Newton's Laws

$$\overrightarrow{F_{net}} = \sum \vec{F} = m\vec{a}$$

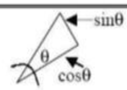
$$F_{net,x} = \sum F_x = ma_x \quad F_{net,y} = \sum F_y = ma_y \quad F_{net,z} = \sum F_z = ma_z$$

$$\overrightarrow{F_{\text{on A due to B}}} = -\overrightarrow{F_{\text{on B due to A}}}$$

$$\overrightarrow{F_{AB}} = -\overrightarrow{F_{BA}}$$

Forces	Contact Forces
weight= $\vec{F_g} = m\vec{g}$	normal force = $\overrightarrow{F_N}$
Tension = T	magnitude static friction $=\mu_s F_N$
spring force = -kx	magnitude kinetic friction $=\mu_k F_N$

Math Notes



Surface Area of a sphere with radius R $A = 4\pi R^2$

Volume of a sphere with radius R $V = \frac{4}{3}\pi R^3$

Volume of a cylinder with radius R and height h $V = \pi R^2 h$

Vectors

$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

In 2 dimensions direction of vector is defined by the angle it makes with the positive x-axis

$$\vec{V} = V_x\hat{i} + V_y\hat{j} \quad \text{and} \quad \tan\theta = \frac{V_y}{V_x}$$

Scalar Product (Dot Product)

$$\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y + A_zB_z = |\vec{A}||\vec{B}|\cos\alpha$$

where α is the angle between vectors $\vec{A}$ and $\vec{B}$

Vector Product (Cross Product)

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\alpha$$

where α is the angle between vectors $\vec{A}$ and $\vec{B}$

Average Power

$$P_{av} = \frac{\Delta W}{\Delta t} \quad P = \lim_{\Delta t\to 0}\frac{\Delta W}{\Delta t} = \frac{dW}{dt} \quad P = \vec{F}\bullet\vec{v}$$

Rotational Kinematics

$$arclength: s = r\Delta\theta$$

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad \omega = \frac{d\theta}{dt}$$

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$v = r\omega$ (relation between linear and angular speed)

$$a_{tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \qquad a_{rad} = \frac{v^2}{r} = \omega^2 r$$

Constant Angular Acceleration

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Kinematics

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \qquad \overrightarrow{a_{av}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\overrightarrow{v(t)} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\overrightarrow{a(t)} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

Motion with Uniform Acceleration

$$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 \qquad y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$$

$$v_x = v_{0x} + a_xt \qquad v_y = v_{0y} + a_yt$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

Projectile Motion

For an object launched from x_o, y_o with initial speed v and initial angle θ relative to the x-axis

$$x = x_o + v_o\cos\theta t \qquad y = y_o + v_o\sin\theta t - \frac{1}{2}gt^2$$

$$v_x = v_o\cos\theta \qquad v_y = v_o\sin\theta - gt$$

$$-2g(y - y_o) = v_y^2 - v_{oy}^2$$

Work-Kinetic Energy

$$\text{Hooke's Law}: \vec{F}_{spring} = -k\cdot\vec{\Delta x}$$

$$W = \vec{F}\cdot\vec{d} \qquad W = Fd\cos\theta$$

$$W = \int_{P_1}^{P_2}\vec{F}\cdot d\vec{l} = \int_{x_1}^{x_2}F_x dx + \int_{y_1}^{y_2}F_y dy + \int_{z_1}^{z_2}F_z dz$$

$$K = \frac{1}{2}mv^2$$

$$W_{tot} = K_2 - K_1 = \Delta K$$

Energy

$$\Delta U = -W$$

$K_1 + U_1 = K_2 + U_2$ (if only conservative forces)

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$U_g(y) = mgy$ (close to earth) $U_{spring}(x) = \frac{1}{2}kx^2$

$F_x(x) = -\frac{dU(x)}{dx}$ (force from potential energy, one dimension)

Conversions

1 in (inch) = 2.54 cm (by definition)
1 foot = 12 in
3 ft = 1 yd (yard)
5280 ft = 1 mi (mile)
1 mi = 1.609 km
1 slug = 14.59 kg
1 gal = 3.786 l (liter)
1 l = 10^{-3} m³
1 ml = 10^{-3} = 1 cm³ = 1 cc
ON EARTH
2.2 lb (pound) = 1 kg
note lb is a unit of WEIGHT, not mass
on earth weight = mg where g = 9.8 m/s² = 32 ft/s²

Useful Constants

Speed of light	c = 2.997 x 10^8 m/s
Gravitational Constant	G = 6.673 x 10^{-11} Nm²/kg²
	g = 9.80 m/s²

Uniform Circular Motion

T = period =2πR/v f = frequency = 1/T
ω = angular frequency =2πf
Centripetal acceleration = a = v²/R

Rotational Dynamics

$$I = m_1r_1^2 + m_2r_2^2 + \cdots = \sum m_ir_i^2$$

$I_P = I_{cm} + Mh^2$ (parallel axis theorem)

$\vec{\tau} = \vec{r}\times\vec{F}$ (definition of torque vector)

$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$ (magnitude of torque)

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

$$\tau_z = I\alpha_z$$

$$K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$Power = P = \tau_z\omega_z$$

$$\tau_z = I_{cm}\alpha_z$$

$v_{com} = R\omega$ and $a_{com} = R\alpha$ (condition for rolling without slipping)

$W = \int_{\theta_1}^{\theta_2}\tau_z d\theta$ (work done by a torque)

$W = \tau_z(\theta_2 - \theta_1) = \tau_z\Delta\theta$ (work done by a constant torque)

$$W_{tot} = \int_{\omega_1}^{\omega_2}I\omega_z\, d\omega_z = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$$

(angular momentum of a particle)

$$\vec{L} = \vec{r}\times\vec{p} = \vec{r}\times m\vec{v}$$

(for a rigid body rotating around a symmetry axis)

$$\vec{L} = I\vec{\omega}$$

$\vec{\tau} = \frac{d\vec{L}}{dt}$ (for any system of particles)

Momentum and Collisions

$\vec{p} = m\vec{v}$ (definition of momentum)

$\vec{J} = \int_{t_i}^{t_f}\vec{F}\,dt$ (general definition of impulse)

$\vec{J} = \Delta\vec{p} = \vec{p}_2 - \vec{p}_1$ (impulse-momentum theorem)

$\vec{J} = \vec{F}\Delta t$

(assuming constant net force)

For any system of particles

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = \vec{p}_1 + \vec{p}_2 + \vec{p}_3\cdots = \vec{P}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

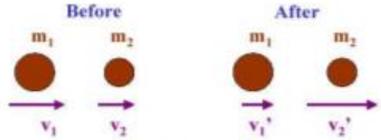
Conservation of Momentum

If total force is zero

$$\vec{\mathbf{P}} = \sum_j\vec{\mathbf{p}}_j = \text{constant}$$

$$\vec{\mathbf{P}}_i = \vec{\mathbf{P}}_f$$

1D Elastic Collisions



$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_1 + m_2}v_2$$

(center of mass)

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

$\vec{F}_{ext} = M\vec{a}_{cm}$ (body or collection of particles)

Conservation of Angular Momentum

If total torque is zero

$$\vec{L} = \sum_i\vec{L}_i = \text{constant}$$

$$\overrightarrow{L_i} = \overrightarrow{L_f}$$

Static Equilibrium

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum \vec{\tau} = 0 \quad \text{about any point}$$

Determination of center of mass:

$$\overrightarrow{r_{cm}} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m_i\vec{r_i}}{\sum m_i}$$

Young's Modulus

$$E = \frac{\textit{Tensile stress}}{\textit{Tensile strain}}$$

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

Shearing Modulus

$$G = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\frac{F}{A} = G\frac{\Delta x}{L}$$

Bulk Modulus

$$B = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$\frac{F}{A} = B\frac{\Delta V}{V}$$

Simple Harmonic Motion

$$F = -kx$$

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$x(t) = x_o\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$E_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_o^{\ 2} = \text{constant}$$

Simple Pendulum

$$\omega = \sqrt{\frac{g}{l}}$$

Physical Pendulum

$$\omega = \sqrt{\frac{mgd}{I}}$$

Torsional Pendulum

$$\omega = \sqrt{\frac{\kappa}{I}}$$

Gravitation

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$U = -\frac{Gm_E m}{r}$$

Kepler's Law of periods

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Damped Oscillations

$$\sum F_x = -kx - bv = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$$

$$\text{where } \omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_0^{\ 2} - \left(\frac{b}{2m}\right)^2}$$