Algebraic meme equation analysis in $\mathbb{Z}^2$ using recreational mathematics

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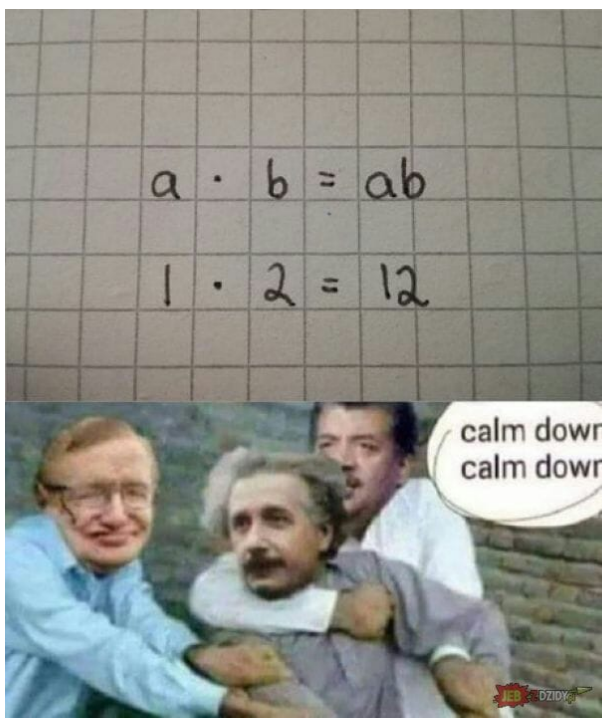
1 Meme



$$a \cdot b = ab \quad (multiplication)$$
$$1 \cdot 2 = 12 \quad (concatenation)$$

Since multiplication (ab) and concatenation ($\overline{ab}$) are two different operations, the second equation is not a logical conclusion from the first one, yet to some uncareful readers it might seem like it, because the operations look almost the same.

2 Equation

Let us consider for which values of $a, b \in \mathbb{Z}$ the following equation will be satisfied.

$$ab = \overline{ab} \tag{1}$$

The equation can be easily shown to be equivalent to

$$ab - 10a = b \tag{2}$$

since $\overline{ab} = 10a + b$. Our goal is to find such integers a and b, so that the multiplication operation and concatenation operation should yield the same result, and we will do that by finding all integer solutions to equation (2).

3 Solution

We can manipulate our equation to obtain value of one unknown in terms of the other.

$$ab - 10a = b$$
$$a(b - 10) = b$$

After dividing by non-zero value ($b \neq 10$) we get the following

$$a = \frac{b}{b - 10} \tag{3}$$

which is equivalent to

$$a = 1 + \frac{10}{b - 10}$$

But both a and b are integers, therefore $\frac{10}{b-10} \in \mathbb{Z}$. From this we conclude

$$(b - 10) \mid 10 \tag{4}$$

Since conditions (3) and (4) have been naturally derived from our first equation and assumptions, they must be met by any pair (a, b) which is a solution to our question; these are the *necessary* conditions.

(4):

$$\begin{aligned} (b-10) &\mid 10 \\ (b-10) &\in d(10) \\ (b-10) &\in \{-10,-5,-2,-1,1,2,5,10\} \\ b &\in \{0,5,8,9,11,12,15,20\} \end{aligned}$$

(3):

$$(a,b) \in \{(0,0);(-1,5);(-4,8);(-9,9);(11,11);(6,12);(5,15);(2,20)\}$$

By the necessity of our conditions, if a solution exists, it has to be one of the pairs listed above. Simple check will rule out all of the options except the trivial solution $(0,0)$.

4 Conclusion

By the proof by exhaustion we find that no non-trivial solutions exist, so there is no non-zero pair $(a,b) \in \mathbb{Z}^2$ that satisfies the equation $ab = \overline{ab}$.