Section 2: Integrating factors

Exercise level 1

1. In each case find the general solution of the exact differential equation.

   (i) $x^2 \dfrac{dy}{dx} + 2xy = \cos x$

   (ii) $x^3 e^y \dfrac{dy}{dx} + 3x^2 e^y = 4$

   (iii) $\dfrac{2y}{x} \dfrac{dy}{dx} - \dfrac{y^2}{x^2} = e^x$

   (iv) $\sin x \dfrac{dy}{dx} + y \cos x = x^2$

2. Use an integrating factor to find the general solution of each of these differential equations.

   (i) $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{1}{x}$

   (ii) $\dfrac{dy}{dx} + y \cos x = e^{-\sin x}$

   (iii) $\dfrac{dy}{dx} + \dfrac{y}{x+2} = x - 3$

   (iv) $x \dfrac{dy}{dx} - y = x^3 e^{2x}$

   (v) $\dfrac{dy}{dx} + y \cot x = x \operatorname{cosec} x$

   (vi) $x \dfrac{dy}{dx} + 2y = x + 1$

3. Find the particular solutions of these differential equations for the given conditions.

   (i) $\dfrac{dy}{dx} + \dfrac{y}{x} = e^x$ \qquad $y = 1$ when $x = 1$

   (ii) $\dfrac{dy}{dx} + 2xy = e^{-(x-2)^2}$ \qquad $y = 0$ when $x = 1$

10/10/18 © MEI