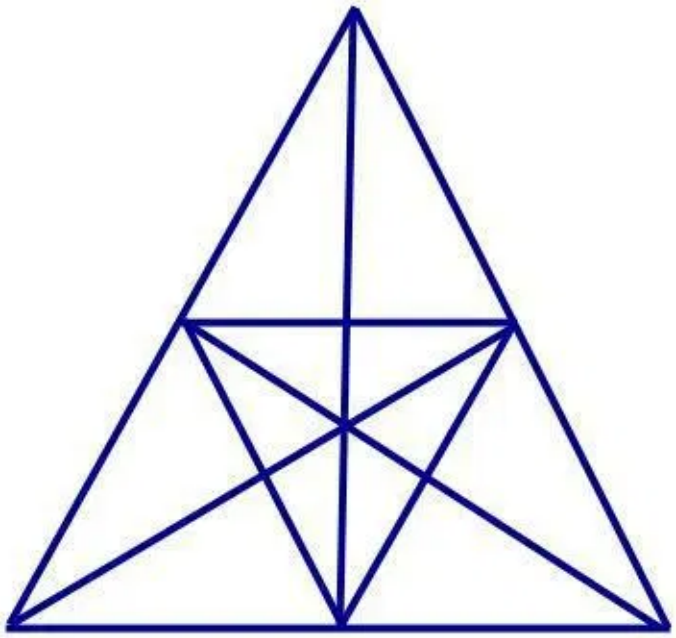
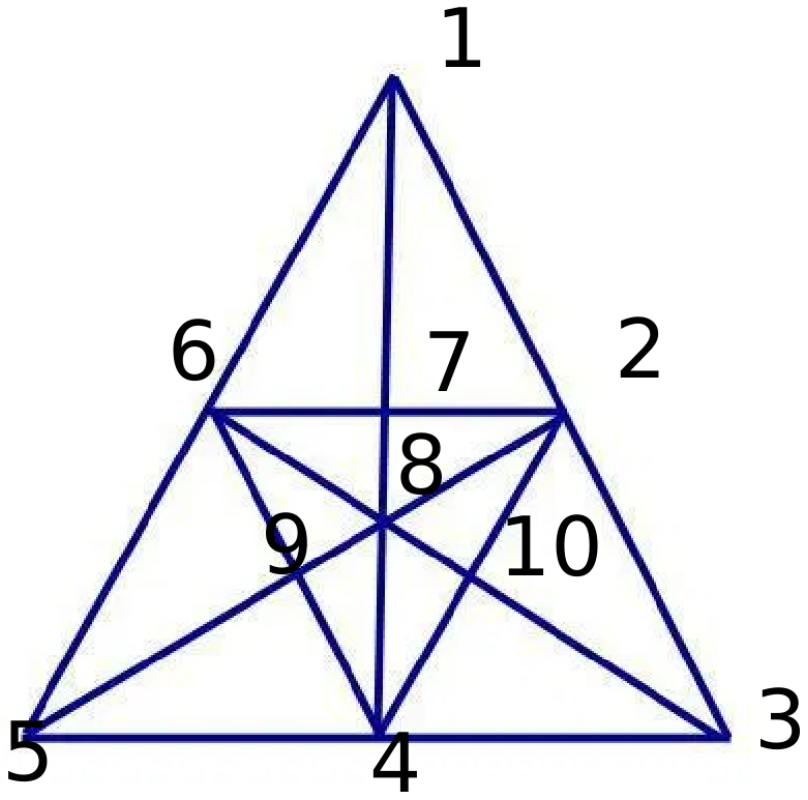


How many triangles are there? Label the intersection points, and form a graph where two vertices have an edge if the corresponding points lie on the same line.

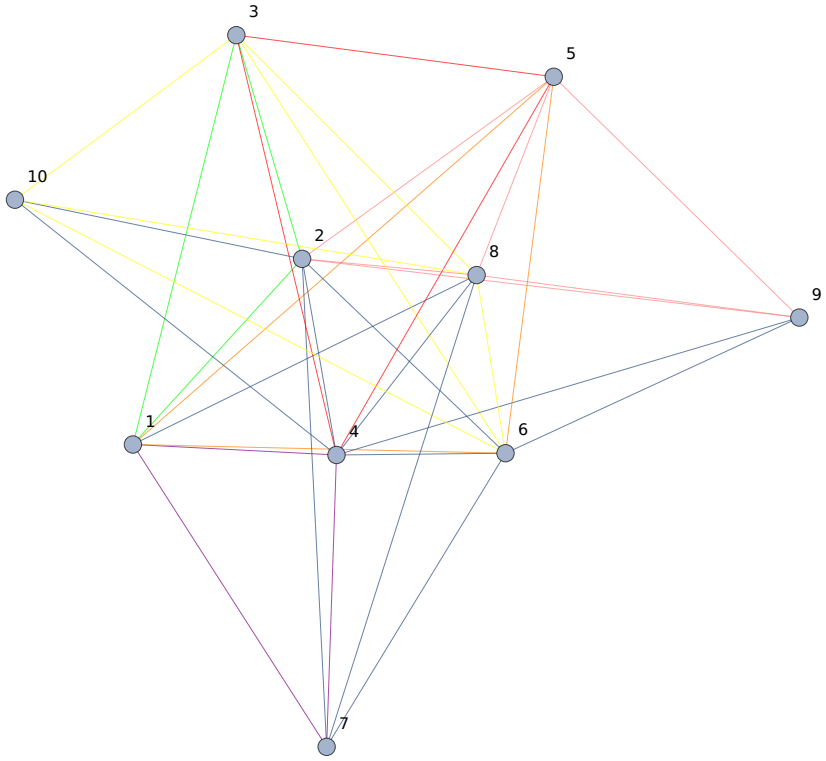


```
Clear[graph, vertices, edges];
vertices = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
edges = {1 ⟷ 2, 1 ⟷ 3, 1 ⟷ 4, 1 ⟷ 5, 1 ⟷ 6, 1 ⟷ 7, 1 ⟷ 8,
   2 ⟷ 3, 2 ⟷ 4, 2 ⟷ 5, 2 ⟷ 6, 2 ⟷ 7, 2 ⟷ 8, 2 ⟷ 9, 2 ⟷ 10, 3 ⟷ 4, 3 ⟷ 5,
   3 ⟷ 6, 3 ⟷ 8, 3 ⟷ 10, 4 ⟷ 5, 4 ⟷ 6, 4 ⟷ 7, 4 ⟷ 8, 4 ⟷ 9, 4 ⟷ 10, 5 ⟷ 6,
   5 ⟷ 8, 5 ⟷ 9, 6 ⟷ 7, 6 ⟷ 8, 6 ⟷ 9, 6 ⟷ 10, 7 ⟷ 8, 8 ⟷ 9, 8 ⟷ 10};
```

Any selection of 3 connected points which do not all lie on a common line forms a triangle. A group of vertices lying on a common line is given a distinct color.

In[216]:=
```
graph = Graph[vertices, edges, VertexLabels → "Name", EdgeStyle →
   {1 ⟷ 2 → Green, 1 ⟷ 3 → Green, 2 ⟷ 3 → Green, 1 ⟷ 6 → Orange, 1 ⟷ 5 → Orange,
    6 ⟷ 5 → Orange, 5 ⟷ 4 → Red, 5 ⟷ 3 → Red, 3 ⟷ 4 → Red, 1 ⟷ 7 → Purple,
    1 ⟷ 4 → Purple, 4 ⟷ 7 → Purple, 5 ⟷ 8 → Pink, 5 ⟷ 9 → Pink, 9 ⟷ 8 → Pink,
    9 ⟷ 2 → Pink, 5 ⟷ 2 → Pink, 2 ⟷ 8 → Pink, 3 ⟷ 6 → Yellow, 3 ⟷ 8 → Yellow,
    3 ⟷ 10 → Yellow, 10 ⟷ 8 → Yellow, 10 ⟷ 6 → Yellow, 8 ⟷ 6 → Yellow}]
```

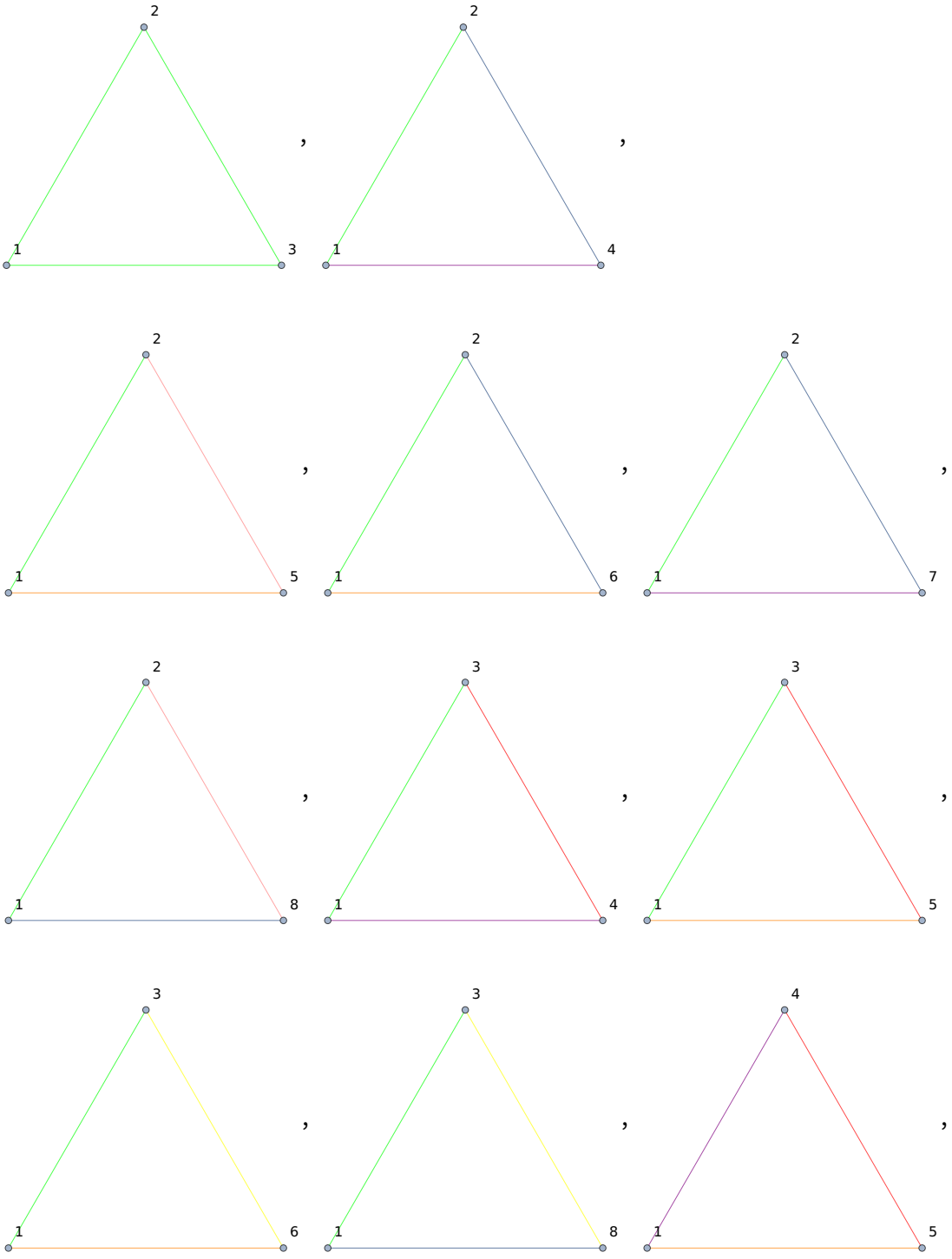
Out[216]=



We can now list all the triangles in the graph, and manually count/subtract all the triangles with all 3 edges the same color.

In[217]:=
```
s = Subsets[Range[VertexCount[graph]], {VertexCount[CompleteGraph[3]]}];
Select[Subgraph[graph, #] & /@ s, IsomorphicGraphQ[#, CompleteGraph[3]] &]
Length[Select[Subgraph[graph, #] & /@ s, IsomorphicGraphQ[#, CompleteGraph[3]] &]] - 16
```

Out[218]=

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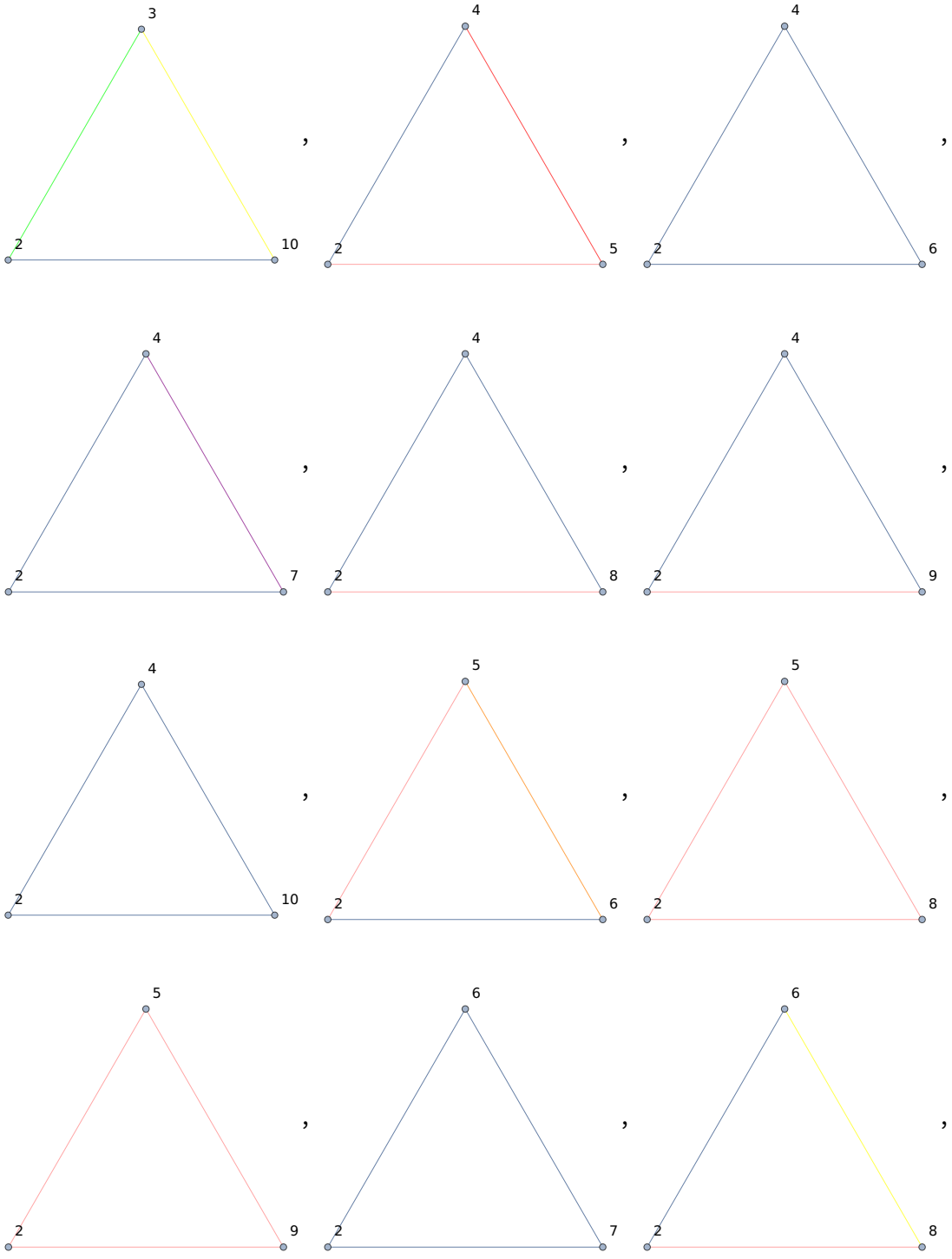
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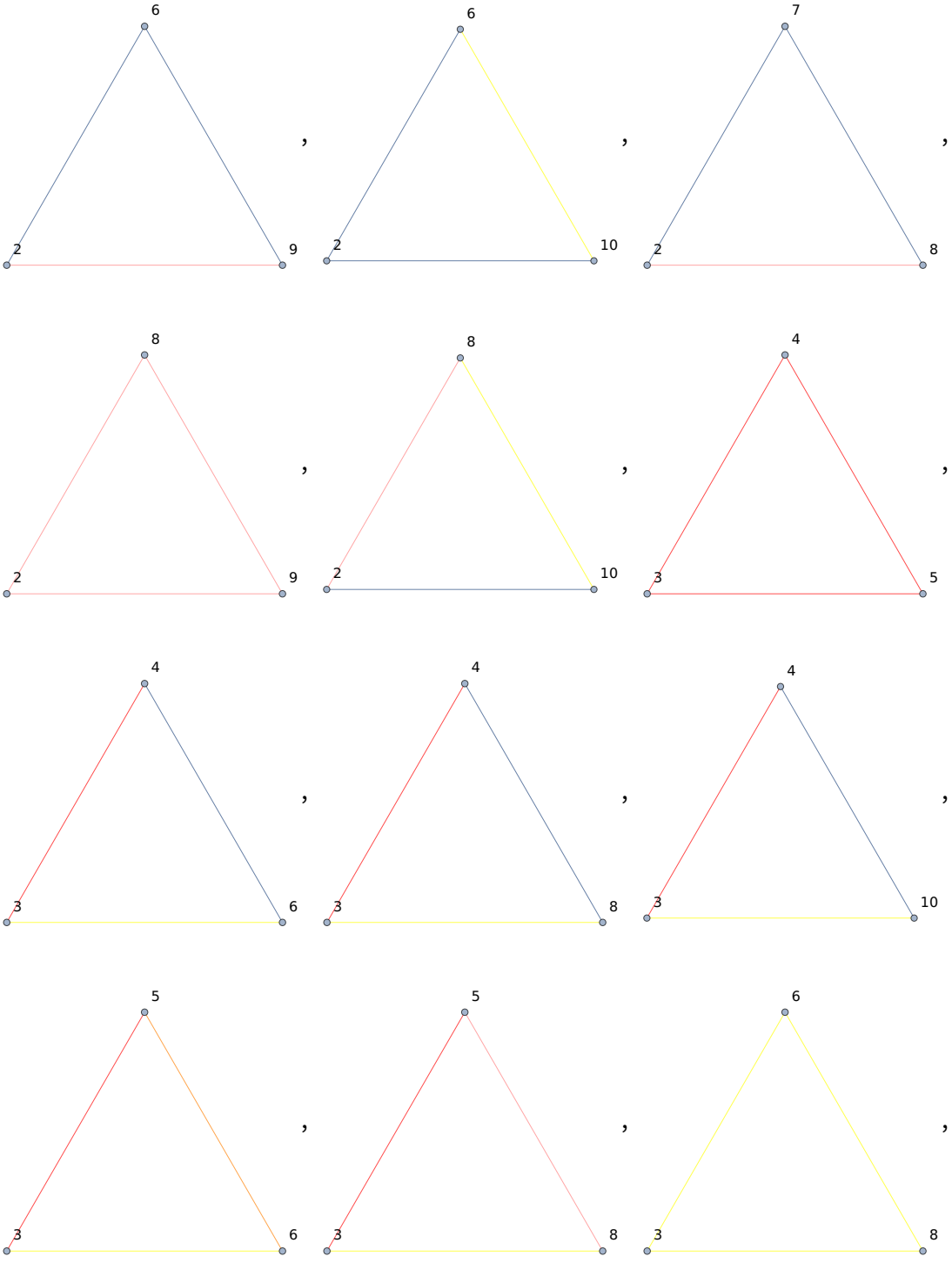
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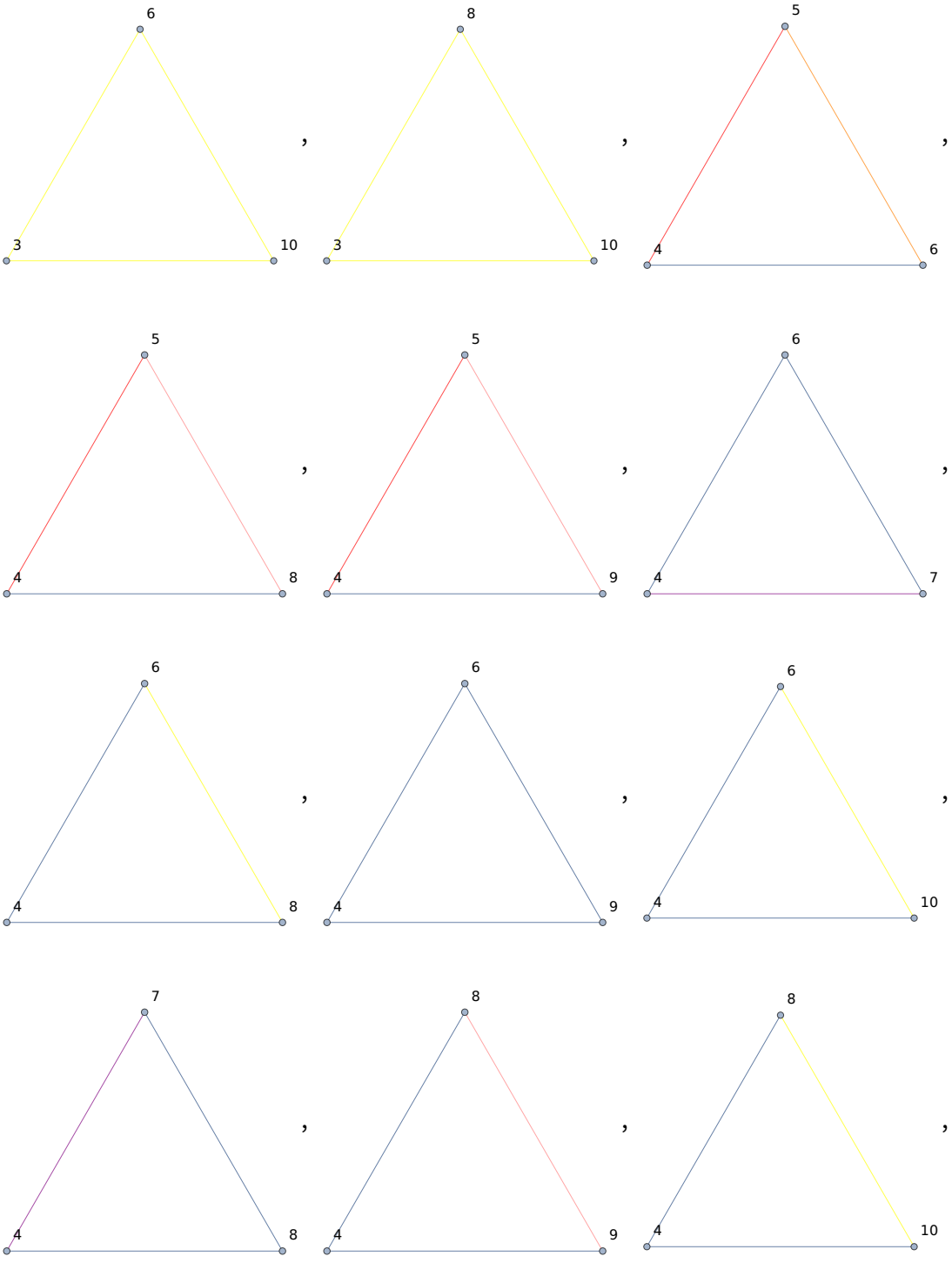
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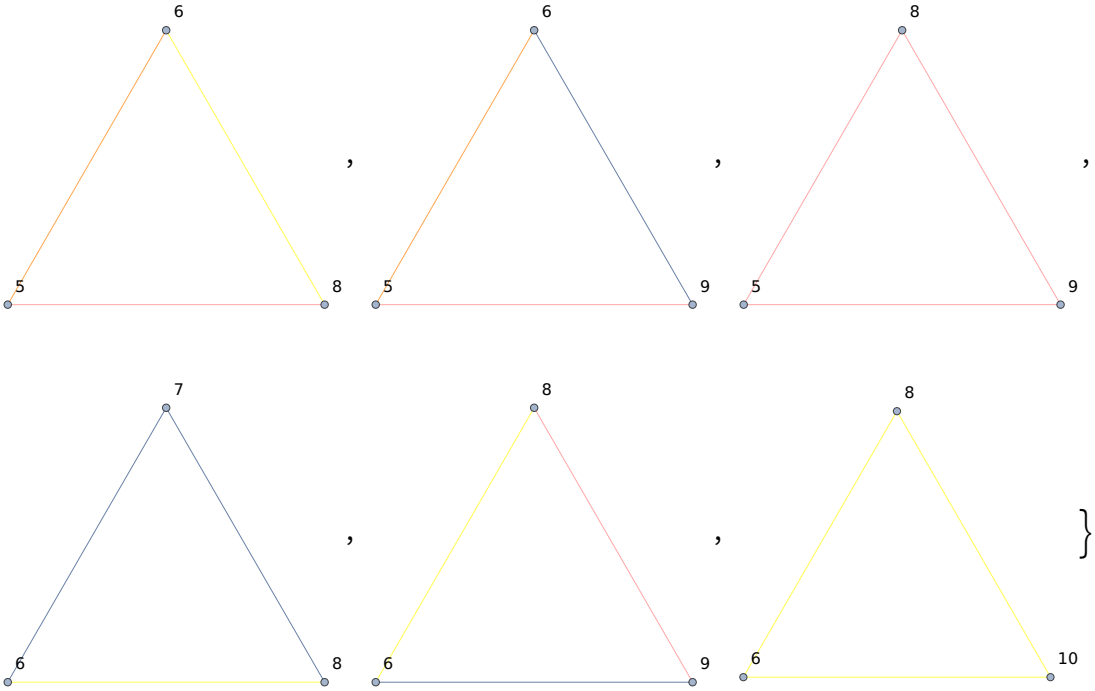

Out[219]= 49