# Year 10 Mathematics

## Sample Resources

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# Quadratics Lesson Guidelines
# Property of Prestige Tuition

## QUADRATIC EQUATIONS FUNDAMENTALS

- Linear equations have their pronumeral or variable with an index or power of 1. $x^1$
- Quadratic Equations are simply those where the highest index is 2.
- E.g

I. $x^2 + 4x = 10$
II. $3x^2 - 5x = 0$,
III. $(x+3)(x-4) = 0$ ← x will multiply with x to make $x^2$

## IDENTIFYING QUADRATIC EQUATIONS

You may need to
I. expand to remove the grouping symbols
II. Collect like terms
III. for equations with fractions, find the common denominator.

1. Identify each of the following equations as linear, quadratic or 'other'. Do not solve them.

(a) $6(x - 2) - 3x = 12$ Quick deduction, we will have 6x -3x = 3x, the only x's with $x^1$. Thus linear.

(b) $x(x - 2) - 3x = 12$ Quick deduction, x will multiply with x, giving $x^2$, hence quadratic.

(c) $\frac{x^2}{2} + 3 = \frac{5}{x}$
LCD is 2x
$\frac{x^3}{2x} + \frac{6x}{2x} = \frac{10}{2x}$
*Multiply LHS and RHS by* $2x$
$x^3 + 6x = 10$
Highest is $x^3$ so neither quadratic or linear, other.

(d) $2^x = 4x$ Highest power is not x, other

## THE STANDARD FORM

The standard form, similar to the general form of linear equations is $ax^2 + bx + c = 0$

1. Rearrange each of the following quadratic equations so that they are in standard form.

(a) $5x^2 - 2x + 3 = 2x^2 + 4x - 12$
rearrange, and collect like terms.
and collect like terms.
$x^2$-6x+15=0

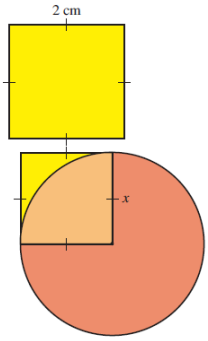
(b) $\frac{x^2}{2} - \frac{1}{6} = x\left(\frac{x}{3}\right) - 4$
LCD = 6.
Collect like terms and rearrange
$x^2$+23=0

(c) $\frac{1}{x} + \frac{x}{2} = \frac{x}{3} + \frac{1}{2}$ LCF = 6x, $x^2$-3x+6 = 0

(d) $5x^2 - 4x + 5 = 2(x^2 - 9)$ $3x^2$-4x+23 = 0

(e) $x^2 - 4x - 16 = 1 - 4x$ $x^2$-17

2. A first square has sides which measure *x* cm. Its area is equal to the difference between the area of a circle whose radius is *x* cm and a second square with sides measuring 2 cm.




Square Two

Square One

(a) Write the formula for the area of the square with side length x. $A = x^2$

(b) Write the formula for the area of the circle. $A = \pi x^2$

(c) Find the difference between the area of the circle and the area of the second square.
$Difference = \pi x^2 - 4$

(d) Write an equation where the left-hand side is the area of the first square and the right-hand side is the difference between the area of the circle and the area of the second square.
$x^2 = \pi x^2 - 4$

(e) Rearrange this equation so that it is of the form: $ax^2 + bx + c = 0$.
$x^2(1 - \pi) + 4 = 0$

## SOLVING QUADRATIC EQUATIONS USING FACTORS

- The null factors law is something MANY students do not understand, but it is very simple.
- Consider the equation $a \times b = 0$.
It is reasonable to say that if this equation is true, it is true that either:
$a = 0$ or $b = 0$

1. Solve each of the following using quadratic equations

(a) $(x - 2)(2x + 1) = 0$
Either $x - 2 = 0$ or $2x + 1 = 0$
$x = 2$ or $2x = -1$
$x = 2$ or $x = -\frac{1}{2}$

(b) $(4 - 3x)(6 + 11x) = 0$ Same as above, $x = -\frac{6}{11}$ or $1\frac{1}{3}$

(c) $x(x - 3) = 0$ $x = 0$, or $x = 3$

(d) $(x - 1)^2 = 0$ x = 1 only

## Difference of two square rule

- First expand (a-b)(a+b)

…

Commit to memory, $a^2 - b^2 = (a - b)(a + b)$

- We may find difference of two squares when factorizing quadratics
- You can divide both sides by constants, but not by x's. You will lose solutions if you divide by x.

Remember to explain that they will use this rule for the next two years. Each rule is a tool in their bag, and the more tools they have the easier things will be.

1. Solve

(a) $x^2 - 1 = 0$ (x+1)(x-1) = 0, x = -1 or 1
(b) $2x^2 - 8 = 0$ $2(x^2-4) = 0$, x = -2 or 2
(c) $(100 - x^2) = 0$ x = 10 or -10
(d) $(2x^2 + 18) = 0$ Ignore brackets, x = 3 or -3
(e) $x^2 = 0$ x -=0
(f) $16x^2 = 25$ x = 5/4 or -5/4

## Standard Factorisation

1. Solve
   (a) $x^2 + 4x = 0$ x(x+4) =0, x = 0 or -4
   (b) $-2x^2 - 4x = 0$ -2x(x+2), x=0 or -2
   (c) $-x^2 + 18x = 0$ x(18-x), x = 0 or 18
   (d) $-2x^2 - 24x = 0$ -2x(x+12) =0, x = 0 or -12

## SOLVING QUADRATICS WITH THREE TERMS (PSF METHOD)

- In most cases, quadratic equations will have three terms.
- Always put in the standard form before solving
- PSF means products, sums and factors
- We will use the PSF method

I. The PRODUCTS = constant term multiplied by the coefficient of $x^2$
II. The SUMS = the coefficient of the middle term.
III. The FACTORS = Add up to the middle term and are a factor term of the products
IV. Then rewrite in factorised form (x+f1)(x+f2) = 0

1. Solve the following quadratics

(a) $x^2 - 5x - 6 = 0$

P: -6 *1 = -6

S: -5

F: -6 and 1

(x-6)(x+1) = 0, x = 6 or -1

(b) $x^2 + 14x = 15$

rearrange to $x^2$+14x-15 = 0

P: -15

S: 14

F: 15 and -1

(x+15)(x-1) = 0, x = -15 or 1

(c) $x^2 + 6x + 5 = 0$ x = -1, x= -5
(d) $x^2 + 4x + 4 = 0$ x=-2
(e) $x^2 - x - 12 = 0$ x=-3, x= 4
(f) $x^2 + x = 20$ x=4, x= -5
(g) $x(x - 2) = 8$ x=4, x= -2

## HARDER QUADRATICS: POLYNOMIC COEFFICIENTS OF $X^2$ (PSF METHOD)

- Quadratics where the coefficient of $x^2$ is not 1, are a bit more difficult. They just require an extra step.

I. We must split the middle x term into our factors
II. Then factorise in pairs.

- YOU MUST PAY CLOSE ATTENTION TO THE COEFFICIENT OF $x^2$!!!!

1. Solve

(a) $2x^2 + 7x + 3 = 0$

P: 6

S:7

F: 6 and 1.

NOW, our goal is to split 7x into 6x and 1x. That's why we calculated the factors.

$$2x^2 + 7x + 3 = 0$$
$$2x^2 + 1x + 6x + 3 = 0$$

We made the SMART choice where 6x goes with 3, and 1x goes with 2. We do this, because we can factor out more with 6 and 3.

$$x(2x + 1) + 3(2x + 1) = 0$$

Now we factor (2x+1) from both those big terms.

$$(2x + 1)(x + 3) = 0, x = -\frac{1}{2}\ or\ -3$$

(b) $2x^2 + 7x + 3 = 0$ x=-3, x= -1/2
(c) $2x^2 + 8x + 6 = 0$ x=-2, x=-1
(d) $5x^2 = 22x + 21 = 0$ $x=1\frac{2}{5}$, x = 3
(e) $7x^2 = 33x + 20 = 0$ x= 5/8, x = 4
(f) $(3x - 2)^2 = (2x + 1)^2$ x= 1/5, x = 3
(g) $2x - x(x + 4) = (x + 5)(x + 2)$ x = -5/2, = -2

## QUADRATIC WORDED EQUATIONS

- We can form a mathematical equation by replacing variables or unknowns with a letter such as x. We can then manage and solve for x!
- When looking at worded equations, replace the key sus words 'something', 'some number', etc by x
- Remember, all equations need an equal sign, meaning a LHS and RHS

1. If the square of a number is multiplied by 5, the answer is 45. Find the number.

replace some number by x.

$$x^2 \times 5 = 45$$
$$5x^2 = 45$$
$$x^2 = 9$$
$$x = 3\ or\ -3$$

2. When a positive number is subtracted from its square, the result is zero. Write the equation and solve.

$$x^2 - x = 0.$$
$$x(x-1) = 0$$

x= 0 or 1

10. Angelo Vertucci designed a scarf and sought the endorsement of pop star Kylie Ciccone. Calculating manufacturing, warehousing, distribution and taxation costs in addition to Kylie's enormous fee, Angelo found that he would have to sell 1 000 000 scarves at $64 each before he made any profit. The gross income from sales was expected to be the square of Kylie Ciccone's fee.

(a) Use a pronumeral to define Kylie Ciccone's fee. $\$x$
(b) What is the expected gross income in terms of this pronumeral? $\$x^2$
(c) How many dollars is the expected gross income? $\$64\ 000\ 000$
(d) Write an equation relating the gross income to the square of Kylie Ciccone's fee. $x^2 = 64\ 000\ 000$
(e) Use the equation to find Kylie Ciccone's fee. $8000

# Quadratics
# Sheet 1
# Property of Prestige Tuition

**Solving by factors**

1. Expand each of the following equations
   a. $(x+2)^2$ $x^2+4x+4$
   b. $(x-3)^2$ $x^2-6x+9$
   c. $(x-8)^2$ $x^2-16x+64$
   d. $(x+6)^2$ $x^2+12x+36$
2. Solve each of the following quadratic equations
   a. $(x+3)(x-2)=0$ x= -3,2
   b. $(2x-5)(4x+3)=0$ x = 5/2,-3/4
   c. $(2x+1)(3-x)=0$ x= -1/2, 3
   d. $(x-2)^2=0$ x=2

3. Solve each of the following quadratics
   a. $x^2-9=0$ x= -3,3
   b. $3x^2-27=0$ x= -3,3
   c. $x^2=4$ x=2,-2
   d. $16x^2=25$ x=5/4,-5/4

4. Solve each of the following equations
   a. $x^2+6x=0$ x=0,-6
   b. $2x^2-15x=0$ x=0,15/2
   c. $-x^2-5x=0$ x= 0,-5
   d. $x^2+x=0$ x= 0,-1
   e. $4x^2-x=0$ x=0,1/4

5. Solve each of the following quadratic equations
   a. $x^2-6x+8=0$ x=2,4
   b. $x^2+x-6=0$ x=-3,2
   c. $x^2+2x-24=0$ x=-6,4
   d. $x^2+13x+12=0$ x=-12,-1
   e. $x^2+6x+8=0$ x= -6,-2
   f. $x(x-15)=50$ x=5,10
   g. $(x+7)^2=-3(5x+17)$

6. Solve each of following quadratic equations
   a. $2x^2+7x+3=0$ x=-3,-1/2
   b. $2x^2+8x+6=0$ x=-3,-1
   c. $5x^2-22x+21=0$ x=7/5,3
   d. $7x^2-33x+20=0$ x=5/7,4
   e. $(3x-2)^2=(2x+1)^2$ x=1/5,3
   f. $2x-x(x+4)=(x+5)$

# Quadratics
# Sheet 2
# Property of Prestige Tuition



**Completing the square**

1. To complete the square, the term which should be added to $x^2 + 4x$ is; b
   a. 16
   b. 4
   c. 4x
   d. 2
2. To complete the square, the term which should be added to $x^2 - 3x + 1$ is; d
   a. 9
   b. 3
   c. 3/2
   d. 9/4
3. Complete the square for each of the following expressions. Show all working out
   a. $x^2 + 10x$ $(x+5)^2$
   b. $x^2 + 16x$ $(x+8)^2$
   c. $x^2 - 14x$ $(x-7)^2$
   d. $x^2 + 6x$ $(x+3)^2$
   e. $x^2 - 20x$ $(x-10)^2$
   f. $x^2 + \frac{5x}{3}$ $\left(x+\frac{5}{6}\right)^2$
   g. $x^2 - 4x$ $(x-2)^2$
4. Solve each of the following by using the completing the square method. Give exact answers
   a. $x^2 - 4x + 2 = 0$ $x = 2 \pm \sqrt{2}$
   b. $x^2 - 8x + 4 = 0$ $x = 4 \pm 2\sqrt{3}$
   c. $x^2 + 2x - 5 = 0$ $x = -1 \pm \sqrt{6}$
   d. $x^2 + 2x - 2 = 0$ $x = -1 \pm \sqrt{3}$
   e. $x^2 - 10x + 1 = 0$ $x = 5 \pm 2\sqrt{6}$
   f. $x^2 + 4x - 6 = 0$ $x = -2 \pm \sqrt{10}$
5. Solve each of the following. Give answers in exact form (2 D.P). Use the completing the square method
   a. $x^2 - 3x - 1 = 0$ x=0.38,2.62
   b. $x^2 + 5x - 1 = 0$ x=-5.19,0.19
   c. $x^2 + 6x - 1 = 0$ x=0.63,6.37
   d. $x^2 - 8x + 4 = 0$ x=-1.79,2.79
   e. $x^2 - 11x + 1 = 0$ x=0.09,10.91
   f. $x^2 + x = 1$ x=-1.62,0.62
   g. $x^2 + 3x - 7 = 0$ -4.54,1.54
6. Solve each of the following by first looking for a common factor and then using the completing he square method. Give answers correct to 2 decimal places
   a. $2x^2 + 4x - 6 = 0$ x=-3,1
   b. $3x^2 + 12x - 3 = 0$ x=-4.24,0.24
   c. $5x^2 - 10x - 15 = 0$ x=-1,3
   d. $4x^2 - 8x - 8 = 0$ x=-0.73,2.73

# Quadratics
# Sheet 3
# Property of Prestige Tuition

**The quadratic formula**

1. State the values for a,b and c in each of the following equations of the form $ax^2 + bx + c = 0$
   a. $3x^2 - 4x + 1 = 0$ a=3,b=-4,c=1
   b. $x^2 + 7 - 5x = 0$ a=1,b=-5,c=7
   c. $12x^2 - 29x + 103 = 0$
   d. $2 + 7x^2 - 12x = 0$

2. Use the quadratic formula to solve each of the following equations. Give exact answers
   a. $x^2 + 2x + 1 = 0$ $x = -1$
   b. $x^2 + 3x - 1 = 0$ $x = \frac{-3 \pm \sqrt{13}}{2}$
   c. $x^2 - 5x + 2 = 0$ $x = \frac{5 \pm \sqrt{17}}{2}$
   d. $x^2 - 4x - 9 = 0$ $x = 2 \pm \sqrt{13}$
   e. $x^2 + 2x - 11 = 0$ $x = -1 \pm 2\sqrt{3}$
   f. $x^2 - 7x + 1 = 0$ $x = \frac{7 \pm 3\sqrt{5}}{2}$
   g. $x^2 - 9x + 2 = 0$ $x = \frac{9 \pm \sqrt{73}}{2}$
   h. $x^2 - 6x - 3 = 0$ $x = 3 \pm 2\sqrt{3}$

3. Use the quadratic formula to solve each of the following equations. Give approximate answers rounded to 2 decimal places
   a. $3x^2 - 4x - 3 = 0$ x=-0.54,1.87
   b. $4x^2 - x - 7 = 0$ x=-1.20,1.45
   c. $2x^2 + 7x - 5 = 0$ x=-4.11,0.61
   d. $7x^2 + x - 2 = 0$ x=-.61,0.47
   e. $5x^2 - 8x + 1 = 0$ x=0.14,1.46
   f. $2x^2 - 13x + 2 = 0$ x=0.16,6.34
   g. $-3x^2 + 2x + 7 = 0$ x=-1.23,1.90
   h. $-7x^2 + x + 8 = 0$ x=-1,1.14

4. Solve each of the following equations using any suitable method. Round to 3 decimal places where appropriate
   a. $2x^2 - 7x + 3 = 0$ x= 0.5,3
   b. $x^2 - 5x = 0$ x=0,5
   c. $x^2 - 2x - 3 = 0$ x=-1,3
   d. $x^2 - 3x + 1 = 0$ x=0.382,2.168
   e. $x^2 - 6x + 8 = 0$ x=0.14,1.46
   f. $x^2 - 5x + 8 = 0$ x=0.16,6.34
   g. $x^2 = 2x - 9 = 0$ x=-1.23,1.90
   h. $3x^2 + 3x - 6 = 0$ x=-1,1.14
   i. $2x^2 + 11x - 21 = 0$ x=-0.83,0.91
   j. $7x^2 - 2x + 1 = 0$ x=-0.64,1.31

5. Consider each of the following equations find;
   i. The values of a,b,c
   ii. The value of the discriminant
   iii. How many real solutions are there for the respective equation
   a. $3x^2 + 2x + 7 = 0$
   b. $-6x^2 + x + 3 = 0$
   c. $x^2 - 4x - 5 = 0$

# Quadratics
# Sheet 4
# Property of Prestige Tuition

**Applications**

1. The distance travelled by a car that is accelerating is given by the formula $d = 5t + t^2$, where t is the time in seconds and d is the distance in metres. If the distance travelled was 150, for how long was the car travelling for? 10s

2. The height of a soccer ball when kicked is determined by the formula $y = -0.1x^2 + 3x$ , where x is the horizontal distance from the striker
   a. How far is the kicked ball from the striker when it hits the ground? 30m
   b. What is the horizontal the ball covers when the height of the ball first reaches 20m? 10m

3. When a certain number is multiplied by 15 and added to its square, the result is zero. Find the number, which is not zero. -15

4. Three times the square of a certain positive number is equal to two more than five times the number. Find the numbers. 2 or 5

5. Mathilda is being pushed on a swing in her backyard. The swing follows the path given by the formula

$h = \frac{1}{4}(x^2 - 3x + 4)$, where h is the height of the swing above ground, x metres from the point where first pushed.

   a. Find the height of the swing when she is first pushed 1m
   b. Find the horizontal distance that the swing has travelled when it reaches the original height 3m

6. A rectangular piece of timber x metres wide is being used to make chairs. It is divided into three pieces, one 2 metres long, and 2 square pieces
   a. Find an algebraic expression for the area of the piece that is 2 metres long 2x
   b. Find an algebraic expression for the sum of the areas of the two squares $2x^2$
   c. If the area of the original piece of timber is 12 square metres, use a quadratic equation to find the width of the timber 2m

7. When two consecutive even numbers are multiplied, the result is 48. Find the numbers 6 and 8

8. Five times a number is added to two times its square. If the result is 168, find the number. 8,-10.5

9. If the length of the paddock is 2 metres more than its width and the area is $48m^2$, find the length and width of the paddock 8m,6m

Name

# Year 10 Mathematics

## Statistics
## 2017

**General Instructions**

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- Draw diagrams in pencil
- Board approved calculators may be used
- Write your name at the top of this page
- Answer on loose leaf paper

**Total marks – 40 marks**

**Part A – 40 marks**
Attempt questions 1 – 11
Allow about 55 minutes for this part

1. Find the mean, median and mode for each of the following sets of data [9 marks]
   a. 7,15,8,8,20,14,8,10,12,6,19
   b.

| *Stem* | *Leaf* |
|---|---|
| 1 | 2 6 |
| 2 | 1 7 8 |
| 3 | 0 3 3 4 6 8 |
| 4 | 0 1 1 5 9 |
| 5 | 1 3 6 |

   c.

| Score ($x$) | Frequency ($f$) |
|---|---|
| 70 | 2 |
| 71 | 6 |
| 72 | 9 |
| 73 | 7 |
| 74 | 4 |

2. A sample of 30 people was selected at random from those attending a local swimming pool. Their ages (in years) were recorded as follows;

   19,7,58,41,17,23,62,55,40,37,32,29,21,18,16,10,40,36,33,59,65,68,15,9,20,29,38,24,10,30

   a. Comment on the location of the data
   b. Group the data into class intervals of 10 and complete a frequency distribution table
      [3 marks]
   c. Calculate the cumulative frequency and, hence, plot a graph of the cumulative frequency (the ogive) [2 marks]

3. The following back to back stem and leaf plot shows the typing speed in words per minute of 30 Year 8 and Year 10 students

| *Leaf Year 8* | *Stem* | *Leaf Year 10* |
|---|---|---|
| 9 9 | 0 | |
| 9 8 6 5 4 2 0 | 1 | 7 9 |
| 9 8 8 6 4 2 1 0 0 | 2 | 2 3 6 8 9 |
| 9 7 7 6 4 1 0 | 3 | 0 2 4 5 5 7 8 8 |
| 8 6 5 2 0 | 4 | 1 2 5 8 8 9 9 |
| | 5 | 0 3 5 7 8 |
| | 6 | 0 0 3 |

   a. Construct a five number summary for each year [2 marks ]
   b. Construct a comparative box plot, clearly marking the necessary information [2 mark]

   c. Find the mean, variance and IQR for each each set of data [3 marks]
   d. Compare the two distributions, using your answer from the earlier parts [2 marks]

4. Find the standard deviation of the following data set without the use of your calculator. Show all working out [3 marks ]

   58,12,98,45,60,34,42

   i. Briefly explain how the formula for the standard deviation was generated [2 marks]

5. Draw histograms that are left and right skewed. Clearly labelling which one is which. [2 marks]

6. Define the two variables in statistics and provide an example of each [2 marks]

7. Distinguish between a census and a sample using an example to assist your answer [2 marks]

8. Briefly identify why we require the sample of a data to be random [1 mark]

9. Habib compares his English and Maths marks. The results of the 8 tests in each subject are shown below

   English; 76,64,90,67,83,60,85,37
   Maths; 80,56,92,84,65,58,55,62

   Based on the data above, in which subject is Habib more consistent in. Provide a brief explanation for your judgement [3 marks]

10. A data set has a mean of 75 and a standard deviation of 5. Another score of 50 is added to the data set. Which of the following will occur [1 mark]
    a. The mean will increase and the standard deviation will increase
    b. The mean will increase and the standard deviation will decrease
    c. The mean will decrease and the standard deviation will increase
    d. The mean will decrease and the standard deviation will decrease

11. The data set has a mean of 60 and a standard deviation of 10. A score of 100 is added to the data set. This score becomes the highest score in the data set. Which of the following will increase [1 mark]
    a. Mean
    b. Range
    c. Standard deviation
    d. IQR