The spin of particles and the uncertainty principle

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Introduction

In 1921, on the basis of the experimental proof of the magnetic moment of the silver atom by Otto Stern, it was postulated that electrons have a self-rotation impulse[1]. In the Gerlach-Stern experiment[2] of 1922 the measurements of the angular momentum gave a much higher value than the theoretically calculated angular momentum value, which spoke for the then new theory of directional quantization[3], postulated in 1916 by Peter Debye and Arnold Sommerfeld. The view of a measurement error was abandoned very quickly since some physical effects such as the Zeemann effect were only explainable by this value. In the following experiments, which were also carried out on other particles, a value of exactly $\hbar/2$ ($\hbar$ is the reduced Planck constant) was measured for all investigated particles. The physicists assumed a new undiscovered particle property, which they called spin. The fact that two successive measurements of one and the same particle were always independent of each other in terms of deflection could not be explained by the spin model and initially remained a mystery. Since only a multiple of the half-integer spin was measured for matter particles such as electrons, protons and atomic nuclei as well as for photons and bosons, physicists assumed that the spin is quantized and can only assume certain quantum states[4]. Because of the quantization of the spin, it has since been assumed that particles do not rotate real, although they have a measurable angular momentum.

In a recent work[5] it could be shown by a correct mathematical derivation that a consistent application of the Fourier transformation theory to the derivative of the uncertainty principle requires that the determining term on the right side of Heisenberg's inequality is $h/2$ and not $\hbar/2$, as it is today specified. This was confirmed when the results of the calculations were extended to the Brillouin zones formulation of solid state physics. The uncertainty principle is therefore:

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2} \qquad (1)$$

The derivation of the exact mathematical formulation of the uncertainty principle is rather complicated and depends i.a. on the Nyquist-Shannon theorem[5]. In this article we describe the derivation of the particle spin from the Heisenberg’s uncertainty principle and answer the question, for which conditions a quantization occurs.

Particle spin

The angular momentum of elementary particles is very small due to the small radius and mass of elementary particles and would not be measurable, if the spin measurements were not influenced by conditions described in the following. According to the uncertainty principle[5,6] (HUP) particles with a substructure like protons, electrons, neutrons, quarks and neutrinos have a measurable angular momentum (spin) s of:

$$\Delta L \Delta\varphi \geq \frac{h}{2}; \Delta\varphi_{max} = 2\pi; \Delta L = \frac{\Delta E_{rot}}{\frac{1}{2}\omega} = \frac{\frac{1}{2} h f_{rw}}{\frac{\bar{v}_r}{2r}} = \frac{h\bar{v}_r}{\lambda_{rw}} \cdot \frac{r}{v_r} = \frac{h v_r}{2\lambda_{rw}} \cdot \frac{r}{v_r} \qquad (2)$$

$$\frac{hr}{2\lambda_{rw}} < \frac{h}{4\pi} \rightarrow \frac{hr}{2\lambda_{rw}} := \frac{h}{4\pi} \qquad (3)$$

$$r' = \lambda_{rw}/2\pi \rightarrow \Delta L = \frac{\hbar}{2}; \hat{s} = \frac{\hbar}{2}$$

($\overline{v_r}$ is the mean rotation velocity, φ is the error of the measurements of the rotation angle, s is the spin of the particle, λ_{rw} is the wavelength of the rotation wave, f is the frequency of the particle, annex D). For punctual self-rotating particle like photons and gluons the expression can be written as:

$$\Delta L \Delta\varphi \geq \frac{h}{2}; \Delta\varphi_{max} = 2\pi; \Delta L = \frac{\Delta E_{rot}}{\frac{1}{2}\omega} = \frac{h f_{rw}}{\frac{v_r}{2r}} = \frac{2hv_r}{\lambda_{rw}} \cdot \frac{r}{v_r} \qquad (4)$$

$$\frac{2hr}{\lambda_{rw}} < \frac{h}{4\pi} \rightarrow \frac{2hr}{\lambda_{rw}} := \frac{\hbar}{2} \qquad (5)$$

$$r' = 2\lambda_{rw}/\pi \rightarrow \Delta L = \frac{\hbar}{2}; \hat{s} = \frac{\hbar}{2}$$

v_r is the rotation velocity, $\overline{v_r} = v_r$, annex D). W- and Z-Bosons have most probably a homogenous (high) mass and therefore also a spin of $\hbar$.

For bosons like photons, Z and W bosons and gluons this is not true. In fact, because of the quantized rotational energy, the actual quantized angular momentum equals the measured spin and only happens to be twice as large as the spin of the fermions.

$$\Delta L \Delta\varphi = \Delta E \Delta t; \Delta L = \frac{\Delta E \Delta t}{\Delta\varphi} = \frac{hf\Delta t}{2\pi} = \frac{h}{2\pi} = \hbar \qquad (6)$$

$$\Delta L = \hbar; \hat{s} = \hbar \qquad (7)$$

Quantization in these cases means the same as a rotation around an accordingly larger radius r ', as the rotation velocity cuts out in the equation (4) and the wavelength λ_{rw} is exactly defined by $\lambda_{rw}=\sqrt{h/m_v f}$. The resulting rotation frequency $v_r/2\pi r$ ' is rather low and explicitly not too high to be considered real. Important is that the rotation velocity v_r does not increase if the particle is accelerated; the rotation speed increases only under special circumstances, i.e. through applying an external magnetic field. Under well-defined conditions the frequency of wave particles decreases accordingly, since the "quantized" spin is accomplished by energy that originates form the wave energy of the particle. If only the rotation frequency or velocity would decrease in such a case, the HUP would not be fulfilled any more.

A particle has a different potential energy depending on the orientation of its spin. Therefore, in an atom, interactions occur between different electrons. This interaction is technically exploited in electron spin resonance. Therefore, in cases where the spin becomes energetically relevant, i.e. in case of (more than one) electrons orbiting the nucleus in an atom, the rotation velocity as well the velocity of the electron has to decrease accordingly by the same factor.

$$E_{mw}=E_w=\frac{1}{2}hf=\frac{1}{2}\frac{hv_r}{\lambda_{rw}}=\frac{1}{2}\frac{hv_v}{\lambda}\rightarrow\frac{v_r}{\lambda_{rw}}=\frac{v_v}{\lambda};f_{rw}=f \qquad (8)$$

(v_v is the velocity of the electron, $v_r$is the rotation velocity of the electron, f_{rw} is the frequency of the rotation wave). In order to test this notion, we took the example of Helium, since ^{4}He has 2 electrons in the 1s orbit and compared it to hydrogen ^{1}H with only one electron (the spin interactions between the nucleus and electron are left out of consideration, since they are very small and the same in the two atoms). From equation (4) results that an energy value $E_n=1/2L\omega$ can be assigned to the quantized spin of a particle. Assuming the hypothesized larger radius $r'=\lambda_{rw}/2\pi$according to equation (5), this energy E_n is calculated as:

$$E_n=\frac{1}{2}L\omega=\frac{2\pi}{2}Lf'=\frac{1}{2}\frac{1}{2}\frac{h}{2\pi}\frac{2\pi v'}{\lambda}=\frac{m_v v\lambda}{8\pi}\cdot\frac{2\pi v'}{\lambda}=\frac{m_v v v'}{4};f_{rw}=\frac{v_r}{2\pi r'}=\frac{v_r}{\lambda_{rw}}=\frac{v'}{\lambda} \qquad (9)$$

(v' is the velocity of one of the electrons in the 1s orbit, $v_r$is the rotation velocity of the electron, v is the velocity of the only electron in the 1s orbit in the hydrogen atom, where the spin of the electron has no (energetic) interaction to another electron,f_{rw} is the rotation frequency). Therefore, due to the principle of energy conservation the distance between the electrons and the nucleus in the ^{4}He atom is calculated as:

$$v' = 2v_p' = a\,v_p = av\,;E = E' + E_n\,;m_v v_p'^2 + \frac{2m_v v_p v_p'}{4} = m' v_p^2\,; \tag{10}$$

$$2v_p'^2 + v_p v_p' - 2v_p^2 = 0$$

$$v' = 2\frac{-v_p \pm \sqrt{v_p^2 + 16v_p^2}}{4} = \frac{2v_p\left(\sqrt{17} - 1\right)}{4} = 2\cdot 0.780776\,v_p \tag{11}$$

$$v^2 r = \frac{e^2}{4\pi\varepsilon_0} = const \rightarrow r = \frac{r_0}{2\cdot\sqrt{0.780776}} = 2.994385\cdot 10^{-11}\,m \tag{12}$$

(13)

¿ is the reduced kinetic wave energy of the electron due to the needed energy E_n, the factor 2 results from the fact that the velocity v' of an electron in an [4]He atom is twice as high as the phase velocity v_p'for the wave packet of two electrons, whereas $v_p = v$ for hydrogen, $a = \left(\sqrt{17} - 1\right)/4$). In hydrogen the distance between the proton and electron is $r_0 = 5.29177\cdot 10^{-11}$m (Bohr's radius). Therefore, the distance from the nucleus to the 1s orbital with two electrons in [4]He is calculated as $2.994385\cdot 10^{-11}$m. This corresponds exactly to the radius of [4]He ($3\cdot 10^{-11}$ m). Hence, the hypothesis of a "quantized" spin, with the (lower) rotation frequency$f_r = v_r/2\pi r$', is therefore verified. The rotation velocity and the velocity of particles are thereby not equal; both, the "quantized" rotation energy and the particle energy have the same energy $E_{rot} = E_{rw} = \frac{1}{2} I\,\omega^2 = \frac{1}{2} hf$, but might have different velocities. Hence, in a He atom the electrons move with $2.92415\cdot 10^6$ m/s and electrons in hydrogen with 2.18 $\cdot 10^6$ m/s.

Beside special requirements for instance in an atom the measurement of the spin also leads to its quantization. During such a measurement, the magnetic moment produced by the self-rotation of particles leads to a deflection in an external magnetic field due to the force F_M. The work done by the force F_M in a Stern-Gerlach experiment with silver atoms corresponds to the spin, since:

$$m_v \Delta v dz = \frac{1}{2} a \Delta t^2 \Delta v\, m_v = \frac{1}{2}\frac{F_M}{m_v}\Delta t^2 \Delta v\, m_v = \frac{1}{2} F_M\, dz\,\Delta t = \Delta L \cdot 2\pi \geq \frac{h}{2} \tag{14}$$

$$F_M = \nabla\left(\mu_s \times B\right) = g_s \mu_S \frac{\delta B_z}{\delta z} = g_s \frac{e\hbar}{2m_v}\frac{2m_v \overline{v_r} r}{\hbar}\frac{\delta B_z}{dz} = g_s \frac{e\overline{v_r}\delta B_z}{dz} r\,;\Delta L = \frac{1}{2} g_s\, e\delta B_z r^2 \geq \frac{\hbar}{2} \tag{15}$$

(dz is the deflection in an spin measurement system). Thereby, quantization onto an "effective radius" $r \rightarrow r'$ is the only way to fulfill the HUP requirement.

The quantized radius $r' = \frac{\lambda_{rw}}{2\pi}$ can be found even in measurements of molecular rotation spectra. Depending on the value of the rotation velocity

$$E_{rot} = \frac{h^2}{4\pi^2 I} J(J+1) = \frac{m_v^2 \lambda_{rw}^2 v_r^2}{8\pi^2 m_v r^2} J(J+1) = \frac{4\pi^2 r'^2}{4\pi^2 r^2} \frac{m_v v_r^2}{2} J(J+1) = \frac{1}{2} I' \omega^2 J(J+1) \quad (16)$$

$$I' = m r'^2 = I \frac{r'^2}{r^2} \quad (17)$$

(I' is the quantized moment of inertia accounting for the quantized radius r', v_r is the rotation velocity, f_{rw} is the rotation frequency, λ_{rw} is the rotation wavelength). For a long time, molecular rotations could not be observed directly. Only atomic resolution techniques enabled the rotation of a single molecule to be detected. For example, the rotation of a hexa- (tert-butyl)decacyclene molecule adsorbed on a Cu_{100} surface or Pentacen ($C_{22}H_{14}$) by means of scanning tunneling microscopy could be observed directly in the local area[4], recording vibration frequencies of $\approx 10^9$ Hz that corresponds to a (vibration) velocity of 0.3 m/s (which is $\ll c$¿,with which the molecule vibrates back and forth. Since the angular velocity ω is determined by the given real rotation, the only remaining variable in the equation of the rotation energy (16) is the moment of inertia I, which must be therefore quantized (I') by means of rotation spectra measurements. Hence, the quantized variable in case of a quantized spin (equation 4) or quantized rotation energy (equation 16) can only be the radius of particles or molecules.

I already showed that under circumstances, where the angular momentum of particles is coupled with a particle wave (particle motion), this might have consequences on a combined effect. Accordingly, the abnormal Zeeman effect[7] shows impressively that the particle spin and the orbital spin are interdependent. This effect is referred to when the angular momentum and the magnetic moment of the two terms between which the optical transition takes place cannot be described by either one of the two quantum numbers s or l, but by both. This is the general case that atomic magnetism is a superposition of spin and orbit magnetism.

The angular momentum, due to the uncertainty, has a range of $\hbar/2$ for itself. Angular momenta, therefore, must be at least this distance to be distinguishable. Therefore, a real rotation is more conceivable and viable than the notion of a somehow unexplainable “untrue rotation”. For instance, the spin operator has all the properties of an angular momentum operator and can therefore be treated analogously. In the Einstein-de-Haas effect, the change in the direction of the electron spins in an iron rod displaces the latter into a macroscopic rotational motion. After all, the effects of the particle self-rotation are used in chemistry, biology and medicine (magnetic resonance tomography) for the detailed investigation of materials, tissues and processes.

Hence, I showed that the notion of an effective radius r ', as such, leads to the correct value of the quantized spin or rotation energy. On the other hand, the rotation velocity of a particle is solely defined by (attracting) processes inside the particle or molecule (covalent forces) and not by the spin (equation 4) or the rotation energy (equation 16); hence, the rotation velocity is not quantized. This variable has a well-defined value, leading to a real self-rotation in all known particles except the Higgs-boson.

As I showed above special requirements for instance in an atom lead to a quantization of the spin, which is durable, except the electron flies out of the atom. But how does the quantized spin gained by a measurement behave afterwards? In any conceivable state, a spin ½ particle has in any direction a well-defined and always the same value (the greatest possible at all) to the square of the component of its spin. In the two states "(anti-)parallel" alignment with the z-axis, the two components perpendicular are therefore twice as large as the component along the alignment axis. A normal vector with these properties would not be parallel to the z-axis, but even closer to the xy-plane perpendicular to it. However, experimentally proven is that the spin direction is always the same as the reference direction of the spin measurement system and that the z-spin has the full value of $\hbar/2$. Ozawa et al.[8] has related this phenomenon to an uncertainty according to Heisenberg related to the noise and disturbance of the measurement system. By means of controlled manipulations of the measuring device[9], it was possible to determine statistically how the different sources of the uncertainty are related to each other using neutrons. We already mentioned that a defined energy $E_n=1/2L\omega$ is supplied for the quantization of the spin according to the HUP, which has no direction (since it is energy). The mysterious phenomenon attained by multiple spin measurements supports our assumptions in regard of the spin of particles made above and can be explained as follows: Without a measurement (or without relevance such as for electrons orbiting a nucleus), a particle generally has a very small and not measurable angular moment (spin). By means of a measurement, the “quantized” spin is accomplished by energy of the particle and determined in its full value with respect to the reference direction of the spin meter. Due to the HUP the value of the measured spin in the measured reference direction must be namely at least $\hbar/2$. Each new measurement generates an new quantized spin by deflecting the particles accordingly. Therefore, each generated spin is not determined with respect to other reference directions for further measurements.

The same applies for the quantization of the rotation energy. This energy is also very small due to the small radius of particles and only quantized if measured or because of another energetic relevance (such as emission or absorption of photons by rotating molecules). An exception from this is the self-rotation energy of photons ($\frac{1}{2} m_v c^2$, as described earlier. This explains the observed real rotation of molecules[10], although they are supposed to have a quantized rotation energy and no true rotation, why in the derivation of the equivalence of mass and energy the sum of the "pure" kinetic energy (and not the double value) and the rest mass energy yields the total energy of a moving particle for $v \ll c$ and why the pure kinetic energy of atoms or molecules is temperature-determining. For molecule vibrations where the amplitude is given by the microscopically observable vibration[11], the vibrational frequency measured by scanning tunneling microscopy might be higher observed than it is in reality, since the measurement is subjected to the HUP. The De Broglie equation is only true if measured (if $\Delta E \Delta t = m_v v^2 \Delta t \leq h/2$ by virtue of the small mass and/or if the measuring time Δt of the velocity using modern technologies is chosen to small) or if the particle is involved in an energetic exchange process (such as two electrons orbiting the nucleus in the same electron shell).

In the radioactive α-decay the energy of the lost mass (mass defect $\Delta m ¿$ is the same as the kinetic energy of the emitted α-particle. Therefore, this process is also subjected to the HUP-Nyquist principle and quantized.

$$\Delta E \Delta t = \Delta m c^2 \Delta t = \frac{h}{2} \rightarrow \Delta E = \Delta m c^2 = \frac{1}{2} hf = \frac{1}{2} m_v v^2 = E_{kin,\alpha} \quad (18)$$

From all these examples we conclude that "quantization" (driven by the HUP) is not a measurement principle per se, neither a "true" principle; this principle finds application in quantum physics if a) the wave particle is generated or emitted in a time $\Delta t = \Delta x / v$ (i.e. photons, radioactive α-decay, gluons, W- and Z-bosons), b) the variable has a relevance in an (energetic) exchange process (i.e. spin interaction), c) the variable is coupled with another energetic condition, which must be fulfilled in the same time (i.e. the rotation of photons, which is created in the same time the wave packet (photon) is generated), d) the variable does not fulfill the HUP requirement (section 3, velocities greater than the light speed), or e) by means of a measurement in quantum physics. In measurements the quantization is a pseudo quantization, if the conditions a) - c) are not present, since the value and direction of the measured variable are not retained after the measurement and only generated within the time Δt ($h/2 \Delta E$in which the measurement is executed.

Conclusion

In conclusion, the concept of spin presented here might help to understand that particles have, other than usually assumed a real self-rotation ≤ c. The "unusual high and constant spin" is accomplished by energy which originates from the energy of particles. In cases where the spin becomes relevant, i.e. in case of electrons orbiting the nucleus in an atom or spin measurements, the rotation velocity even decreases. Quantization in this case means the same as a rotation around an accordingly larger radius. Hence, the quantization does not imply that the velocity is not determined or undefined, as this notion of a quantized spin is usually associated with the idea that the particles have no conceivable or real self-rotation. This is in accordance to our previous calculations[12] using data from hadron collisions and magnetic resonance imaging (MRI) that revealed a quit low rotation frequency of protons of 2072.180437Hz. The argument that such a "high" spin requires that the particles, if they would have a true self-rotation, would turn faster than with light speed, which is not possible, hence they do not rotate, is therefore redundant.

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