AUSTRALIAN MATHEMATICS TRUST

AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

NAME

YEAR TEACHER

2018

SENIOR DIVISION

AUSTRALIAN SCHOOL YEARS 11 and 12

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

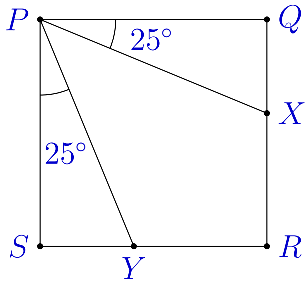
INTEGRITY OF THE COMPETITION

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Senior Division

Questions 1 to 10, 3 marks each

1. In the diagram, $PQRS$ is a square. What is the size of $\angle XPY$?



(A) 25° (B) 30° (C) 35° (D) 40° (E) 45°

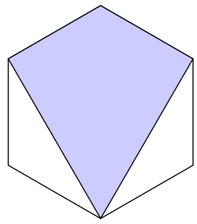
2. The *Great North Walk* is a 250 km long trail from Sydney to Newcastle. If you want to complete it in 8 days, approximately how far do you need to walk each day?

(A) 15 km (B) 20 km (C) 30 km (D) 40 km (E) 80 km

3. Half of a number is 32. What is twice the number?

(A) 16 (B) 32 (C) 64 (D) 128 (E) 256

4. What fraction of this regular hexagon is shaded?



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{4}{5}$

5. The value of $9 \times 1.2345 - 9 \times 0.1234$ is

(A) 9.9999 (B) 9 (C) 9.0909 (D) 10.909 (E) 11.1111

6. What is $2^0 - 1^8$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 10

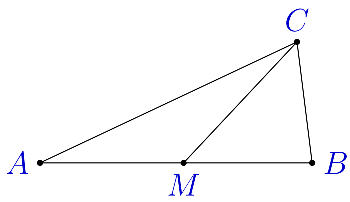
7. 1000% of a number is 100. What is the number?

(A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000

8. The cost of feeding four dogs for three days is \$60. Using the same food costs per dog per day, what would be the cost of feeding seven dogs for seven days?

(A) \$140 (B) \$200 (C) \$245 (D) \$350 (E) \$420

9. In the triangle ABC, M is the midpoint of AB.
Which one of the following statements must be true?



(A) $\angle CAM = \angle ACM$ (B) $\angle CMB = 2\angle CAM$

(C) $AC = 2\,BC$ (D) $CM = BC$

(E) Area $\triangle AMC$ = Area $\triangle MBC$

10. The sum of the numbers from 1 to 100 is 5050. What is the sum of the numbers from 101 to 200?

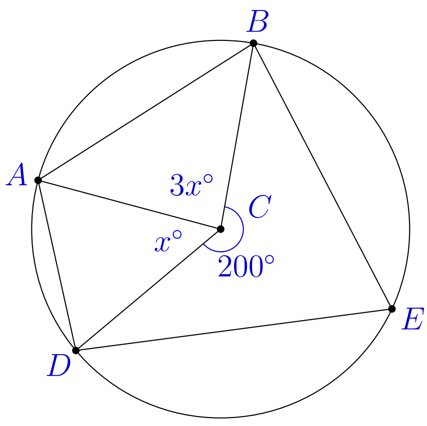
(A) 15 050 (B) 50 500 (C) 51 500 (D) 150 500 (E) 505 000

Questions 11 to 20, 4 marks each

11. Leila has a number of identical equilateral triangle shaped tiles. How many of these must she put together in a row (edge to edge) to create a shape which has a perimeter ten times that of a single tile?

(A) 14 (B) 20 (C) 25 (D) 28 (E) 30

12. In the circle shown, C is the centre and A, B, D and E all lie on the circumference.
Reflex $\angle BCD = 200°$, $\angle DCA = x°$ and $\angle BCA = 3x°$ as shown.
The ratio of $\angle DAC : \angle BAC$ is



(A) 3 : 1 (B) 5 : 2 (C) 8 : 3

(D) 7 : 4 (E) 7 : 3

13. Instead of multiplying a number by 4 and then subtracting 330, I accidentally divided that number by 4 and then added 330. Luckily, my final answer was correct. What was the original number?

(A) 220 (B) 990 (C) 144 (D) 374 (E) 176

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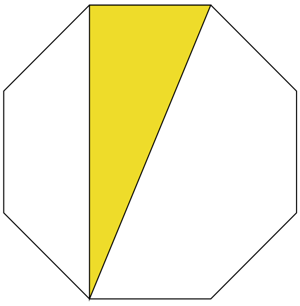
14. The diagram shows a regular octagon of side length 1 metre. In square metres, what is the area of the shaded region?

(A) 1 (B) $\sqrt{2}$ (C) 2

(D) $3-\sqrt{2}$ (E) $\dfrac{1+\sqrt{2}}{2}$



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15. A netball coach is planning a train trip for players from her two netball clubs, Panthers and Warriors.

The two clubs are in different towns, so the train fares per player are different. For the same cost she can either take 6 Panthers and 7 Warriors or she can take 8 Panthers and 4 Warriors.

If she takes only members of the Warriors on the train journey, the number she could take for the same cost is

(A) 11 (B) 13 (C) 16 (D) 20 (E) 25

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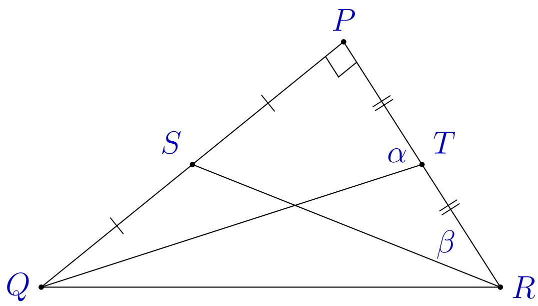
16. The triangle PQR shown has a right angle at P. Points T and S are the midpoints of the sides PR and PQ, respectively. Also $\angle QTP = \alpha$ and $\angle SRP = \beta$.

The ratio $\tan\alpha : \tan\beta$ equals

(A) $3:1$ (B) $4:1$ (C) $5:1$

(D) $7:2$ (E) $9:2$



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17. Three fair 6-sided dice are thrown. What is the probability that the three numbers rolled are three consecutive numbers, in some order?

(A) $\frac{1}{6}$ (B) $\frac{1}{9}$ (C) $\frac{1}{27}$ (D) $\frac{7}{36}$ (E) $\frac{1}{54}$

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18. How many digits does the number 20^{18} have?

(A) 24 (B) 38 (C) 18 (D) 36 (E) 25

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19. In this subtraction, the first number has 100 digits and the second number has 50 digits.

$$\underbrace{111\ldots\ldots111}_{100 \text{ digits}} - \underbrace{222\ldots222}_{50 \text{ digits}}$$

What is the sum of the digits in the result?

(A) 375 (B) 420 (C) 429 (D) 450 (E) 475

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20. I have two regular polygons where the larger polygon has 5 sides more than the smaller polygon. The interior angles of the two polygons differ by 1°. How many sides does the larger polygon have?

(A) 30 (B) 40 (C) 45 (D) 50 (E) 60

Questions 21 to 25, 5 marks each

21. How many solutions (m, n) exist for the equation $n = \sqrt{100 - m^2}$ where both m and n are integers?

(A) 4 (B) 6 (C) 7 (D) 8 (E) 10

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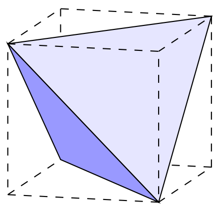
22. A tetrahedron is inscribed in a cube of side length 2 as shown. What is the volume of the tetrahedron?



(A) $\frac{8}{3}$ (B) 4 (C) $\frac{16}{3}$ (D) $\sqrt{6}$ (E) $8 - 2\sqrt{2}$

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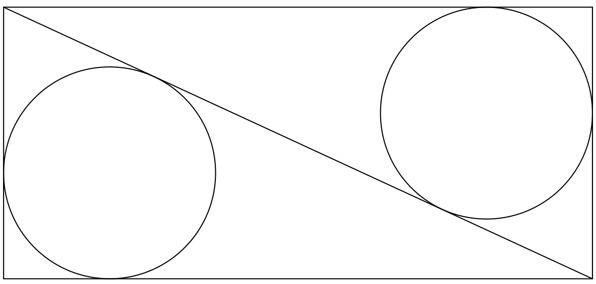
23. A rectangle has sides of length 5 and 12 units.

A diagonal is drawn and then the largest possible circle is drawn in each of the two triangles.

What is the distance between the centres of these two circles?



(A) $\sqrt{60}$ (B) 8 (C) $\sqrt{65}$ (D) $\sqrt{68}$ (E) 9

24. In the equation $\underbrace{\sqrt{\sqrt{\ldots\sqrt{256}}}}_{60} = 2^{(8^x)}$ the value of x is

(A) -17 (B) -19 (C) -21 (D) -23 (E) 16

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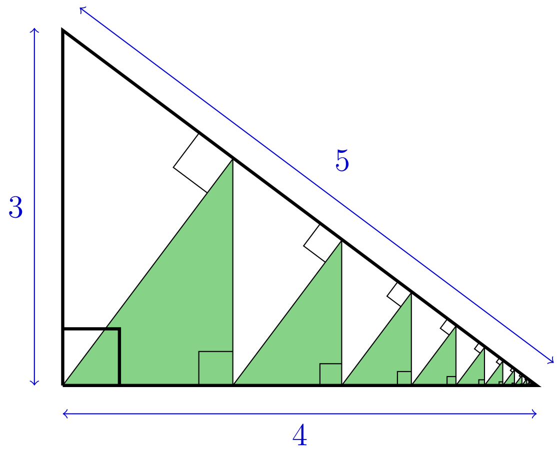
25. A right-angled triangle with sides of length 3, 4 and 5 is tiled by infinitely many right-angled triangles, as shown. What is the shaded area?

(A) $\frac{18}{7}$ (B) $\frac{54}{25}$ (C) $\frac{8}{3}$

(D) $\frac{27}{17}$ (E) $\frac{96}{41}$



For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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26. Let A be a 2018-digit number which is divisible by 9. Let B be the sum of all digits of A and C be the sum of all digits of B. Find the sum of all possible values of C.

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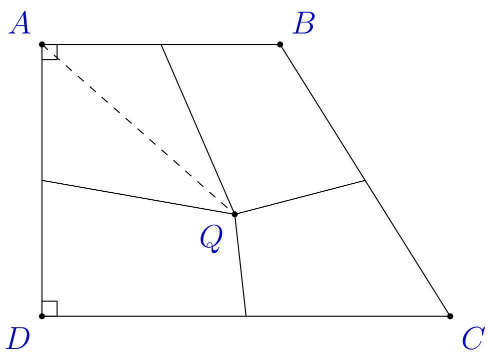
27. The trapezium $ABCD$ has $AB = 100$, $BC = 130$, $CD = 150$ and $DA = 120$, with right angles at A and D.

An interior point Q is joined to the midpoints of all 4 sides. The four quadrilaterals formed have equal areas.

What is the length AQ?



28. Donald has a pair of blue shoes, a pair of red shoes, and a pair of white shoes. He wants to put these six shoes side by side in a row. However, Donald wants the left shoe of each pair to be somewhere to the left of the corresponding right shoe. How many ways are there to do this?

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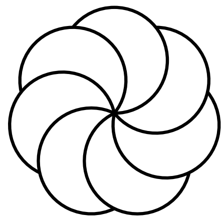
29. For $n \geq 3$, a pattern can be made by overlapping n circles, each of circumference 1 unit, so that each circle passes through a central point and the resulting pattern has order-n rotational symmetry.

For instance, the diagram shows the pattern where $n = 7$.

If the total length of visible arcs is 60 units, what is n?



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30. Consider an $n \times n$ grid filled with the numbers $1, \ldots, n^2$ in ascending order from left to right, top to bottom. A *shuffle* consists of the following two steps:

- Shift every entry one position to the right. An entry at the end of a row moves to the beginning of the next row and the bottom-right entry moves to the top-left position.
- Then shift every entry down one position. An entry at the bottom of a column moves to the top of the next column and again the bottom-right entry moves to the top-left position.

An example for the 3×3 grid is shown. Note that the two steps shown constitute *one* shuffle.

1	2	3
4	5	6
7	8	9

$\longmapsto$ shift right

9	1	2
3	4	5
6	7	8

$\longmapsto$ shift down

8	6	7
9	1	2
3	4	5

What is the smallest value of n for which the $n \times n$ grid requires more than 20 000 shuffles for the numbers to be returned to their original order?

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