Integral of $(dx)^2$

We wish to evaluate the integral $\int_a^b (dx)^2$. I do not know any integration techniques that can help us evaluate this integral. However, we can write the integral as an infinite sum. We can think of the integral as adding up the areas of an infinite number of squares with side length dx. So, we have:

$$\sum_{i=1}^{\infty} (dx)^2.$$

Now, since infinite sums can be tricky to deal with, let us say we are adding together the areas of n squares, rather than an infinite number. Then, the side length of each square will be $(b-a)/n$, since the side lengths of all squares must add up to the length of the interval $[a, b]$. Then, we have:

$$\sum_{i=1}^{n} \left(\frac{b-a}{n}\right)^2.$$

Now, to get back to the infinite number of the squares, we will take the limit of the above sum as n goes to infinity:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b-a}{n}\right)^2.$$

Everything that is being summed is a constant, so we can pull it out of the sum:

$$\lim_{n\to\infty} \left(\frac{b-a}{n}\right)^2 \sum_{i=1}^{n} 1.$$

For the remaining sum, we are just adding up bunch of $1s$. How many? n of them. So this sum is just equal to n. So, we have

$$\lim_{n\to\infty} \left(\frac{b-a}{n}\right)^2 \times n,$$

which simplies to

$$\lim_{n\to\infty} \frac{(b-a)^2}{n}$$

For any finite interval $[a, b]$, this clearly converges to 0. If you are interested, then perhaps you can come up with some exciting stuff by letting $b-a$ go to infinity.