Conclusion :→

(i) Electromagnetic wave in free space travell with the speed of light
(ii) Electromagnetic waves are transverse in nature.
(iii) $\vec{E}$ & $\vec{H}$ are in same phase
(iv) Electromagnetic energy is transported along the direction of propagation because $\vec{S}$ is along $\hat{n} = \hat{k}$
(v) Electrostatic energy density is equal to the magnetic energy density.

2012

Plane electromagnetic wave in non-conducting isotropic medium (Isotropic dielectric) :→

A medium which is non-conducting and has some properties in all directions is called an isotropic dielectric. In such a medium we have,

$$\vec{D} = \epsilon \vec{E}, \quad \vec{B} = \mu \vec{H}, \quad \vec{J} = \sigma \vec{E} = 0$$
$$\& \ \rho = 0$$

Hence maxwell's equ^n becom

$$\left.\begin{array}{ll} \text{div}\, \vec{E} = 0 & \text{— (a)} \\ \text{div}\, \vec{H} = 0 & \text{— (b)} \\ \text{Curl}\, \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} & \text{— (c)} \\ \text{Curl}\, \vec{H} = \epsilon \dfrac{\partial \vec{E}}{\partial t} & \text{— (d)} \end{array}\right\} (1)$$

from equ^n (1)(c) ⇒

$$\text{Curl curl}\, \vec{E} = -\mu \frac{\partial}{\partial t}(\text{Curl}\, \vec{H})$$

$$= -\mu \frac{\partial}{\partial t}\left(\epsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\text{curl curl } \vec{E} = -\mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (2)$$

similarly, we can obtain

$$\text{curl curl } \vec{H} = -\mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \longrightarrow (3)$$

Therefore equ^n (2) becomes

$$\nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \longrightarrow (4)$$

similarly equ^n (3) becomes

$$\nabla^2 \vec{H} - \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \longrightarrow (5)$$

Each of the 6 components of $\vec{E}$ and $\vec{H}$ seperately satisfies the same scaler wave equ^n —

$$\nabla^2 \varphi - \mu\epsilon \frac{\partial^2 \varphi}{\partial t^2} = 0 \longrightarrow (6)$$

Where φ stands for any of the $\{E_x\ E_y\ E_z$ or $H_x\ H_y\ H_z\}$.

comparing equ^n (4), (5) & (6) with general wave equ^n →

$$\nabla^2 \varphi - \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2} = 0 \longrightarrow (7)$$

We conclude that the field vector $\vec{E}$ & propagate in isotropic dielectric as wave with a velocity v is given by

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}}$$

$\epsilon_r = \frac{\epsilon}{\epsilon_0}$

$\mu_r = \frac{\mu}{\mu_0}$

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{n} \longrightarrow (8)$$

Where 'c' is the speed of light in free space and $n = \sqrt{\mu_r\epsilon_r}$ is the refractive index of the medium.
Since

$$n > 1$$

$$\therefore v < c$$

Hence the speed of e-m wave in an isotropic dielectric is less than the speed of electromagnetic wave in free space
For a non-magnetic material

$$\mu_r = 1$$

$$n = \sqrt{\epsilon_r}$$

$$\boxed{n^2 = \epsilon_r = K_e}$$

Where K_e is called dielectric constant of the medium.

Now wave equ^n (4) & (5) may be written as -

$$\nabla^2\vec{E} - \frac{1}{v^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \longrightarrow (9)$$

$$\nabla^2\vec{H} - \frac{1}{v^2}\frac{\partial^2\vec{H}}{\partial t^2} = 0 \longrightarrow (10)$$

Planes wave equ^n of (9) & (10) are respectively -

$$\vec{E}(r,t) = \vec{E_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \longrightarrow (11)$$

$$\vec{H}(r,t) = \vec{H_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \longrightarrow (12)$$

Where E_0, H_0 are complex amplitude & are constant in space and time.
Where propagation vector -

$$\vec{K} = K\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{\omega}{v}\hat{n} \longrightarrow (13)$$

(I) Relative direction of $\vec{E}$ & $\vec{H}$:-

$$\vec{\nabla}\cdot\vec{E} = 0$$

& $\vec{\nabla}\cdot\vec{H} = 0$ will give $\vec{K}\cdot\vec{E} = 0$ and $\vec{K}\cdot\vec{H} = 0$

$\vec{E}$ & $\vec{H}$ are $\perp^r$ to the direction of propagation.

This means that electromagnetic waves in istropic dielectric are transverse in nature.

Further $\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$

& $\vec{\nabla}\times\vec{H} = \epsilon\frac{\partial\vec{E}}{\partial t}$ will gives us-

$$\vec{K}\times\vec{E} = \mu\omega\vec{H} \longrightarrow (14)$$

$$\& \quad \vec{K}\times\vec{H} = -\epsilon\omega\vec{E} \longrightarrow (15)$$

From these eq.^n we conclude that $\vec{E}$, $\vec{K}$ & $\vec{H}$ are mutually $\perp^r$. They are also $\perp^r$ to the direction of propagation. Then in a plane e-m wave in isotropic dielectric, vectors ($\vec{E}$, $\vec{H}$ & $\vec{K}$) from a set of orthogonal vector and represent a right handed co-ordinate system.

(II) Wave impedence (z) :→

$$\vec{H} = \frac{1}{\mu\omega}(\vec{K}\times\vec{E}) = \frac{k}{\mu\omega}(\hat{n}\times\vec{E})$$

$$\vec{H} = \frac{1}{\mu v}(\hat{n}\times\vec{E}) = \sqrt{\frac{\epsilon}{\mu}}(\hat{n}\times\vec{E})$$

∴ 'Z' is defined as

$$Z = \left|\frac{\vec{E}}{\vec{H}}\right| = \left|\frac{E_0}{H_0}\right| = \sqrt{\frac{\mu}{\epsilon}} = \text{Real quatity} \longrightarrow (16)$$

Thus the field vectors $\vec{E}$ & $\vec{H}$ are in same phase i.e. They have same relative magnitudes at all points at all times.

'z' has a unit of 'ohm'..

Now

$$Z = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} = \sqrt{\frac{\mu_r}{\epsilon_r}}\, Z_0$$

where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is called impedence of free space.

(III) Poynting Vector :→

$$\vec{S} = (\vec{E}\times\vec{H})$$

$$\vec{S} = \vec{E}\times\sqrt{\frac{\epsilon}{\mu}}(\hat{n}\times\vec{E})$$

$$= \sqrt{\frac{\epsilon}{\mu}}\{\vec{E}\times(\hat{n}\times\vec{E})\}$$

$$= \sqrt{\frac{\epsilon}{\mu}}\{(\vec{E}\cdot\vec{E})\hat{n} - (\vec{E}\cdot\hat{n})\vec{E}\}$$

$$= \frac{1}{Z}(E^2\hat{n}) \qquad (\vec{E}\cdot\hat{n} = 0)$$

$$\therefore \vec{S} = \frac{E^2}{Z}\hat{n}$$

Now average of poynting vector.

$$\langle \vec{S} \rangle = \langle \vec{E} \times \vec{H} \rangle = \langle \frac{E^2}{Z} \hat{n} \rangle$$

$$\langle \vec{S} \rangle = \frac{1}{Z} \langle \{E_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)}\}^2 \rangle \hat{n}$$

$$= \frac{E_0^2}{Z} \langle \cos(\omega t - \vec{K}\cdot\vec{r}) \rangle \hat{n}$$

$$= \frac{E_0^2}{Z} \times \frac{1}{2} \hat{n}$$

$$\boxed{\langle \vec{S} \rangle = \left(\frac{E_{rms}^2}{Z}\right) \hat{n}} \longrightarrow (17)$$

$$\langle S \rangle = \sqrt{\frac{\epsilon_r}{\mu_r}} \frac{1}{Z_0} (E_{rms}^2) \hat{n}$$

$$\langle S \rangle = \frac{\sqrt{\mu_r \epsilon_r}}{\mu_r} \frac{1}{Z_0} (E_{rms}^2) \hat{n}$$

$$\langle S \rangle = \frac{n}{\mu_r} \frac{E_{rms}^2}{Z_0} \hat{n} \longrightarrow (18)$$

Where $n = \sqrt{\mu_r \epsilon_r}$ is refractive index. equ^n (17) & (18) show that the flow of energy by electromagnetic wave takes place along the direction of propagation.

$$\boxed{S = \frac{n}{\mu_r} \langle S \rangle_{\text{free space}}}$$

(IV) Power flow and energy density :→ Ratio of electrostatic and magnetostatic densities in an electromagnetic field is →

$$\frac{U_E}{U_m} = \frac{\frac{1}{2}\epsilon E^2}{\frac{1}{2}\mu H^2} = \frac{\epsilon}{\mu}\frac{E^2}{H^2} = \frac{\epsilon}{\mu} Z^2$$

$$\frac{U_e}{U_m} = \frac{\epsilon}{\mu}\frac{\mu}{\epsilon} = 1$$

$$\boxed{U_e = U_m}$$

Summry :-

(1) E-m wave in isotropic dielectric travell with a speed less than the speed of Light.

(2) Field vectors $\vec{E}$ & $\vec{H}$ are mutually $\perp^r$ and are also $\perp^r$ to the direction of propagation of wave.

(3) Vectors $\vec{E}$ & $\vec{H}$ are in same phase. Electro-magnetostatic energy density is equal to electrostatic energy density. Energy flow per unit area per second is given by —

$$\boxed{\langle S \rangle = \frac{E^2_{rms}}{Z}\hat{n}}$$

★ Plane Electromagnetic waves in a conducting Medium :→

Let us assume that the medium is linear and isotropic with permitivity (ϵ) and permeability (μ) and ~~conducting~~ Conductivity (σ). The change density is