ME 274 DYNAMICS notes

CHAPTER 1:

→ Cartesian description

- components are described in x + y, usually by giving an eqn $y = y(x)$

$$\boxed{\vec{r} = x\hat{i} + y\hat{j}} \quad \boxed{\vec{V} = \frac{d\vec{r}}{dt} = \dot{x}\hat{i} + \dot{y}\hat{j}} \quad \boxed{\vec{a} = \frac{d^2\vec{r}}{dt^2} = \ddot{x}\hat{i} + \ddot{y}\hat{j}}$$

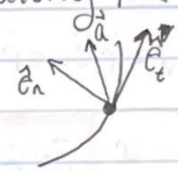
[assuming $\hat{i}$ + $\hat{j}$ represent constant directions]

- if x + y are explicit functions, x(t) + y(t)), then the cartesian components for velocity + acceleration vectors are found directly by time differentiation of these functions
- if the path of the point is given by $y = f(x)$, y is an implicit function of time, then we have to use the chain rule of differentiation

* $\boxed{\text{Chain rule}: \dot{y} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \dot{x}\frac{dy}{dt}}$

→ Path description

- position is known in terms of a distance "s" measured along the path of the particle



$$\boxed{\vec{V} = v\frac{d\vec{r}}{dt}} \rightarrow \boxed{\vec{V} = v\hat{e}_t} \qquad \vec{a} = \frac{d\vec{v}}{dt} \text{ when implicit} \rightarrow \boxed{\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n}$$

$\vec{V}$ = velocity
v = speed
ρ = radius of curvature
$\hat{e}_t$ = unit vector tangent to motion
$\hat{e}_n$ = unit vector normal to motion (centripital)

- velocity of a point is always tangent to the path
- |velocity| = speed
- v^2/ρ is the centripital acceleration component, $\dot{v}$ is the tangential acceleration component
- constant speed ≠ 0 acceleration [centripital accel]

* ρ [radius of curvature] CARTESIAN $\boxed{\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}}$

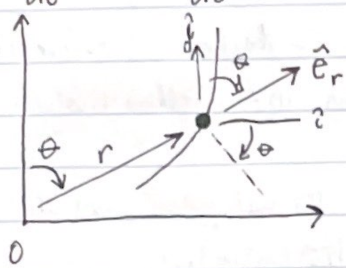
- magnitude of acceleration includes both tangential + normal components
- rate of change of speed = $\dot{v}$

* magnitude of acceleration: $\boxed{|\vec{a}| = \sqrt{\dot{v}^2 + \left(\frac{v^2}{\rho}\right)^2}}$

→ Polar Kinematics

- $\hat{e}_r$: pointing from O to point P
  $\hat{e}_\theta$: perpendicular to $\hat{e}_r$ + pointing in the "positive θ direction"
- positional vector: $\boxed{\vec{r} = r\hat{e}_r}$

$$\vec{v} = \frac{d}{dt}(r\hat{e}_r) = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt} \rightarrow \boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$$



$$\boxed{\begin{aligned}\hat{e}_r &= \sin\theta\,\hat{i} + \cos\theta\,\hat{j} \\ \hat{e}_\theta &= \cos\theta\,\hat{i} - \sin\theta\,\hat{j}\end{aligned}}$$

- $\frac{d\hat{e}_r}{d\theta} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j} \Rightarrow \boxed{\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta}$

  $\frac{d\hat{e}_\theta}{d\theta} = -\sin\theta\,\hat{i} - \cos\theta\,\hat{j} \Rightarrow \boxed{\frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r}$

* $$\boxed{\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}(\hat{e}_r) + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}(\hat{e}_\theta)}$$

or $$\boxed{\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta}$$

C

Velocity Vector		acceleration vector
$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$	Cartesian	$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$
$= v\hat{e}_t$	path	$= \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$
$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	polar	$= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

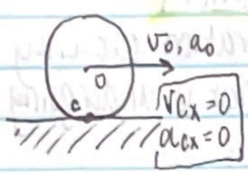
- there are 2 critical steps in solving for components for one kinematic description in terms of another
  i) write unit vectors of one description in terms of the other description
  ii) using either projection methods or coefficient balencing for like kinematic descriptions

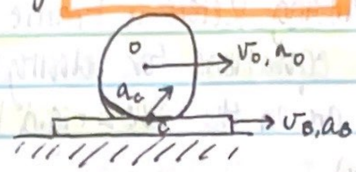
D
- $\boxed{\vec{r}_{B/A} = \text{the vector which points FROM A to B}}$ A is start, B is end $\boxed{\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A}$

⇒ differentiation of this vector will produce the relative velocity + relative acceleration vectors of B with respect to A

CHAPTER 2: Planar Rigid Body Kinematics:

A
- $\vec{R}_{B/A} = \vec{R}_B - \vec{R}_A$
- * $\boxed{\vec{V}_B = \vec{V}_A + (\vec{W} \times \vec{r}_{B/A})}$ V = velocity; ω = angular velocity; r = radius B with respect to A
- * $\boxed{\vec{a}_B = \vec{a}_A + (\vec{\alpha} \times \vec{r}_{B/A}) - \omega^2 \vec{r}_{B/A}}$ $\vec{a}$ = acceleration; α = angular acceleration
- α + ω are in $\hat{k}$ direction!
- relative velocity $\vec{V}_{B/A}$ is always perpendicular to the line connecting points A + B. HOWEVER, this is NOT the same with acceleration
- Rolling without slipping: $\boxed{V_{ox} = 0 + a_{ox} = 0}$ ← {x direction = 0!}



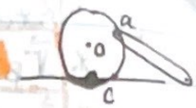


↳ therefore $V_{oy} = 0$ + $a_{cx} = 0$ ↳ $V_{cx} = V_B$ + $a_{cx} = a_b$
$V_{oy} = 0$, $a_{cy} \neq 0$

B Instantaneous centers of rotation:
- Since C is the center of rotation, $\vec{V}_c = 0$
- the speed of any point is proportional to the distance from that point to C

Instant Centers method:
1. Locate points A + B. Know the direction of both's velocity, and the magnitude of A
2. on a sketch of the body, draw the directions of its velocity vectors $\vec{V}_A + \vec{V}_B$
3. draw lines that are perpendicular to $\vec{V}_A + \vec{V}_B$
4. the intersection of these two lines is the instant center C of the body
5. ↳ From this we can find:
   - the magnitude of the angular velocity of the body: $\boxed{\omega = \frac{|\vec{V}_A|}{|\vec{r}_{A/C}|}}$
   - if the body is rotating clockwise or counterclockwise about pt. C
6. the velocity of any point D on the body is ⊥ to the line connecting C + D. the speed of D is found from $\boxed{|\vec{V}_D| = |\vec{\omega}||\vec{r}_{D/C}|}$

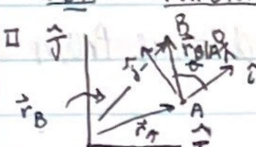
- disk rolling without slipping with rod attached
  - → Velocity eqn #1) two circumference points [AC]
    eqn #2) circumference point + point on rod [AB]



  - → Acceleration eqn #1) two circumference points [AC]
    eqn #2) circumference point + point on rod [AB]
    eqn #3) non-slipping circumference point + center pt [CO]

C
- given angular velocities + accelerations must point in +/- $\hat{k}$ direction propperly
- $\vec{r}_{B/A} = \vec{r}_{B-A}$ points from A to B

CHAPTER 3: Moving Reference Frame Kinematics

A
- the previous equations for velocity & acceleration are only true if A+B are on the SAME rigid body that is undergoing planar motion
- this chapter focuses on relating the motion of two points that are NOT on the SAME rigid body
- the "observer" on a "moving reference frame" has both translational + rotational motion
- 

  → note, unit vectors $\hat{i} + \hat{j}$ have time-varying orientations (+ therefore not constant vectors) since they are rotating with the moving reference frame.

  → $\hat{I} + \hat{J}$ are constant

- $\vec{\omega} = \dot{\theta}\hat{k}$ → the angular velocity of the observer
- $\frac{d\hat{i}}{dt} = \dot{\theta}\hat{j} \rightarrow \frac{d\hat{i}}{dt} = \dot{\theta}(\hat{k} \times \hat{i}) = \dot{\theta}\hat{k} \times \hat{i} = \boxed{\vec{\omega} \times \hat{i} = \frac{di}{dt}}$

  $\frac{d\hat{j}}{dt} = -\dot{\theta}\hat{i} \rightarrow \frac{d\hat{j}}{dt} = -\dot{\theta}(-\hat{k} \times \hat{j}) = \dot{\theta}\hat{k} \times \hat{j} = \boxed{\vec{\omega} \times \hat{j} = \frac{dj}{dt}}$

- recall relative position eqn: $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$
  ↳ $\vec{r}_{B/A} = x\hat{i} + y\hat{j}$

- $\boxed{\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{rel} + (\vec{\omega} \times \vec{r}_{B/A})}$

  $\boxed{\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + (\alpha \times \vec{r}_{B/A}) + (2\vec{\omega} \times (v_{B/A})_{rel}) + [\vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})]}$

- $\omega + \alpha$: ___ of the observer

- $\boxed{\begin{array}{l}(V_{B/A})_{rel} = \dot{x}\hat{i} + \dot{y}\hat{j} \\ (a_{B/A})_{rel} = \ddot{x}\hat{i} + \ddot{y}\hat{j}\end{array}}$ "velocity of B as seen by the moving observer" "acceleration of B as seen by the moving observer"
- $\vec{V}_A, \vec{V}_B, \vec{a}_A, \vec{a}_B$ are the properties of A+B as seen by the fixed observer

CHAPTER 4: Particle Kinetics + CHAPTER 5: RIGID Kinetics

A Cartesian: $\Sigma \vec{F} = m\vec{a} \Rightarrow \Sigma \vec{F}_x = m\ddot{x}$ & $\Sigma \vec{F}_y = m\ddot{y}$

path: $\Sigma \vec{F} = m\vec{a} \Rightarrow \Sigma F_t = m\dot{v}$ & $\Sigma F_n = m\, v^2/\rho$

polar: $\Sigma \vec{F} = ma \Rightarrow \Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) + \Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

- Net force explicitly dependent on time: $\boxed{v_2 = v_1 + \frac{1}{m}\int_{t_1}^{t_2} F(t)\,dt}$

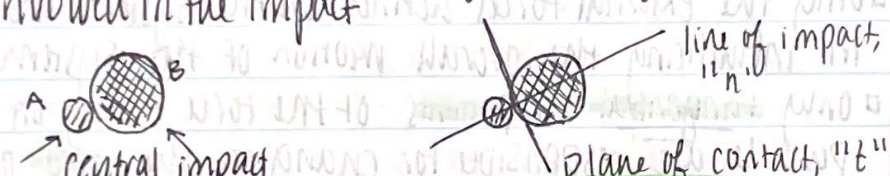
  Net force explicitly dependent on position: $\boxed{v_2 = \sqrt{v_1^2 + \frac{2}{m}\int_{s_1}^{s_2} F(s)\,ds}}$

  Net force explicitly dependent on speed: $\boxed{t_2 = t_1 + m\int_{v_1}^{v_2} \frac{dv}{F(v)}}$ $\boxed{s_2 = s_1 + m\int_{v_1}^{v_2} \frac{v\,dv}{F(v)}}$
- only the external forces acting on the system are effective in influencing the overall motion of the system

B
- Only tangential components of the force acting on the particle are responsible for changing the speed of the particle
- the normal components are responsible for changing the direction of the motion
- $R_t = m\frac{dv}{dt} = mv\frac{dv}{ds}$
- $U_{\text{Non conservative}} = \int_1^2 (\vec{R}\cdot\hat{e}_t)\,ds$
- potential energy of a spring is ALWAYS ≥ 0
- the work-energy equation is a SCALAR

C
- linear impulse momentum - relates the change in velocity of a particle as a result of forces on the particle over an elapsed period of time

- linear impulse momentum: $\int_1^2 \vec{R}\,dt = mv_2 - mv_1$
  - ↳ this relates the **change in velocity** of the particle to a **change in time**
- in contrast to work-energy eqn, LIM is a **vector** equation
- if the resulting force in a given direction is zero, $\int R\,dt$, the linear momentum is conserved (if $\Sigma F_{net} = 0$)
- **conservation of momentum** does **NOT imply** or **result from** conservation of **energy**
- it is advised to make the system as **big as possible**
- **Plane of contact**: a plane that is tangent to the contact surfaces of A + B ("t")
- **line of impact**: a line that is perpendicular to the plane of contact for A + B ("n")
- **Central Impact**: an impact in which the line of impact passes through the centers of masses of each body involved in the impact



- **Coefficient of Restitution** (e):

$$\boxed{e = -\frac{v_{B_{n2}} - v_{A_{n2}}}{v_{B_{n1}} - v_{A_{n2}}}}$$

- during impact of **impulsive forces, non-impulsive forces can be ignored** (ex: gravity, spring etc.)
  - ↳ reaction forces can be impulsive
- NEVER use work-energy eqn during impact
- the moment of $\vec{R}$ about point O, $\vec{M}_O$ is

$$\boxed{\vec{M}_O = \vec{r}_{P/O} \times \vec{R}}$$

R = force

r = radius from points P+O

- "Angular Momentum" of P about the fixed point O:

$$\boxed{\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P} \Rightarrow \boxed{\cancel{\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P}}$$

- "Angular Impulse Momentum"

$$\int_1^2 \vec{M}_O\,dt = \int_1^2 d\vec{H}_O = \boxed{H_{O2} - H_{O1}}$$

- Angular momentum in CARTESIAN:

$$\boxed{\vec{H}_O = m\left(x_P v_{Py} - y_P v_{Px}\right)\vec{k}}$$ p=point

- Angular momentum in POLAR:

$$\boxed{\vec{H}_O = m\left(r^2\dot{\theta}\right)\vec{k}}$$ ← most helpful form

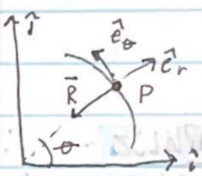
- the angular momentum vector points perpendicular to the plane of motion (into or out of the page)

Central force problem:

↳ a special case where the resultant force $\vec{R}$ on point P points towards/away from the fixed point O.

↳ the moment M of point O due to this resultant force is zero

↳ therefore, the Angular momentum of the particle about the fixed point O is CONSTANT



$$\boxed{\vec{H}_{O2} = H_{O1} + \int_1^2 \vec{M}_O\,dt}$$

- the angular impulse momentum equation relates the time integral of the moment exerted on a particle about a fixed point O to the change in angular momentum of the particle about O
- when angular momentum is conserved (ex central force problems)

$$\boxed{\dot{\theta}_2 = \frac{r_1^2}{r_2^2}\dot{\theta}_1}$$

- angular momentum gives only information about the rotational component, $r\dot{\theta}$, of the particles velocity

↳ if the total velocity of the particle is required, then usually the work-energy equation will be used to find the other component, $\dot{r}$

- $$\boxed{\sum_{i=1}^{N} \vec{M}_A = m\vec{r}_{G/A} \times \vec{a}_A + \sum_{i=1}^{N} \frac{d}{dt}\left[\vec{r}_{i/A} \times (m_i v_{i/A})\right]}$$

G = center of mass of system — m = total mass of system

CHAPTER 5: Planar + Rigid Body Kinetics

NEWTON EULER:
- If point A is either
  - Center of mass
  - Fixed Point on the body
  - $\vec{a}_A$ is PARALLEL to $\vec{r}_{G/A}$

  $\Big\}\ \Sigma M = I\vec{\alpha}$

* radius of Gyration: $\boxed{K_A = \sqrt{I_A/M}}$ ↑ mass

- $\Sigma F = ma$ → acceleration is ALWAYS $\vec{a}_{COM}$

WORK-ENERGY:
- includes multiple bodies

* Parallel Axis Theorem - $\boxed{I_A = I_G + md_{AG}^2}$

- eliminate term $m\vec{v}_P \cdot (\vec{\omega} \times \vec{r}_{G/P})$ through
  - Point is COM
  - Point is FIXED on Body

  $\Big\}\ K_E$ term

IMPULSE-MOMENTUM:
- G has to be COM, A has to be fixed point or COM
- good when momentum is conserved in one direction / AM

* Coefficient of Restitution: $\boxed{e = \dfrac{-V_{BN2} - V_{AN2}}{V_{BN1} - V_{AN1}}}$

KINETICS TABLE:

Relation	method	Body Model	Fundamental Eqn's
forces – acceleration	NEWTON-EULER	Particle	$\Sigma \vec{F} = m\vec{a}$
		rigid body	$\Sigma \vec{F} = m\vec{a}_G$ $\Sigma \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Δ in speed – Δ in position	WORK-ENERGY	particle	$K_1 + U_1 + W_{NC} = K_2 + U_2$ where $K = \frac{1}{2}mv^2$
		rigid body [G = C.M., A pt on body]	$K_1 + U_1 + W_{NC} = K_2 + U_2$ where $K = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times r_{G/A})$
Δ in velocity – Δ in time	Linear IMPULSE-MOMENTUM	particle	$\int_{t_1}^{t_2} \Sigma \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$
		rigid body [G = C.M]	$\int_{t_1}^{t_2} \Sigma \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Δ in (ω) angular velocity – Δ in time	ANGULAR IMPULSE-MOMENTUM	particle [O = fixed pt]	$\int_{t_1}^{t_2} \Sigma \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
		rigid Body [A = C.M. or fixed pt]	$\int_{t_1}^{t_2} \Sigma \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A \vec{\omega}$

KINEMATICS

$$\left.\begin{aligned} &* R_{B/A} = R_B - R_A \\ &* \vec{V}_B = \vec{V}_A + (\vec{\omega} \times \vec{r}_{B/A}) \\ &* \vec{a}_B = \vec{a}_A + (\alpha \times \vec{r}_{B/A}) - \omega^2 \vec{r}_{B/A} \end{aligned}\right\} \text{RIGID BODY}$$

$$\left.\begin{aligned} &* \vec{V}_B = \vec{V}_A + (V_{B/A})_{rel} + (\omega \times \vec{r}_{B/A}) \\ &* \vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + (\alpha \times \vec{r}_{B/A}) + (2\omega \times (V_{B/A})_{rel}) + [\omega \times (\omega \times \vec{r}_{B/A})] \end{aligned}\right\} \text{MOVING REFERENCE FRAME}$$

CHAPTER 6 : Vibrations

A
- when spring is compressed, the spring pushes, + vise versa for stretch
- $\boxed{F_{spring} = Kx}$ + $\boxed{F_{dashpot} = C\dot{x}}$ → "shock absorber"
- when drawing FBD's for Equations of Motion (EOM's), draw in the $+\Delta x$ direction

* $\boxed{\text{General EOM: } M\ddot{x} + C\dot{x} + Kx = f(t)}$

↳ all parts should be the same sign!

[B] measuring from static deformation (not from no Δx): $\boxed{z = x - x_{st}}$

↳ also x_{st} = constant ∴ $\dot{z} = \dot{x}$ $\ddot{z} = \ddot{x}$

▫ general solution to $M\ddot{x} + C\dot{x} + Kx = 0$ is $\boxed{x(t) = Ae^{\lambda t}}$

and substitution gives $\boxed{(\lambda^2 M + \lambda C + K)Ae^{\lambda t} = 0}$

↳ only real solution is $\boxed{\lambda^2 M + \lambda C + K = 0}$

* Dampening ratio ζ: $\boxed{\frac{C}{M} = 2\zeta\omega_n}$

* $\boxed{\omega_n = \sqrt{K/M}}$

▫ $\boxed{x(t) = e^{-\zeta\omega_n t}\left[A\cos\omega_d t + B\sin\omega_d t\right]}$

▫ Underdamped − $0<\zeta<1$,

↳ Damped Natural freq. $\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$

▫ Standard Form for EOM of Single-Degree-of-Freedom System

$$\boxed{\ddot{x} + \frac{C}{M}\dot{x} + \frac{K}{M}x = \frac{1}{M}f(t)}$$

[C] resonance: $\omega/\omega_n = 1$ (unbounded in Theory)