(15) $\frac{dx}{dt} + 5x + \frac{dy}{dt} = 1$ $\quad x(0)=0,\ y(0)=0$

$\frac{dx}{dt} - x + \frac{dy}{dt} - y = e^t$

$x' + 5x + y' = 1$

$x' - x + y' - y = e^t$ $\rightarrow$ $\mathcal{L}\{x'\} + 5\mathcal{L}\{x\} + \mathcal{L}\{y'\} = \mathcal{L}\{1\}$

$\mathcal{L}\{x'\} - \mathcal{L}\{x\} + \mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{e^t\}$

$\downarrow$

$sX(s) - x(0) + 5X(s) + sY(s) - y(0) = \frac{1}{s}$

$sX(s) - x(0) - X(s) + sY(s) - y(0) - Y(s) = \frac{1}{s-1}$

$\downarrow$

$sX(s) + 5X(s) + sY(s) = \frac{1}{s}$

$sX(s) - X(s) + sY(s) - Y(s) = \frac{1}{s-1}$

$\downarrow$

$[\,(s+5)X(s) + sY(s) = \frac{1}{s}\,] \cdot (s-1)$

$[\,(s-1)X(s) + (s-1)Y(s) = \frac{1}{s-1}\,] \cdot -s$

$\downarrow$

$(s+5)(s-1)X(s) + (s-1)(s)Y(s) = \frac{s-1}{s}$ $\qquad (s-1)(s-1) = s^2 - s - s + 1 = s^2 - 2s + 1$

$(-s)(s-1)X(s) - (s-1)(s)Y(s) = \frac{-s}{s-1}$

$(s+5)(s-1) = s^2 - s + 5s - 5 = s^2 + 4s - 5$

$(-s)(s-1) = -s^2 + s$

$(s^2+4s-5)X(s) + (-s^2+s)X(s) = \frac{(s-1)^2}{s(s-1)} - \frac{s(s)}{s(s-1)}$

$= \frac{s^2 - 2s - s + 1}{s(s-1)}$

$= \frac{s^2 - 3s + 1}{s(s-1)}$