Modeling Piezoelectric Materials in ANSYS APDL

VERSION 2, NOVEMBER 2023

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Table of Contents

Introduction to Piezoelectric Materials

1.1 General Introduction

This part presents an introductory of piezoelectric materials and structures. It starts with defining smart materials, the history of piezoelectric materials and their relation to ferroelectric materials. The physics of piezoelectric phenomena are also mentioned followed by presenting common piezoelectric materials. At the end, some applications of piezoelectric materials are stated.

The past 50 years of technology was driven by what is called "Smart Materials". Most smart materials have been discovered or well understood in the previous century, but their utility became clear in this century the 21^{st}. Smart materials defined by Rogers et al. in 1988 in a workshop of the United States Army Research Office as "those materials which possess the ability to change their physical properties in a specific manner in response to specific stimulus input" **Bhalla et al. (2017)**. Stimuli may include pressure, moisture, magnetic field, radiation, and electrical fields, while the responses may include strains, shape changing, stiffness, damping and many other physical aspects. The smart materials group include different materials such piezoelectric materials (ceramics and polymers), Shape memory alloy, optical fibers, and many others.

The word Piezoelectricity was proposed by Hankel after 12 months of the discovery made by the brothers Curie (Jacques and Pierre) when they noticed an electrical potential is developed in the surfaces of a small piece of quartz, when this piece of quartz is under pressure or stress, from there this phenomenon was understood **Arnau (2004)**. The Term Piezoelectricity is derived from the Greek word ‘’ Piezo’’ which mean to press, then the whole word means electricity from press. What has been discovered by Curie brothers is the direct effect of piezoelectric materials. Mathematically, G. Lippman in 1881 proposed the inverse effect of piezoelectric, which is a deformation or stress is developed in piezoelectric materials when electric potential is applied and in the same year Curie brothers proved the inverse effect experimentally.

Until the first decade in 20^{th} century piezoelectric materials did not leave research labs. Put when the Frist World War was looming, developed countries started competing in weaponizing resulting in many inventions, among them was the first application of piezoelectric materials which is the Sonar made by Langevin 1917, **Rupitsch (2018)**. Where quartz was used in sonar at that time there were only two available piezoelectric materials, Rochelle salt and quartz. After that between the two World Wars other applications appeared such electric Oscillator and many others. United states, Russia, and Japan during the second World War individually discovered that Barium Titanate will earn piezoelectric behavior after electric polarization applied to it. From there the Piezoceramics era started by R. B. Gray in 1946 and became ‘The father of Piezoceramics’. The new material was applied in transducers, but it had some limitation such aging and the large temperature coefficient. To overcome these limitations many studies were carried out considering ion replacement to end up with the most famous piezoceramic PZT (Lead Zirconate Titanate) in 1954 by Bernard Jaffe **Uchino (2017)**.

1.1.1 Physics behind Piezoelectric effects:

Dielectric materials showed be defined to understand comprehensively the physics responsible for piezoelectric effects. Dielectric materials have the property of dimensions changing when an external electrical field is applied to them. This occurs as the positive charges and negative ones displace inside the material. In details, considering the material's crystals consist of anions as negative and cations as positive with chemical connections represented as springs, applying electric field will result in displacing cations and anions, one in the direction of the field and the other opposite to it, respectively causing deformation. **Figure 1.1** illustrate that.

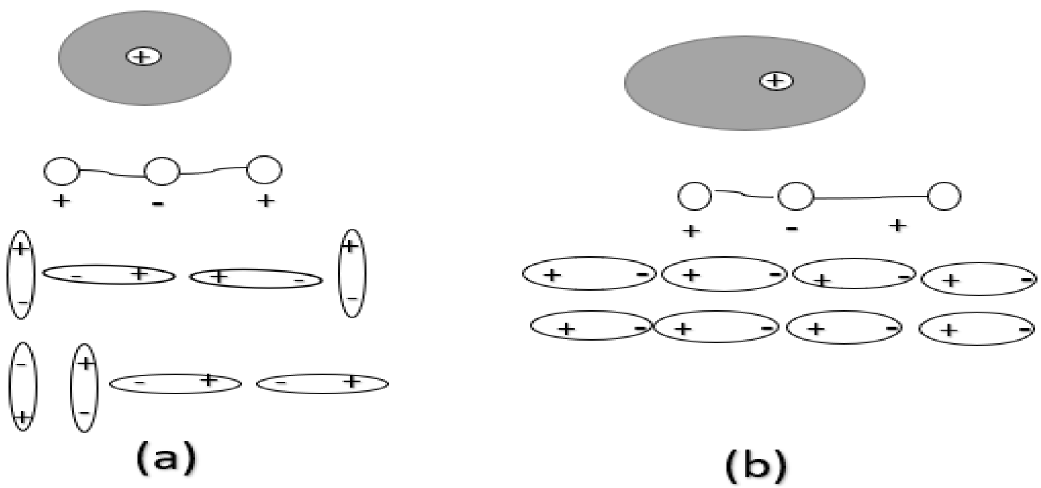


Figure 1.1 (a) shows anions and cations without applied electric field. (b) Shows the displacement in the bonds between cations and anions under electric field.

One of the most significant parameters of Dielectric materials is the electric Dipole Moment (p), which causes the response to outside stimuli. Electric dipole moment is the moment resulting from the distance between centers of positive and negative charges in an atom or molecule. And the summation of these vector moments in a piece of material called Electric Polarization (P) **Vijaya (2012)**.

Based on these parameters Dielectric materials are grouped in two, Polar and Nonpolar Dielectric materials. Nonpolar materials the dipole moment is induced by the applied electric field and disappears with its absence. While for polar ones there are electric dipoles, with net polarization equal to zero. But when they are subjected to electric field the electric dipole will orient at electric field direction and the material will generate finite polarization which rise with the rise of electric field.

In **Figure 1.2**, dielectric materials classified to two types, Centro Symmetric materials includes 11 crystal classes out of the total 32 crystal classes and Non-Centro Symmetric (21 Crystal classes) which known by piezoelectric materials.

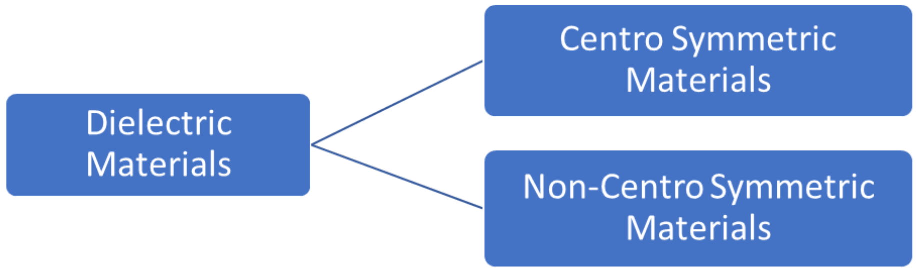


Figure 1.2 Classification of Dielectric Materials.

When Non-Centro Symmetric (piezoelectric materials) is subject to an electric field, the ions will move asymmetrically causing deforming or strain of crystal which is proportionate to the electric field applied. The strain could be compressive or extensive based on material polarity to the field applied polarity, when they are identical in direction tensile /extensive strain will be developed and if opposite a compressive strain is developed and this the Indirect/Inverse Piezoelectric Effect. The inverse piezoelectric effect which known by Actuating Effect is demonstrated in **Figure 1.3** .

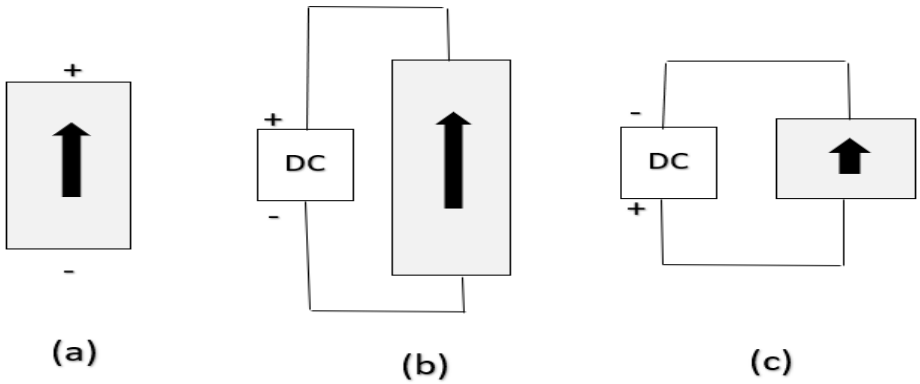


Figure 1.3 (a) polarization direction of piezoelectric material. (b) Extension strain resulted from electric field with identical direction to polarization. (c) Compression strain resulting from electric field with opposite direction to polarization.

Also, these materials have another character in which, showed in **Figure 1.4**, when a stress/pressure is applied externally to them the diploes will orient resulting in developing positive and negative charges across material's surfaces causing an electric field. These unequal charges were created as the electric dipoles forced to align in the same direction by the external stress/pressure at crystal sides where electrons on one side while deficiencies on the other side resulting in voltage generation.
When the stress is compressive on the material, negative voltage will be generated. While tension stress results in positive voltage. Also, known by sensing effect and the below figure illustrate it.

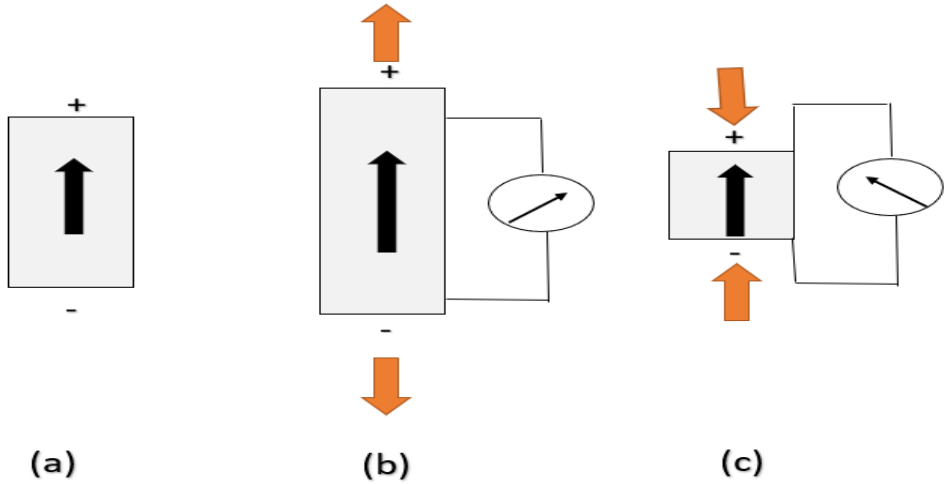


Figure 1.4 (a) polarization direction. (b) Tensile stress resulting in positive voltage. (c) Compressive stress result in negative voltage.

There is another effect that piezoelectric materials produce when they are stimulated by alternate current (AC) where the material extent and contract at the same frequency of the applied alternated field.

1.1.2 Piezoelectric Materials:

In this section some of the most used and common piezoelectric materials are explained. Generally piezoelectric materials divided to two main group: Naturally found and manmade ones. Natural piezoelectric materials such as Quartz, Cane Sugar, Rochelle salt, Topaz, Tourmaline, and Bones (possess some piezoelectric properties due to Collagen). Manufactured piezoelectric materials such as Lead Zirconated Titanate (PZT), Barium Titanate, Lead Titanate, Potassium Niobate, Lithium Niobate, Lithium Tungstate, Sodium Tungstate, Bismuth Ferrite, Sodium Niobate, Polyvinylidene Fluoride (PVDF) and Lead-Free piezoelectric materials.

This huge number of manmade piezoelectric materials is credited to the discovery of Ferroelectric materials which have properties such Spontaneous Polarization and Reversible Polarization. In addition, these materials can be poled, meaning generating permanent polarization in the material by applying appropriate electric field, where the ferroelectric domains start polarizing with the increase of electric field while keeping high temperature but lower than transition temperature, at the end all ferroelectric domains will be polarized at electric field direction. When the electric field is disconnected, and the material reach room temperature most of ferroelectric domains preserve their polarization as shown in **Figure 1.5**.

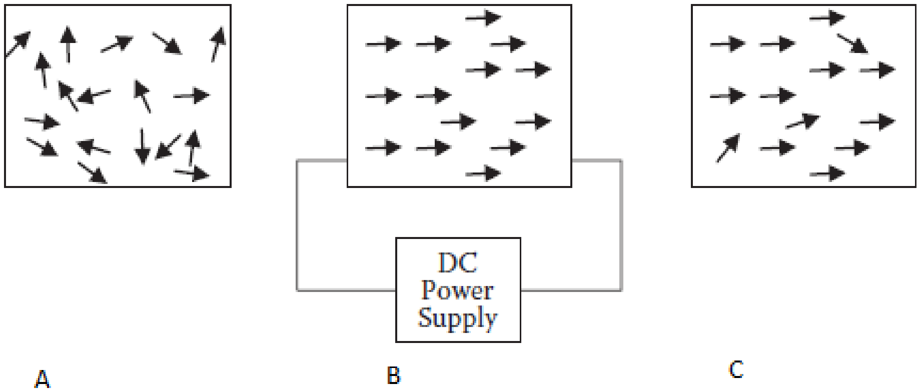


Figure 1.5 (A) Material before poling. (B) Material during Poling. (C) Material After Poling Vijaya (2012).

Quartz, PZT and PVDF will be further explained as following:

1.1.1.1 Quartz

Quartz is silicon dioxide but in crystal form. Quartz is considered from Natural piezoelectric materials. It has properties which are valuable, leading it to be used until now, such the high stiffness constant and long working life as it has good abilities to withstand changing in temperatures and environment. Usually used in the shape of thin plates such tuning forks shown in **Figure 1.6**. Applications that need accurate control of frequency such watches and transmitters, quartz oscillators are very suitable.

Figure 1.6 at the left is the natural Quartz crystal and at the right is the quartz tuning fork (Google).

1.1.1.2 Lead Zirconate Titanate (PZT)

Lead Zirconate Titanate have been discovered by a Japanese group of scientists, let by E.Sawaguchi but its piezoelectric properties have been discovered by the American researcher Bernard Jaffe and with the help of Clevite Corporation they accelerate the work on PZT and developed Hard and Soft versions of the material, thus PZT trademark was possessed by Clevite. PZT is a ferroelectric ceramic and is the most used manufactured piezoelectric material due to its' good mechanical and piezoelectric properties.

PZT is produced in a fine powder form and then formed to other shapes with different techniques, this gives PZT additional advantage. PZT has perovskite structure (3 component Structure) in this form ($A^{2+}B^{4+}X_3^{2-}$) having PbZr(Ti)O_3 chemical formula as it consists of two oxides; Lead Zirconate and Lead Titanate. **Figure 1.7** shows the structure of PZT.

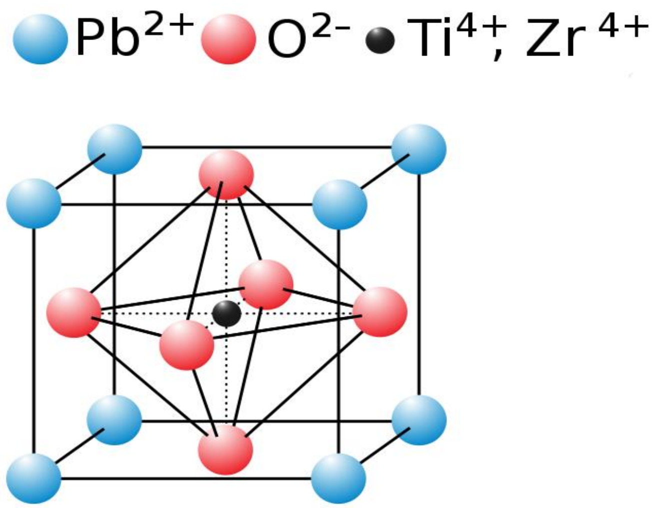


Figure 1.7 the Structure of single cell of PZT(Google).

Manufacturing techniques are used to process PZT and produce it as fine powder such as Solid-state reaction technique and Solgel technique. After that PZT powder can be shaped in certain shapes, mostly disks, thick or thin plates and cylinders, other techniques are used like pressing (Uniaxial pressing or Isostatic pressing), Tape casting for thick films and chemical vapor deposition for thin films and many others. PZT family has the highest piezoelectric coupling factor k with a curie point from $220 - 315c^o$.

1.1.1.3 Polyvinylidene Fluoride (PVDF)

PVDF is the most used and common polymer piezoelectric materials that has this formula $(C_2H_2F_2)_n$. They overpass piezoelectric ceramics in flexibility and the ability to be formed in large-area films. Their polymer properties give PVDF the ability to be manufactured in lower temperatures and with easer forming processes. But PVDF has drawbacks, as they have very low piezoelectric coefficients compared to piezoelectric ceramics, therefore using them as actuators are very limited but widely used as sensors.

1.1.3 Piezoelectric Applications:

Their applications are based on either working as actuators, sensors, or transducers (doing sensing and actuating functions together). For instance, in measurement technology piezoelectric are used as sensors. And in automotive field they are used in controlling fuel injections in diesel engines and parking sensors. While in the production process they are applied in ultrasonic welding and cleaning. In the medical field, piezoelectric materials are found in diagnostics devices such pregnancy tests and therapy devices such Lithotripsy devices.

Most gadgets such speakers, microphones, cameras, and injecting printers are made with piezoelectric parts that usually perform the main function of the device such voice converting to electricity in speakers. Adding to that in sports such reducing vibrations in tennis rackets, music in guitars sound captures.

Furthermore, piezoelectric materials are used in smarts structures which perform certain functions like noise active control and structural health monitoring.

In Structural health monitoring applications piezoelectric materials are used as sensors and actuators, where they sense, observe and analysis structural systems over time to detect changes such high stresses, cracks, and geometry changes due deformations then actuators will be controlled to overcome these stress and changes if possible. **Figure 1.8** shows different types of piezoelectric materials applications.

Also, one of the most important and game changers use of piezoelectric is the energy harvesting device wither from sound, vibration or movement of people, cars, or trains. As it is considered one of the renewable energy sources, which is not affected by environment and has long working life with Zero waste, **Figure 1.9** shows smart shoes with different piezoelectric energy harvester.

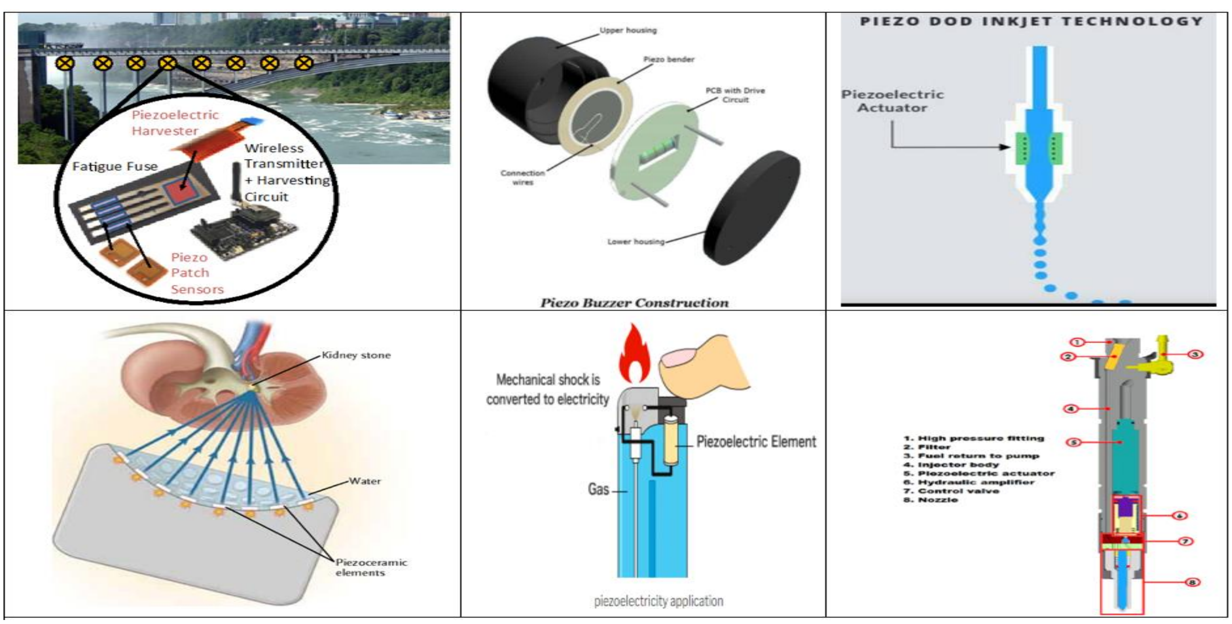


Figure 1.8 Different types of piezoelectric materials applications (Google).

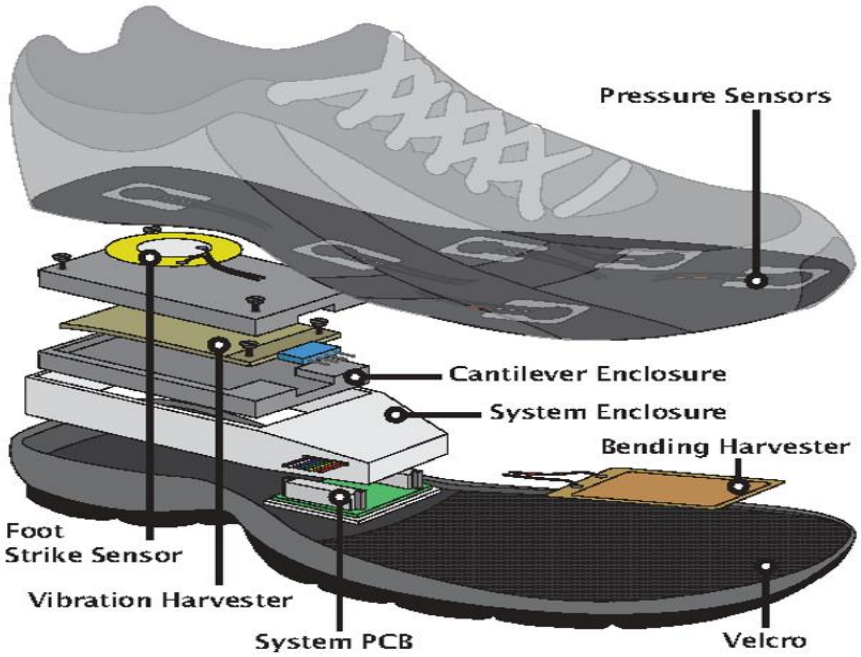


Figure 1.9 Cantilever Vibration piezoelectric Harvester embedded in shoes (Google).

Piezoelectric Physics

2.1 Constitutive Equations of Piezoelectric:

Constitutive equations are the equations that describe the electromechanical properties of piezoelectric materials. These equations used in this work are following IEEE guides on piezoelectricity representation, which is widely used and accepted.

Linear behavior of piezoelectric materials is considered in forming these equations, although at large electric field or stress piezoelectric materials behave nonlinearly to some extent.

Regarding direct piezoelectric effect, when piezoelectric material is under strain a voltage will generate through the material, and if the material is attached to electric electrodes the developed electric energy (voltage) can be collected.

From Hook's law for strain-stress:

$$\sigma = E * \varepsilon \tag{2.1}$$

where E is young's modulus (N/m^2), $\sigma$is stress (N/m^2) and ε is strain (m/m). Taking direct piezoelectric effect, additional electric terms will be added to the equation to fully represent piezoelectric materials (to account for electric behavior), after applying mathematical manipulations we get:

$$\varepsilon_i = S_{ij}^{E}\sigma_j + d_{mi}E_m \tag{2.2}$$

The invers piezoelectric effect can be represented from electric displacement equation, and adding mechanical terms to the equation to fully represent piezoelectric materials (to characterize mechanical behavior), after applying mathematical manipulations we get:

$$D_m = d_{mi}\sigma_i + \xi_{ik}^{\sigma}E_k \tag{2.3}$$

where, σ is stress vector (N/m^2), ε is strain vector (m/m), E is vector of applied electric field (V/m), ξ is permittivity (C/Vm), d is piezoelectric strain constants (m/V), S is compliance (m^2/N) which is equal to the inverse of stiffness $c = E\ young'smodulus$, and D is electric displacement (C/m^2). The E over compliance coefficient refers to constant electric field, while σover permittivity refers to constant stress, and $i, and\ j$ refers to the direction of electric potential. Equations (2.2) and (2.3) are the constitutive equations that governing piezoelectric structures **Vijaya (2012)**.

2.1.1 Piezoelectric Constants and Factors

Those are the constants and characteristics must be found in the material to be able to produce piezoelectric effects.

1- Electromechanical Coupling Factor (K):

It is an indicator of the effectiveness of piezoelectric materials in converting electrical energy into mechanical energy and vice versa.

For direct effect:

$$K^2 = \frac{Mechanical\ Energy\ Converted\ to\ Electrical\ Energy}{Input\ Mechanical\ Energy} \tag{2.4}$$

For inverse effect:

$$K^2 = \frac{Electrical\ Energy\ Converted\ to\ \ Mechanical\ Energy}{Input\ Electrical\ Energy} \tag{2.5}$$

Piezoelectric ceramics able to convert from 25% to 75% between the two forms of energy. But practically it depends on the design of piezoelectric structure and the path of the affecting provocateur.

2- Piezoelectric Strain (Charge) Constant (d):

It is the ratio between electrical charge generated per unit area (D) and the applied force, it is expressed at Coulomb/Newton (C/N).

Direct effect:

$$d = \frac{Charge\ Density}{Applied\ Stress} \tag{2.6}$$

Inverse effect:

$$d^* = \frac{Strain}{Electric\ field} \tag{2.7}$$

$$d = K * \sqrt{\xi_o * \xi * S^E} \tag{2.8}$$

Where, K is Electromechanical coupling factor, $\xi_o$is relative permittivity of free space($8.854 * 10^{-12} F/m$), ξ relative permittivity of the material (Dielectric Constant) and S^E is compliance constant ($10^{-12} m^2/N$).

3- Piezoelectric Voltage Constant (g):

The piezoelectric Voltage constant is the ratio of the electric field (E) produced to the mechanical stress applied and expressed as volt*meter/newton ($V.m/N$).

$$g = \frac{Field\ Develped}{Applied\ Stress} \tag{2.9}$$

$$g^* = \frac{Strain\ Developed}{Applied\ Charge\ Density} \tag{2.10}$$

And there are other piezoelectric coefficients such e, e^*, h and h^*. The following **Table 2.1** summarizes them and the relation between each other. The superscript * and ' to denote the inverse piezoelectric effect but in terms of value they are equal.

The relations between the coefficients are derived from the following equation connecting the charge density with the electric field:

$$D = \xi E \tag{2.11}$$

4- Dielectric Constant ξ:

It is the ratio of the permittivity of the material to the permittivity of free space. And giving by:

$$\xi = \frac{C_o * h_p}{\xi_o * A} \tag{2.12}$$

where A is area of electrodes (C_o) and is the measured capacitance at $1\ KH_z$ (F-Farade), ξ_o is relative permittivity of free space($8.854 * 10^{-12} F/m$)and $h_p$is the height between surfaces. **Figure 2.1** illustrate this equation.

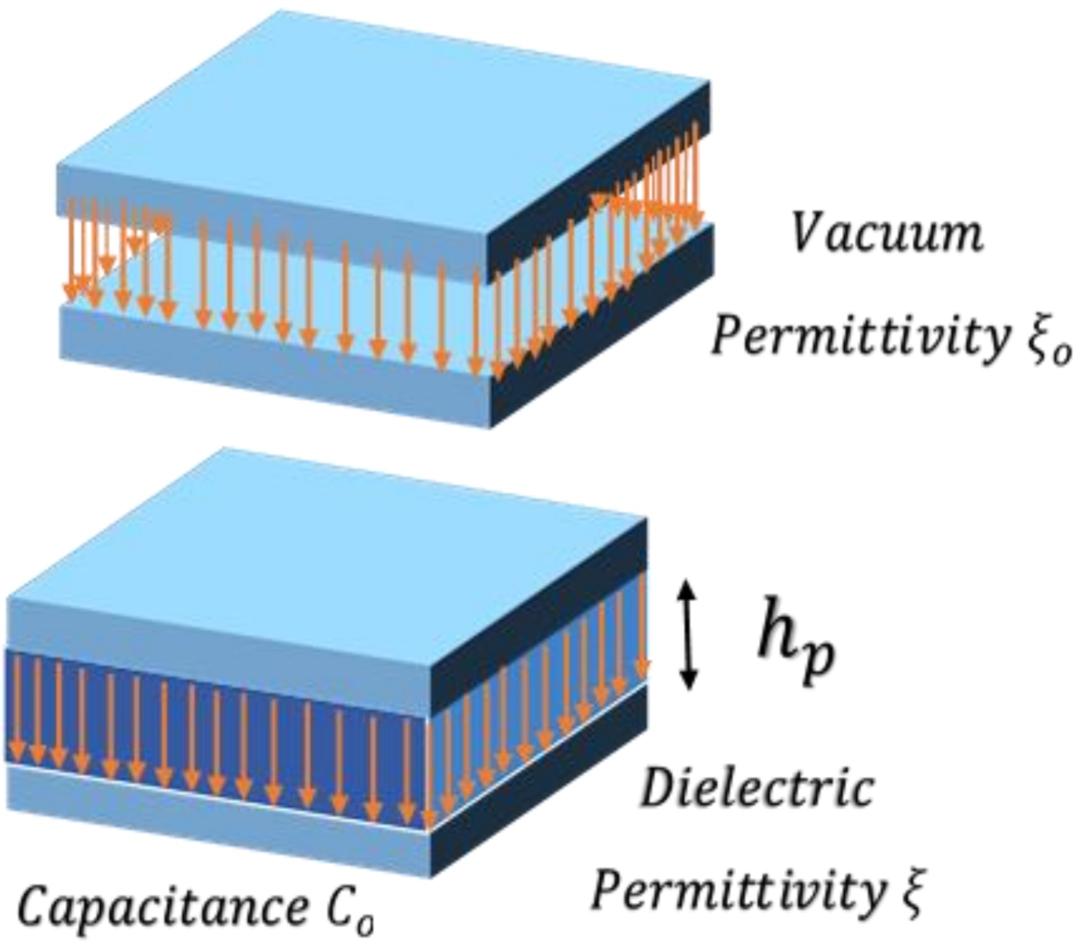


Figure 2.1 shows the dielectric constant and its connection to the capacitance and area.

Table 2.1 shows piezoelectric coefficient and their relation to each other.

Piezoelectric Coefficient	Explanation	Explanation in Equations	Unit	Relation to Other Coefficients
d	$\frac{Charge\ Density}{Applied\ Stress}$	D/σ	C/N	$d = g * \xi$ or $e' * S$
d^*	$\frac{Strain}{Electric\ field}$	ε/E	m/V	$d^* = g^* * \xi$
e	$\frac{Charge\ Density}{Strain}$	D/ε	C/m^2	$e = h * \xi$ or $d' * c$
e^*	$\frac{Stress}{Electric\ Field}$	σ/E	N/V.∗ m	$e^* = h^* * \xi$
g	$\frac{Field\ Develped}{Applied\ Stress}$	E/σ	$V.m/N$	$g = d/\xi$
g^*	$\frac{Strain\ Developed}{Applied\ Charge\ Density}$	ε/D	m^2/C	$g^* = d^*/\xi$
h	$\frac{Electric\ field}{Strain}$	E/σ	V/m	$h = e/\xi$
h^*	$\frac{Stress}{Applied\ Charge\ Density}$	σ/D	N/C	$h^* = e^*/\xi$

Piezoelectric materials have many other constants such as the Mechanical Quality Factor and its inverse Electric Loss Factor, Piezoelectric Voltage Constant, and the Stress-Polarization Factor, but their usage depends on the application of piezoelectric material.

In 3D the constitutive equations can be written in matrix form as:

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}\end{bmatrix} * \begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix} + \begin{bmatrix} d_{11} & d_{21} & d_{31}\\ d_{12} & d_{22} & d_{32}\\ d_{13} & d_{23} & d_{33}\\ d_{14} & d_{24} & d_{34}\\ d_{15} & d_{25} & d_{35}\\ d_{16} & d_{26} & d_{36}\end{bmatrix} * \begin{bmatrix}E_1\\ E_2\\ E_3\end{bmatrix} \tag{2.13}$$

$$\begin{bmatrix}D_1\\ D_2\\ D_3\end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16}\\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26}\\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36}\end{bmatrix} * \begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix} + \begin{bmatrix}\xi_{11}^{\sigma} & \xi_{12}^{\sigma} & \xi_{13}^{\sigma}\\ \xi_{21}^{\sigma} & \xi_{22}^{\sigma} & \xi_{23}^{\sigma}\\ \xi_{31}^{\sigma} & \xi_{32}^{\sigma} & \xi_{33}^{\sigma}\end{bmatrix}\begin{bmatrix}E_1\\ E_2\\ E_3\end{bmatrix} \tag{2.14}$$

Considering a general piezoelectric device shown in **Figure 2.2**. As piezoelectric materials can be poled in only one direction (i.e., usually the thickness direction) and generalizing isotropic properties for piezoelectric material, the above matrices can be written as following:

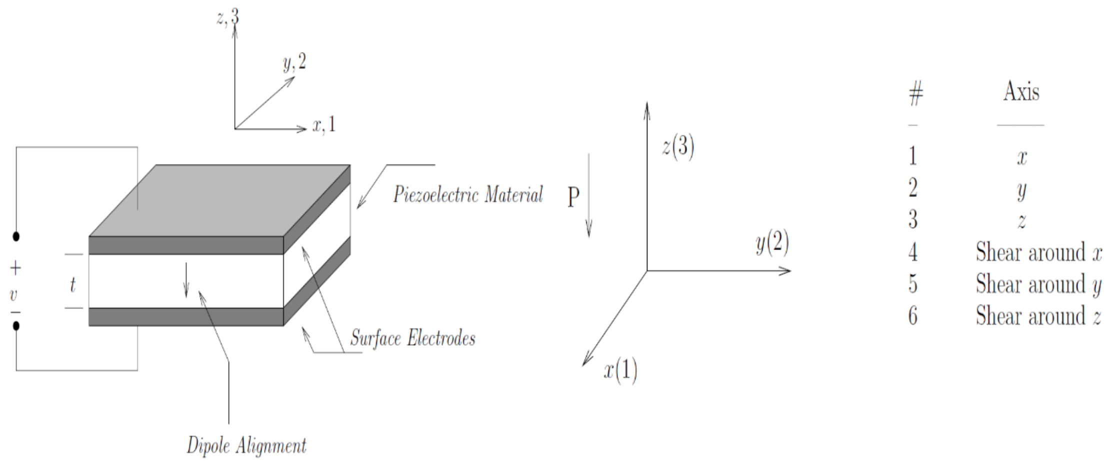


Figure 2.2 General piezoelectric device with its coordinate system Vijaya (2012).

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix}=\begin{bmatrix}S_{11} & S_{12} & S_{13} & 0 & 0 & 0\\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0\\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & S_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & S_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & 2(S_{11}-S_{12})\end{bmatrix}*\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ d_{31} & d_{31} & d_{33}\\ 0 & d_{15} & 0\\ d_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}*\begin{bmatrix}E_1\\ E_2\\ E_3\end{bmatrix} \quad (2.15)$$

$$\begin{bmatrix}D_1\\ D_2\\ D_3\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0 & d_{15} & 0\\ 0 & 0 & 0 & d_{15} & 0 & 0\\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0\end{bmatrix}*\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix}+\begin{bmatrix}\xi_{11}^{\sigma} & 0 & 0\\ 0 & \xi_{22}^{\sigma} & 0\\ 0 & 0 & \xi_{33}^{\sigma}\end{bmatrix}\begin{bmatrix}E_1\\ E_2\\ E_3\end{bmatrix} \quad (2.16)$$

The assumptions made created zero terms and equal terms. The compliance equal terms are:

$$S_{11}=S_{22}, \qquad S_{13}=S_{31}=S_{23}=S_{32}, \qquad S_{12}=S_{21}, \\ S_{44}=S_{55,}, \qquad S_{66}=2(S_{11}-S_{12}) \quad (2.17)$$

The non-zero piezoelectric strain constants:

$$d_{31}=d_{13}, \qquad d_{15}=d_{51} \quad (2.18)$$

The non-zero dielectric coefficients are:

$$\xi_{11}^{\sigma}=\xi_{22}^{\sigma}\neq\xi_{33}^{\sigma} \quad (2.19)$$

These equations are good for all piezoelectric ceramics, but when considering PVDF the term d_{15}in equation (2.16) at the second column should be replace by d_{25}, as PVDF produce non-isotropic electric fields at direction 1 and 2.

The subscript in the piezoelectric coefficient d_{ij} described as following; the second number (j) refers to two things the direction of the external stimulus and the face affected by the stimulus while the first number (i) refers to the face of the piezoelectric device where the response is developed at. And there are three modes. First Mode d_{31} or e_{31} : The Transverse mode when the stimulus effects on face 1 in direction 1 causing response in face 3. Second Mode d_{33} or e_{33}: The Longitudinal mode where the stimulus impacts in direction 3 on face 3 resulting in response on face 3. Lastly, The Shear mode d_{15} or e_{15} where the stimulus is parallel to face 1 but in direction 3 resulting in response on face 1 and this for both direct and inverse piezoelectric effects.

As the piezoelectric coefficients get a subscript to define their status, the piezoelectric coupling factor K also has its own subscript definition which are used to define piezoelectric material properties in materials data sheet. It has K_{33} when the response at direction 3 and stimulus at direction 3 too, but for thin discs when the polarization direction is the thickness direction the subscript becomes K_{t}. K_{31} when the stimulus at direction 1 and response at direction 3. For thin discs when the polarization at thickness direction (direction 3) and the response in thickness direction while the stimulus is along radial direction the superscript becomes K_{p} **Vijaya (2012)**.

2.2 Finite Element method of Piezoelectric Materials

Finite element method (FEM) is a numerical method used to solve structural systems, which solving them using analytical methods is very hard and time consuming.

Finite element method provides a faster way and easier to solve problems, which is based on the concept of dividing the structural system into finite number of elements and solve each element a time and then assembling all elements' solutions into one to give a numerical solution to the whole system.

This method is easy to apply in simple problems but when the problem is complicated or large it will be hard to complete all elements' solutions by hand.

Therefore, the finite element method has been used extensively after the development of finite element solving software such as **ANSYS**.

This software provides the ability of virtual prototyping and play as simulations platform, which is better than running costly experiments.
And taking σ and e as the main character for the constitutive equations we get:

Direct effect:

$$\sigma = c^{E}\varepsilon - \mathrm{e}E \qquad (2.20)$$

Inverse effect:

$$\mathrm{D} = \mathrm{e}\varepsilon + \xi^{\sigma}E \qquad (2.21)$$

Electric potential can be represented by in electric field:

$$E = -\nabla\phi \qquad (2.22)$$

The relationship between displacement and strain tensor can be represented by:

$$\varepsilon = Bu \tag{2.23}$$

Where, B equals:

$$B = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \tag{2.24}$$

While the reaction of the material considering mechanics is:

$$div\ (\sigma) = \rho \frac{\partial^2 u}{\partial t^2} \tag{2.25}$$

Where, density is referred to by ρ.

And the reaction of the material considering electrics is:

$$div\ (D) = 0 \tag{2.26}$$

As the absence of free charges at the medium of piezoelectric. The above equations describe the response of piezoelectric structures and with appropriate boundary conditions can be solved.

When applying the finite element method and dividing piezoelectric structure into a number of very small elements connected to each other by nodes, each nodes has variables known by degrees of freedom (D.O.F). These variables may include displacement, electric potential and so on. These variables can be defined with polynomial interpolation function (N_i) at any point in the structure. **Figure 3** below illustrate a part of piezoelectric structure divide to small elements, using 2D element with 8 nodes **Vijaya (2012)**.

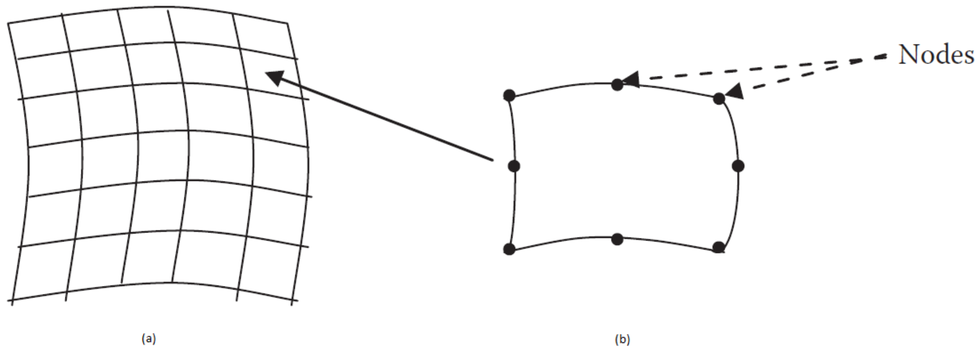


Figure 3 (a) a part of piezoelectric structure divided to many elements, (b) type o the element is 2D with 8-nodes Vijaya (2012).

Considering element with n nodes, the displacement of a point(x, y, z), indicated by $u(x, y, z)$ is represented considering nodal displacement values $\hat{u}(x_i, y_i, z_i)$ by:

$$u(x, y, z) = N_u\hat{u}(x_i, y_i, z_i) \quad i = 1,2, \dots, n \tag{2.27}$$

Where n is nodes' number in the element, N_u is displacement interpolation function defined within element as a set of natural coordinates $\zeta, \eta \; and \phi$.

In the same way, the potential at the same point indicated by $\varphi(x, y, z)$ is represented at nodal potential values by $\hat{\varphi}(x_i, y_i, z_i)$:

$$\varphi(x, y, z) = N_\varphi\hat{\varphi}(x_i, y_i, z_i) \quad i = 1,2, \dots, n \tag{2.28}$$

Where, N_φ is displacement interpolation function.

Equations (2.22) and (2.23), variational method of approximation have been applied to them. And where interpolation functions N_i matching weight function Galerkin approach is used.

From (2.23) and (2.27):

$$\varepsilon = Bu = BN_u\hat{u} = B_u\hat{u} \tag{2.29}$$

And from (2.22) and (2.28):

$$E = -\nabla\phi = -\nabla\left(N_\varphi\hat{\varphi}\right) = -B_\varphi\hat{\varphi} \tag{2.30}$$

Using (2.25) and (2.26) the approximations' residuals are attained, and weighted integrals are developed:

$$\int_\Omega N_u[\nabla * \sigma - \rho\frac{\partial^2 u}{\partial t^2}]d\Omega = 0 \tag{2.31}$$

$$\int_\Omega N_\varphi[\nabla * \mathrm{D}]d\Omega = 0 \tag{2.32}$$

Where N_φ, N_u are weighted functions.

Substitute for σ and D from equations (2.20) and (2.21) using equations (2.27), (2.28), (2.29) and (2.30) in equations (2.31) and (2.32):

$$\int_{\Omega} N_u[\nabla * (Y^E B_u \hat{u} + \mathrm{e} B_\varphi \hat{\varphi})] d\Omega - \int_{\Omega} N_u \rho \frac{\partial^2}{\partial t^2}(N_u \hat{u}) d\Omega = 0 \tag{2.33}$$

$$\int_{\Omega} N_\varphi[\nabla * (\mathrm{e} B_u \hat{u} - \xi^\sigma B_\varphi \hat{\varphi})] d\Omega = 0 \tag{2.34}$$

Integrating equations (2.33) and ((2.34) by parts with boundary condition implemented, resulting in:

$$m\hat{\hat{u}} + d_{uu}\hat{u} + k_{uu}\hat{u} + k_{u\varphi}\hat{\varphi} = f_B + f_S + f_P \tag{2.35}$$

$$k_{u\varphi}\hat{u} + k_{\varphi\varphi}\hat{\varphi} = q_s + q_p \tag{2.36}$$

$\hat{\hat{u}}$, $\hat{u}$, and $\hat{u}$ are the acceleration, velocity, and displacement, respectively. $\hat{\varphi}$ is the nodal electric potential. $d_{uu}\hat{\hat{u}}$ in equation (2.36) is added to account for mechanical damping. m is mass matrix:

$$m = \iiint \rho\, {N_u}^t N_u dV \tag{2.37}$$

d_{uu} is the mechanical damping matrix:

$$d_{uu} = \alpha \iiint \rho\, {N_u}^t N_u dV + \beta \iiint {B_u}^t Y^E B_u dV \tag{2.38}$$

$k_{uu}$is the mechanical stiffness matrix:

$$k_{uu} = \iiint {B_u}^t Y^E B_u dV \tag{2.39}$$

$k_{u\varphi}$ is the piezoelectric stiffness matrix:

$$k_{u\varphi} = \iiint {B_u}^t e B_\varphi dV \tag{2.40}$$

$k_{\varphi\varphi}$is the dielectric stiffness matrix:

$$k_{\varphi\varphi} = \iiint {B_\varphi}^t \xi^\sigma B_\varphi dV \tag{2.41}$$

f_B, f_S and f_P are the body, surface and point external forces affecting the element, correspondingly. While q_s, and q_p are the electrical charges at the surface and point on the element, sequentially.

All the above equations represent only one element. Completing the finite element method by assembling all single elements equation to get the global equations:

$$M\ddot{u} + D_{uu}\dot{u} + K_{uu}u + K_{u\varphi}\varphi = F_B + F_S + F_P \tag{2.42}$$

$$K_{u\varphi}u + K_{\varphi\varphi}\varphi = Q_s + Q_P \tag{2.43}$$

$\ddot{u}$, $\dot{u}$, u, φ, F_B, F_S, F_P, Q_s and Q_P are the assembled global field quantities which are vectors. M, D_{uu}, K_{uu}, $K_{u\varphi}$, and $K_{\varphi\varphi}$ are the assembled matrices.

The above equations are written in the following matrix form:

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} * \begin{Bmatrix} \ddot{u} \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} D_{uu} & 0 \\ 0 & 0 \end{bmatrix} * \begin{Bmatrix} \dot{u} \\ \dot{\varphi} \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{u\varphi} \\ K_{u\varphi} & K_{\varphi\varphi} \end{bmatrix} * \begin{Bmatrix} u \\ \varphi \end{Bmatrix} = \begin{Bmatrix} F_B + F_S + F_P \\ Q_s + Q_P \end{Bmatrix} \quad (2.44)$$

2.3 Piezoelectric in Finite Element Software

Analyzing using FEM in software consists of three steps which are preprocessing followed by solving and ended by post-processing.

A- Preprocessing:

In this step the structure model is created either in 2D or 3D, then a suitable element which can fully represent the physics under study is selected. Table 2 shows different types of elements used with piezoelectric structures. The difference between piezoelectric structures and traditional structures is that they have additional degree of freedom at each node which is the electric potential (V). Meshing is the process of dividing the structure to number of finite elements with the selected element.

Material properties adding is also in this step, for piezoelectric structures the concerned properties are piezoelectric coefficient ($[d]$ or $[e]$), dielectric constant, compliance or stiffness and density.

Normally piezoelectric structures are attached to electrodes to apply or collect electric potential, the electrode process is done here too by adding additional degree of freedom of electric potential to all nodes in the faces attached to the electrodes.

Then boundary conditions are applied either mechanical boundary conditions (stress, displacement or fixed) or electric one (electric potential '$Voltage$' $= 0$ for grounding, or with a value in actuating) or mix of the two. Then selecting analysis type or several different analyses.

For piezoelectric structure 3 analyses are the commonly selected; Modal analysis (to get mode shapes and natural frequencies), Harmonic analysis (system response to external load at specific frequencies), and Transient analysis (system response to external load depending on time).

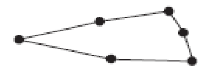
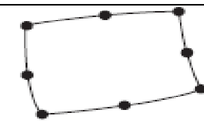
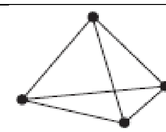
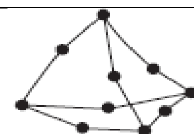
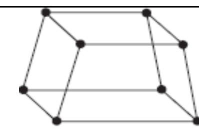
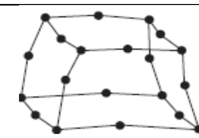
B- Solving:

Here the software takes the post-processing inputs and create all needed equations and solve them considering the boundary conditions.

C- Post-processing:

After solving the problem, the outputs of solution can be obtained in many ways such as graphs, tables, or animations.

Table 2 shows elements used with piezoelectric materials.

No.	Element Type	Shape
1	2D 3-Noded triangular elements	
2	2D 6-Noded curvilinear triangular elements	
3	2D 4-Noded quadrilateral elements	
4	2D 8-Noded curvilinear quadrilateral elements	
5	3D 4-Noded tetrahedral elements	
6	3D 10-Noded curvilinear tetrahedral elements	
7	3D 8-Noded rectangular prism elements (brick elements)	
8	3D 20-Nodedcurvilinear rectangular prism elements (brick elements)	

2.3.1 Piezoelectric in ANSYS APDL:

Piezoelectric materials and structures are one of the coupled field applications. Coupled field is when a finite element that couples the effects of interrelated physics within the element matrices or load vectors, which contain all necessary terms required for coupling. And coupled element has two matrices Strong or Weak (known as Sequential). Piezoelectric element has Strong Matrix type. The coupling in piezoelectric is between structural and electrical fields. Piezoelectric analyses may be:

1- Static: used to determine effects of steady-state of mechanical or electric loads, such deflection of piezoelectric actuator under constant voltage or voltage of piezoelectric sensor under constant strain. SPARSE matrix direct solver is recommended is recommended to be used for solving. For static analyses, piezoelectric materials are characterized by: Structural elasticity, Piezoelectric Coupling, and Dielectric permittivity.

2- Transient.
3- Modal: including prestressed modal (using linear perturbation): used to determine vibration characteristics, including the resonant frequencies and mode shapes. Piezoelectric actuators often driven near resonance for higher efficiency. And if AC is applied near resonance frequency, deflection amplitude increases drastically. Modal analysis is typically used to determine working frequencies of piezoelectric device. Also, it is the starting point for piezoelectric harmonic analysis. Block Lanczos direct solver is recommended is recommended to be used for solving.
4- Harmonic (FULL and MSUP): including prestressed harmonic (using linear perturbation): used to determine response to sinusoidally varying loading to calculates response within specified frequency range or frequency range typically spans resonance frequencies determined from modal analysis. SPARSE matrix direct solver is recommended to be used for solving. For dynamic analyses, additional data are required: Density, Structural damping (Experiments extracted), and Dielectric damping **Vijaya (2012)**.

2.3.2 Steps of Modeling Piezoelectric in ANSYS

1- Activate the multiphase option by choosing Electric and Structural Preferences.
2- Select element types of piezoelectric material from the list below:
  - Plane 13 a 2D 4-Noded quadrilateral element. For 2-dimension squired bodies.
  - Plane 223 a 2D 8-Node element.
  - Solid 226 a 3D 20-Node Coupled-Field Solid element.
  - Solid 227 a 3D 10-Node element.
  - Solid 98 a 3D 10-Node element. For complicated and curved geometries.
  - Solid 5 a 3D 8-Node Element. For cubed geometries.
3- Defining Piezoelectric Material:

This step includes many other steps inside it to define all the properties of the piezoelectric materials including poling direction, as following:

a- Defining Stiffness $[C]$/Compliance $[S]$ matrix:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \; and \; [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \quad (2.45)$$

The matrices in (2.45) are in IEEE form, which is not applicable to the ANSYS APDL, ANSYS APDL form is as follow:

$$[C]=\begin{bmatrix} C_{11} & & & & & \\ C_{21} & C_{22} & & & & \\ C_{31} & C_{32} & C_{33} & & & \\ C_{41} & C_{42} & C_{43} & C_{66} & & \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{44} \end{bmatrix} and [S]=\begin{bmatrix} S_{11} & & & & & \\ S_{21} & S_{22} & & & & \\ S_{31} & S_{32} & S_{33} & & & \\ S_{41} & S_{42} & S_{43} & S_{66} & & \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{44} \end{bmatrix} \quad (2.46)$$

Where interchanging the places of C_{66} with C_{44}, and in compliance matrix S_{66} with S_{44}.

b- Defining Piezoelectric Coefficient stress form $[e]$ or strain form $[d]$:

With them the polarization direction can be defined, in X, Y, or Z as below and ANSYS has different form than **IEEE** form. **IEEE** form is :

$$[e]=\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{41} & e_{42} & e_{43} \\ e_{51} & e_{52} & e_{53} \\ e_{61} & e_{62} & e_{63} \end{bmatrix} \text{ for 3D models and } [e]=\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \\ e_{31} & e_{32} \\ e_{41} & e_{42} \end{bmatrix} \text{ for 2D models} \quad (2.47)$$

$$[d]=\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \\ d_{41} & d_{42} & d_{43} \\ d_{51} & d_{52} & d_{53} \\ d_{61} & d_{62} & d_{63} \end{bmatrix} \text{ for 3D models and } [e]=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \\ d_{41} & d_{42} \end{bmatrix} \text{ for 2D models} \quad (2.48)$$

IEEE to ANSYS Form, interchanging the last 3 rows as fallowing:

$$\begin{matrix} x \\ y \\ z \\ yz \\ xz \\ xy \end{matrix} \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{41} & e_{42} & e_{43} \\ e_{51} & e_{52} & e_{53} \\ e_{61} & e_{62} & e_{63} \end{bmatrix} IEEE ------\rightarrow ANSYS \begin{matrix} x \\ y \\ z \\ xy \\ yz \\ xz \end{matrix} \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{61} & e_{62} & e_{63} \\ e_{41} & e_{42} & e_{43} \\ e_{51} & e_{52} & e_{53} \end{bmatrix} \quad (2.49)$$

And the polarization is defined as following for $[e]$, and the same thing applied for d:

$$\begin{bmatrix} e_{33} & 0 & 0 \\ e_{31} & 0 & 0 \\ e_{31} & 0 & 0 \\ 0 & e_{15} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & e_{15} \end{bmatrix}^{\substack{\text{X} \\ \text{Polarization}}} \rightarrow \begin{bmatrix} 0 & e_{31} & 0 \\ 0 & e_{31} & 0 \\ 0 & e_{33} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & e_{15} \\ 0 & 0 & 0 \end{bmatrix}^{\substack{\text{Y} \\ \text{Polarization}}} \rightarrow \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & 0 & 0 \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \end{bmatrix}^{\substack{\text{Z} \\ \text{Polarization}}} \quad (2.50)$$

In addition, to change the direction of polarization on the same axis, meaning from forward to inverse direction, e.g., from → in x axis to ← in x axis, we just add the value with negative sign in front of them.

$$\begin{bmatrix} -e_{33} & 0 & 0 \\ -e_{31} & 0 & 0 \\ -e_{31} & 0 & 0 \\ 0 & -e_{15} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -e_{15} \end{bmatrix}^{\substack{-\text{X} \\ \text{Polarization}}} \rightarrow \begin{bmatrix} 0 & -e_{31} & 0 \\ 0 & -e_{31} & 0 \\ 0 & -e_{33} & 0 \\ -e_{15} & 0 & 0 \\ 0 & 0 & -e_{15} \\ 0 & 0 & 0 \end{bmatrix}^{\substack{-\text{Y} \\ \text{Polarization}}} \rightarrow \begin{bmatrix} 0 & 0 & -e_{31} \\ 0 & 0 & -e_{31} \\ 0 & 0 & -e_{33} \\ 0 & 0 & 0 \\ 0 & -e_{15} & 0 \\ -e_{15} & 0 & 0 \end{bmatrix}^{\substack{-\text{Z} \\ \text{Polarization}}} \quad (2.51)$$

c- Dielectric (Permittivity):

It defined by matrix in equation (31) which also affected by the polarization direction as following:

$$\begin{bmatrix} \xi_{33}^{\sigma} & 0 & 0 \\ 0 & \xi_{11}^{\sigma} & 0 \\ 0 & 0 & \xi_{11}^{\sigma} \end{bmatrix}^{\substack{\text{X} \\ \text{Polarization}}} \begin{bmatrix} \xi_{11}^{\sigma} & 0 & 0 \\ 0 & \xi_{33}^{\sigma} & 0 \\ 0 & 0 & \xi_{11}^{\sigma} \end{bmatrix}^{\substack{\text{Y} \\ \text{Polarization}}} \begin{bmatrix} \xi_{11}^{\sigma} & 0 & 0 \\ 0 & \xi_{11}^{\sigma} & 0 \\ 0 & 0 & \xi_{33}^{\sigma} \end{bmatrix}^{\substack{\text{Z} \\ \text{Polarization}}} \quad (2.52)$$

In 2D the non-used direction will be omitted.

Notes:

- In many times we may need to calculate piezoelectric factors and constants from the Material Data Sheet (MDS), Appendix (C) include a Code to calculate piezoelectric matrices and constants using the relationships of the constitutive equations of piezoelectric materials and the relationships between the piezoelectric constant.
- The main differences between modeling piezoelectric materials in ANSYS APDL and Ansys Workbench are first in workbench we have the option to select between the IEEE form or the APDL form. Secondly, we can control the direction of polarization of piezoelectric element by changing its coordinate after assigning part coordinate to it (not the general coordinate of all the system, if the system has many piezoelectric parts poled in different directions)

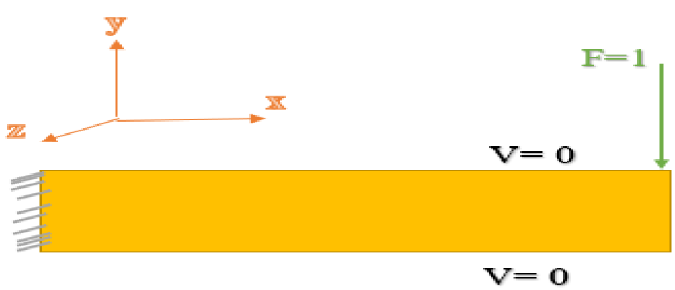
Case Study

We will apply the above concepts in **ANSYS APDL** to model a piezoelectric Actuator & sensor in 3D. The actuator is made of PZT-4 with properties shown in the table below, and it is fixed from one side and free in the other side. The actuator has a rectangular shape with the following dimensions: Length = 10 mm, Thickness = 1.5 mm and for the 3D the Width is 2 mm.

Piezoelectric Material (PZT-4)	
Elastic Stiffness Parameters (GPa)	$c_{11} = 139,$ $c_{12} = 67.8$ $c_{13} = 74.3,$ $c_{33} = 115$ $c_{44} = 2.56$
Permittivity (C/Vm)	$\xi_{11} = 6.45 * 10^{-9}$ $\xi_{33} = 5.62 * 10^{-9}$
Density	$7500\ Kg/m^3$
Piezoelectric Constant (C/m^2)	$e_{31} = -5.2,$ $e_{33} = 15.1$ $e_{15} = 12.7$

For the actuator is under 1 volt current/charge applied on the upper surface while the bottom surface is grounded (V=0). The polarization of the structure is taken in the Y direction.

For the Sensor we will consider the same material & geometry with fixation in one side and free in the other side, but the upper and lower surfaces are grounded (V=0) and there is a downward force at the free side having a value of 1 N.



To See Step by Step Solution in ANSYS APDL Watch this video:
https://www.youtube.com/watch?v=lHSI-uu24qo from minute 27 till the end.

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Appendices

Appendix A: APDL Script of Actuator Example

```
!------------------------------Preference Selection-------------------
KEYW,PR_SET,1
KEYW,PR_STRUC,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAG,1
KEYW,MAGNOD,0
KEYW,MAGEDG,0
KEYW,MAGHFE,0
KEYW,MAGELC,1
KEYW,PR_MULTI,1
!------------------------------Element Selection----------------------
ET,1,SOLID5

KEYOPT,1,1,3
KEYOPT,1,3,0
KEYOPT,1,5,0
!-----------------------------Material Definition---------------------
TB,ANEL,1,1,21,0
TBTEMP,0
TBDATA,,139e+9,67.8e+9,74.3e+9,0,0,0          !Stiffness Matrix
TBDATA,,139e+9,74.3e+9,0,0,0,115e+9
TBDATA,,0,0,0,142.4e+9,0,0
TBDATA,,2.56e+9,0,2.56e+9,,,

TB,PIEZ,1,,,0
TBMODIF,1,1,
TBMODIF,1,2,-5.2                       ! Piezoelectric Matrix
TBMODIF,1,3,
TBMODIF,2,1,
TBMODIF,2,2,-5.2
TBMODIF,2,3,
TBMODIF,3,1,
```

```
TBMODIF,3,2,15.1
TBMODIF,3,3,
TBMODIF,4,1,12.7
TBMODIF,4,2,
TBMODIF,4,3,
TBMODIF,5,1,
TBMODIF,5,2,
TBMODIF,5,3,12.7
TBMODIF,6,1,
TBMODIF,6,2,
TBMODIF,6,3,

MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,PERX,1,,6.45e-9/8.854e-12            ! Dielectric Constant Matrix
MPDATA,PERY,1,,5.62e-9/8.854e-12
MPDATA,PERZ,1,,6.45e-9/8.854e-12
MPTEMP,,,,,,,,

MPTEMP,1,0
MPDATA,DENS,1,,7500                          ! Density
!----------------------------Creating Geometry--------------------
BLOCK,0,0.001,0,.00015,0,0.0002,
!--------------------------Meshing----------------------------
TYPE,   1
MAT,       1
REAL,
ESYS,       0
SECNUM,

CM,_Y,VOLU
VSEL, , , ,       1
CM,_Y1,VOLU
CHKMSH,'VOLU'
CMSEL,S,_Y
MSHAPE,0,3d
MSHKEY,1
```

```
VMESH,_Y1
MSHKEY,0
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
!-----------------------Coupled Field (representing the electrodes)----------
FLST,4,24,1,ORDE,8
FITEM,4,1
FITEM,4,5
FITEM,4,-9
FITEM,4,31
FITEM,4,34
FITEM,4,-38
FITEM,4,61
FITEM,4,-72

CP,1,VOLT,P51X                ! for the upper surface

FLST,4,24,1,ORDE,8
FITEM,4,2
FITEM,4,10
FITEM,4,13
FITEM,4,-16
FITEM,4,25
FITEM,4,-30
FITEM,4,49
FITEM,4,-60

CP,2,VOLT,P51X             ! Bottom surface

!-----------------Boundary Conditions--------------------------
ANTYPE,0               ! Analysis type

FLST,2,16,1,ORDE,12
FITEM,2,1
FITEM,2,-4
FITEM,2,25
```

```
FITEM,2,34
FITEM,2,39
FITEM,2,-40
FITEM,2,49
FITEM,2,-50
FITEM,2,63
FITEM,2,-64
FITEM,2,73
FITEM,2,-76

D,P51X,UX,0, , , ,UY,UZ, , , ,        ! Fixation at one side

FLST,2,24,1,ORDE,8
FITEM,2,2
FITEM,2,10
FITEM,2,13
FITEM,2,-16
FITEM,2,25
FITEM,2,-30
FITEM,2,49
FITEM,2,-60

D,P51X,VOLT,0           ! Grounding at bottom surface

FLST,2,24,1,ORDE,8
FITEM,2,1
FITEM,2,5
FITEM,2,-9
FITEM,2,31
FITEM,2,34
FITEM,2,-38
FITEM,2,61
FITEM,2,-72

D,P51X,VOLT,1           ! applying 1 v at upper surface
!-------------------Solve---------------
SOLVE
```

Appendix B: APDL Script of Sensor Example

```
!------------------------------Preference Selection-------------------
KEYW,PR_SET,1
KEYW,PR_STRUC,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAG,1
KEYW,MAGNOD,0
KEYW,MAGEDG,0
KEYW,MAGHFE,0
KEYW,MAGELC,1
KEYW,PR_MULTI,1
!------------------------------Element Selection----------------------
ET,1,SOLID5

KEYOPT,1,1,3
KEYOPT,1,3,0
KEYOPT,1,5,0
!-----------------------------Material Definition---------------------
TB,ANEL,1,1,21,0
TBTEMP,0
TBDATA,,139e+9,67.8e+9,74.3e+9,0,0,0          !Stiffness Matrix
TBDATA,,139e+9,74.3e+9,0,0,0,115e+9
TBDATA,,0,0,0,142.4e+9,0,0
TBDATA,,2.56e+9,0,2.56e+9,,,

TB,PIEZ,1,,,0
TBMODIF,1,1,
TBMODIF,1,2,-5.2                    ! Piezoelectric Matrix
TBMODIF,1,3,
TBMODIF,2,1,
TBMODIF,2,2,-5.2
TBMODIF,2,3,
TBMODIF,3,1,
TBMODIF,3,2,15.1
TBMODIF,3,3,
TBMODIF,4,1,12.7
```

```
TBMODIF,4,2,
TBMODIF,4,3,
TBMODIF,5,1,
TBMODIF,5,2,
TBMODIF,5,3,12.7
TBMODIF,6,1,
TBMODIF,6,2,
TBMODIF,6,3,

MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,PERX,1,,6.45e-9/8.854e-12            ! Dielectric Constant Matrix
MPDATA,PERY,1,,5.62e-9/8.854e-12
MPDATA,PERZ,1,,6.45e-9/8.854e-12
MPTEMP,,,,,,,,

MPTEMP,1,0
MPDATA,DENS,1,,7500                 ! Density
!----------------------------Creating Geometry--------------------
BLOCK,0,0.001,0,.00015,0,0.0002,
!---------------------------Meshing----------------------------
TYPE,   1
MAT,       1
REAL,
ESYS,       0
SECNUM,

CM,_Y,VOLU
VSEL, , , ,       1
CM,_Y1,VOLU
CHKMSH,'VOLU'
CMSEL,S,_Y

MSHAPE,0,3d
MSHKEY,1
VMESH,_Y1
MSHKEY,0
```

```
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2

!----------------------Coupled Field (representing the electrodes)----------
FLST,4,24,1,ORDE,8
FITEM,4,1
FITEM,4,5
FITEM,4,-9
FITEM,4,31
FITEM,4,34
FITEM,4,-38
FITEM,4,61
FITEM,4,-72

CP,1,VOLT,P51X                ! for the upper surface

FLST,4,24,1,ORDE,8
FITEM,4,2
FITEM,4,10
FITEM,4,13
FITEM,4,-16
FITEM,4,25
FITEM,4,-30
FITEM,4,49
FITEM,4,-60

CP,2,VOLT,P51X            ! bottom surface

!----------------Boundary Conditions-------------------------
ANTYPE,0                ! Analysis type

FLST,2,16,1,ORDE,12
FITEM,2,1
FITEM,2,-4
FITEM,2,25
```

```
FITEM,2,34
FITEM,2,39
FITEM,2,-40
FITEM,2,49
FITEM,2,-50
FITEM,2,63
FITEM,2,-64
FITEM,2,73
FITEM,2,-76

D,P51X,UX,0, , , ,UY,UZ, , , ,        ! Fixation at one side

FLST,2,24,1,ORDE,8
FITEM,2,2
FITEM,2,10
FITEM,2,13
FITEM,2,-16
FITEM,2,25
FITEM,2,-30
FITEM,2,49
FITEM,2,-60

D,P51X,VOLT,0           ! Grounding at bottom surface

FLST,2,24,1,ORDE,8
FITEM,2,1
FITEM,2,5
FITEM,2,-9
FITEM,2,31
FITEM,2,34
FITEM,2,-38
FITEM,2,61
FITEM,2,-72

D,P51X,VOLT,0            ! applying 0 v at upper surface

FLST,2,1,4,ORDE,1
```

```
FITEM,2,11

DL,P51X, ,UY,-1          ! Applying Downwards displacement

!--------------------------------------Solve--------------------------
FINISH
SOLVE
```

Appendix C: Piezoelectric Material Properties Calculation Code (MATLAB)

(Y is Young's Modulus, v is the Poisson's ratio, G is Shear Modulus).

```
E1=input('Enter the Value of Y11 : ');
E2=input('Enter the Value of Y11 : ');
E3=input('Enter the Value of Y33 : ');
E5=input('Enter the Value of Y55 : ');
v=input('Enter the Value of V : ');
d31=input('Enter the value of d31: ');
d32=input('Enter the value of d31: ');
d33=input('Enter the value of d33: ');
d15=input('Enter the value of d15: ');
%Y11=E1;
%E1=E2;
%Y33=E3;
%Y55=G13;
%G13=E5;
E6=E1/(2*(1+v));
%E6=G12*
%------------Compliance Matrix S------------------!!
S=[(1/E1),((-v)/E2),-(v/E3),0,0,0;
    -(v/E2),(1/E2),-(v/E3),0,0,0;
    -(v/E3),-(v/E3),(1/E3),0,0,0;
    0,0,0,(1/(2*E5)),0,0;
    0,0,0,0,(1/(2*E5)),0;
    0,0,0,0,0,(1/(E6))];
%-------------------Stiffness Matrix C--------------------
C= inv(S);
disp(S)
disp(C)
%--------------------Piezoelectric Coefficient [d]--------
d=[0,0,0,0,d15,0;
    0,0,0,d15,0,0;
    d31,d32,d33,0,0,0];
D=d';
%disp(D)
%--------------------Piezoelectric Coefficient [e]--------
e=C*D;
%or e=d*C and then bringing e transpose by e' because Ansys take it as
6*3
%matrix
disp(e)
%-------------------------or--------------
%e=[0,0,0,0,e15,0;0,0,0,e24,0,0;e31,e32,e33,0,0,0]
% where e15=e24 and e31=e32
% and to get [d] from [e] using d=e*S or d=e*inv(C)
```