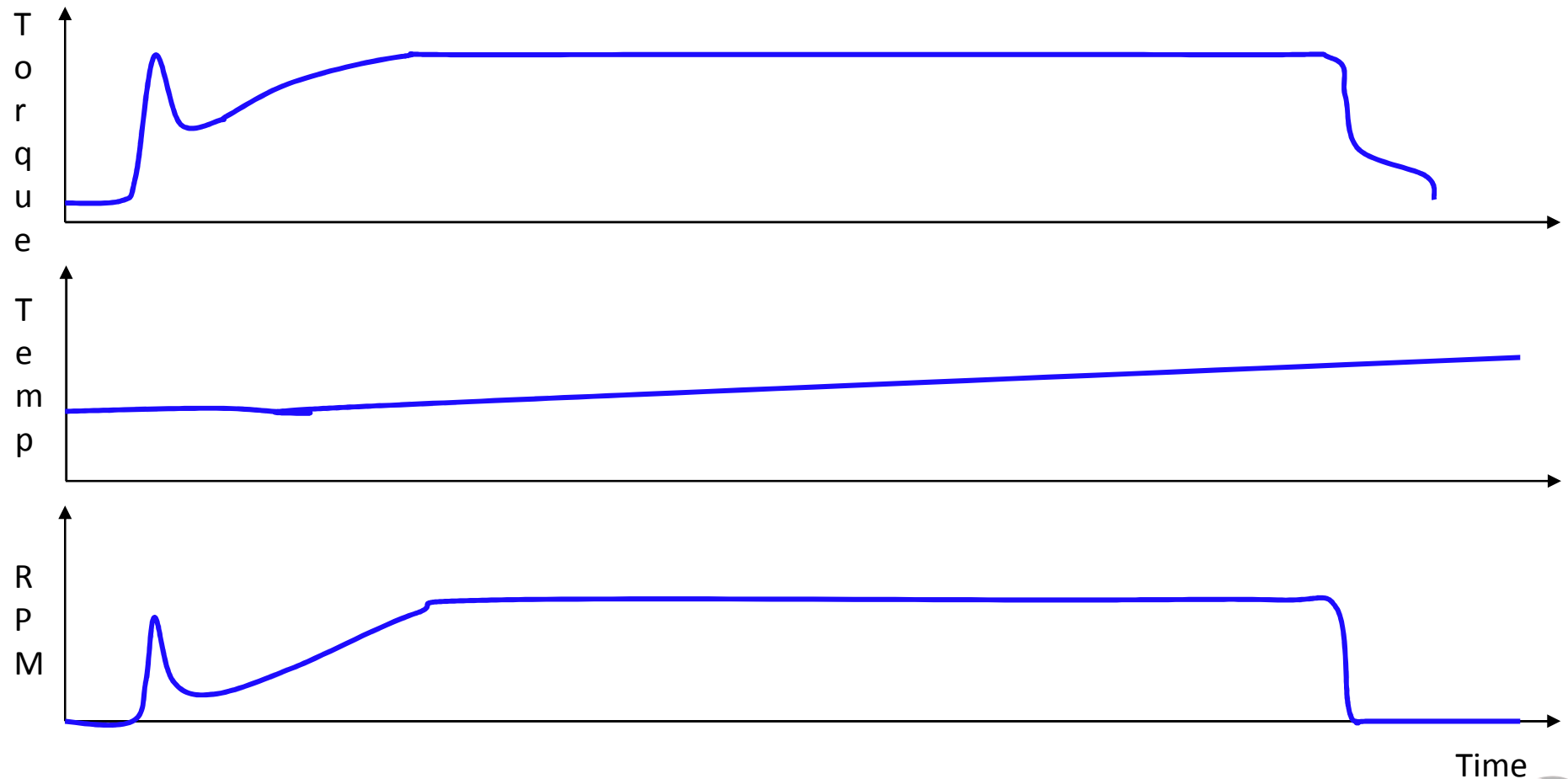
- Number of plates
- Number of configuration of slots and holes
- Amount of fluid
- Alteration of the fluid's characteristics



The Limited Slip Differential

Viscous Differential

Typical Calibration Curve for a Viscous Differential



Within the designed operating range the viscous coupling is very stable. Temperature, however, plays an important part in a condition termed the "hump mode."



The Limited Slip Differential

Viscous Differential

Advantages:

- Compact, Cheap, Easily adjustable.
- Consistent transition from open to nearly locked.
- Progressive locking depending on speed difference of axles.
- Accommodates small speed differences (ie- different tire diameters, etc).
- If hump-mode is avoided component wear will be minimal.

Disadvantages:

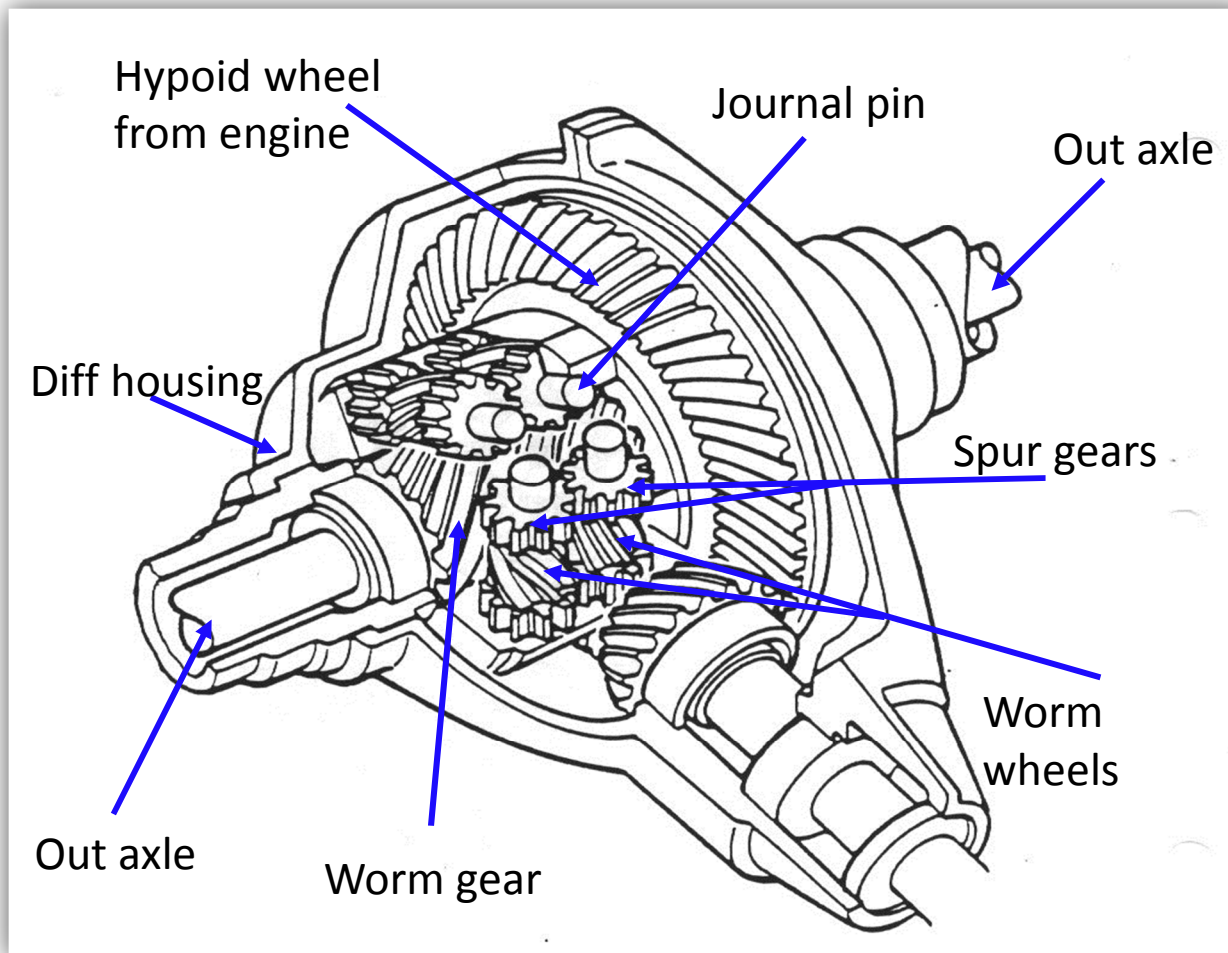
- Changes characteristics with temperature variation.
- Must be assembled in a clean room environment.
- Metal to metal contact in hump-mode permanently locks diff.



The Limited Slip Differential

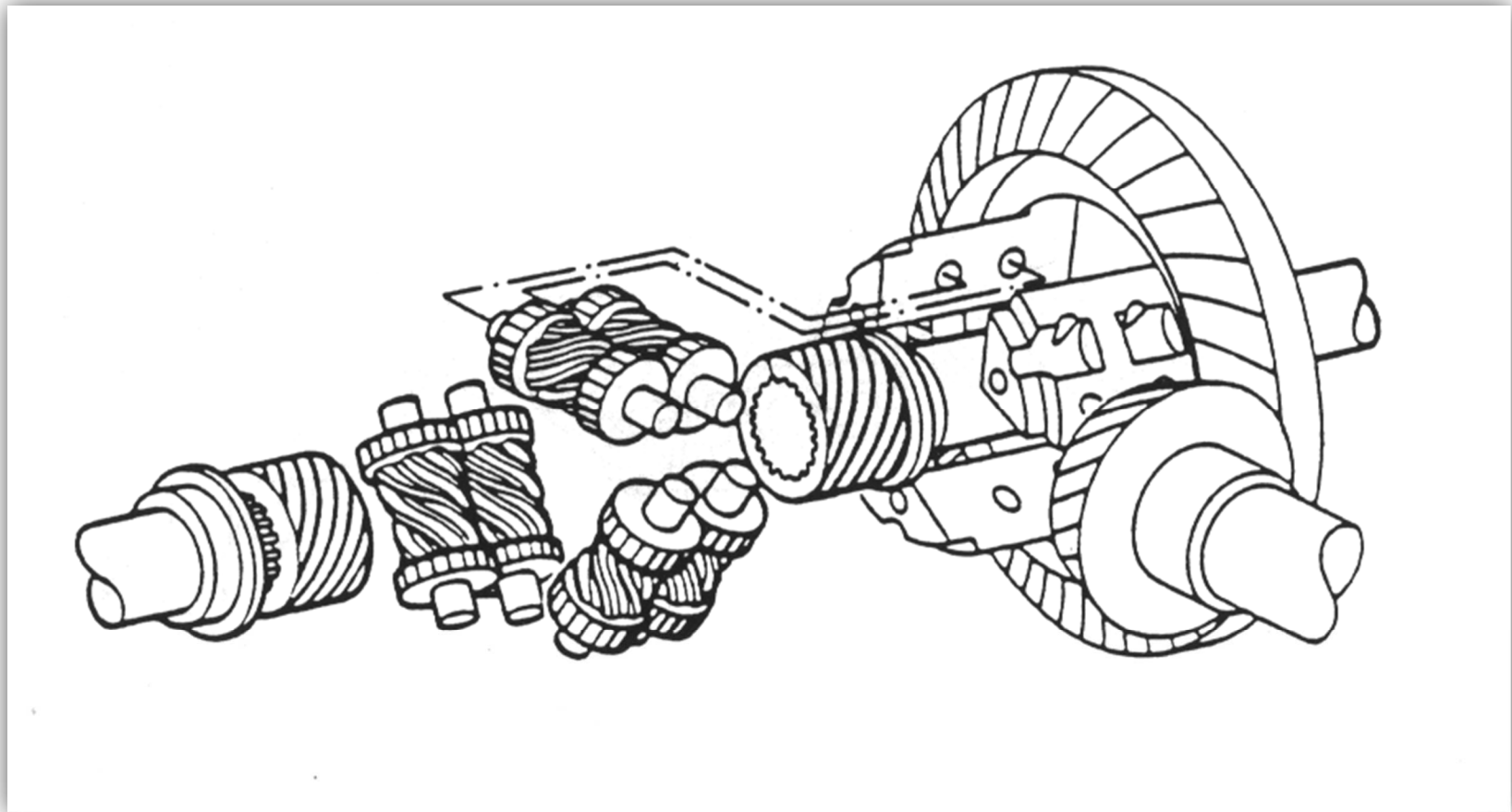
TORSEN Differential

The Torsen (TORque SENsing) differential was invented in 1958 by American Vern Gleasman



The Limited Slip Differential

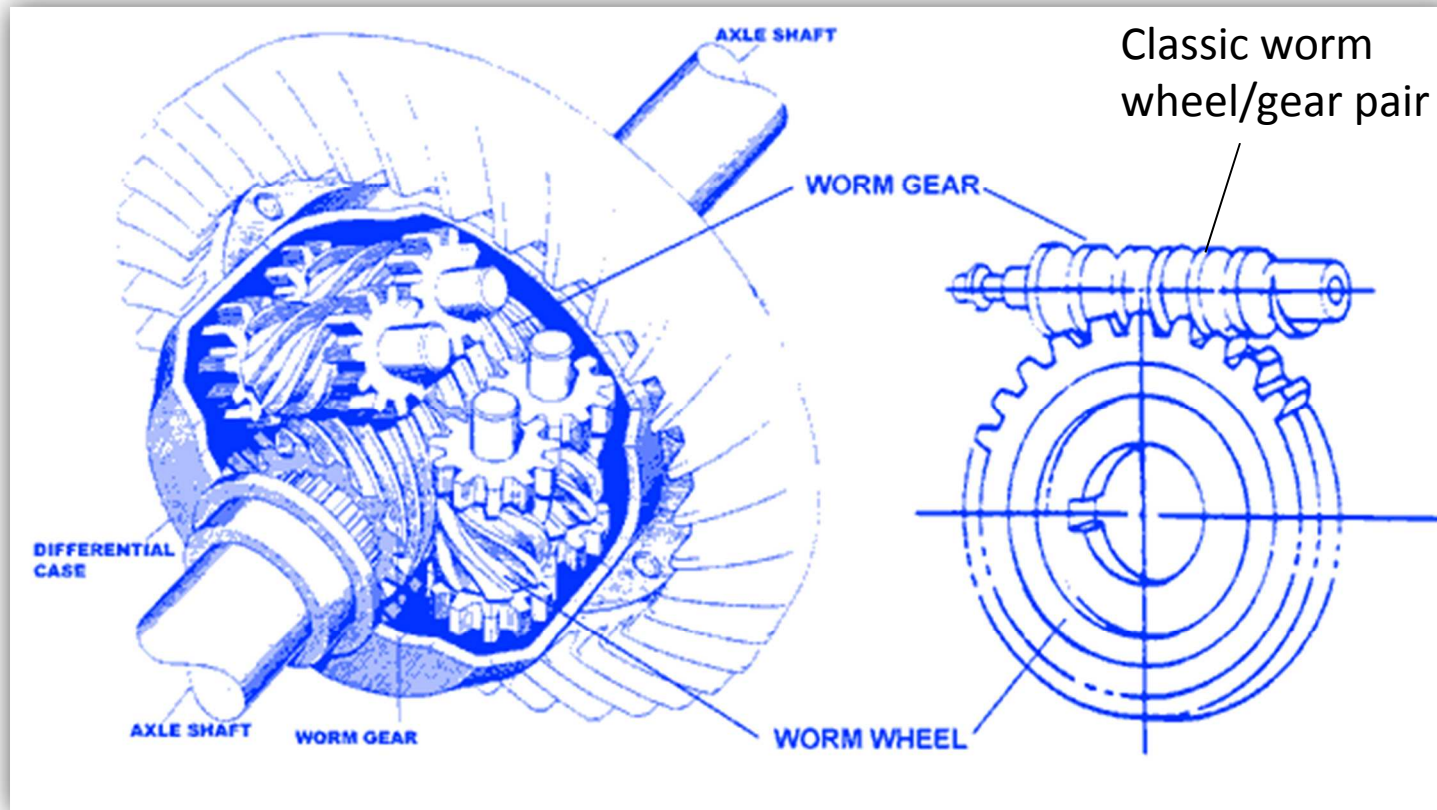
TORSEN Differential



The Limited Slip Differential

TORSEN T-1 Differential

The gear configuration of the Torsen was completely revolutionary upon its inception. It functions based on the properties of a worm gear-worm wheel pair. A worm gear is capable of turning a worm wheel; however, the reverse cannot occur. Gleason was the first person to create a worm wheel that is smaller than the worm gear that turns it.



The Limited Slip Differential

TORSEN T-1 Differential

How it Works:

The worm wheels in a Torsen T-1 are connected to the diff housing via journal pins. Each Torsen T-1 has two or three pairs of worm wheels, depending on the size of the differential. The two wheels making up a pair are positioned side-by-side, connected to each other with spur gears at a 1:1 ratio. They are also splined to the worm gears (side gears in a LS) at a high-helix angle. The left worm gear is splined to the corresponding left worm wheel, and the right gear with the right wheel. Since the worm wheel pairs are connected via a 1:1 ratio, they effectively connect the worm gears with the same ratio.

When torque is applied to the differential, it is transferred from the ring and pinion to the diff housing, causing it to rotate. The rotation of the housing forces the worm wheels to follow suit since they are attached. The inability of the worm wheels to turn the worm gears causes them to “lock” together, which rotates the axles since the worm gears are splined to the axles.

The Torsen makes the differential action and torque distribution of the differential independent of one another, which accommodates both differentiation and proper torque allocation in all situations without sacrificing one for the other.



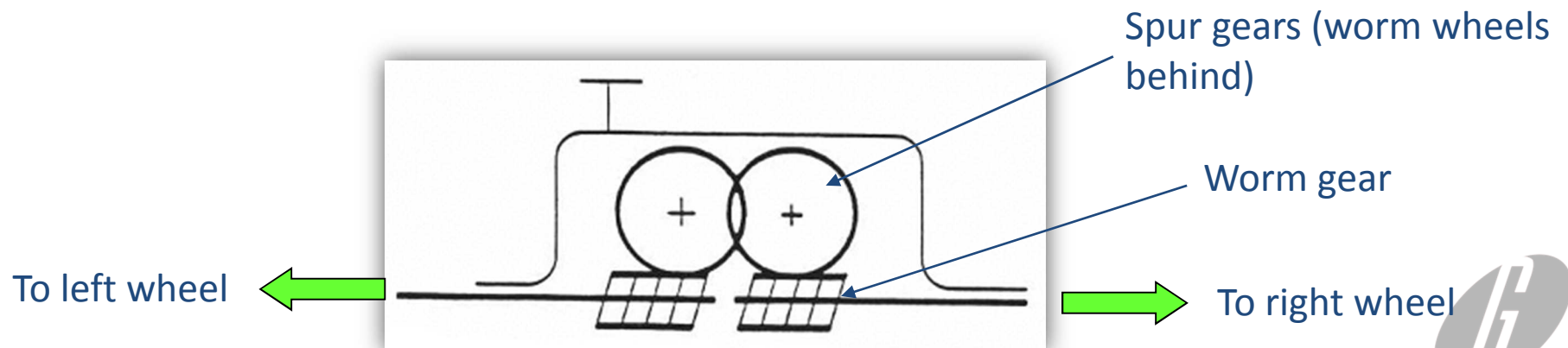
The Limited Slip Differential

TORSEN T-1 Differential

The Differentiation while Cornering:

Since the outside wheel must travel a larger distance through a corner it must increase its RPM relative to the inside wheel. With a center-mounted diff this can easily be achieved by increasing the outside wheel by the same RPM that the inside wheel is decreasing.

A Torsen functions based on this 1:1 ratio, and should therefore be mounted in the center between the driving wheels. In a left-hand corner the right wheel will increase its RPMs relative to the diff, which causes the right worm gear to turn the right worm wheels. Those worm wheels effectively turn their corresponding left worm wheels at the same rate in the opposite direction, which reduces the RPMs of the left worm gear by the same amount the right has increased.



The Limited Slip Differential

TORSEN T-1 Differential

Traction Loss by One Wheel:

When one wheel on the driven axle loses traction compared to the other, the Torsen will prevent the wheel with the loss of traction from spinning, while an open differential would send all of the torque to that wheel inducing wheel spin. This is due to the worm wheel/gear interaction.

When one wheel begins to lose traction it will start to turn its worm gear faster than the diff, which forces it to turn its corresponding worm wheel. This worm wheel then tries to turn its mating worm wheel via the spur gears connecting them. However, since a worm wheel cannot turn a worm gear the tire with traction continues to rotate with the same RPMs as before.

This combined with the large magnitude of friction found in the worm wheel/gear mesh **prevents** the wheel with a loss of traction from spinning by biasing the torque toward the wheel with traction.



The Limited Slip Differential

TORSEN T-1 Differential

Traction Loss by One Wheel in a Corner:

This is the true “magic” of the Torsen. It will still allow the differentiation described earlier because of the different distances traveled by the wheels while cornering. However, it will still prevent wheel spin in the wheel with less traction exactly as described in the previous slide.

The Torsen can accomplish this because it has separated the two primary functions of a differential: torque distribution and differentiation. With these two functions acting independent of each other you can maximize the properties of both without sacrificing one for the other.

Torsen = No tire slip

When one wheel on the driven axle loses traction compared to the other in a corner, the Torsen will prevent the wheel with the loss of traction from spinning, yet still allow proper differentiation.

The Torsen is not a locking differential, the worm wheel/gear pairs never completely lock together. Even though the worm wheels cannot turn the worm gears, and they cannot lock together, torque is transferred from worm wheels to worm gears.



The Limited Slip Differential

TORSEN T-1 Differential

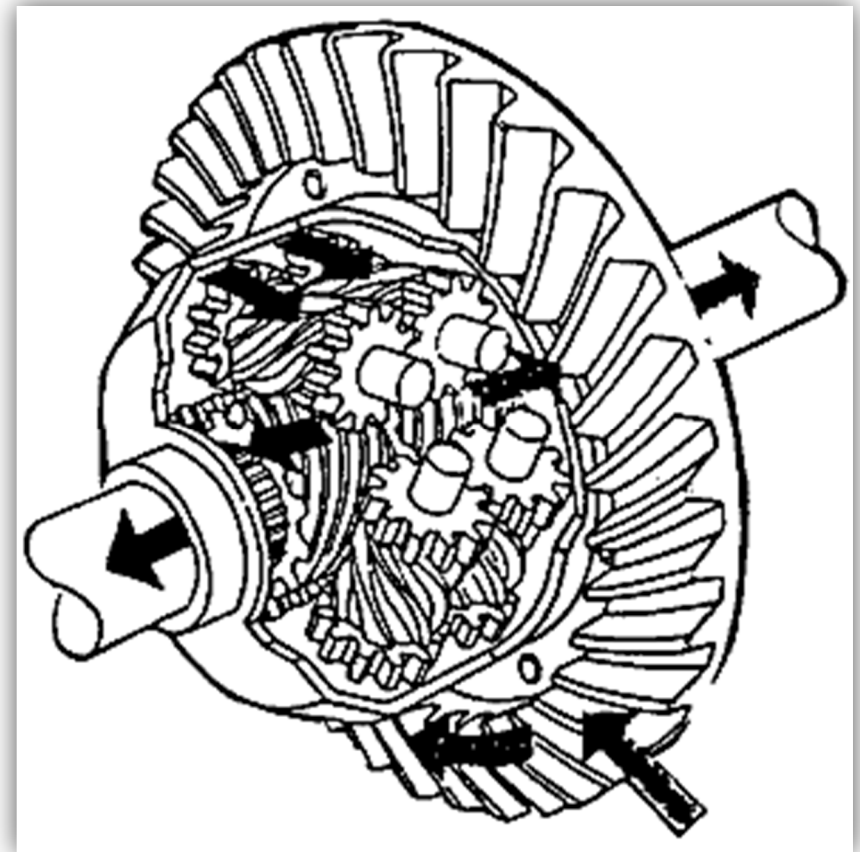
Torque Bias Ratio:

This is the ratio of torque sent to the axle (wheel) with good traction vs. the axle that is on the verge of slipping.

$$TBR = \frac{T_{HighTraction}}{T_{LowTraction}}$$

For example: You have a differential with a 4:1 TBR. That means that 4 times the amount of torque the slipping wheel can maintain (80% of total torque) will be placed on the wheel with good traction.

Therefore, if the wheel that is on the verge of slipping can achieve 100 ft*lbs of torque, the wheel with good traction will have 400 ft*lbs of torque applied to it.



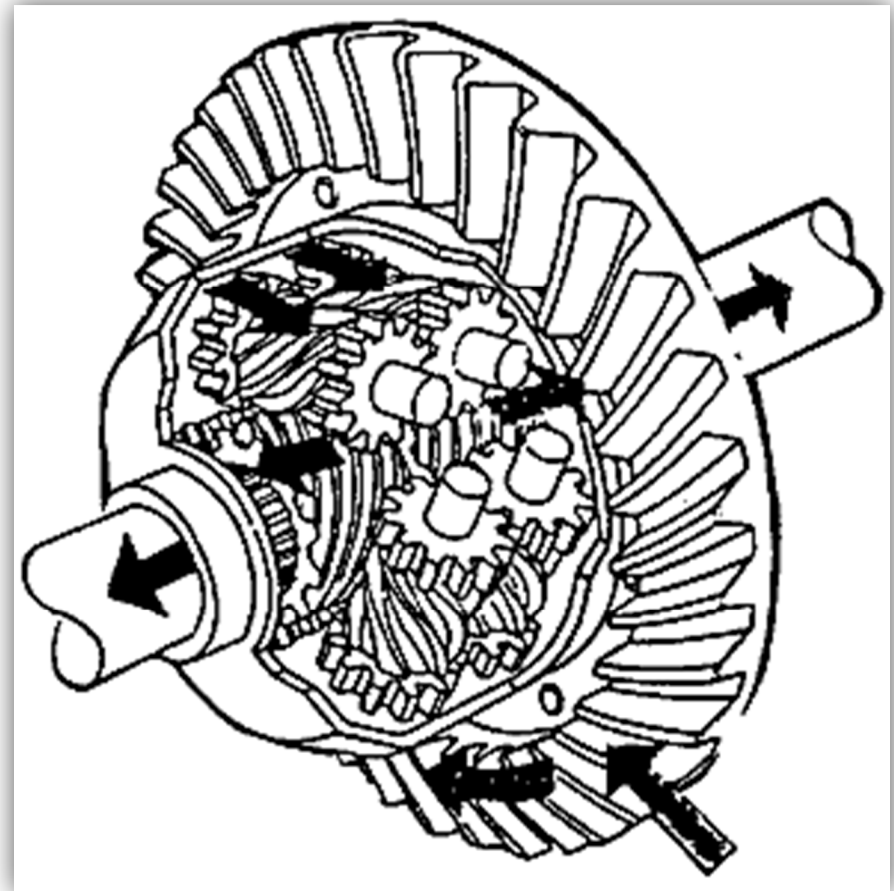
The Limited Slip Differential

TORSEN T-1 Differential

Torque Bias Ratio continued:

The Torsen I differential can have a torque bias ratio as high as approximately 7:1, and as low as approximately 2.5:1.

The bias ratio can be adjusted by the choice of gear angles, gear surface treatments, and the type of bearings and friction washers used. However, the majority of these adjustments must be done by the manufacturer.



The Limited Slip Differential

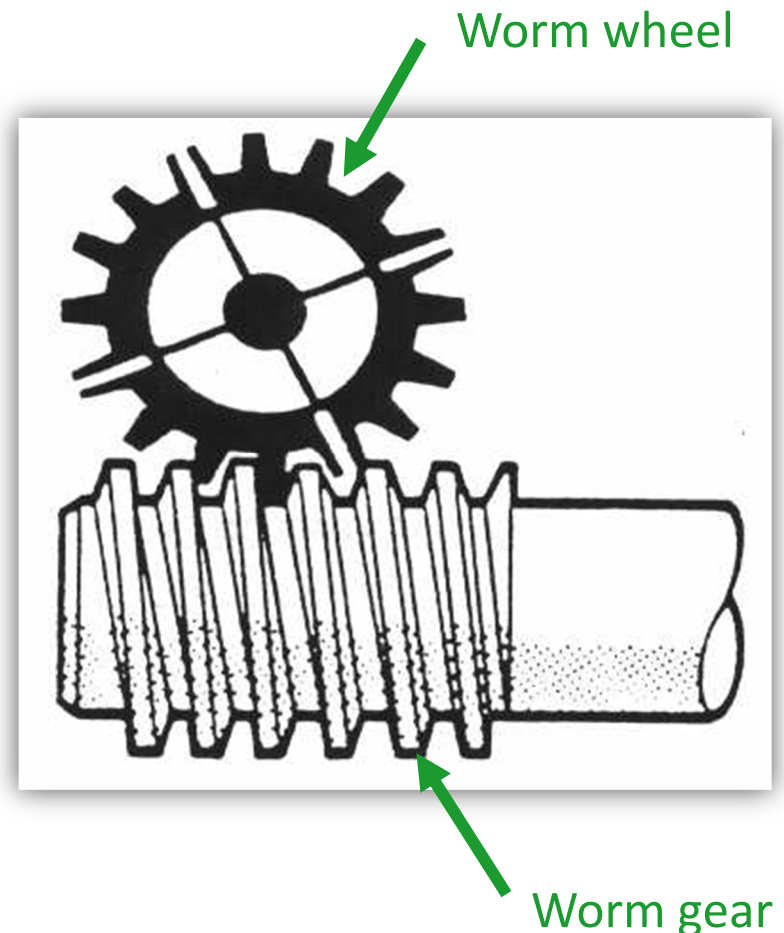
TORSEN T-1 Differential

Advantages:

- Driving wheels are always receiving some torque.
- Instant reaction to tire slip
- Linear lock-up characteristics
- Torque distribution is independent of differentiation

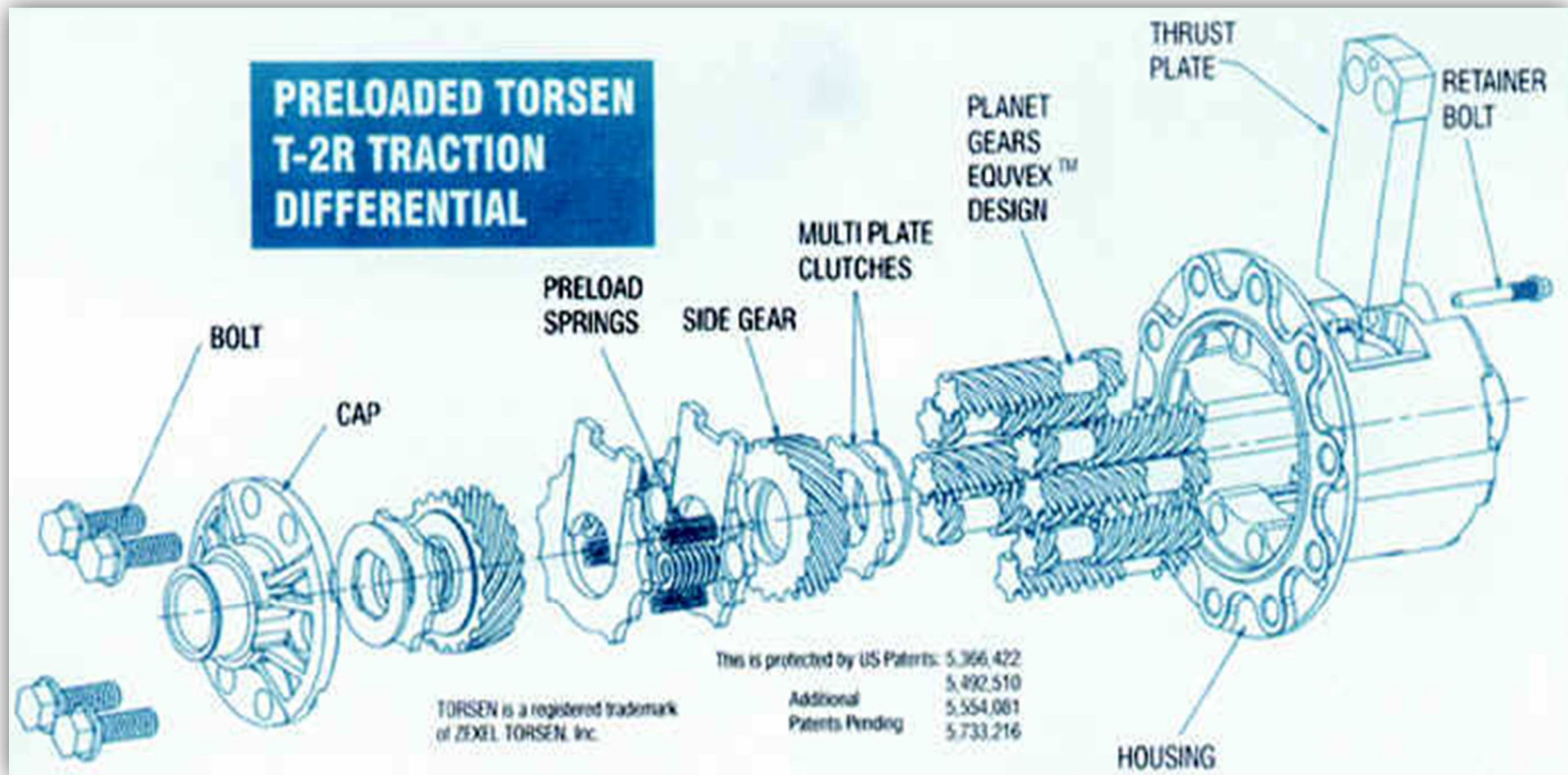
Disadvantages:

- Expensive, heavy
- Planetary gears must be properly timed to side gears
- No preload setting (just gear friction)
- Torque bias ratio is difficult to adjust
- Tendency for corner exit oversteer



The Limited Slip Differential

TORSEN T-2R Differential



The Limited Slip Differential

TORSEN T-2R Differential

Why are we investigating the T-2R instead of the T-2?

The T-2R is the Torsen Type 2 Racemaster. It is designed for racing and high performance purposes. The T-2 has a low torque bias ratio, and is primarily used as a front or center differential for that reason. The T-2R has a higher torque bias ratio than the T-2, which accommodates rear differentiation for racing purposes.

The T-2R also offers a preload function that the T-2 does not. A spring system is fitted between the side gears giving them an initial locking factor, which prevents the wheels from losing all torque even when one wheel becomes airborne.

How is the Torsen T-2R different than the Torsen T-1?

Although the T-2R has a completely different look than the T-1 it functions relatively similar.

The T-2R uses parallel axis helical gearing as opposed to the perpendicular axis high-helix angle of the original.



The Limited Slip Differential

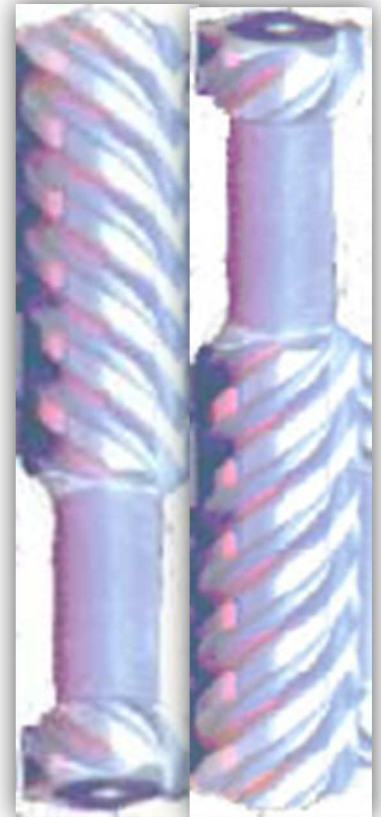
TORSEN T-2R Differential

The T-2R consists of four sets of planetary gears, which surround and connect the two side gears.

Each planet gear is mated with one side gear while its corresponding planet gear is mated to the opposite side gear. The reason each planet has a space that is not splined is to prevent it from direct contact with both side gears.

The planetary and side gears are splined with Torsen's *equivex* parallel axis design, which accommodates quiet operation, low backlash, and provides better management of gear mesh separation forces.

The new parallel axis gear alignment reduces the magnitude of friction in the gear mesh, and thus reduces the torque bias ratio. However, the T-2R has the addition of preload and steel friction discs between the side gears and differential housing, which can raise its TBR up to approximately 4:1. This reduction of friction allows the engine's torque output to be used more effectively, delivering more horsepower to the road.



1 set of planetary gears

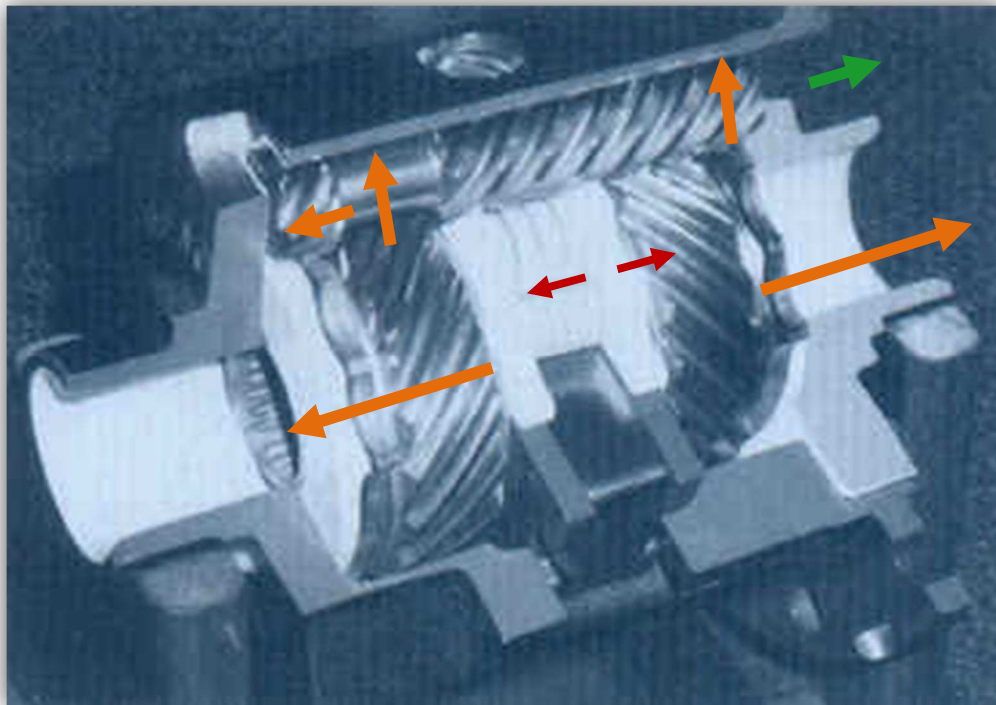


The Limited Slip Differential

TORSEN T-2R Differential

The Torsen T-2R is a torque sensing differential. It is capable of instantly sensing a change in torque, and biasing the engine torque to compensate and prevent the wheels from slipping.

The T-2R relies upon the spreading forces of the gears in the differential. These spreading forces press the gears against the diff housing locking them together, forcing them to rotate with the same velocity as the housing.



Axial spreading force from mating planetary gear

Axial and radial spreading forces

Preload forces

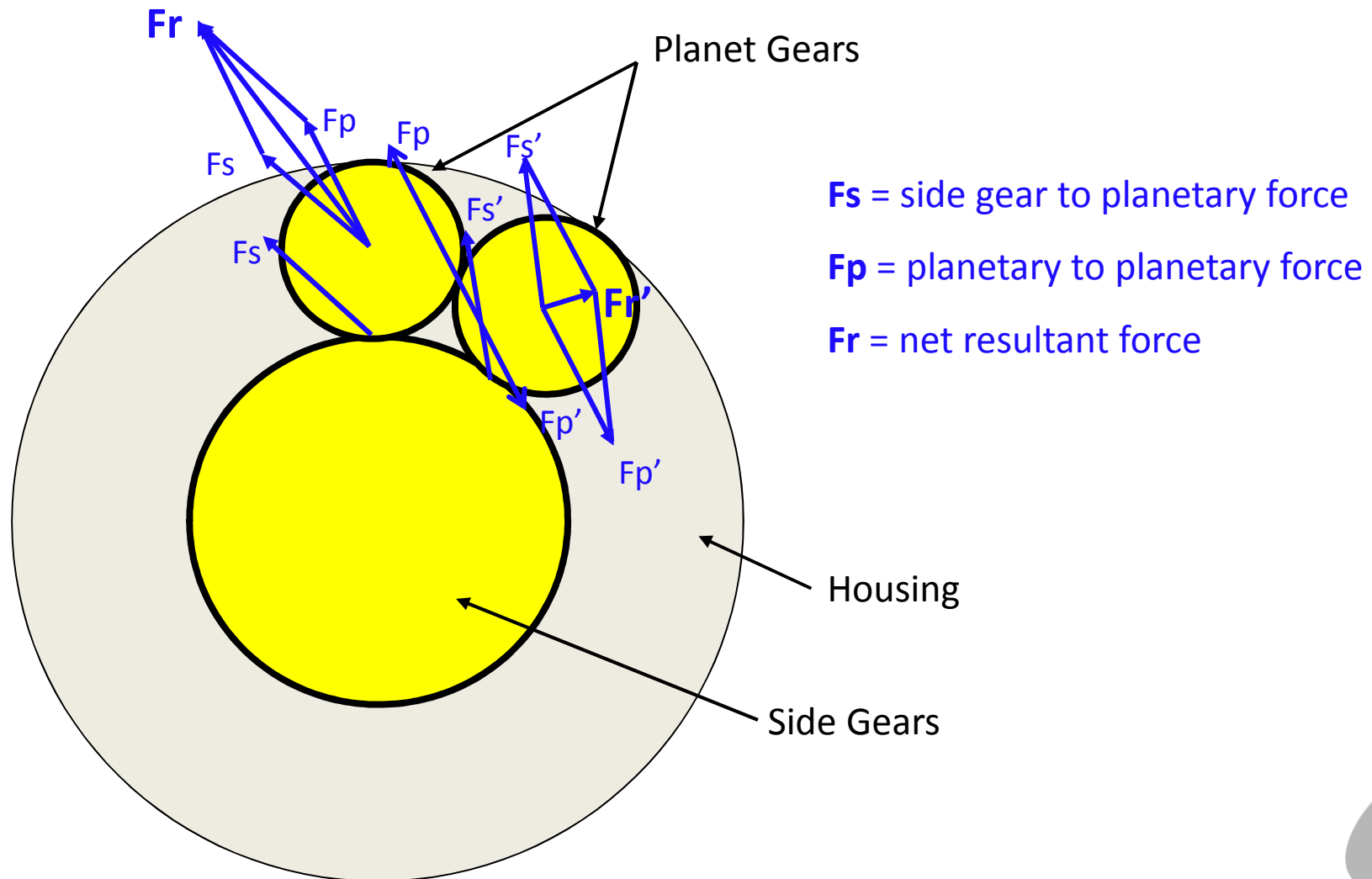
The magnitude of the combined spreading force is directly proportional to the torque difference between the wheels, and the gear helix angles and material properties.



The Limited Slip Differential

TORSEN T-2R Differential

A detailed examination of the gear separating forces.



The Limited Slip Differential

TORSEN T-2R Differential

The total tractive effort in the Torsen T-2R is an improvement over previous Torsen differentials, which allows the T-2R to reach its maximum torque biasing ratio quicker; thereby, preventing wheel spin while still delivering the engine's torque to the wheels.

The *Equivex* parallel axis design of the gearing in the T-2R reduces the amount of friction in the differential allowing the engine's torque to be efficiently translated to the wheels.

Due to the reduced friction in the T-2R differential automatic transmission fluid can be used in place of gear oil.

Due to its gear-based design, the T-2R's torque biasing will remain constant for the lifetime of the diff, and it requires no maintenance.

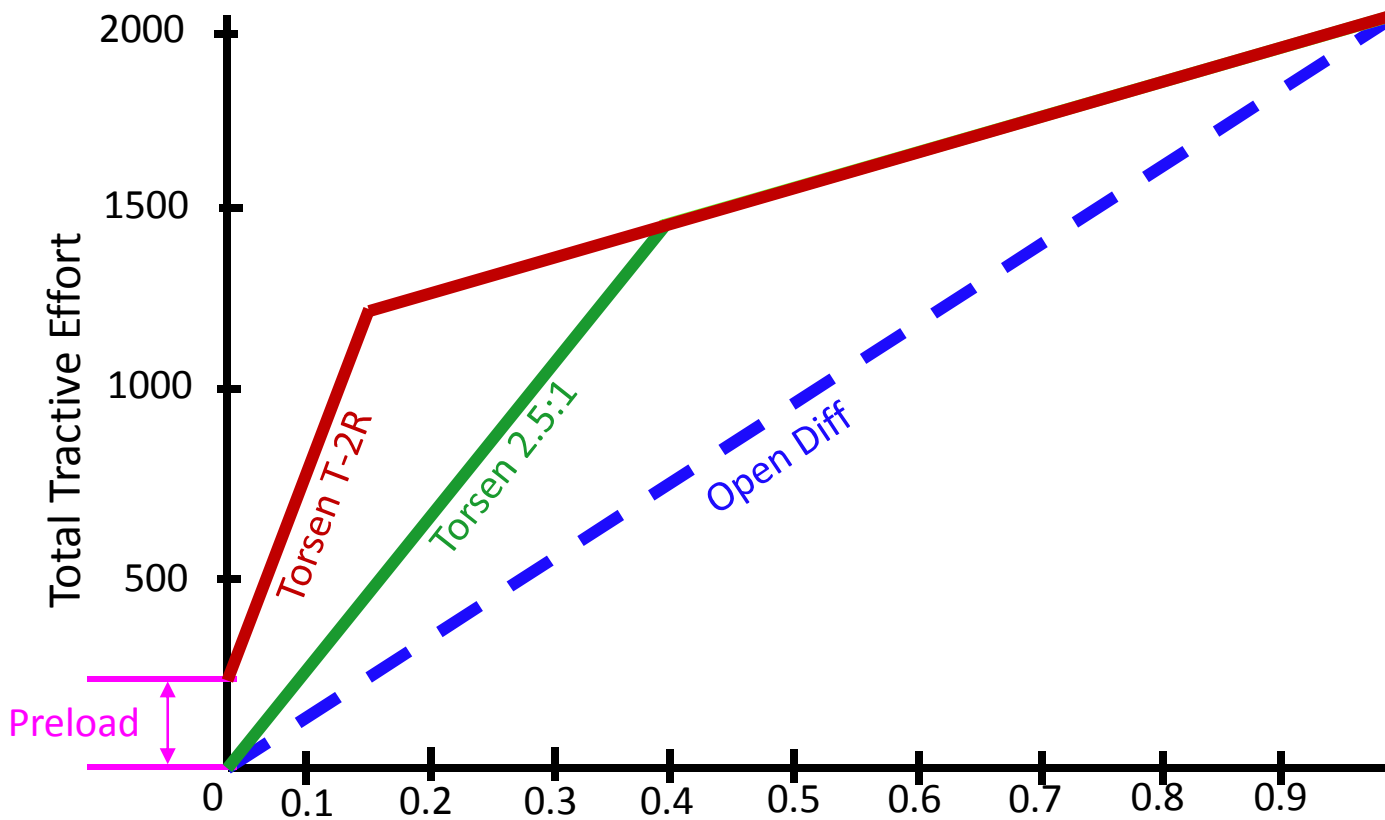
The combination of improved traction and engine torque efficiency found in the Torsen T-2R provides the car with better handling and improved acceleration.



The Limited Slip Differential

TORSEN T-2R Differential

Typical Split Coefficient Performance Curve



Surface coefficient: Variable Low Traction Wheel;
High Traction Wheel = 1.0μ



The Limited Slip Differential

TORSEN T-2R Differential

Advantages:

Easier to adjust torque bias ratio compared to T-1

Preload adjustment. T-1 has no preload

Less friction = less heat (can use automatic transmission fluid as opposed to heavy-duty gear oil)

Torque biasing does not change like it does in a LS

Torque sensing differential; prevents wheel spin

Compact

Disadvantages:

Expensive, heavy

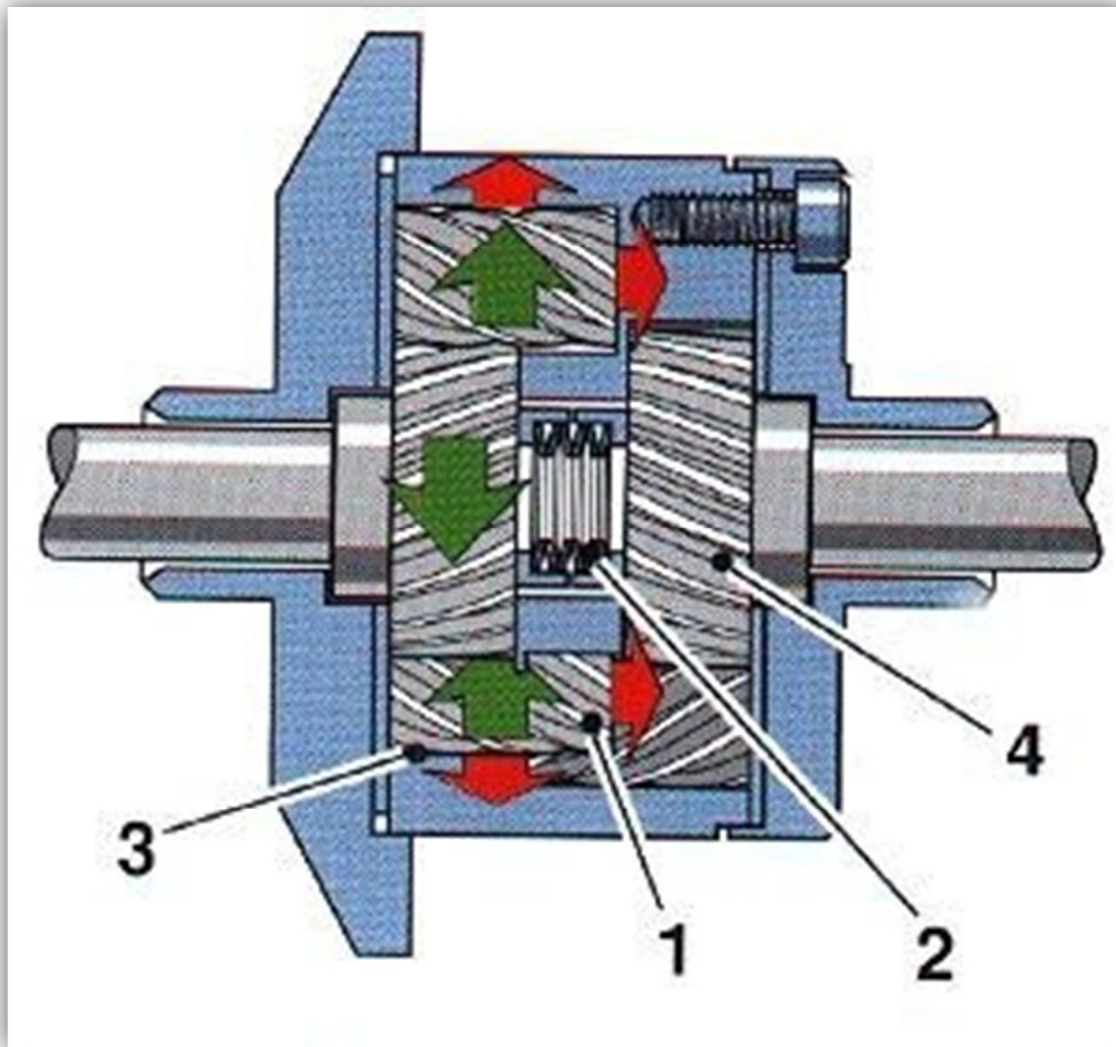
Torque bias ratio is still difficult to adjust even though it is easier than T-1

Tendency for corner exit oversteer



The Limited Slip Differential

Quaife Differential



1. Helical gear pinions
2. Center spring disc
3. Pocket
4. Sun gear



The Limited Slip Differential

Quaife Differential



The Quaife Limited Slip Differential is an Automatic gear-operated Torque Biasing differential, similar to the Torsen.

The functionality of the Quaife can be closely related to that of the Torsen T-2R without the friction discs.

The planetary gears are splined to a side (sun) gear, and the a mating planetary, which is splined to the opposite side gear in a parallel helix-gear format. The planetary gears also rest in precision made pockets in the differential housing, which forces them to rotate with the housing.

The preload spring simply adds an initial axial load to the side gears. This preload allows the Quaife to limit wheel spin even if one wheel becomes completely unloaded because the force of the preload must be overcome before differentiation can occur.

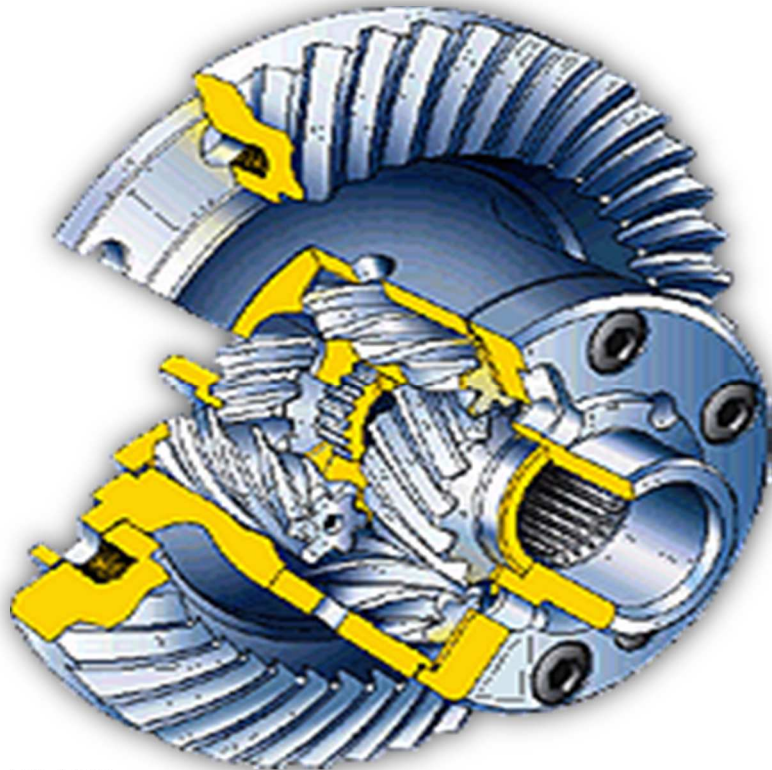
The torque bias ratio of the Quaife is a function of the helix angle and the surface treatment of the gears.



The Limited Slip Differential

Quaife Differential

- Sets of floating helical gear pinions are meshed to provide the normal speed differential action.
- A center spring provides a pre-load force on the side gears.
- When a torque imbalance occurs, torque bias is generated by the axial and radial thrusts of the pinions in their pockets and the axial thrust of the side gears.



- The resultant friction force enables the driving road wheel and side gear to transmit a greater proportion of the torque. This effect is progressive, but at no stage does the differential lock solid.
- Similar to the Torsen, the Quaife's torque bias ratio remains constant throughout the life of the differential.



The Limited Slip Differential

Quaife Differential

Advantages:

- Provides constant and infinitely variable drive.
- Power is transferred automatically without the use of normal friction pads or plates seen in other limited-slip designs.
- Strong and durable.
- No plates or clutches that wear out and need costly replacement.
- Biases torque and virtually eliminates wheel spin.
- Adjustable preload setting

Disadvantages:

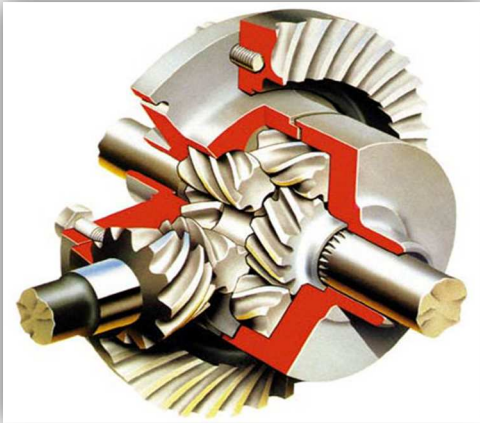
- Expensive, heavy
- Torque bias ratio is very difficult to adjust (must be done by changing helix angle of gears or surface treatment)
- Tendency to induce corner exit oversteer



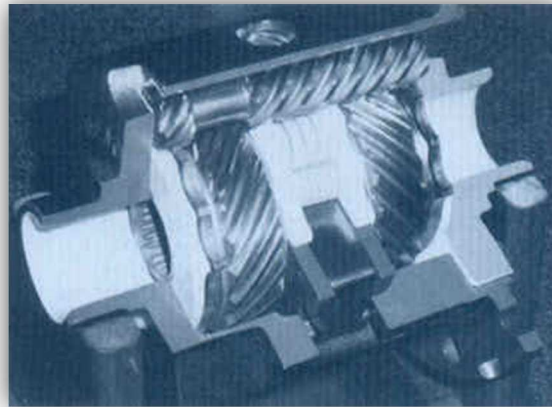
The Limited Slip Differential

Torque Sensing Differentials

There are currently several different torque sensing differentials on the market that are being used for racing purposes, including the Torsen, Quaife, DPI Black and Gold, etc. . .



Quaife



Torsen T-2R



DPI Black & Gold

A common characteristic of all torque sensing differentials is the tendency for power-on oversteer. Such oversteer is due to the fact that torque sensing diffs send more torque to the tire experiencing more load; the outside tire.



The Central Differential

Two Functions of a Central Differential

Used on four wheel drive cars such as a WRC. These cars have central differential and differential on each axle.

This unit has two functions, the same as a differential on the axle.

1) Transmitting the torque between the front and the rear axle:

For the axle differential in the straight line position, the torque is split 50/50 to each wheel. If it were not 50/50 the vehicle would have a natural tendency to turn away from the wheel with the most torque.

In a central differential, it is not necessary to consider turning, so it is possible to vary torque split to bias the torque towards the front and the rear of the car. The gear system used to split the torque can either be a bevel gear arrangement (i.e. plate differential).

2) Acting as a limited slip device:

This can be achieved using exactly the methods used for the axle differential with plates and ramps, viscous or hydraulic active.



The Central Differential

Purpose of Locking on Central Differential

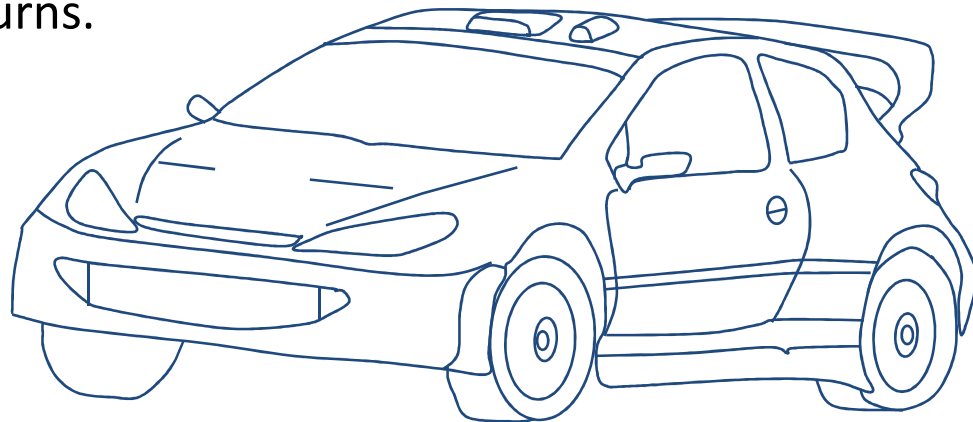
Reduce wheel spin at one end of the car.

Improved braking:

For example, if the rear tires begin to skid because of brake locking the differential allows the braking force from the rear brakes to be directed to the front wheels, effectively acting as an anti-lock brake system.

Provides an element of anti-lock:

If the rear wheels are connected to the front wheels, and the front wheels are still rotating, the rear wheels cannot lock...so must be disconnected for handbrakes turns.



Hydraulic and Active Differentials

This is the most complex differential to design and engineer; however, the way it functions is quite simple.

The differential plates and discs are locked by an external force instead of being locked by ramps, viscous fluid, etc. . . on non active differentials.

This external force is applied by a hydraulic piston, so any racing car that is using this device must have a hydraulic power supply and control system.

The external force can supply a designated force at any time, using relevant parameters such as wheel speed, drive shaft torque,..., to provide the differential characteristic that are desired.

Basically, active differentials are a multi-plate clutch, enabling variable torque between the axles it connects. The torque is controlled by a computer, which gathers the necessary information from several sensors. Depending on the design and software, some systems allow very precise control of traction during hard cornering, some achieve desirable understeer / oversteer, some can even make the best use of traction for acceleration and braking during normal conditions.



Hydraulic and Active Differentials

Let's take an example: *Porsche 959's PSK (Porsche-Steuer Kupplung) system.*

This system uses variable torque split for maximum traction under normal conditions. Most of the time, torque split between front and rear is 40:60, similar to the weight distribution of the car.

During hard acceleration, weight transfer to the rear axle increases traction in the rear tires while reduces traction in the front. Therefore, the system will transfer up to 80% of the engine torque to the rear axle for better traction.

On slippery road, 50:50 torque is used. A computer determines the torque split ratio by analyzing parameters such as throttle position, steering angle, G-Force and even turbo boost.

This system provides optimum traction under all conditions and not only when tire slip occurs.



Hydraulic and Active Differentials

The PSK continued:

The multi-plate clutch has 6 pairs of frictional plate, each pair is independently controlled by computer and actuated by hydraulic pressure. This is equivalent to 6 independent clutches.

How does it work?:

- The front and rear drive shafts must run at different speeds in normal operating conditions (959 front tires are 1% larger in diameter than the rears). Due to the speed difference between front and rear drive shafts, the frictional plates of each independent clutch are rotating at different speeds.
- When applying hydraulic pressure to the first clutch, a small amount of torque will transfer to the front axle.

Note: the 2 drive shafts cannot be fully locked up unless all 6 clutches are locked simultaneously.

Locking 2, 3, or more clutches...and the torque will be proportionally increased to the drive shaft with less torque, subsequently, the torque split could be 50:50 if all of the clutches are fully engaged.



Hydraulic and Active Differentials

Today, active differentials are used for yaw control, traction control, and much. . .much more.

The ECU (electronic control unit) is the computer, or brain of today's cars. For active differentials, the ECU judges the surface frictional coefficient, the actual vehicle behavior, and the driver's intentions to determine the optimum transfer of driving torque. Then it signals the hydraulic actuator to adjust the differential accordingly.

Some of the inputs used by the ECU to adjust a differential are: steering wheel angle, accelerator stroke, all four wheel speeds, longitudinal acceleration, and lateral acceleration.

The algorithms, or conditions that are used by the ECU to adjust the active differentials (front, center, and rear diffs) can be altered to conform to a driver's particular style of driving.



MoTeC M800 ECU



Differential Set-Up

➤ Open Differential:

We can find an Open Diff in Formula Ford, Formula Vee and Formula Ford 2000. These cars often have problems with wheel spin due to the fact that there is no lock up of the differential.

- Reduce Wheel Spin:

- Maximum Front Roll stiffness

Some good teams are using up to 80 % front roll stiffness. This allows most of the lateral weight transfer to take place at the front and keeps the inside rear in better contact with the ground, whereby reducing the wheel spin. Be careful, a front roll stiffness of 80% will cause the front tires to lift before the rear.

- Using a droop limiter at the front to help reduce inner rear wheel spin

You can adjust it to allow only a fraction of an inch of droop travel before the inside front tire is lifted off the ground

- Perfect corner weight



Differential Set-Up

Limited Slip Differential:

Contrary to Open Differentials, cars using this type of device run 50% to 60% front roll stiffness.

- Corner entry understeer:
 - Too much locking to allow proper wheel speed variation
Use fewer shims, larger angle on the coast ramp, less preload.
- Wheel spin in slow to medium corner:
 - More lockup
 - Less rear roll stiffness if you are already to a fully locked diff



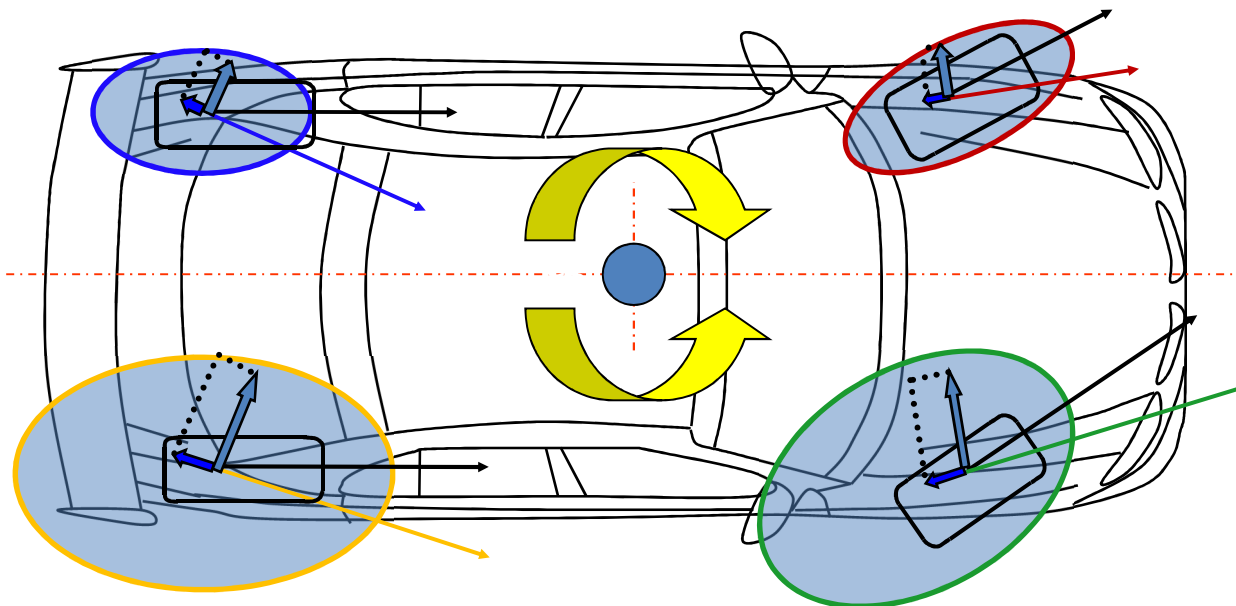
Differential Set-Up

It appears on the sketch below that an excess of torque on either rear tire will create a yawing moment around the center of gravity of the car.

While many people have a good comprehension of roll steer and bump steer, few people are confident with the steering action of various differentials.

This yaw moment will “steer” the car in the direction away from the tire with the most torque.

Understeer Torque



Oversteer Torque

Excess torque on the inside tire will cause understeer, and oversteer on the outside tire.

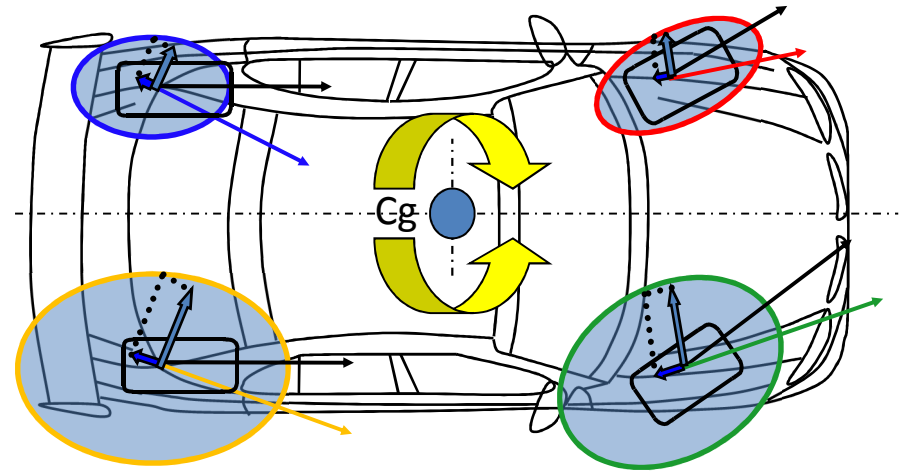
In the over-run or off-throttle mode, excess drag on the outside wheel will cause understeer, and oversteer if excess drag is on inside wheel.



Differential Set-Up

Open differential:

In this case, the driving torque on each wheel should be the same, and the steering effect will be negligible. When wheel spin occurs, the driving tire will steer the car towards the spinning wheel. Watch the start of a F3 or Toyota Atlantic race (cars without traction control), or try it with you own car.



Locked differential:

In this case (with no stagger), the outside tire in a corner “wants” to rotate faster than the inside. The longitudinal forces on the tires are consequently forward on the inner wheel and rearward on the outer wheel. This causes an understeer moment about the center of gravity, and that’s why oval track racers are using tire stagger.



Linear Differential Model

Yaw Moment vs Lateral G

The following simple linear model has been developed to describe the effect of the differential on the vehicle dynamics. (ref. S. Kloppenborg). In spool mode the equation is the following.

$$N_{spool} = \left[\frac{2V\rho hY}{r} - t^2 \left(F_{z0} + \frac{hX}{2w} \right) \right] \frac{r \left(X + 2C_x \left(F_{z0} + \frac{hX}{2w} \right) \right)}{2V \left(F_{z0} + \frac{hX}{2w} \right) - \rho hY} - 2C_x \rho hY$$

Where: **X** = longitudinal force
Y = lateral force
h = cg height
C_x = tire cornering stiffness (linear tire assumed)
F_{z0} = static wheel load on each tire
ρ = fraction of anti roll torque on the driven axle
w = wheelbase
t = track
r = yaw rate



Linear Differential Model

Yaw Moment vs Lateral G

When the torque difference between the two wheels reaches T_c the differential switches to differentiating mode and can no longer support a difference in torque between the two wheels.

The Equation that describes the differentiating mode of a Salisbury differential is the following.

$$N_{diff} = \frac{-tT_c}{R_t} - \frac{t(CXR_t + B)}{R_t}$$

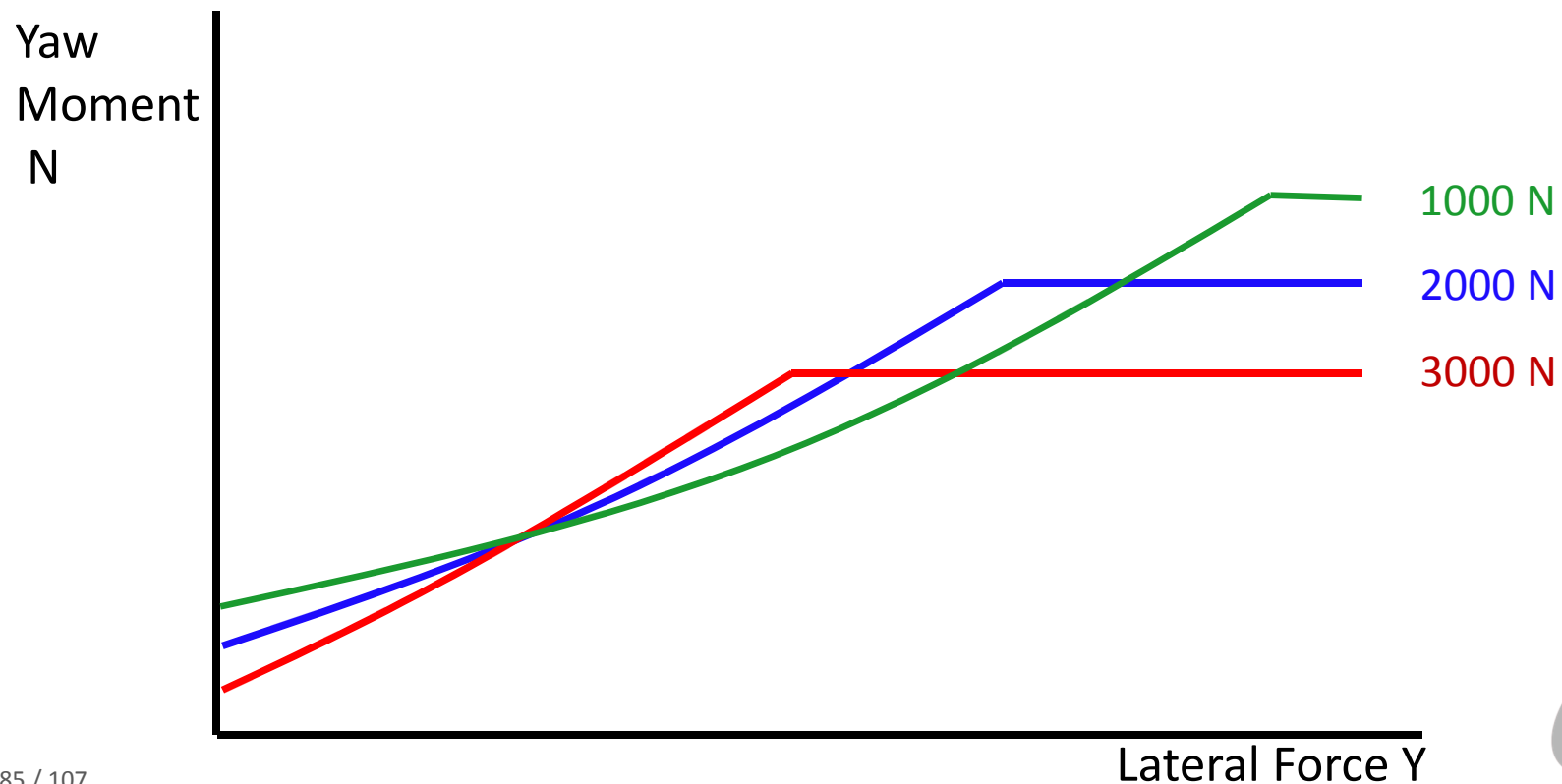
Where: **C** = ramp coefficient of differential (the gradient of T_c - T_{app})
B = preload torque
R_t = Tire Radius



Linear Differential Model

Yaw Moment vs Lateral G

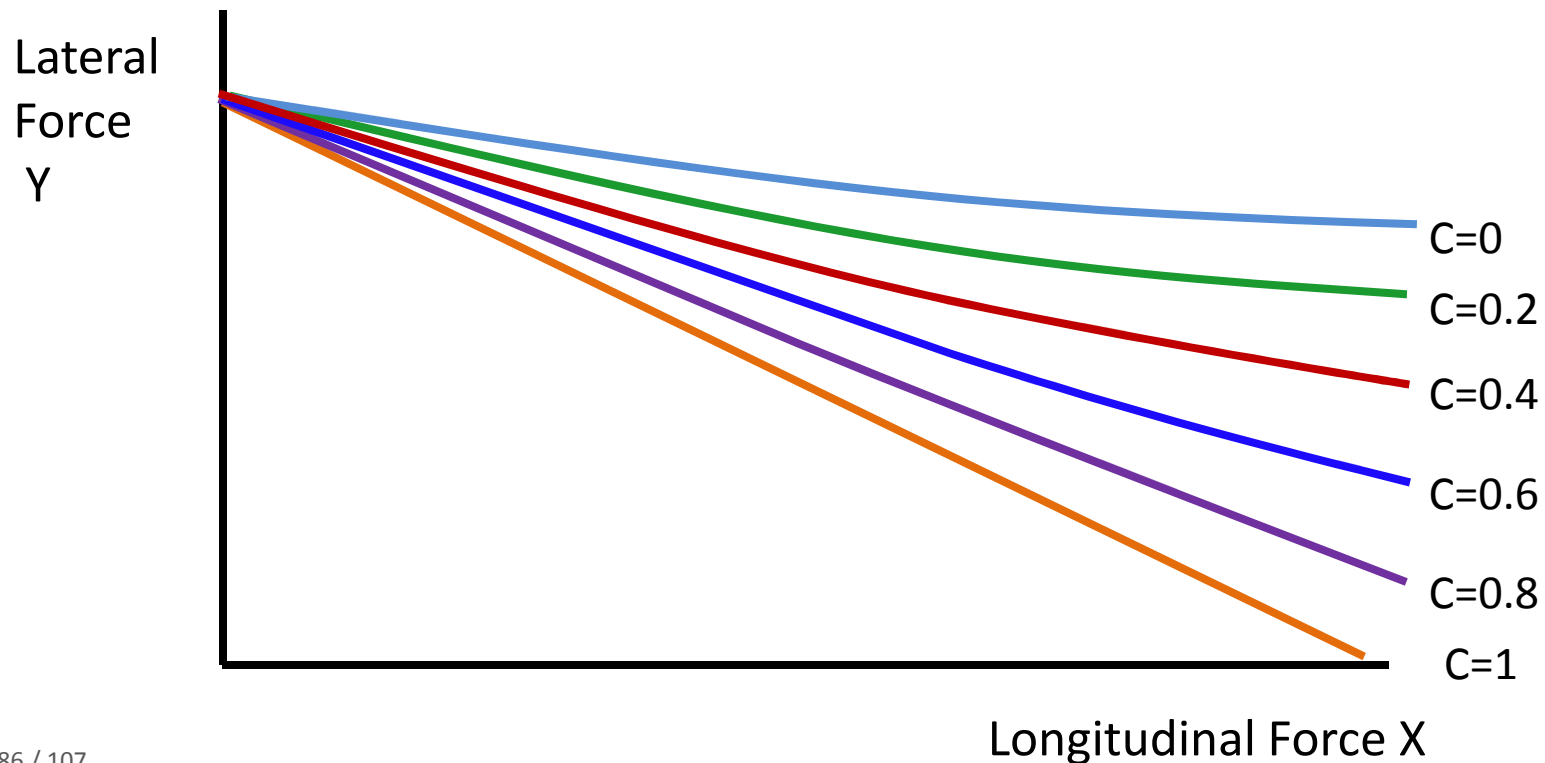
- The following example shows the Yaw Moment vs Lateral G for a Salisbury differential. The following is plot for a turn radius of 20m at a number of different longitudinal forces.
- We can see that the value of T_c may also be considered a maximum yaw moment.



Linear Differential Model

Critical Lateral G vs Longitudinal G

- The following example plots the critical lateral force where the diff transitions from spool to differentiating mode against Longitudinal force for a range of ramp coefficients.
- Decreasing the ramp coefficient causes the diff to transition between modes at higher lateral force.



Linear Differential Model

Track Example determining C & D from track data

The total drive torque of a differential is described by the following relationship assuming the right tire has a lower maximum torque.

$$T_{app}^{\max} = T_r^{\max} + \min \left\{ \begin{array}{l} \left(\frac{1+C}{1-C} \right) \left[T_r^{\max} + \frac{B}{1+C} \right] \\ \frac{T_r^{\max}}{\lambda} \end{array} \right.$$

Where: $\lambda = \frac{T_r^{\max}}{T_l^{\max}}$

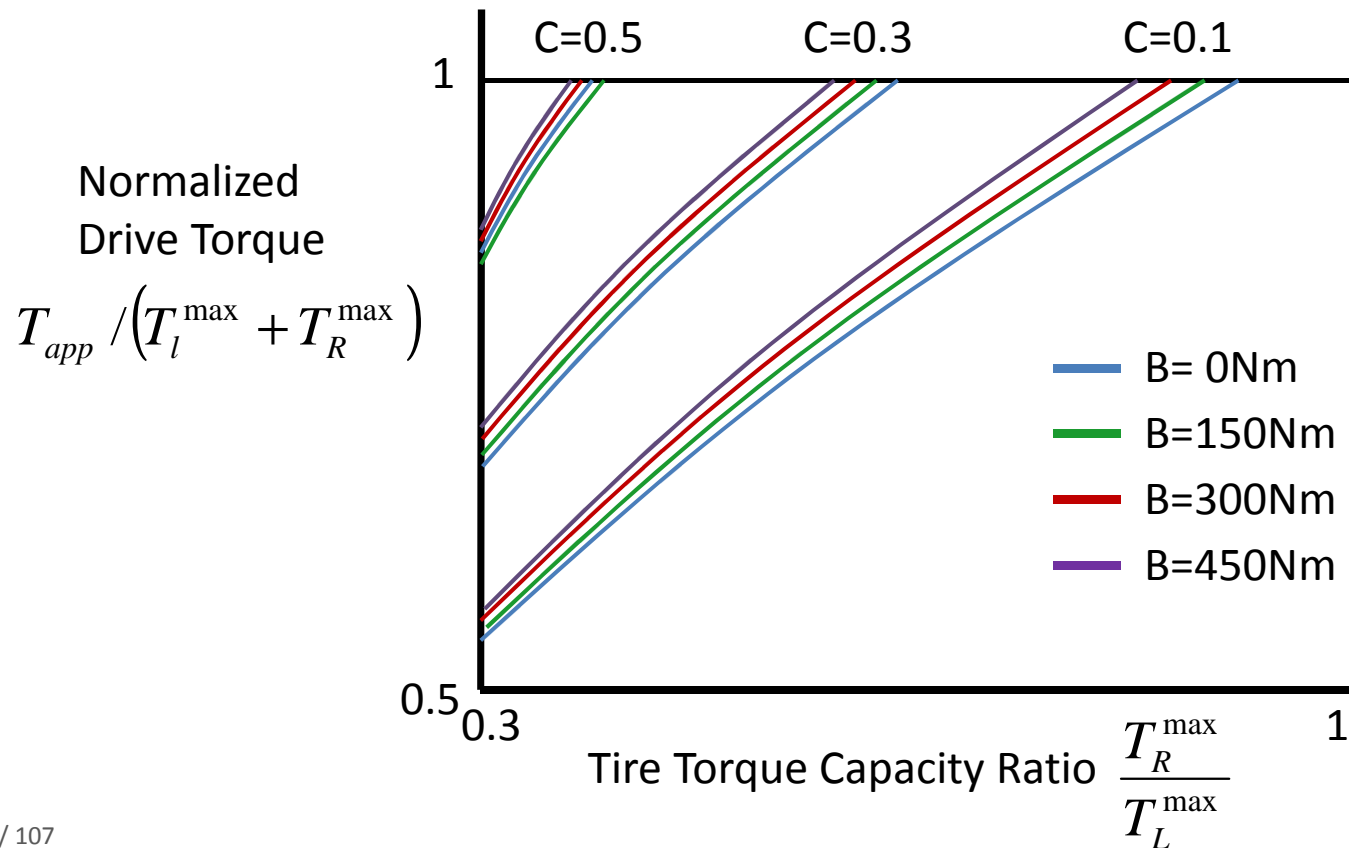
This relationship can be plotted to illustrate the effect of the differential parameters B and C on the maximum drive torque.



Linear Differential Model

Maximum Drive Torque

- The following example shows how the maximum drive torque and hence longitudinal acceleration is affected by the diff parameters C and B.
- It is clear that the more lock the differential has (high C high B) the greater the differential is able to utilize the traction available from the tires.



Linear Differential Model

Consequences of the Linear Model

From the previous analysis of the linear model we can draw the following conclusions:

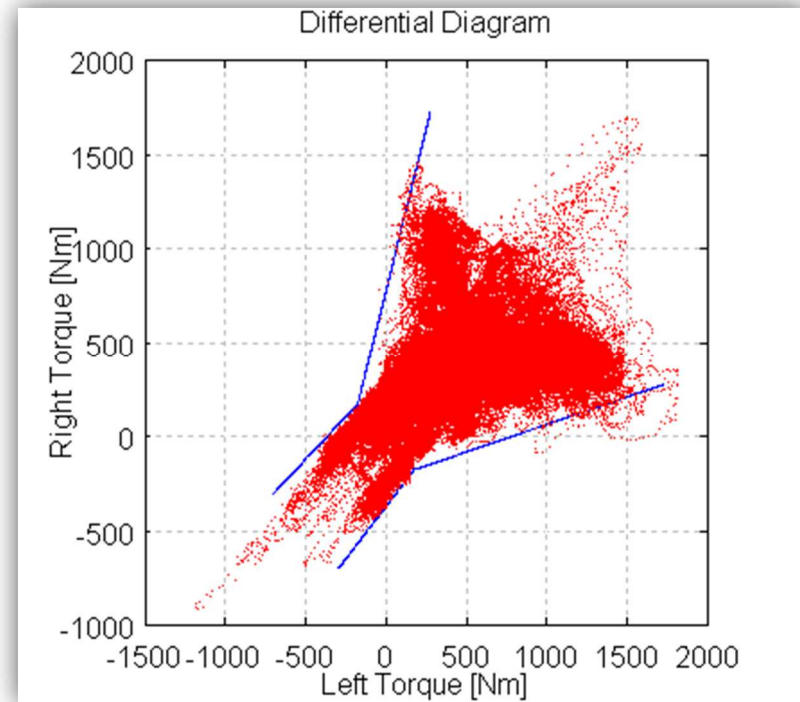
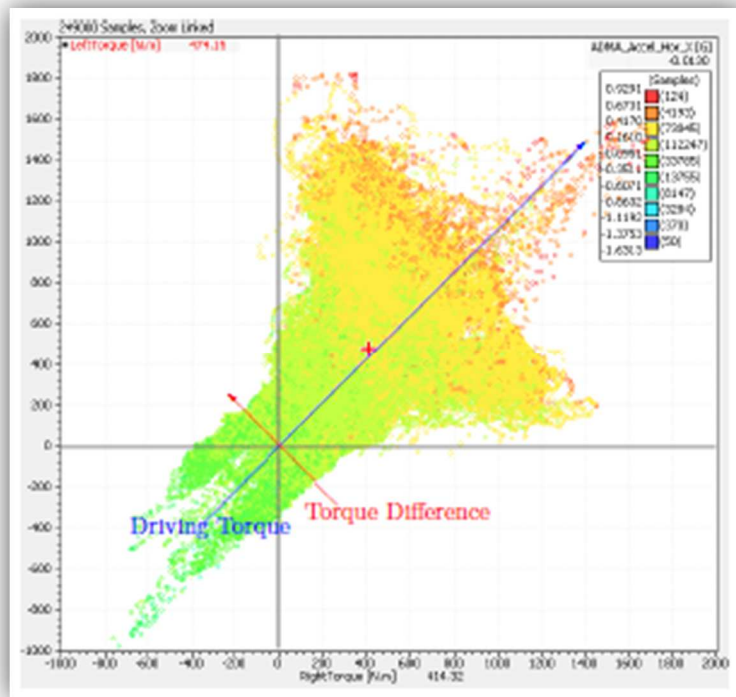
- The more lock the greater the ability of the car to utilize the combined traction available from the tires
- However the more lock the greater the oversteer moment
- The driver may not be able to apply the throttle as early on corner exit as it will cause the car to spin particularly for small radius turns. A more experienced driver may tolerate more lock as it allows a greater degree of throttle steer.
- For large radius turns the gains in longitudinal acceleration outweigh the additional oversteer moment so more lock would be favorable.



Linear Differential Model

Track Example determining C & D from track data

The following plots were obtained from a front wheel drive touring car fitted with wheel force transducers on the drive wheels.



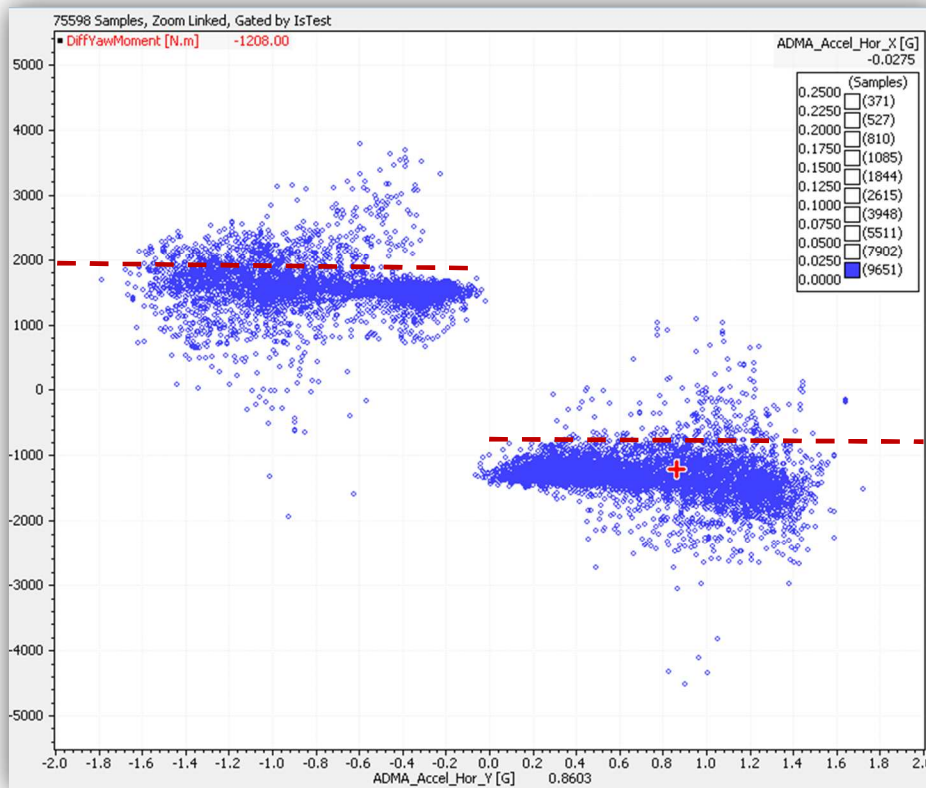
The value of B can be determined from the width of the “waist” of the plot. The value of C can be obtained by overlaying the equations from the previous section. (B = 350Nm and C = 0.55)



Linear Differential Model

Track Example determining C & D from track data

The following test performed on a 50m skid pad predicts that the diff will be operating in differentiating mode which we can see from the following plot of the yaw moment vs lateral acceleration. However there is considerable scatter in the data.

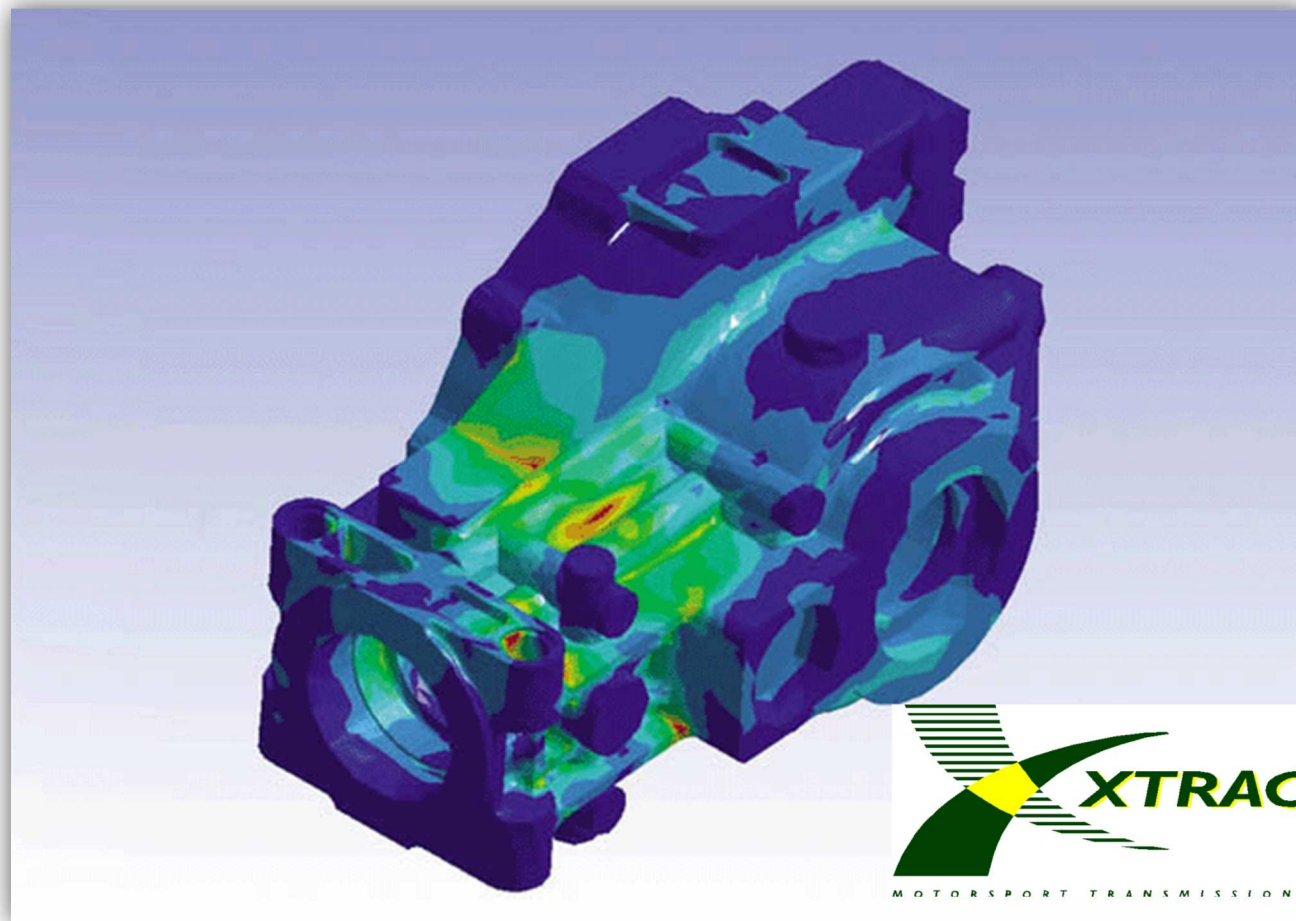


$$T_r - T_l = \text{sgn}(\Omega_l - \Omega_r)(CT_{app} + B)$$

The linear model used provides a good approximation however in reality there will be significant scatter in the results due particularly to tire non-linearity and dynamic effects.



Calculation of Locking for Frictional Contact Differentials



Data provided by XTRAC Transmissions Ltd



Calculation of Locking for Frictional Contact Differentials

Engine

Ri

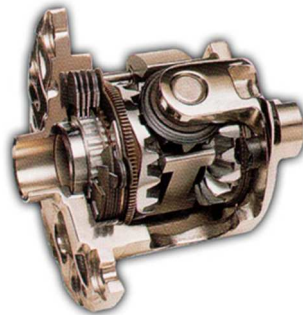
Gear train 21/29

Final Gear R0 (9/31)

Differential

Net ratio before gear train
 $R_i=1$ (ratio defined as output speed/input speed)

Engine torque $T_e = 550 \text{ N.m}$



Cluster gear data

No. teeth 1st wheel: $Z_1 = 21$

No. teeth 2nd wheel: $Z_2 = 29$

Net gear ratio after cluster is $R_o = (31 / 9) = 3.44$

Differential data

Ramp angle: $A = 65 \text{ deg}$

Effective radius ramp: $R_{\text{ramp}} = 50 \text{ mm}$

$L_o = 4.25 \text{ mm}$

Z_u (# of friction surfaces) = 6

Outside diameter of friction plates: $D_u = 97 \text{ mm}$

Inside diameter of friction plates: $d_u = 59 \text{ mm}$

Dynamic coefficient of friction: $\mu = 0.08$

Static coefficient of friction: $\mu_0 = 0.12$

Belleville washer data

Belleville washer deflection: $f = 0.4112 \text{ mm}$

Belleville washer outside diameter: $D_o = 87 \text{ mm}$

Belleville washer inside diameter: $D_i = 67 \text{ mm}$

Thickness of washer: $e = 2.25 \text{ mm}$

Cone height of unloaded washer: $h = 2 \text{ mm}$

Young modulus: $E = 206 \text{ GPa}$

Poisson ratio: $\nu = 0.292$



Calculation of Locking for Frictional Contact Differentials

Calculation of ramp loads

Overall ratio engine to differential: $U = R_i \cdot (Z_2 / Z_1) \cdot R_o = 1 \cdot (29 / 21) \cdot 3.44 \rightarrow U = 4.75$

Torque at differential: $T_d = (T_e \cdot U) = (550 \cdot 4.75) \rightarrow T_d = 2612.5 \text{ Nm}$

Force at effective ramp radius: $F_{t\text{-tot}} = (T_d / R_{\text{ramp}}) = (2612.5 / 50 \text{ mm}) \rightarrow F_{t\text{-tot}} = 52250 \text{ N}$

Hence force at each ramp: $F_t = F_{t\text{-tot}} / 2 = 52250 / 2 \rightarrow F_t = 26125 \text{ N}$

Induced axial load at ramp contact with pin: $F_a = F_t / \tan(A) = 26125 / \tan(65) \rightarrow F_a = 12182.3 \text{ N}$

Calculation of dynamic locking torque

Let's say that the pressure is uniform over a friction disc surfaces

Dynamic friction torque per side: $T_f = \frac{F_a \cdot u}{3} \cdot \left(\frac{D_u^3 - d_u^3}{D_u^2 - d_u^2} \right) \cdot Z_u \rightarrow T_f = 232.56 \text{ Nm}$

Hence total locking torque (2 sides): $T_{f\text{-tot}} = 2 * T_f \rightarrow T_{f\text{-tot}} = 465.12 \text{ Nm}$

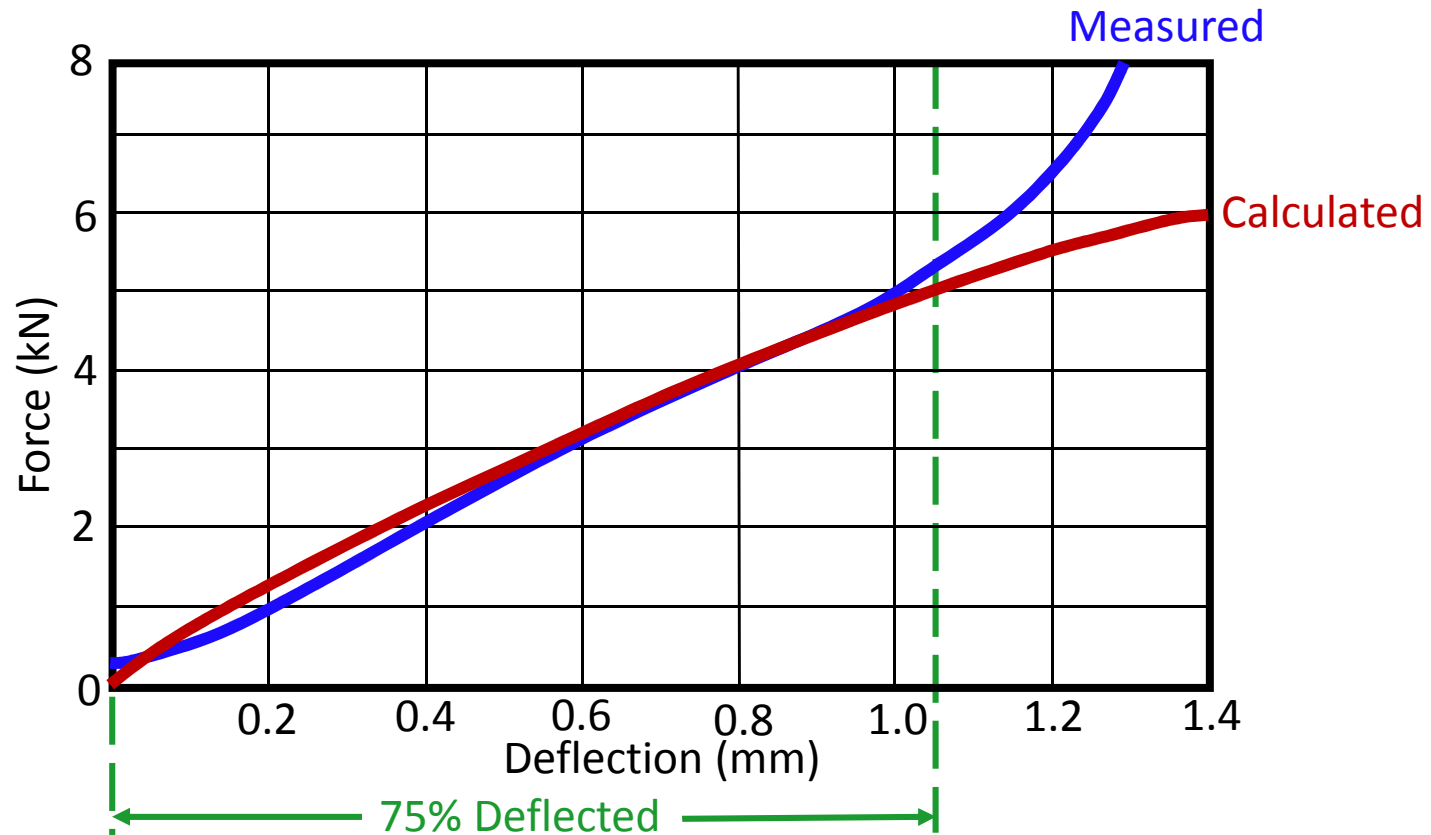
Static friction torque per side: $T_{f0} = \frac{F_a \cdot u_0}{3} \cdot \left(\frac{D_u^3 - d_u^3}{D_u^2 - d_u^2} \right) \cdot Z_u \rightarrow T_{f0} = 348.84 \text{ Nm}$

Total static locking torque (2 sides): $T_{f0\text{-tot}} = 2 * T_{f0} \rightarrow T_{f0\text{-tot}} = 697.69 \text{ Nm}$



Belleville Washers

Calculated Characteristics vs. Actual Test Results

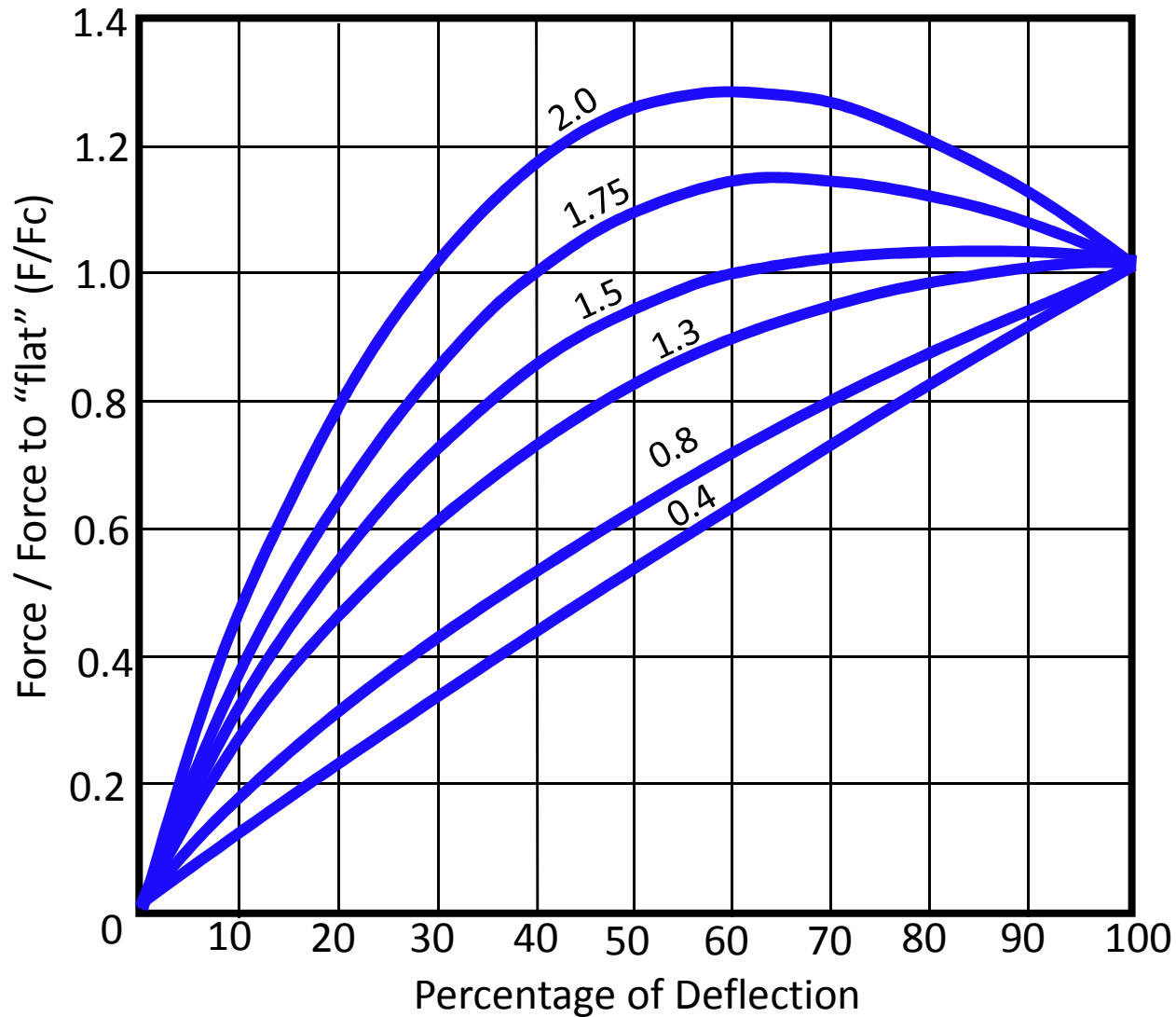


This characteristic is typical for most disc springs. Due to the sharply increasing force and stress above 75% of the total deflection, the maximum deflection used should be limited to 75% of the total available deflection.



Belleville Washers

Examples of Varying the Ratio of Cone Height / Thickness



Belleville Washers

Belleville washers, due to their design, have a nonlinear stiffness factor. This nonlinear stiffness can be calculated via the following equation:

$$R = C \times \Omega \times \frac{e}{D_o^2} \times \left(h^2 - 3.h.f + \frac{3}{2}.f^2 + e^2 \right)$$

Where R is the washer stiffness (N/m²), and:

$$C = \pi \cdot \left(\frac{\delta}{\delta - 1} \right)^2 \cdot \left(\frac{\delta + 1}{\delta - 1} - \frac{2}{\ln(\delta)} \right) \quad \delta = \frac{D_o}{D_i} \quad \Omega = \frac{4.E}{1 - \nu^2}$$

$$\delta = 1.2985$$

$$C = 2.575$$

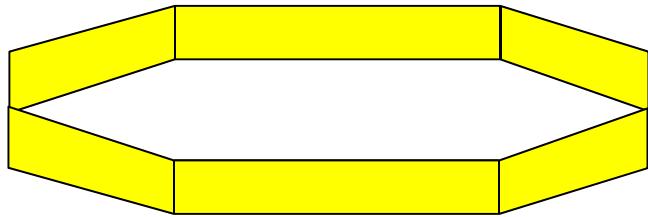
$$\Omega = 900.806 \text{ GPa}$$

$$R = 4722514.952 \text{ N/m}$$

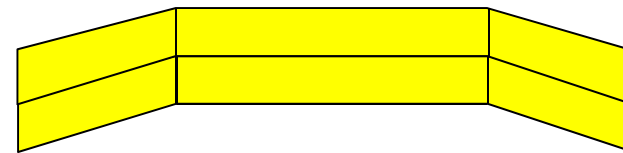


Belleville Washers

You have found the stiffness of a single Belleville washer. However, Belleville washers are often stacked together in both, series and parallel.



Series Connection (*2 sets in series*)



Parallel Connection (*2 washers in set*)

Two washers in series double the amount of deflection while the spring stiffness remains the same.

Two washers in parallel; however, double the value of the spring stiffness, while the deflection remains the same.

To properly calculate the total washer stiffness in a stack of washers you must account for the parallel and series connections in the stack with this equation:

$$\text{Stack Stiffness} = \frac{\text{Single Washer Stiffness} \times \text{Washers in One Parallel Set} \times \text{Friction Factor}}{\text{Sets of Washers in Series}}$$

The friction factor represents the friction between the plates. ($FF > 1.0$)



Belleville Washers

The spring stiffness of the Belleville washers can be adjusted by using different quantities of washers and different stack configurations.

If the stack configuration does not have parallel sets with all of the same number of plates, you must use the following equation to determine your total spring stack stiffness:

$$\text{Total Stack Stiffness} = \frac{1}{\frac{1}{\text{1st Stack Stiff}} + \frac{1}{\text{2nd Stack Stiff}} + \dots + \frac{1}{\text{n Stack Stiff}}}$$

As you can see, the stack stiffness is not necessarily dependent upon the thickness of the stack (number of plates). However, the thickness can be calculated via the following equation:

$$\text{Stack Width} = (\# \text{ of sets in series} * (\text{washer height} + (\# \text{ of washers in parallel set} - 1) * \text{washer thickness}))$$

Washer height is the sum of the washer thickness and the cone height.

In the case when all of the parallel sets do not contain the same number of washers the stack width can be found by adding the widths of each series.



Belleville Washers

The preload force in the differential due to the Belleville washers can be calculated via a simple spring force equation, when using the correct spring stiffness for a given displacement.

$$F_{BW} = R * f$$

Therefore, in the case of one washer: $F_{BW} = 1941.898 \text{ N}$



The preload force can then be used to find the static and dynamic preload torque.



Calculation of Locking for Frictional Contact Differentials

Dynamic friction torque from spring preload per side:

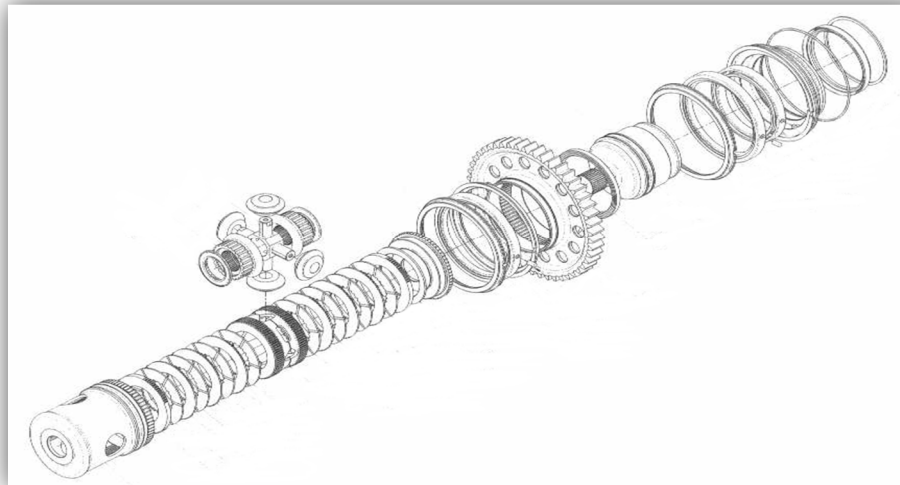
$$T_{fp} = \frac{F_{BW} \cdot u}{3} \cdot \left(\frac{Du^3 - du^3}{Du^2 - du^2} \right) \cdot Zu \quad \text{so } T_{fp} = 37.07 \text{ N.m}$$

Hence total locking torque from preload: $T_{fp-tot} = 2 \cdot T_{fp}$ so $T_{fp-tot} = 74.14 \text{ N.m}$

Static friction torque from spring preload per side:

$$T_{fp0} = \frac{F_{BW} \cdot u0}{3} \cdot \left(\frac{Du^3 - du^3}{Du^2 - du^2} \right) \cdot Zu \quad \text{so } T_{fp0} = 55.60 \text{ N.m}$$

Hence total locking torque from preload: $T_{fp0-tot} = 2 \cdot T_{fp0}$ so $T_{fp0-tot} = 111.20 \text{ N.m}$



Calculation of Locking for Frictional Contact Differentials

Resultant locking effect of differential

If locking from ramps is greater than load from preload spring then locking will occur at the ramp value:

i.e. *Dynamic* $T_{f-tot} = 465.12 \text{ N.m}$

Static $T_{f0-tot} = 697.69 \text{ N.m}$

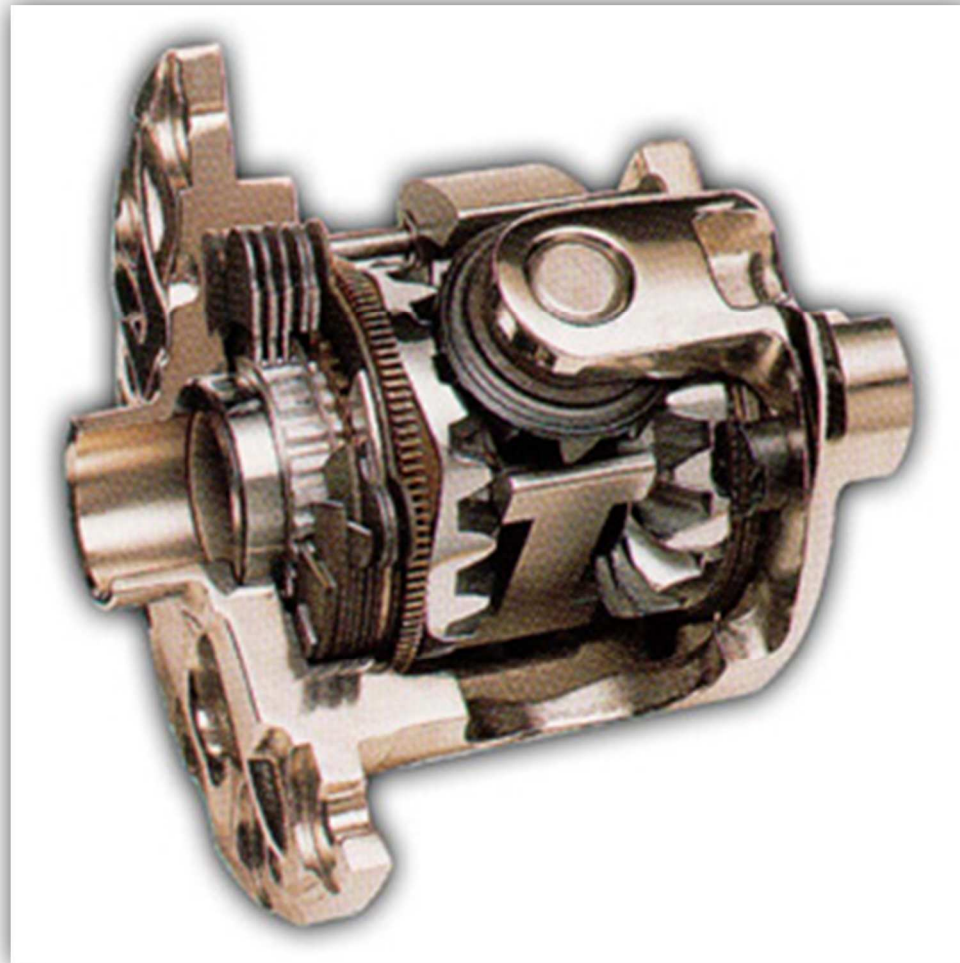
However, if locking from ramps is less than load from preload spring then locking will occur at the spring value:

i.e. *Dynamic* $T_{fp-tot} = 74.14 \text{ N.m}$

Static $T_{fp0-tot} = 111.20 \text{ N.m}$



Locking Percentage Rating



Data provided by HEWLAND Engineering Limited



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Locking Percentage Rating

Ignoring the effect of preload, the total clutch plate stack clamping force is the sum of the side gear reaction force and the axial component of the ramp force. This is defined as a locking percentage in the following equation:

$$\text{Percentage of lock} = \left[\frac{48}{\tan(\text{Ramp Angle})} + 17 \right] \times \frac{\text{Number of Plate Surface Used}}{\text{Max Possible Number of Working Surfaces}}$$

This formula gives a comparison between chosen settings, NOT a definitive locking torque value. The 30 degree ramp was used as the baseline, since it is the most aggressive commonly used ramp angle. Therefore, a 30 degree ramp with the maximum number of working surfaces will yield a value of 100%.

The ramp axial force is 4.882 times that of the side gear reaction force with a 30 degree ramp. Therefore, 17% of the clutch plate clamping force is due to the reaction forces of the side bevel gears. And $48/\tan(30) = 83\%$. $83 + 17 = 100\%$ of the reference locking force.

The second part of the equation: the plate number ratio, is used to show that locking percentage is also a function of the number of active clutch plates. You can see that the more active clutches you have in the diff the more lock you will achieve.



Locking Percentage Rating

Ramp Angle	Number of friction surfaces					
	2	4	6	8	10	12
30	17	33	50	67	83	100
40	12	25	37	49	62	74
50	10	19	29	38	48	57
60	7	15	22	30	37	45
70	6	11	17	23	29	34
80	4	8	13	17	21	25

Calculated percentages of lock using a 30 degree ramp with 12 friction surfaces as the 100% reference value.

Torque Transferable Between Wheels

Equation is specific to the Hewland FTC Differential

$$TTBW = \text{Crownwheel Torque} \times \left\{ 0.378 + \left[\frac{0.415}{\tan(\text{Ramp Angle})} \right] \right\} \times \mu \times \text{Number Of Plate Surface Used}$$

$\mu = 0.1$ is the friction coefficient between steel plates.



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Locking Percentage Rating



Percentage rating	Ramp angle	Number of friction surfaces
4	80	2
7	60	2
8	80	4
11	45	2
13	80	6
15	60	4
17	80	8
17	30	2
21	80	10
22	60	6
22	45	4
25	80	12
30	60	8
33	45	6
33	30	4
37	60	10
43	45	8
45	60	12
50	30	6
54	45	10
65	45	12
67	30	8
83	30	10
100	30	12



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Differentials Conclusion

To get the most out of your car you want the driving wheels to:

1. Allow differentiation when the throttle is lifted.
2. Smoothly and progressively lock as throttle is reapplied.
3. Avoid understeer / oversteer characteristics once throttle returns to wide open.

**The result will be a car that is stable,
responsive and easy to drive at its maximum limit.**

