AUSTRALIAN MATHS TRUST

2020 AMC

AUSTRALIAN MATHEMATICS COMPETITION

Senior Years 11–12

(Australian school years)

THURSDAY 30 JULY 2020

NAME _____

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

General

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

The answer sheet

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

Integrity of the competition

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Reminder: You may sit this competition once, in one division only, or risk no score.

Senior Division

Questions 1 to 10, 3 marks each

1. What is the value of $2020 \div 20$?

(A) 2000 (B) 2040 (C) 11 (D) 101 (E) 1001

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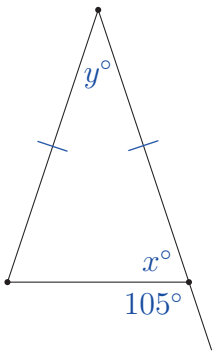
2. In the diagram provided, find the sum of x and y.

(A) 30 (B) 75 (C) 95 (D) 105 (E) 180



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3. Evaluate $\sqrt{7 + 18 \div (10 - 1^5)}$.

(A) $\frac{5}{3}$ (B) 9 (C) 3 (D) 5 (E) $\frac{1}{27}$

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4. Sebastien is thinking of two numbers whose sum is 26 and whose difference is 14. The product of Sebastien's two numbers is

(A) 80 (B) 96 (C) 105 (D) 120 (E) 132

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5. If $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{\star}{7}$ of $\frac{7}{8}$ is equal to 1, then the value of $\star$ is

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

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6. A square garden of area $10\,000\,\text{m}^2$ is to be enlarged by increasing both its length and width by 10%. The increase in area, in square metres, is

(A) 1000 (B) 2000 (C) 2100 (D) 2400 (E) 4000

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7. Given that $f(x) = 2x^2 - 3x + c$ and $f(2) = 6$, then c is equal to

(A) 4 (B) 3 (C) 6 (D) 8 (E) 12

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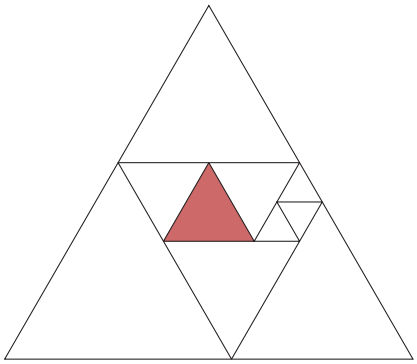
8. An equilateral triangle is subdivided into a number of smaller equilateral triangles, as shown. The shaded triangle has side length 2. What is the perimeter of the large triangle?



(A) 24 (B) 27 (C) 30 (D) 33 (E) 36

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9. If $a \neq 0$, then $\dfrac{a^{x+y}}{a^x}$ is equivalent to

(A) a^y (B) $\dfrac{1}{a^y}$ (C) $-a^y$ (D) a^{1+y} (E) $1 + a^y$

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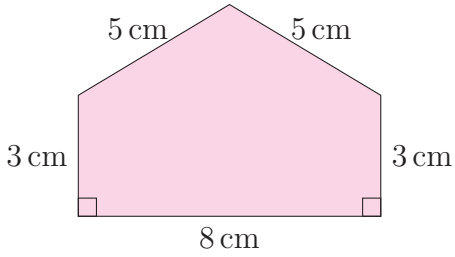
10. What is the area of the pentagon shown?

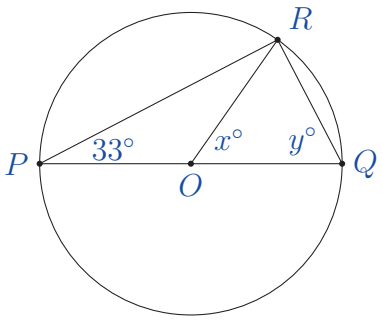


(A) $32\,\text{cm}^2$ (B) $36\,\text{cm}^2$ (C) $42\,\text{cm}^2$ (D) $56\,\text{cm}^2$ (E) $64\,\text{cm}^2$

Questions 11 to 20, 4 marks each

11. In the diagram, PQ is a diameter of the circle, OR is a radius, and $\angle OPR = 33°$.

The value of $x + y$ is



(A) 99 (B) 113 (C) 115 (D) 123 (E) 137

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12. This diagram is composed entirely of semicircles. The diameter of each of the eight smallest semicircles is exactly one-quarter of the diameter of the two biggest semicircles.

What fraction of the large circle is shaded?

(A) $\dfrac{9}{16}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{5}{8}$

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13. In Paradise, all days are either fine or wet.

If today is fine, the probability of tomorrow being fine is $\frac{3}{4}$.

If today is wet, the probability of tomorrow being fine is $\frac{1}{3}$.

Today is Friday and it is fine. I am having a BBQ on Sunday. What is the probability that it will be fine on Sunday?

(A) $\frac{25}{48}$ (B) $\frac{29}{48}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{31}{48}$

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14. Given that x and y are both integers and $2^{x+1} + 2^x = 3^{y+2} - 3^y$, the value of $x + y$ is

(A) 0 (B) 1 (C) 4 (D) 7 (E) 9

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15. A bag contains exactly 50 coins. The coins are either worth 10 cents, 20 cents or 50 cents, and there is at least one of each. The total value of the coins is \$10.

How many different ways can this occur?

(A) 2 (B) 4 (C) 8 (D) 12 (E) 16

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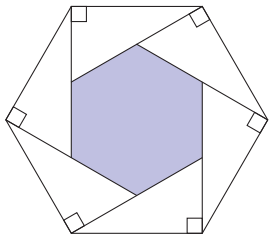
16. A regular hexagon is partially covered by six right-angled triangles, as shown.

What fraction of the hexagon is not covered?



(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

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17. In a paddock of sheep, there are 4 times as many male sheep as female sheep. In another paddock, there are 5 times as many females as males. When the two flocks of sheep are combined, there are equal numbers of males and females. What is the smallest possible total number of sheep?

(A) 20 (B) 26 (C) 30 (D) 38 (E) 42

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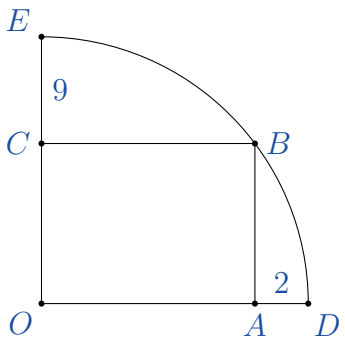
18. The rectangle $OABC$ is drawn in the quadrant of a circle ODE, so that $AD = 2$ and $CE = 9$.
What is the radius of the circle?

(A) 11 (B) 13 (C) 15 (D) 17 (E) 20



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19. The minimum value of the function $f(x) = 2^{x^2-2x-3}$ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$ (E) $\frac{1}{16}$

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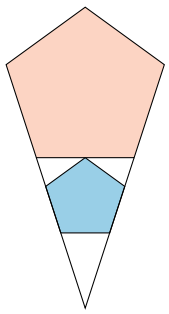
20. Two sides of a regular pentagon are extended to create a triangle. Inside this triangle, a smaller regular pentagon is drawn, as shown.
In area, how many times bigger is the larger pentagon than the smaller pentagon?

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $\frac{\sqrt{5}+3}{2}$ (E) $\sqrt{5}$



Questions 21 to 25, 5 marks each

21. For $n \geq 1$, s_n is defined to be the number consisting of n consecutive ones, so $s_1 = 1$, $s_2 = 11$, $s_3 = 111$, and so on.
Which one of the following numbers is divisible by 7?

(A) s_{902} (B) s_{903} (C) s_{904} (D) s_{905} (E) s_{906}

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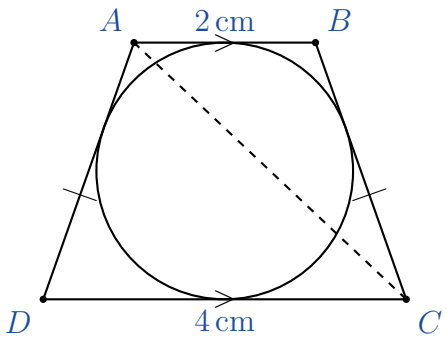
22. A circle is inscribed in the quadrilateral $ABCD$ so that it touches all four sides, as shown. Sides AB and DC are parallel with lengths 2 cm and 4 cm, respectively, and sides AD and BC have equal length.
What, in centimetres, is the length of AC?

(A) $\sqrt{17}$ (B) $2\sqrt{5}$ (C) $\sqrt{13}$ (D) 5 (E) $3\sqrt{2}$



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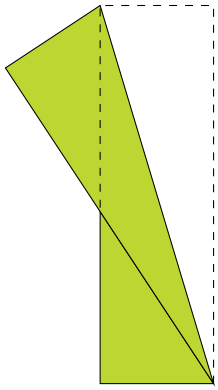
23. A rectangular sheet of paper that is three times as tall as it is wide is folded along one diagonal, making the pentagon shown.



What is the ratio of the area of this pentagon to the area of the original rectangle?

(A) 13 : 18 (B) 3 : 4 (C) 7 : 12 (D) 2 : 3 (E) $\sqrt{10} : 4$

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24. Alex writes down the value of the following sum, where the final term is the number consisting of 2020 consecutive nines:

$$9 + 99 + 999 + 9999 + \cdots + \underbrace{99\ldots9}_{\text{2019 nines}} + \underbrace{99\ldots9}_{\text{2020 nines}}$$

How many times does the digit 1 appear in the answer?

(A) 0 (B) 2016 (C) 2018 (D) 2020 (E) 2021

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25. Three real numbers a, b and c are such that

$$a+b+c=4 \quad \text{and} \quad \frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=5$$

Then, $\dfrac{c}{a+b}+\dfrac{a}{b+c}+\dfrac{b}{c+a}$ is equal to

(A) $\dfrac{3}{2}$ (B) $\dfrac{4}{5}$ (C) 2 (D) 20 (E) 17

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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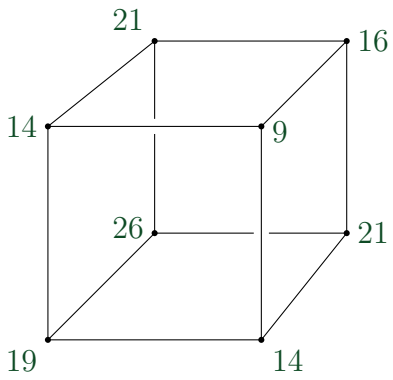
26. A different integer from 1 to 10 is placed on each of the faces of a cube. Each vertex is then assigned a number which is the sum of the numbers on the three faces which touch that vertex.

Only the vertex numbers are shown here.

What is the product of the 4 smallest face numbers?



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27. The coefficients of a polynomial function $P(x)$ are all non-negative integers. Given that $P(2) = 40$ and $P(40) = 2\,688\,008$, what is the value of $P(3)$?

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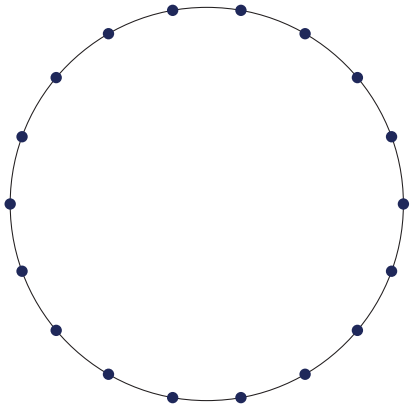
28. This circle has 18 equally spaced points marked. There are 816 ways of joining 3 of these points to form a triangle.

How many of these triangles have a pair of angles that differ by 30°?



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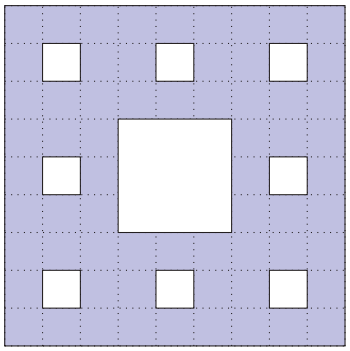
29. Starting with a $9 \times 9 \times 9$ cube, Steve mined out nine square tunnels through each face so that the resulting solid shape had front view, top view and side view all the same, as shown.

Going from the original cube to the perforated cube, how much did the surface area increase?



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30. When I drive to school every day, I pass eight traffic lights, each either green, yellow, or red. I find that, because of synchronization, a green light is always followed immediately by a yellow, and a red light is never immediately followed by a red.

Thus a sequence of lights may start with GYRY, but not RRGG.

How many possible sequences of the eight lights are there?

2020 AMC — SENIOR