

# MATHEMATICS LEVEL 2 TEST

The top portion of the page of the answer sheet that you will use to take the Mathematics Level 2 Test must be filled in as illustrated below. When your supervisor tells you to fill in the circle next to the name of the test you are to take, mark your answer sheet as shown.

Literature	<input type="radio"/> Mathematics Level 1	<input type="radio"/> German	<input type="radio"/> Chinese Listening	<input type="radio"/> Japanese Listening
Biology E	<input checked="" type="radio"/> Mathematics Level 2	<input type="radio"/> Italian	<input type="radio"/> French Listening	<input type="radio"/> Korean Listening
Biology M	<input type="radio"/> U.S. History	<input type="radio"/> Latin	<input type="radio"/> German Listening	<input type="radio"/> Spanish Listening
Chemistry	<input type="radio"/> World History	<input type="radio"/> Modern Hebrew		
Physics	<input type="radio"/> French	<input type="radio"/> Spanish		

Background Questions:  1  2  3  4  5  6  7  8  9

After filling in the circle next to the name of the test you are taking, locate the Background Questions section, which also appears at the top of your answer sheet (as shown above). This is where you will answer the following Background Questions on your answer sheet.

## BACKGROUND QUESTIONS

Answer Part I and Part II below by filling in the appropriate circle in the Background Questions box on your answer sheet. The information you provide is for statistical purposes only and will not affect your test score.

I. Which of the following describes a mathematics course you have taken or are currently taking? (FILL IN ALL CIRCLES THAT APPLY.)

- Algebra I or Elementary Algebra OR Course I of a college preparatory mathematics sequence —Fill in circle 1.
- Geometry OR Course II of a college preparatory mathematics sequence —Fill in circle 2.
- Algebra II or Intermediate Algebra OR Course III of a college preparatory mathematics sequence —Fill in circle 3.
- Elementary Functions (Precalculus) and/or Trigonometry OR beyond Course III of a college preparatory mathematics sequence —Fill in circle 4.
- Advanced Placement Mathematics (Calculus AB or Calculus BC) —Fill in circle 5.

II. What type of calculator did you bring to use for this test? (FILL IN THE ONE CIRCLE THAT APPLIES. If you did not bring a scientific or graphing calculator, do not fill in any of circles 6-9.)

- Scientific —Fill in circle 6.
- Graphing (Fill in the circle corresponding to the model you used.)
  - Casio 9700, Casio 9750, Casio 9800, Casio 9850, Casio 9860, Casio FX 1.0, Casio CG-10, Sharp 9200, Sharp 9300, Sharp 9600, Sharp 9900, TI-82, TI-83, TI-83 Plus, TI-83 Plus Silver, TI-84 Plus, TI-84 Plus Silver, TI-85, TI-86, TI-Nspire, or TI-Nspire CX —Fill in circle 7.
  - Casio 9970, Casio Algebra FX 2.0, HP 38G, HP 39 series, HP 40 series, HP 48 series, HP 49 series, HP 50 series, TI-89, TI-89 Titanium, TI-Nspire CAS, or TI-Nspire CX CAS —Fill in circle 8.
  - Some other graphing calculator —Fill in circle 9.

When the supervisor gives the signal, turn the page and begin the Mathematics Level 2 Test. There are 100 numbered questions on the answer sheet and 50 questions in the Mathematics Level 2 Test. Therefore, use only circles 1 to 50 for marking your answers.

## MATHEMATICS LEVEL 2 TEST

## REFERENCE INFORMATION

THE FOLLOWING INFORMATION IS FOR YOUR REFERENCE IN ANSWERING SOME OF THE QUESTIONS IN THIS TEST.

Volume of a right circular cone with radius  $r$  and height  $h$ :  $V = \frac{1}{3}\pi r^2 h$

Volume of a sphere with radius  $r$ :  $V = \frac{4}{3}\pi r^3$

Volume of a pyramid with base area  $B$  and height  $h$ :  $V = \frac{1}{3}Bh$

Surface Area of a sphere with radius  $r$ :  $S = 4\pi r^2$

DO NOT DETACH FROM BOOK.

MATHEMATICS LEVEL 2 TEST

For each of the following problems, decide which is the BEST of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding circle on the answer sheet.

Notes: (1) A scientific or graphing calculator will be necessary for answering some (but not all) of the questions in this test. For each question you will have to decide whether or not you should use a calculator.

(2) For some questions in this test you may have to decide whether your calculator should be in the radian mode or the degree mode.

(3) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(4) Unless otherwise specified, the domain of any function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number. The range of  $f$  is assumed to be the set of all real numbers  $f(x)$ , where  $x$  is in the domain of  $f$ .

(5) Reference information that may be useful in answering the questions in this test can be found on the page preceding Question 1.

USE THIS SPACE FOR SCRATCH WORK.

1. If  $x = 2$ , for which of the following is the value of  $y$  greatest?

(A)  $y = \frac{1}{x-1}$

(B)  $y = |x|$

(C)  $y = x^2$

(D)  $y = x^3 - 1$

(E)  $y = 2^x$

DBC

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## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

2. If  $f(x) = \sqrt{x^3 + 8}$ , how much does  $f(x)$  increase as  $x$  increases from 4 to 5?

- (A) 0.94
- (B) 1.00
- (C) 3.05
- (D) 3.18
- (E) 7.81

3. The speedometer on Jim's car is inaccurate. The relationship between the actual speed of the car,  $a$ , in miles per hour, and the reading on Jim's speedometer,  $r$ , is given by  $a = 12 + 1.7r$ . What is the actual speed of Jim's car when the reading on the speedometer is 15?

- (A) 25.5 mph
- (B) 27.0 mph
- (C) 33.7 mph
- (D) 37.5 mph
- (E) 45.9 mph

4. If  $x = 5^{-2}$ , then  $\sqrt[3]{x} =$

- (A) -2.92
- (B) -2.15
- (C) 0.34
- (D) 1.31
- (E) 2.92

## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

- 5 What is the third term of an arithmetic sequence whose second term is 7 and whose sixth term is 23?

(A) 19

(B)  $12\frac{1}{3}$

(C) 12

(D) 11

(E) 4

- 6 In the  $xy$ -plane, which of the following equations represents a circle of radius 5 centered at the origin?

(A)  $x + y = 5$

(B)  $|x| + |y| = 5$

(C)  $x^2 + y^2 = 5$

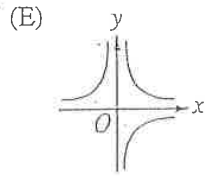
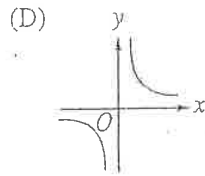
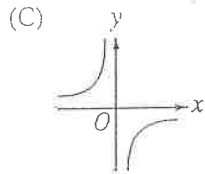
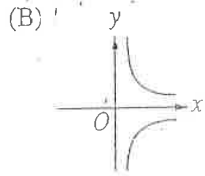
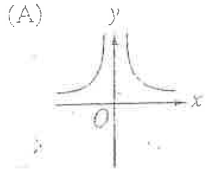
(D)  $x^2 + y^2 = 25$

(E)  $(x - 5)^2 + (y - 5)^2 = 25$

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

7. Which of the following graphs is symmetric with respect to the  $x$ -axis?



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## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

8 For all real numbers  $a$  and  $b$  such that  $a^2 > b^2$ , which of the following must be true?

I  $a^2 - b^2 > 0$

II  $a > b$

III  $ab^2 > 0$

(A) I only

(B) II only

(C) III only

(D) I and II

(E) II and III

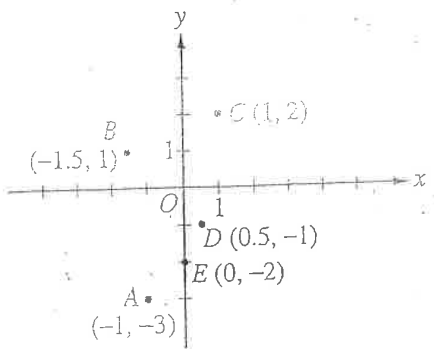
9.

What is the least positive integer  $n$  for which  $6n + 1$  is NOT prime?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.



10. Which of the points in the figure above satisfies the relation  $y > 2x^2 - 1$ ?
- (A) A (B) B (C) C (D) D (E) E

11. The senior class at North High designed a T-shirt as a class fund raiser. They had to pay \$35 to set up the design and the total cost to make each finished shirt was \$7.50. They sold the shirts for \$10 each. If  $x$  is the number of shirts made and  $y$  is the number of shirts sold, which of the following conditions represents a profit for the senior class?
- (A)  $10y - 7.5x + 35 > 0$   
(B)  $10y - 7.5x + 35 < 0$   
(C)  $10y - 7.5x - 35 > 0$   
(D)  $10y - 7.5x - 35 < 0$   
(E)  $10y - (35 + 7.5)x > 0$

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## MATHEMATICS LEVEL 2 TEST—Continued

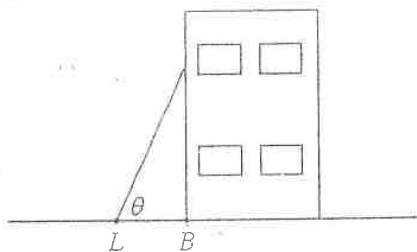
USE THIS SPACE FOR SCRATCH WORK.

12.

$y = 3^{-x} + 1$ , what value does  $y$  approach as  $x$  increases infinitely large?

- (A)  $-1$   
(B)  $0$   
(C)  $\frac{1}{3}$   
(D)  $1$   
(E) The value of  $y$  increases without bound.

13.



Note: Figure not drawn to scale.

Safety officials have determined that a ladder placed against a building as shown in the figure above remains stable if  $\theta$ , the angle it makes with the ground, is between  $70^\circ$  and  $80^\circ$ ,

inclusive. Which of the following distances from the base of the ladder  $L$  to the base of the building  $B$  would be considered safe for a ladder 21 feet long?

- (A) 19.7 feet  
(B) 18.8 feet  
(C) 10.6 feet  
(D) 6.5 feet  
(E) 3.4 feet

## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

14. For which of the following functions is there a value of  $x$  such that  $f(x) = -1,000$ ?

(A)  $f(x) = 2^x$

(B)  $f(x) = \sin x$

(C)  $f(x) = (x - 2)^2$

(D)  $f(x) = |x - 4|$

(E)  $f(x) = 2x + 3$

15. Box I and box II each contain only red marbles and blue marbles. The probability of selecting a red marble from box I is  $\frac{2}{3}$ , and the probability of selecting a red marble from box II is  $\frac{3}{8}$ . If one marble is to be randomly selected from each box, what is the probability that both marbles will be blue?

(A)  $\frac{5}{24}$

(B)  $\frac{8}{24}$

(C)  $\frac{15}{24}$

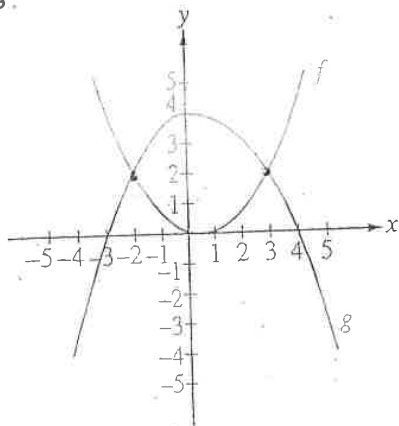
(D)  $\frac{19}{24}$

(E)  $\frac{23}{24}$

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

16.



The figure above shows the graphs of functions  $f$  and  $g$ . Which of the following is closest to a value of  $x$  such that  $f(x) - g(x) = 0$ ?

- (A) -3 (B) -2 (C) 1 (D) 2 (E) 4

17. A total of 500 tickets were sold for a school play, generating total sales of \$1,700. If tickets cost \$4 for adults and \$2 for children, how many adult tickets were sold?

- (A) 83  
 (B) 150  
 (C) 283  
 (D) 350  
 (E) 425

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MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

18. If 3 is a root of

$$x^5 - x^4 - 11x^3 + 21x^2 - 11x + k = 0,$$

then  $k$  is

- (A) -23
- (B) -21
- (C) 4
- (D) 14
- (E) 20

19. If  $e^x = 4$ , then  $x =$

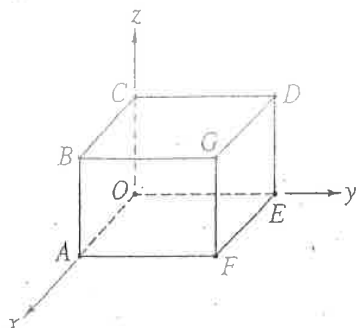
- (A) 1.4
- (B) 1.5
- (C) 2.0
- (D) 4.0
- (E) 54.6

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## MATHEMATICS LEVEL 2 TEST—Continued

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20. In the figure above, vertex  $G$  of the rectangular solid shown has coordinates  $(5, 6, 4)$ . What is the length of the diagonal  $\overline{CF}$ ?

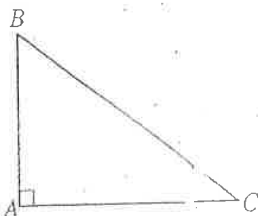
(A) 11.81  
(B) 8.77  
(C) 7.81  
(D) 7.21  
(E) 6.40

21. For which of the following equations is the product of its roots NOT a negative number?

(A)  $x^2 + 6x = 0$   
(B)  $x^2 - 25 = 0$   
(C)  $x^2 - 10x - 24 = 0$   
(D)  $x^2 - 6x - 27 = 0$   
(E)  $x^2 + 7x - 18 = 0$

## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.



22. In right triangle  $ABC$  above,  $\sin C = \frac{3}{5}$  and

$AC = 12$ . What is the length of  $BC$ ?

- (A) 5 (B) 9 (C) 15 (D) 20 (E) 25

23. What is the domain of  $f$  if  $f(x) = \frac{x-5}{x^2-5x}$ ?

- (A) All real numbers  
(B) All real numbers except 0  
(C) All real numbers except 5  
(D) All real numbers except 0 and  $-5$   
(E) All real numbers except 0 and 5

24. For which of the following does the property  $f(f(x)) = x$  hold for all  $x$ ?

- (A)  $f(x) = -x$   
(B)  $f(x) = x^2$   
(C)  $f(x) = 2^x$   
(D)  $f(x) = \cos x$   
(E)  $f(x) = \sin x$

## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

25)

If  $\log_{10} \sqrt{x} = a$ , then  $\log_{10} x^2 =$ 

- (A)  $a^2$   
(B)  $4a$   
(C)  $2a$   
(D)  $a$   
(E)  $\sqrt{a}$

26

If  $f$  is a linear function such that  $f(-1) = 2.5$  and  $f(1) = -3$ , what is the value of  $f(4)$ ?

- (A) 2.75  
(B) -8.25  
(C) -8.5  
(D) -11.25  
(E) -14

27. If  $f(x) = x^2 - 1$  and  $g(x) = x + 1$ , thenwhen  $x \neq -1$ ,  $\frac{f(x)}{g(x)} =$ 

- (A)  $g(-x)$   
(B)  $g(x-2)$   
(C)  $g(x-1)$   
(D)  $g(x)$   
(E)  $g(x+1)$

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

28. What is the period of the function  $f$  given by

$$f(x) = |\sin x|?$$

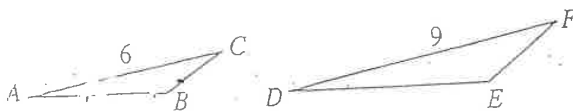
- (A)  $\frac{\pi}{4}$   
 (B)  $\frac{\pi}{2}$   
 (C)  $\pi$   
 (D)  $2\pi$   
 (E)  $4\pi$

29. For all values of  $x$ ,

$$x^2 + 2x + 4 = (x + 2)^2 + a(x + 2) + b,$$

where  $a$  and  $b$  are constants. What is the value of  $a$ ?

- (A)  $-2$  (B)  $-1$  (C)  $0$  (D)  $1$  (E)  $2$



30.  $\triangle ABC$  and  $\triangle DEF$ , shown above, are similar. If

in  $\triangle ABC$ ,  $\frac{\sin B}{\sin A} = 3$ , then in  $\triangle DEF$  the length

of  $\overline{EF}$  is

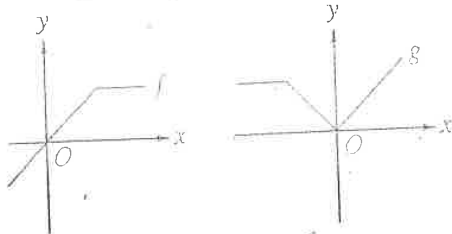
- (A)  $\frac{1}{3}$  (B)  $\frac{2}{3}$  (C)  $2$  (D)  $3$  (E)  $6$



MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

31.



The graphs of the functions  $f$  and  $g$  are shown above. Which of the following is true?

- (A)  $g(x) = |f(-x)|$
- (B)  $g(x) = |f(x)|$
- (C)  $g(x) = f(-x)$
- (D)  $g(x) = -f(-x)$
- (E)  $g(x) = -f(x)$

32.

What is the ratio of the volume of a sphere of radius  $r$  to the volume of a sphere of radius  $2r$ ?

- (A) 1:2
- (B) 1:3
- (C) 1:4
- (D) 1:6
- (E) 1:8

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## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

33. In the  $xy$ -plane,  $g$  is the inverse function of  $f$  and  $(5, 0)$  is a point on the graph of  $y = f(x)$ . Which of the following is a point on the graph of  $y = g(x)$ ?

(A)  $(-5, 0)$

(B)  $(0, -5)$

(C)  $(0, 5)$

(D)  $(\frac{1}{5}, 0)$

(E)  $(5, 0)$

34. Using data collected from several donut shops, a linear regression was used to generate a model relating the price of a dozen donuts,  $p$ , in dollars, and the quantity sold,  $d$ , in dozens of donuts. The resulting equation was  $d = -26.97p + 342.80$ . What is the meaning of  $-26.97$  in this model?

(A)  $d$  decreases by 26.97 dozen for every \$1 increase in  $p$ .

(B)  $d$  increases by 26.97 dozen for every \$1 increase in  $p$ .

(C)  $p$  decreases by \$26.97 for every one dozen decrease in  $d$ .

(D)  $p$  decreases by \$26.97 for every one dozen increase in  $d$ .

(E)  $p$  increases by \$26.97 for every one dozen decrease in  $d$ .

## MATHEMATICS LEVEL 2 TEST—Continued

35.

USE THIS SPACE FOR SCRATCH WORK.

In the  $xy$ -plane, the circles with equations  $(x - 3)^2 + y^2 = 11$  and  $(x + 2)^2 + y^2 = r^2$  are externally tangent. What is the value of  $r$ ?

- (A) 0.3
- (B) 1.3
- (C) 1.7
- (D) 2.8
- (E) 6.0

36.

What is the sum of

$$2 + \left( \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \dots + \frac{1}{5^n} + \dots \right)?$$

- (A)  $2\frac{1}{4}$
- (B)  $2\frac{1}{2}$
- (C)  $2\frac{4}{5}$
- (D)  $3\frac{1}{4}$
- (E) The sum is infinite.

## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

37. If  $f(x) = -x^3 + 4x^2 - 2x + 2$ , the maximum value of  $f(x)$  on the interval  $[-2, 4]$  occurs when  $x =$

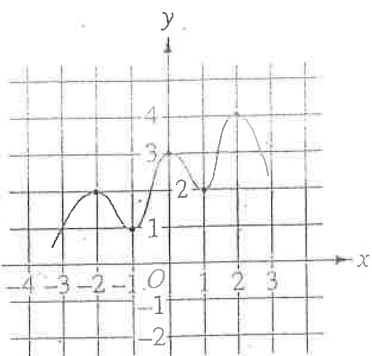
- (A) -2
- (B) 0.28
- (C) 2.39
- (D) 4
- (E) 6.42

38. An equilateral triangle with a perimeter of 6 has one vertex at the origin of the  $xy$ -coordinate plane and one vertex on the positive  $x$ -axis. Which of the following could be the coordinates of the third vertex of the triangle?

- (A)  $(0, 2)$
- (B)  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- (C)  $(1, -1)$
- (D)  $(1, -\sqrt{3})$
- (E)  $(2, 2)$

## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.



39. In the figure above, a portion of the graph of  $y = f(x)$  is shown. What are the coordinates of the  $y$ -intercept of the graph of  $y = f(x + 2)$ ?

(A) (0, 1)  
(B) (0, 2)  
(C) (0, 3)  
(D) (0, 4)  
(E) (0, 5)

40. What is the average (arithmetic mean) of the first 225 positive integers?

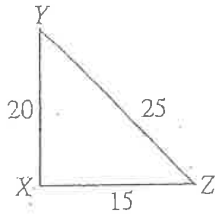
(A) 56.25  
(B) 112  
(C) 112.5  
(D) 113  
(E) 226

## MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

41. If  $n$  is a positive integer, then  $n!$  is divisible by 9 if and only if

- (A)  $n \geq 3$
- (B)  $n \geq 6$
- (C)  $n \geq 9$
- (D)  $n$  is a multiple of 3
- (E)  $n$  is a multiple of 9



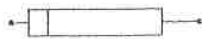
42. What is the volume of the solid obtained by rotating the triangle in the figure above about the side  $\overline{XY}$ ?

- (A)  $375\pi$
- (B)  $1,250\pi$
- (C)  $1,500\pi$
- (D)  $1,875\pi$
- (E)  $2,000\pi$

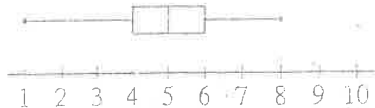
MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

Data Set A



Data Set B



43. The boxplots for data set  $A$  and data set  $B$  are shown above. Which of the following statements must be true about  $A$  and  $B$ ?

- I. At least 50 percent of the values in  $A$  are greater than 50 percent of the values in  $B$ .
- II. The standard deviation of  $A$  is greater than the standard deviation of  $B$ .
- III. The range of  $A$  is less than the range of  $B$ .

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) I and III

44. If  $\frac{\pi}{2} < \theta < \pi$  and  $\csc \theta = 2$ , what is  $\tan \theta$ ?

- (A) -1.7321
- (B) -0.5774
- (C) 0.5774
- (D) 1.7321
- (E) There is no such angle  $\theta$ .

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

45. Jamie earned \$32.50 from baby-sitting and caring for pets. He earned \$3.75 per hour baby-sitting and \$2.00 per hour caring for pets. Which of the following describes the number of hours worked baby-sitting as a function of the number of hours  $x$  worked caring for pets?

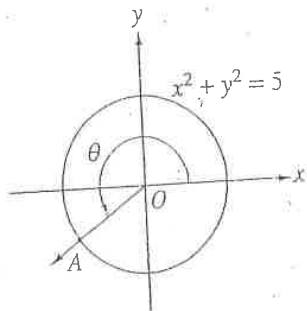
(A)  $f(x) = \frac{-3.75x + 32.50}{2}$

(B)  $f(x) = \frac{-2x + 32.50}{3.75}$

(C)  $f(x) = -(2 + 3.75)x + 32.50$

(D)  $f(x) = (2 \cdot 3.75)x + 32.50$

(E)  $f(x) = \frac{2x + 3.75x}{32.50}$



46. In the figure above, point A is on the circle  $x^2 + y^2 = 5$ . If point A has coordinates  $(x, -1)$ , what is the degree measure of  $\theta$ ?

- (A) 245.9  
 (B) 243.4  
 (C) 221.8  
 (D) 206.6  
 (E) 204.1

GO ON TO THE NEXT PAGE



## MATHEMATICS LEVEL 2 TEST—Continued

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47. If a point  $(x, y)$  is chosen at random from the set of points where  $-1 \leq x \leq 1$  and  $-1 \leq y \leq 1$ , what is the probability that the distance from the point  $(x, y)$  to the origin is less than or equal to 1?

(A)  $\frac{1}{2}$

(B)  $\frac{\pi}{8}$

(C)  $\frac{\pi}{6}$

(D)  $\frac{\pi}{4}$

(E)  $\pi$

48. If  $h(x) = f(g(x))$ , where  $h(x) = \sin x$  and  $f(x) = \cos x$ , which of the following could define the function  $g$ ?

(A)  $g(x) = \sqrt{x^2 - 1}$

(B)  $g(x) = \sqrt{1 - x^2}$

(C)  $g(x) = \frac{\pi}{2} + x$

(D)  $g(x) = \frac{\pi}{2} - x$

(E)  $g(x) = \tan x$

MATHEMATICS LEVEL 2 TEST—Continued

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49. Two numbers have a sum of 10 and a product of 20. What is the sum of the reciprocals of the two numbers?

- (A)  $\frac{1}{2}$
- (B)  $\frac{1}{5}$
- (C)  $\frac{3}{20}$
- (D)  $\frac{1}{10}$
- (E)  $\frac{1}{20}$

50. A parallelogram has sides of length 50 centimeters and 75 centimeters. If the smaller angle of the parallelogram measures  $45^\circ$ , what is the length of the shorter diagonal, to the nearest centimeter?

- (A) 50
- (B) 53
- (C) 71
- (D) 90
- (E) 116

**STOP**

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