

# AIEEE Mathematics Quick Review

## COMPLEX NUMBERS AND DEMOIVRES THEOREM

- General form of Complex numbers  $x + iy$  where  $x$  is Real part and  $y$  is Imaginary part.
- Sum of  $n^{\text{th}}$  root of unity is zero
- Product of  $n^{\text{th}}$  root of unity  $(-1)^{n-1}$
- Cube roots of unity are  $1, \omega, \omega^2$
- $1 + \omega + \omega^2 = 0, \omega^3 = 1,$   
 $\omega = \frac{-1 + \sqrt{3}i}{2}, \omega^2 = \frac{-1 - \sqrt{3}i}{2}$
- $\text{Arg } z = \tan^{-1} \frac{y}{x}$  principle value of  $\theta$  is  $-\pi < \theta \leq \pi$
- $\text{Arg of } x + iy$  is  $\theta = \tan^{-1} \frac{y}{x}$  for every  $x > 0, y > 0$
- $\text{Arg of } x - iy$  is  $\theta = -\tan^{-1} \frac{y}{x}$  for every  $x > 0, y > 0$
- $\text{Arg of } -x + iy$  is  $\theta = \pi - \tan^{-1} \frac{y}{x}$  for every  $x > 0, y > 0$
- $\text{Arg of } -x - iy$  is  $\theta = -\pi + \tan^{-1} \frac{y}{x}$  for every  $x > 0, y > 0$
- $\text{Arg } z_1 z_2 = \text{Arg } z_1 + \text{Arg } z_2$
- $\text{Arg } \frac{z_1}{z_2} = \text{Arg } z_1 - \text{Arg } z_2$
- $\text{Arg } \bar{z} = -\text{Arg } z$
- $i = \sqrt{-1}, \frac{1+i}{1-i} = i, \frac{1-i}{1+i} = -i, (1+i)^2 = 2i, (1-i)^2 = -2i$   
 $\sqrt{a+ib} = \sqrt{\frac{x+a}{2}} + i\sqrt{\frac{x-a}{2}}, \sqrt{a-ib} = \sqrt{\frac{x+a}{2}} - i\sqrt{\frac{x-a}{2}}$  where  $x = \sqrt{a^2 + b^2}$
- $(1 + \sqrt{3}i)^n + (1 - \sqrt{3}i)^n = 2^n \cos \frac{n\pi}{3}$
- $(1+i)^n + (1-i)^n = 2^{\frac{n+1}{2}} \cos \frac{n\pi}{4}$
- $|z_1 + z_2| \leq |z_1| + |z_2|$ ;  
 $|z_1 - z_2| \geq |z_1| - |z_2|$ ;  
 If three complex numbers  $Z_1, Z_2, Z_3$  are collinear then  

$$\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} = 0$$
- Area of triangle formed by  $Z, iZ, Z + Zi$  is  $\frac{1}{2} Z^2$
- Area of triangle formed by  $Z, \omega Z, Z + \omega Z$  is  $\frac{\sqrt{3}}{4} Z^2$
- If  $Z_1^2 - Z_2 Z_3 + Z_3^2 = 0$  then origin,  $Z_1, Z_2$  forms an equilateral triangle
- If  $Z_1, Z_2, Z_3$  forms an equilateral triangle and  $Z_0$  is circum center then  $Z_1^2 + Z_2^2 + Z_3^2 = 3Z_0^2$ ,
- If  $Z_1, Z_2, Z_3$  forms an equilateral triangle and  $Z_0$  is circum center then  $Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$

- Distance between two vertices  $Z_1, Z_2$  is  $|z_1 - z_2|$
- $|z - z_0| = p$  is a circle with radius  $p$  and center  $z_0$
- $z\bar{z} + \bar{\alpha}z + \alpha\bar{z} + \beta = 0$  Represents circle  
 With radius  $\sqrt{\alpha^2 - \beta}$  where  $\alpha$  is nonreal complex and  $\beta$  is constant
- If  $\left| \frac{z - z_1}{z - z_2} \right| = k$  ( $k \neq 1$ ) represents circle with ends of diameter  $\frac{kz_2 \pm z_1}{k \pm 1}$   
 If  $k = 1$  the locus of  $z$  represents a line or perpendicular bisector.
- $|z - z_1| + |z - z_2| = k, k > |z_1 - z_2|$  then locus of  $z$  represents Ellipse and if  $k < |z_1 - z_2|$  it is less, then it represents hyperbola
- $A(z_1), B(z_2), C(z_3)$ , and  $\theta$  is angle between  $AB, AC$  then  $\left| \frac{z_1 - z_2}{z_1 - z_3} \right| = \frac{AB}{AC} e^{i\theta}$
- $e^{i\theta} = \cos\theta + i\sin\theta = \cos\theta, e^{i\pi} = -1,$   
 $e^{\frac{\pi}{2}i} = i, \log i = \frac{\pi}{2}i$
- $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
- $\cos\theta + i\sin\theta = \text{Cis}\theta,$   
 $\text{Cis}\alpha \cdot \text{Cis}\beta = \text{Cis}(\alpha + \beta),$   
 $\frac{\text{Cis}\beta}{\text{Cis}\alpha} = \text{Cis}(\beta - \alpha)$
- If  $x = \cos\theta + i\sin\theta$  then  $\frac{1}{x} = \cos\theta - i\sin\theta$   
 $\Rightarrow x + \frac{1}{x} = 2\cos\alpha \Rightarrow x - \frac{1}{x} = 2i\sin\alpha$   
 $\Rightarrow x^n + \frac{1}{x^n} = 2\cos n\alpha$   
 $\Rightarrow x^n - \frac{1}{x^n} = 2i\sin n\alpha$
- If  $\Sigma \cos\alpha = \Sigma \sin\alpha = 0$   
 $\Sigma \cos 2\alpha = \Sigma \sin 2\alpha = 0$   
 $\Sigma \cos 2^n \alpha = \Sigma \sin 2^n \alpha = 0,$   
 $\Sigma \cos^2 \alpha = \Sigma \sin^2 \alpha = 3/2$   
 $\Sigma \cos 3\alpha = 3\cos(\alpha + \beta + \gamma),$   
 $\Sigma \sin 3\alpha = 3\sin(\alpha + \beta + \gamma)$   
 $\Sigma \cos(2\alpha - \beta - \gamma) = 3,$   
 $\Sigma \sin(2\alpha - \beta - \gamma) = 0,$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

## Quadratic Expressions

- Standard form of Quadratic equation is  $ax^2 + bx + c = 0$   
 Sum of roots =  $-\frac{b}{a}$ , product of roots  $\frac{c}{a}$ , discriminate =  $b^2 - 4ac$   
 If  $\alpha, \beta$  are roots then Quadratic equation is  $x^2 - x(\alpha + \beta) + \alpha\beta = 0$
- If the roots of  $ax^2 + bx + c = 0$  are  $1, \frac{c}{a}$  then  $a + b + c = 0$
- If the roots of  $ax^2 + bx + c = 0$  are in ratio  $m : n$  then  $mnb^2 = (m + n)^2 ac$
- If one root of  $ax^2 + bx + c = 0$  is square of the other then  $ac^2 + a^2c + b^3 = 3abc$
- If  $x > 0$  then the least value of  $x + \frac{1}{x}$  is 2
- If  $a_1, a_2, \dots, a_n$  are positive then the least value of

- $(a_1 + a_2 + \dots + a_n) \left( \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right)$  is  $n^2$
- If  $a^2 + b^2 + c^2 = K$  then range of  $ab + bc + ca$  is  $\left[ \frac{-K}{2}, K \right]$
  - If the two roots are negative, then a, b, c will have same sign
  - If the two roots are positive, then the sign of a, c will have different sign of 'b'
  - $f(x) = 0$  is a polynomial then the equation whose roots are reciprocal of the roots of  $f(x) = 0$  is  $f\left(\frac{1}{x}\right) = 0$  increased by 'K' is  $f(x - K)$ , multiplied by K is  $f(x/K)$
  - For a, b, h  $\in \mathbb{R}$  the roots of  $(a - x)(b - x) = h^2$  are real and unequal
  - For a, b, c  $\in \mathbb{R}$  the roots of  $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$  are real and unequal
  - Three roots of a cubical equation are A.P, they are taken as a - d, a, a + d
  - Four roots in A.P, a-3d, a-d, a+d, a+3d
  - If three roots are in G.P  $\frac{a}{r}, a, ar$  are taken as roots
  - If four roots are in G.P  $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$  are taken as roots
  - For  $ax^3 + bx^2 + cx + d = 0$ 
    - $\Sigma \alpha^2 \beta = (\alpha\beta + \beta\gamma + \gamma\alpha) (\alpha + \beta + \gamma) - 3\alpha\beta\gamma = s_1 s_2 - 3s_3$
    - $\alpha^2 + \beta^2 + \gamma^2 = s_1^2 - 2s_2$
    - $\alpha^4 + \beta^4 + \gamma^4 = s_1^4 - 4s_1^2 s_2 + 4s_1 s_3 + 2s_2^2$
    - $\alpha^3 + \beta^3 + \gamma^3 = s_1^3 - 3s_1 s_2 + 3s_3$
    - In  $ax^n + bx^{n-1} + cx^{n-2} + \dots = 0$  to eliminate second term roots are diminished by  $-\frac{b}{na}$

### Binomial Theorem And Partial Fractions

- Number of terms in the expansion  $(x + a)^n$  is  $n + 1$
- Number of terms in the expansion  $(x_1 + x_2 + \dots + x_r)^n$  is  ${}^{n+r-1}C_{r-1}$
- In  $(x + a)^n$ ,  $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r}$
- For  $\left( ax^p + \frac{b}{x^q} \right)^n$  independent term is  $\frac{np}{p+q} + 1$
- In above, the term containing  $x^s$  is  $\frac{np-s}{p+q} + 1$
- $(1 + x)^n - 1$  is divisible by x and  $(1 + x)^n - nx - 1$  is divisible by  $x^2$ .
- Coefficient of  $x^n$  in  $(x+1)(x+2)\dots(x+n) = n$
- Coefficient of  $x^{n-1}$  in  $(x+1)(x+2)\dots(x+n)$  is  $\frac{n(n+1)}{2}$
- Coefficient of  $x^{n-2}$  in above is  $\frac{n(n+1)(n-1)(3n+2)}{24}$

- If  $f(x) = (x + y)^n$  then sum of coefficients is equal to  $f(1)$
- Sum of coefficients of even terms is equal to  $\frac{f(1) - f(-1)}{2}$
- Sum of coefficients of odd terms is equal to  $\frac{f(1) + f(-1)}{2}$
- If  ${}^n C_{r-1}, {}^n C_r, {}^n C_{r+1}$  are in A.P  $(n-2r)^2 = n + 2$
- For  $(x+y)^n$ , if n is even then only one middle term that is  $\left(\frac{n}{2} + 1\right)^{th}$  term.
- For  $(x + y)^n$ , if n is odd there are two middle terms that is  $\frac{n+1}{2}^{th}$  term and  $\frac{n+3}{2}^{th}$  term.
- In the expansion  $(x + y)^n$  if n is even greatest coefficient is  ${}^n C_{\frac{n}{2}}$
- In the expansion  $(x + y)^n$  if n is odd greatest coefficients are  ${}^n C_{\frac{n-1}{2}}, {}^n C_{\frac{n+1}{2}}$  if n is odd
- For expansion of  $(1 + x)^n$  General notation  ${}^n C_0 = C_0, {}^n C_1 = C_1, {}^n C_r = C_r$
- Sum of binomial coefficients  $C_0 + C_1 + C_2 + \dots + C_n = 2^n$
- Sum of even binomial coefficients  $C_0 + C_2 + C_4 + \dots = 2^{n-1}$
- Sum of odd binomial coefficients  $C_1 + C_3 + C_5 + \dots = 2^{n-1}$

### MATRICES

- A square matrix in which every element is equal to '0', except those of principal diagonal of matrix is called as diagonal matrix
- A square matrix is said to be a scalar matrix if all the elements in the principal diagonal are equal and Other elements are zero's
- A diagonal matrix A in which all the elements in the principal diagonal are 1 and the rest '0' is called unit matrix
- A square matrix A is said to be Idem-potent matrix if  $A^2 = A$ ,
- A square matrix A is said to be Involu-ntary matrix if  $A^2 = I$
- A square matrix A is said to be Symm-etric matrix if  $A = A^T$   
A square matrix A is said to be Skew symmetric matrix if  $A = -A^T$
- A square matrix A is said to be Nilpotent matrix If there exists a positive integer n such that  $A^n = 0$  'n' is the index of Nilpotent matrix
- If 'A' is a given matrix, every square mat-rix can be expressed as a sum of symme-tric and skew symmetric matrix where  
Symmetric part  $= \frac{A + A^T}{2}$   
unsymmetric part  $= \frac{A - A^T}{2}$
- A square matrix 'A' is called an ortho-gonal matrix if  $AA^T = I$  or  $A^T = A^{-1}$
- A square matrix 'A' is said to be a singular matrix if  $\det A = 0$
- A square matrix 'A' is said to be non singular matrix if  $\det A \neq 0$
- If 'A' is a square matrix then  $\det A = \det A^T$
- If  $AB = I = BA$  then A and B are called inverses of each other
- $(A^{-1})^{-1} = A, (AB)^{-1} = B^{-1}A^{-1}$
- If A and  $A^T$  are invertible then  $(A^T)^{-1} = (A^{-1})^T$
- If A is non singular of order 3, A is invertible, then  $A^{-1} = \frac{\text{Adj}A}{\det A}$
- If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$  if  $ad-bc \neq 0$
- $(A^{-1})^{-1} = A, (AB)^{-1} = B^{-1}A^{-1}, (A^T)^{-1} = (A^{-1})^T, (ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

$A^{-1}$ . If A is a  $n \times n$  non-singular matrix, then

- a)  $A(\text{Adj}A) = |A|I$
- b)  $\text{Adj} A = |A| A^{-1}$
- c)  $(\text{Adj} A)^{-1} = \text{Adj} (A^{-1})$
- d)  $\text{Adj} AT = (\text{Adj} A)^T$
- e)  $\text{Det} (A^{-1}) = (\text{Det} A)^{-1}$
- f)  $|\text{Adj} A| = |A|^{n-1}$
- g)  $|\text{Adj} (\text{Adj} A)| = |A|^{(n-1)^2}$
- h) For any scalar 'k'  
 $\text{Adj} (kA) = k^{n-1} \text{Adj} A$

19. If A and B are two non-singular matrices of the same type then

- (i)  $\text{Adj} (AB) = (\text{Adj} B) (\text{Adj} A)$
- (ii)  $|\text{Adj} (AB)| = |\text{Adj} A| |\text{Adj} B|$   
 $= |\text{Adj} B| |\text{Adj} A|$

20. To determine rank and solution first convert matrix into Echolon form

$$\text{i.e. } A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 2 \\ 3 & 2 & 1 & 0 \end{bmatrix} \quad \text{Echolon form of } A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & x & y & z \\ 0 & 0 & k & 1 \end{bmatrix}$$

No of non zero rows = n = Rank of a matrix

If the system of equations  $AX=B$  is consistent if the coeff matrix A and augmented matrix K are of same rank

Let  $AX = B$  be a system of equations of 'n' unknowns and ranks of coeff matrix =  $r_1$  and rank of augmented matrix =  $r_2$

If  $r_1 \neq r_2$ , then  $AX = B$  is inconsistent,

i.e. it has no solution

If  $r_1 = r_2 = n$  then  $AX=B$  is consistent, it has unique solution

If  $r_1 = r_2 < n$  then  $AX=B$  is consistent and it has infinitely many number of solutions

### Random Variables- Distributions & Statistics

1. For probability distribution if  $x=x_i$  with range  $(x_1, x_2, x_3 \dots)$  and  $P(x=x_i)$  are their probabilities then

mean  $\mu = \sum x_i P(x=x_i)$

Variance  $= \sigma^2 = \sum x_i^2 P(x=x_i) - \mu^2$

Standard deviation =  $\sqrt{\text{variance}}$

2. If n be positive integer p be a real number such that  $0 \leq p \leq 1$  a random variable X with range  $(0,1,2,\dots,n)$  is said to follow binomial distribution.

For a Binomial distribution of  $(q+p)^n$

i) probability of occurrence = p

ii) probability of non occurrence = q

iii)  $p + q = 1$

iv) probability of 'x' successes

$P(x=x_i) = nC_x q^{n-x} p^x$

v) Mean =  $\mu = np$

vi) Variance = npq

vii) Standard deviation =  $\sqrt{npq}$

3. If number of trials are large and probability of success is very small then poisson distribution is used and given as

$$P(x=k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

4. i) If  $x_1, x_2, x_3, \dots, x_n$  are n values of variant

x, then its Arithmetic Mean  $\bar{x} = \frac{\sum x_i}{n}$

ii) For individual series If A is assumed

average then A.M  $\bar{x} = A + \frac{\sum (x_i - A)}{n}$

iii) For discrete frequency distribution:

$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$  where  $(d_i = x_i - A)$

$$\text{iv) Median} = l + \left( \frac{\frac{N}{2} - F}{f} \right) \times C$$

where  $l$  = Lower limit of Median class

$f$  = frequency

$N = \sum f_i$

$C$  = Width of Median class

$F$  = Cumulative frequency of class just preceding to median class

v) First or lower Quartile deviation

$$Q_1 = l + \left( \frac{\frac{N}{4} - F}{f} \right) C$$

where  $f$  = frequency of first quartile class

$F$  = cumulative frequency of the class just preceding to first quartile class

vi) upper Quartile deviation

$$Q_3 = l + \left( \frac{\frac{3N}{4} - F}{f} \right) C$$

vii) Mode  $Z = l + \left( \frac{f_m - f_1}{2f_m - f_1 - f_2} \right) \cdot C$  where

$l$  = lower limit of modal class with maximum frequency

$f_1$  = frequency preceding modal class

$f_2$  = frequency successive modal class

$f_3$  = frequency of modal class

viii) Mode =  $3\text{Median} - 2\text{Mean}$

ix) Quartile deviation =  $\frac{Q_3 - Q_1}{2}$

x) coefficient of quartile deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

xi) coefficient of Range

$$= \frac{\text{Range}}{\text{Maximum} + \text{Minimum}}$$

### VECTORS

1. A system of vectors  $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$  are said to be linearly independent if there exists scalars  $x_1, x_2, \dots, x_n$

Such that  $x_1 \vec{a}_1 + x_2 \vec{a}_2 + \dots + x_n \vec{a}_n = 0$

$\Rightarrow x_1 = x_2 = x_3 = \dots = x_n = 0$

2. Any three non coplanar vectors are linearly independent

A system of vectors  $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$  are said to be linearly dependent if there

$x_1 \vec{a}_1 + x_2 \vec{a}_2 + \dots + x_n \vec{a}_n = 0$

atleast one of  $x_i \neq 0, i=1, 2, 3, \dots, n$

And determinant = 0

3. Any two collinear vectors, any three coplanar vectors are linearly dependent. Any set of vectors containing null vectors is linearly independent

4. If ABCDEF is regular hexagon with center 'G' then  $AB + AC + AD + AE + AF = 3AD = 6AG$

5. Vector equation of sphere with center at  $\vec{c}$  and radius a is  $(\vec{r} - \vec{c})^2 = a^2$  or  $\vec{r}^2 - 2\vec{r} \cdot \vec{c} + \vec{c}^2 = a^2$

6.  $\vec{a}, \vec{b}$  are ends of diameter then equation of sphere  $(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = 0$

7. If  $\vec{a}, \vec{b}$  are unit vectors then unit vector along bisector of  $\angle AOB$  is

$$\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} \quad \text{or} \quad \frac{(\vec{a} + \vec{b})}{\pm |\vec{a} + \vec{b}|}$$

8. Vector along internal angular bisector is

$$\pm \lambda \left( \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} \right)$$

9. If 'I' is in centre of  $\Delta^{ic} ABC$  then,

$$|\overline{BC}| |\overline{IA}| + |\overline{CA}| |\overline{IB}| + |\overline{AB}| |\overline{IC}| = 0$$

10. If 'S' is circum centre of  $\Delta^{le}$  ABC then,  $\overline{SA} + \overline{SB} + \overline{SC} = \overline{SO}$
11. If 'S' is circum centre, 'O' is orthocenter of  $\Delta^{le}$  ABC then,  $\overline{OA} + \overline{OB} + \overline{OC} = 2\overline{OS}$
12. If  $\vec{a} = (a_1, a_2, a_3)$  & if axes are rotated through an  
 i) x - axis  
 $(a_1, a_2 \cos \alpha + a_3 \sin \alpha, a_2 \cos \alpha + a_3 \sin(90 - \alpha))$   
 ii) y - axis  $(a_3 \cos(90 + \alpha) + a_1 \sin(90 + \alpha), a_3 \cos \alpha + a_1 \sin \alpha)$   
 iii) z - axis  $(a_1 \cos \alpha + a_2 \sin \alpha, a_1 \cos(90 + \alpha) + a_2 \sin(90 + \alpha), a_3)$   
 If 'O' is circumcentre of  $\Delta^{le}$  ABC then  

$$\Sigma \overline{OA} \sin 2A = \frac{\sqrt{3}}{2} (\overline{OA} + \overline{OB} + \overline{OC})$$
 (Consider equilateral  $\Delta^{le}$ )
13.  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$  where  $0^\circ \leq \theta \leq 180^\circ$   
 i)  $\vec{a} \cdot \vec{b} > 0 \Rightarrow 0 < \theta < 90^\circ \Rightarrow \theta$  is acute  
 ii)  $\vec{a} \cdot \vec{b} < 0 \Rightarrow 90^\circ < \theta < 180^\circ \Rightarrow \theta$  is obtuse  
 iii)  $\vec{a} \cdot \vec{b} = 0 \Rightarrow \theta = 90^\circ \Rightarrow$  two vectors are  $\perp$  to each other.
14. In a right angled  $\Delta^{le}$  ABC, if AB is the hypotenuse and  $AB = P$  then  $\overline{AB} \cdot \overline{BC} + \overline{BC} \cdot \overline{CA} + \overline{CA} \cdot \overline{AB} = P^2$
15.  $\Delta ABC$  is equilateral triangle of side 'a' then  $\overline{AB} \cdot \overline{BC} + \overline{BC} \cdot \overline{CA} + \overline{CA} \cdot \overline{AB} = \frac{3a^2}{2}$
16.  $(\vec{a} \cdot \vec{i})^2 + (\vec{a} \cdot \vec{j})^2 + (\vec{a} \cdot \vec{k})^2 = a^2$   
 $(\vec{a} \times \vec{i})^2 + (\vec{a} \times \vec{j})^2 + (\vec{a} \times \vec{k})^2 = 2a^2$
17. Vector equation. of a line passing through the point A with P.V.  $\vec{a}$  and parallel to 'b' is  $\vec{r} = \vec{a} + t\vec{b}$
18. Vector equation of a line passing through  $\vec{A}(\vec{a}), \vec{B}(\vec{b})$  is  $\vec{r} = (1-t)\vec{a} + t\vec{b}$
19. Vector equation. of line passing through  $\vec{a}$  &  $\perp$  to  $\vec{b}, \vec{c}$   
 $\vec{r} = \vec{a} + t(\vec{b} \times \vec{c})$
20. Vector equation. of plane passing through a pt  $\vec{A}(\vec{a})$  and parallel to non-collinear vectors  $\vec{b}$  &  $\vec{c}$  is  $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$ ,  $s, t \in \mathbb{R}$  and also given as  
 $[\vec{r} - \vec{a}, \vec{b}, \vec{c}] = [\vec{r}, \vec{b}, \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$
21. Vector equation. of a plane passing through three non-collinear Points.  
 $\vec{A}(\vec{a}), \vec{B}(\vec{b}), \vec{C}(\vec{c})$  is  $[\overline{AB}, \overline{AC}, \overline{AP}] = 0$   
 i.e.  $\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t(\vec{c} - \vec{a})$   
 $= (1-s-t)\vec{a} + s\vec{b} + t\vec{c} = [\vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}] = 0$
22. Vector equation. of a plane passing through pts  $\vec{A}(\vec{a}), \vec{B}(\vec{b})$  and parallel to  
 $\vec{C}(\vec{c})$  is  $[\overline{AP}, \overline{AB}, \vec{c}] = 0$
23. Vector equation of plane, at distance p ( $p > 0$ ) from origin and  $\perp$  to  $\vec{n}$  is  $\vec{r} \cdot \vec{n} = p$
24. Perpendicular distance from origin to plane passing through a, b, c  

$$\frac{|\vec{a} \cdot \vec{b} \times \vec{c}|}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|}$$
25. Plane passing through a and parallel to b, c is  $[\vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}] = 0$  and  $[\vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]$
26. Vector equation of plane passing through A, B, C with position vectors  $\vec{a}, \vec{b}, \vec{c}$  is  $[\vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}] = 0$  and  $\vec{r} \cdot [\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}] = [\vec{a}, \vec{b}, \vec{c}]$

27. Let,  $a \neq 0$  b be two vectors. Then  
 i) The component of b on a is  $b \cdot \hat{a}$   
 ii) The projection of b on a is  $(b \cdot \hat{a}) \hat{a}$
28. i) The component of b on a is  $\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}$   
 ii) the projection of b on a is  $\frac{(\vec{b} \cdot \vec{a}) \vec{a}}{|\vec{a}|^2}$   
 iii) the projection of b on a vector perpendicular to 'a' in the plane generated by  
 a, b is  $\vec{b} - \frac{(\vec{b} \cdot \vec{a}) \vec{a}}{|\vec{a}|^2}$
29. If a, b are two nonzero vectors then  

$$\cos(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$
30. If a, b are not parallel then  $\vec{a} \times \vec{b}$  is perpendicular to both of the vectors a, b.
31. If a, b are not parallel then a, b,  $\vec{a} \times \vec{b}$  form a right handed system.
32. If a, b are not parallel then  $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin(\angle a, b)$  and hence
33. If a is any vector then  $\vec{a} \times \vec{a} = 0$
34. If a, b are two vectors then  $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ .
35.  $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$  is called anticommutative law.
36. If a, b are two nonzero vectors, then  

$$\sin(\angle a, b) = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|}$$
37. If ABC is a triangle such that  $\overline{AB} = a, \overline{AC} = b$  then the vector area of  $\Delta ABC$  is  
 $\frac{1}{2}(\vec{a} \times \vec{b})$  and scalar area is  $\frac{1}{2} |\vec{a} \times \vec{b}|$
38. If a, b, c are the position vectors of the vertices of a triangle, then the vector area of the triangle  

$$= \frac{1}{2}(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$$
39. If ABCD is a parallelogram  $\overline{AB} = a, \overline{BC} = b$  and then the vector area of ABCD is  $\vec{a} \times \vec{b}$
40. The length of the projection of b on a vector perpendicular to a in the plane generated by a, b is  $\frac{|\vec{a} \times \vec{b}|}{|\vec{a}|}$
41. The perpendicular distance from a point P to the line joining the points A, B is  $\frac{|\overline{AP} \times \overline{AB}|}{|\overline{AB}|}$
42. Torque: The torque or vector moment or moment vector M of a force F about a point P is defined as  $M = \vec{r} \times \vec{F}$  where r is the vector from the point P to any point A on the line of action L of F.
43. a, b, c are coplanar then  $[\vec{a}, \vec{b}, \vec{c}] = 0$
44. Volume of parallelepiped =  $[\vec{a}, \vec{b}, \vec{c}]$  with a, b, c as coterminus edges.
45. The volume of the tetrahedron ABCD is  

$$\pm \frac{1}{6} [\overline{AB}, \overline{AC}, \overline{AD}]$$
46. If a, b, c are three conterminous edges of a tetrahedron then the volume of the tetrahedron =  $\pm \frac{1}{6} [abc]$
47. The four points A, B, C, D are coplanar if  

$$[\overline{AB}, \overline{AC}, \overline{AD}] = 0$$
48. The shortest distance between the skew lines  $\vec{r} = \vec{a} + s\vec{b}$  and  $\vec{r} = \vec{c} + t\vec{d}$  is  $\frac{|\vec{a} - \vec{c} \cdot \vec{b} - \vec{d}|}{|\vec{b} \times \vec{d}|}$
49. If i, j, k are unit vectors then  $[\vec{i}, \vec{j}, \vec{k}] = 1$
50. If a, b, c are vectors then  $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$

51.  $[a \times b, b \times c, c \times a] = (abc)^2$

52.  $\Sigma ix(a \times i) = 2a$

53.  $|\overline{a \times b}|^2 + |\overline{a \times c}|^2 = |\overline{a}|^2 |\overline{b}|^2 + |\overline{a}|^2 |\overline{c}|^2$

54.  $(\overline{a \times b}) \cdot (\overline{c \times d}) = \begin{vmatrix} \overline{a \cdot c} & \overline{a \cdot d} \\ \overline{b \cdot c} & \overline{b \cdot d} \end{vmatrix}$

55. If A, B, C, D are four points, and

$$|\overline{AB} \times \overline{CD} + \overline{BC} \times \overline{AD} + \overline{CA} \times \overline{BD}| = 4(\Delta ABC)$$

56.  $a^{-1} = \frac{b \times c}{[abc]}, b^{-1} = \frac{c \times a}{[abc]}, c^{-1} = \frac{a \times b}{[abc]}$

are called reciprocal system of vectors

57. If a, b, c are three vectors then  $[a \ b \ c] = [b \ c \ a] = [c \ a \ b] = -[b \ a \ c] = -[c \ b \ a] = -[a \ c \ b]$

58. Three vectors are coplanar if  $\det = 0$ If  $ai + bj + ck, i + bj + k, i + j + ck$  where  $a \neq b \neq c \neq 1$  are coplanar then

i)  $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

ii)  $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = 2$

### Preparation Tips - Mathematics

- Memorizing landmark problems (remembering standard formulae, concepts so that you can apply them directly) and being strong in mental calculations are essential (Never use the calculator during your entire AIEEE preparation. Try to do first and second level of calculations mentally)
- You are going to appear for AIEEE this year, you must be very confident, don't panic, it is not difficult and tough. You need to learn some special tips and tricks to solve the AIEEE questions to get the top rank.
- Don't try to take up new topics as they consume time, you will also lose your confidence on the topics that you have already prepared.
- Don't try to attempt 100% of the paper unless you are 100% confident: It is not necessary to attempt the entire question paper, Don't try if you are not sure and confident as there is negative marking. If you are confident about 60% of the questions, that will be enough to get a good rank.
- Never answer questions blindly. Be wise, preplanning is very important.
- There are mainly three difficulty levels, simple, tough and average. First try to finish all the simple questions to boost your confidence.
- Don't forget to solve question papers of previous years AIEEE before the examination. As you prepare for the board examination, you should also prepare and solve the last year question papers for AIEEE. You also need to set the 3 hours time for each and every previous year paper, it will help you to judge yourself, and this will let you know your weak and strong areas. You will gradually become confident.
- You need to cover your entire syllabus but don't try to touch any new topic if the examination is close by.
- Most of the questions in AIEEE are not difficult. They are just different & they require a different approach and a different mindset. Each question has an element of surprise in it & a student who is adept in tackling 'surprise questions' is most likely to sail through successfully.
- It is very important to understand what you have to attempt and what you have to omit. There is a limit to which you can improve your speed and strike rate beyond which what becomes very important is your selection of question. So success depends upon how judiciously one is able to select the questions. To optimize your performance you should quickly scan for easy questions and come back

to the difficult ones later.

- Remember that cut-off in most of the exams moves between 60 to 70%. So if you focus on easy and average question i.e. 85% of the questions, you can easily score 70% marks without even attempting difficult questions. Try to ensure that in the initial 2 hours of the paper the focus should be clearly on easy and average questions, After 2 hours you can decide whether you want to move to difficult questions or revise the ones attempted to ensure a high strike rate.

### Topic-wise tips

#### Trigonometry:

In trigonometry, students usually find it difficult to memorize the vast number of formulae. Understand how to derive formulae and then apply them to solving problems. The more you practice, the more ingrained in your brain these formulae will be, enabling you to recall them in any situation. Direct questions from trigonometry are usually less in number, but the use of trigonometric concepts in Coordinate Geometry & Calculus is very profuse.

#### Coordinate Geometry:

This section is usually considered easier than trigonometry. There are many common concepts and formulae (such as equations of tangent and normal to a curve) in conic sections (circle, parabola, ellipse, hyperbola). Pay attention to Locus and related topics, as the understanding of these makes coordinate Geometry easy.

#### Calculus:

Calculus includes concept-based problems which require analytical skills. Functions are the backbone of this section. Be thorough with properties of all types of functions, such as trigonometric, algebraic, inverse trigonometric, logarithmic, exponential, and signum. Approximating sketches and graphical interpretations will help you solve problems faster. Practical application of derivatives is a very vast area, but if you understand the basic concepts involved, it is very easy to score.

#### Algebra:

Don't use formulae to solve problems in topics which are logic-oriented, such as permutations and combinations, probability, location of roots of a quadratic, geometrical applications of complex numbers, vectors, and 3D-geometry.

### AIEEE 2009 Mathematics Section Analysis of CBSE syllabus

Of all the three sections in the AIEEE 2009 paper, the Mathematics section was the toughest. Questions were equally divided between the syllabi of Class XI and XII. Many candidates struggled with the Calculus and Coordinate Geometry portions.

#### Class XI Syllabus

Topic	No. of Questions
Trigonometry	1
Algebra (XI)	6
Coordinate Geometry	5
Statistics	3
3-D (XI)	1

#### Class XII Syllabus

Topic	No. of Questions
Calculus	8
Algebra (XII)	2
Probability	2
3-D (XII)	1
Vectors	1