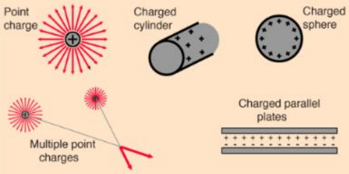
Electric Field

Electric field is defined as the electric force per unit charge. The direction of the field is taken to be the direction of the force it would exert on a positive test charge. The electric field is radially outward from a positive charge and radially in toward a negative point charge.



Click on any of the examples above for more detail.

Electric field in N/C or volts/m. $\vec{E} = \frac{\vec{F}}{q}$ electric force in Newtons; charge in Coulombs

Since the measured electric field can depend upon your reference frame, a more general definition of the electric field comes from the Lorentz force law. The electric field can be defined as the electromagnetic force per unit charge in the rest frame of the charge.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

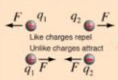
Electric force; Magnetic force. A charge that is moving relative to the source will experience part of the force as a magnetic force.

Lorentz force law

Coulomb's Law

Like charges repel, unlike charges attract.

The electric force acting on a point charge q_1 as a result of the presence of a second point charge q_2 is given by Coulomb's Law:



$$F = \frac{kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \quad \text{Coulomb's Law}$$

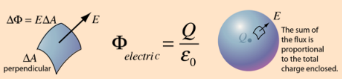
where ε_0 = permittivity of space

Note that this satisfies Newton's third law because it implies that exactly the same magnitude of force acts on q_2. Coulomb's law is a vector equation and includes the fact that the force acts along the line joining the charges. Like charges repel and unlike charges attract. Coulomb's law describes a force of infinite range which obeys the inverse square law, and is of the same form as the gravity force.

$$k = \frac{1}{4\pi\varepsilon_0} \approx 9x10^9 N\cdot m^2/C^2 = \text{Coulomb's constant}$$

Gauss's Law

The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.



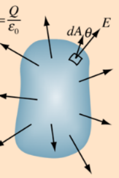
$$\Phi_{electric} = \frac{Q}{\varepsilon_0}$$

The electric flux through an area is defined as the electric field multiplied by the area of the surface projected in a plane perpendicular to the field. Gauss's Law is a general law applying to any closed surface. It is an important tool since it permits the assessment of the amount of enclosed charge by mapping the field on a surface outside the charge distribution. For geometries of sufficient symmetry, it simplifies the calculation of the electric field.

Another way of visualizing this is to consider a probe of area A which can measure the electric field perpendicular to that area. If it picks any closed surface and steps over that surface, measuring the perpendicular field times its area, it will obtain a measure of the net electric charge within the surface, no matter how that internal charge is configured.

Gauss' Law, Integral Form

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$



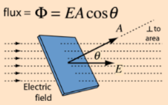
The area integral of the electric field over any closed surface is equal to the net charge enclosed in the surface divided by the permittivity of space. Gauss' law is one of Maxwell's equations, the four fundamental equations for electricity and magnetism.

Gauss' law permits the evaluation of the electric field in many practical situations by forming a symmetric Gaussian surface surrounding a charge distribution and evaluating the electric flux through that surface.

Electric Flux

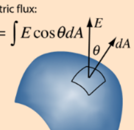
The concept of electric flux is useful in association with Gauss' law. The electric flux through a planar area is defined as the electric field times the component of the area perpendicular to the field. If the area is not planar, then the evaluation of the flux generally requires an area integral since the angle will be continually changing.

$$\text{flux} = \Phi = EA\cos\theta$$



Electric flux:

$$\Phi = \int E\cos\theta\, dA$$



When the area A is used in a vector operation like this, it is understood that the magnitude of the vector is equal to the area and the direction of the vector is perpendicular to the area.

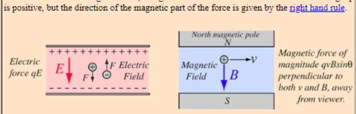
Lorentz Force Law

Both the electric field and magnetic field can be defined from the Lorentz force law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Electric force; Magnetic force

The electric force is straightforward, being in the direction of the electric field if the charge q is positive, but the direction of the magnetic part of the force is given by the right hand rule.



Relation of Electric Field to Charge Density

Since electric charge is the source of electric field, the electric field at any point in space can be mathematically related to the charges present. The simplest example is that of an isolated point charge. For multiple point charges, a vector sum of point charge fields is required. If we envision a continuous distribution of charge, then calculus is required and things can become very complex mathematically.

One approach to continuous charge distributions is to define electric flux and make use of Gauss' law to relate the electric field at a surface to the total charge enclosed within the surface. This involves integration of the flux over the surface.

Another approach is to relate derivatives of the electric field to the charge density. This approach can be considered to arise from one of Maxwell's equations and involves the vector calculus operation called the divergence. The divergence of the electric field at a point in space is equal to the charge density divided by the permittivity of space.

$$\nabla\cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field
ρ = charge density
ε_0 = permittivity

In a charge-free region of space where $\rho = 0$, we can say

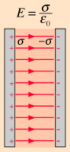
$$\nabla\cdot E = 0$$

While these relationships could be used to calculate the electric field produced by a given charge distribution, the fact that E is a vector quantity increases the complexity of that calculation. It is often more practical to convert this relationship into one which relates the scalar electric potential to the charge density. This gives Poisson's equation and LaPlace's equation.

Electric Field: Parallel Plates

If oppositely charges parallel conducting plates are treated like infinite planes (neglecting fringing), then Gauss' law can be used to calculate the electric field between the plates. Presuming the plates to be at equilibrium with zero electric field inside the conductors, then the result from a charged conducting surface can be used:

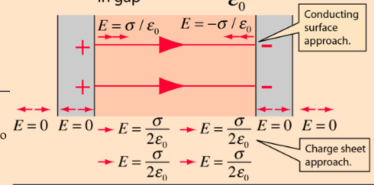
$$E = \frac{\sigma}{\varepsilon_0}$$



This is also consistent with treating the charge layers as two charge sheets with electric field

$$E = \frac{\sigma}{2\varepsilon_0}$$

in both directions.



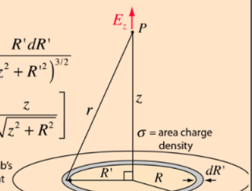
Electric Field:Disc of Charge

The electric field of a disc of charge can be found by superposing the point charge fields of infinitesmal charge elements. This can be facilitated by summing the fields of charged rings. The integral over the charged disc takes the form

$$E_z = k\sigma 2\pi z\int_0^R \frac{R'dR'}{\left(z^2+R'^2\right)^{3/2}}$$

$$E_z = k\sigma 2\pi\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]$$

$$k = \frac{1}{4\pi\varepsilon_0} = \text{Coulomb's constant}$$

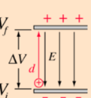


Voltage Difference and Electric Field

The change in voltage is defined as the work done per unit charge against the electric field. In the case of constant electric field when the movement is directly against the field, this can be written

$$V_f - V_i = \frac{Fd}{q} = -Ed$$

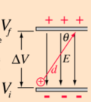
Moving a positive charge from the bottom to the top plate requires work and raises voltage.



If the distance moved, d, is not in the direction of the electric field, the work expression involves the scalar product:

$$V_f - V_i = \frac{\vec{F}\cdot\vec{d}}{q} = -\vec{E}\cdot\vec{d} = -Ed\cos\theta$$

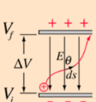
Moving a positive charge along the slanted line gives the same change in voltage.



In the more general case where the electric field and angle can be changing, the expression must be generalized to a line integral:

$$V_f - V_i = -\int \vec{E}\cdot d\vec{s}$$

Moving a positive charge along the curved path indicated would require the integral to calculate, but in this case would give the same voltage difference.

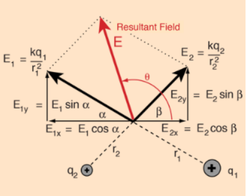


Multiple Point Charges

The electric field from multiple point charges can be obtained by taking the vector sum of the electric fields of the individual charges.

$$E_y = E_{1y} + E_{2y}$$
$$E_x = E_{1x} + E_{2x}$$
$$E = \sqrt{E_x^2 + E_y^2}$$
$$\tan\theta = \frac{E_y}{E_x}$$

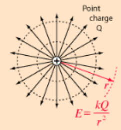
After calculating the individual point charge fields, their components must be found and added to form the components of the resultant field. The resultant electric field can then be put into polar form. Care must be taken to establish the correct quadrant for the angle because of ambiguities in the arctangent.



Electric Field of Point Charge

The electric field of a point charge can be obtained from Coulomb's law:

$$E = \frac{F}{q} = \frac{kQ_{source}q}{qr^2} = \frac{kQ_{source}}{r^2}$$

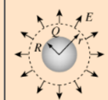


The electric field is radially outward from the point charge in all directions. The circles represent spherical equipotential surfaces.

The electric field from any number of point charges can be obtained from a vector sum of the individual fields. A positive number is taken to be an outward field; the field of a negative charge is toward it.

This electric field expression can also be obtained by applying Gauss' law.

Electric Field of Conducting Sphere



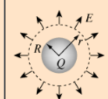
The electric field of a conducting sphere with charge Q can be obtained by a straightforward application of Gauss' law. Considering a Gaussian surface in the form of a sphere at radius **r > R** , the electric field has the same magnitude at every point of the surface and is directed outward. The electric flux is then just the electric field times the area of the spherical surface.

$$\Phi = EA = E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

The electric field is seen to be identical to that of a point charge Q at the center of the sphere. Since all the charge will reside on the conducting surface, a Gaussian surface at r< R will enclose no charge, and by its symmetry can be seen to be zero at all points inside the spherical conductor

For $r > R$
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$
For $r < R$
$$E = 0$$

Electric Field: Sphere of Uniform Charge



The electric field of a sphere of uniform charge density and total charge charge Q can be obtained by applying Gauss' law. Considering a Gaussian surface in the form of a sphere at radius **r > R**, the electric field has the same magnitude at every point of the surface and is directed outward. The electric flux is then just the electric field times the area of the spherical surface.

$$\Phi = EA = E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

The electric field outside the sphere (**r > R**)is seen to be identical to that of a point charge Q at the center of the sphere.

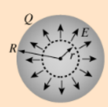
For $r > R$
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

For a radius **r < R**, a Gaussian surface will enclose less than the total charge and the electric field will be less. Inside the sphere of charge, the field is given by:

For $r < R$
$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

Show

Inside a Sphere of Charge



The electric field inside a sphere of uniform charge is radially outward (by symmetry), but a spherical Gaussian surface would enclose less than the total charge Q. The charge inside a radius r is given by the ratio of the volumes:

$$\frac{Q'}{Q} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} \quad \text{or} \quad Q' = Q\frac{r^3}{R^3}$$

The electric flux is then given by $\Phi = E4\pi r^2 = \frac{Qr^3}{\varepsilon_0 R^3}$

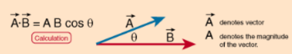
and the electric field is $E = \frac{Qr}{4\pi\varepsilon_0 R^3}$

Note that the limit at r= R agrees with the expression for r >= R. The spherically symmetric charge outside the radius r does not affect the electric field at r. It follows that inside a spherical shell of charge, you would have zero electric field.

Scalar Product of Vectors

The scalar product and the vector product are the two ways of multiplying vectors which see the most application in physics and astronomy. The scalar product of two vectors can be constructed by taking the component of one vector in the direction of the other and multiplying it times the magnitude of the other vector. This can be expressed in the form:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$



If the vectors are expressed in terms of unit vectors i, j, and k along the x, y, and z directions, the scalar product can also be expressed in the form:

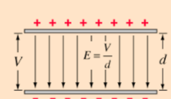
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{where} \quad \vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}, \quad \vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$$

Applications

The scalar product is also called the "inner product" or the "dot product" in some mathematics texts.

Work and Voltage: Constant Electric Field

The case of a constant electric field, as between charged parallel plate conductors, is a good example of the relationship between work and voltage.



The electric field is by definition the force per unit charge, so that multiplying the field times the plate separation gives the work per unit charge, which is by definition the change in voltage.

$$Ed = \frac{Fd}{q} = \frac{W}{q} = \Delta V$$

For constant electric field.

$$\text{Units:} \quad \frac{N}{C}\,m = \frac{N\,m}{C} = \frac{\text{Joule}}{C} = \text{Volts}$$

This association is the reminder of many often-used relationships:

$$E = \frac{F}{q} \qquad W = q\Delta V$$

General definition relationships

$$E = \frac{V}{d} \qquad V = Ed$$

Constant field special case relationships

Electric Field:Cylinder of Charge

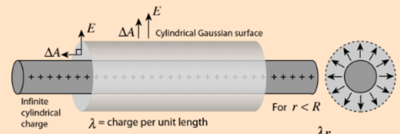
The electric field of an infinite cylinder of uniform volume charge density can be obtained by a using Gauss' law. Considering a Gaussian surface in the form of a cylinder at radius **r > R**, the electric field has the same magnitude at every point of the cylinder and is directed outward. The electric flux is then just the electric field times the area of the cylinder.

$$\Phi = E2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

For $r \geq R$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

This expression is a good approximation for the field close to a long cylindrical charge.

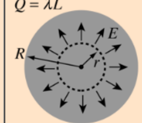


For $r < R$

$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$

Inside a Cylinder of Charge

$$Q = \lambda L$$



The electric field inside an infinite cylinder of uniform charge is radially outward (by symmetry), but a cylindridal Gaussian surface would enclose less than the total charge Q. The charge inside a radius r is given by the ratio of the volumes:

$$\frac{Q'}{Q} = \frac{\pi r^2 L}{\pi R^2 L} \quad \text{or} \quad Q' = Q\frac{r^2}{R^2}$$

The electric flux is then given by $\Phi = E2\pi rL = \frac{\lambda L r^2}{\varepsilon_0 R^2}$

and the electric field is $E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$

Note that the limit at r= R agrees with the expression for r >= R.

Electric Field: Conducting Cylinder

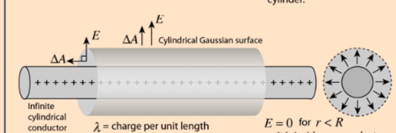
The electric field of an infinite cylindrical conductor with a uniform linear charge density can be obtained by using Gauss' law. Considering a Gaussian surface in the form of a cylinder at radius **r > R**, the electric field has the same magnitude at every point of the cylinder and is directed outward. The electric flux is then just the electric field times the area of the cylinder.

$$\Phi = E2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

For $r \geq R$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

This expression is a good approximation for the field close to a long conducting cylinder.



$E = 0$ for $r < R$ as it is inside any conductor at equilibrium.

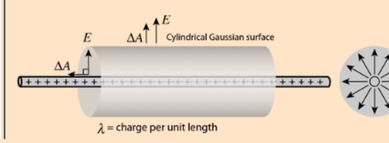
Electric Field of Line Charge

The electric field of an infinite line charge with a uniform linear charge density can be obtained by a using Gauss' law. Considering a Gaussian surface in the form of a cylinder at radius **r**, the electric field has the same magnitude at every point of the cylinder and is directed outward. The electric flux is then just the electric field times the area of the cylinder.

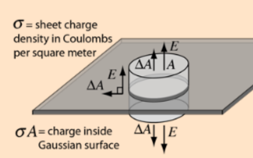
$$\Phi = E2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0}$$

This expression is a good approximation for the field close to a long line of charge.



Electric Field: Sheet of Charge

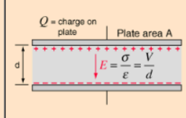


$$\Phi = E2A = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

For an infinite sheet of charge, the electric field will be perpendicular to the surface. Therefore only the ends of a cylindrical Gaussian surface will contribute to the electric flux . In this case a cylindrical Gaussian surface perpendicular to the charge sheet is used. The resulting field is half that of a conductor at equilibrium with this surface charge density.

Capacitance of Parallel Plates



The electric field between two large parallel plates given by

$$E = \frac{\sigma}{\varepsilon} \quad \text{where} \quad \sigma = \text{charge density}, \ \varepsilon = \text{permittivity}$$

and $\sigma = \frac{Q}{A}$

Show

The voltage difference between the two plates can be expressed in terms of the work done a positive test charge q when it moves from the positive to the negative plate.

$$V = \frac{\text{work done}}{\text{charge}} = \frac{Fd}{q} = Ed$$

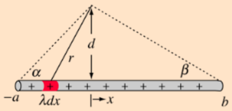
It then follows from the definition of capacitance that

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q\varepsilon}{\sigma d} = \frac{QA\varepsilon}{Qd} = \frac{A\varepsilon}{d}$$

Potential of Line Charge

The potential of a line of charge can be found by superposing the point charge potentials of infinitesimal charge elements. It is an example of a continuous charge distribution.

$$V=\int\frac{kdq}{r}=\int_{-a}^{b}\frac{k\lambda dx}{r}=\int_{-a}^{b}\frac{k\lambda dx}{\sqrt{x^2+d^2}}=k\lambda\ln\left[\frac{b+\sqrt{b^2+d^2}}{-a+\sqrt{a^2+d^2}}\right]$$



Potential for Disc of Charge

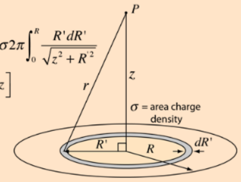
The potential of a disc of charge can be found by superposing the point charge potentials of infinitesimal charge elements. It is an example of a continuous charge distribution. The evaluation of the potential can be facilitated by summing the potentials of charged rings. The integral over the charged disc takes the form:

$$V=k\sigma 2\pi\int_0^R\frac{R'dR'}{r}=k\sigma 2\pi\int_0^R\frac{R'dR'}{\sqrt{z^2+R'^2}}$$

$$V=k\sigma 2\pi\left[\sqrt{z^2+R^2}-z\right]$$

$k=\frac{1}{4\pi\varepsilon_0}$ = Coulomb's constant

σ = area charge density



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$$V=k\sigma 2\pi\left[\sqrt{z^2+R^2}-z\right]$$

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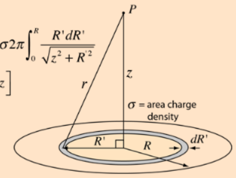
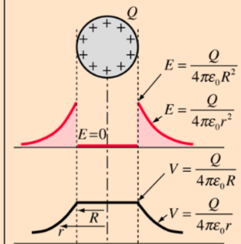


Potential: Charged Conducting Sphere



The use of Gauss' law to examine the electric field of a charged sphere shows that the electric field environment outside the sphere is identical to that of a point charge. Therefore the potential is the same as that of a point charge:

$$E=\frac{Q}{4\pi\varepsilon_0 R^2}\qquad E=\frac{Q}{4\pi\varepsilon_0 r^2}\qquad E=0$$

$$V=\frac{kQ}{r}=\frac{Q}{4\pi\varepsilon_0 r}$$

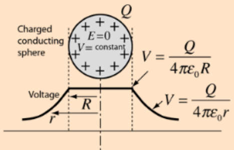
The electric field inside a conducting sphere is zero, so the potential remains constant at the value it reaches at the surface:

$$V=\frac{Q}{4\pi\varepsilon_0 R}\qquad V=\frac{Q}{4\pi\varepsilon_0 r}$$

$$V_{inside}=\frac{kQ}{R}=\frac{Q}{4\pi\varepsilon_0 R}$$

Potential of a Conductor

When a conductor is at equilibrium, the electric field inside it is constrained to be zero.



$$V=\frac{Q}{4\pi\varepsilon_0 R}\qquad V=\frac{Q}{4\pi\varepsilon_0 r}$$

Since the electric field is equal to the rate of change of potential, this implies that the voltage inside a conductor at equilibrium is constrained to be constant at the value it reaches at the surface of the conductor. A good example is the charged conducting sphere, but the principle applies to all conductors at equilibrium.

Potential for Ring of Charge

The potential of a ring of charge can be found by superposing the point charge potentials of infinitesimal charge elements. It is an example of a continuous charge distribution. The ring potential can then be used as a charge element to calculate the potential of a charged disc.

Since the potential is a scalar quantity, and since each element of the ring is the same distance r from the point P, the potential is simply given by

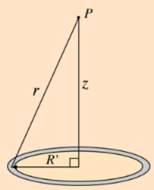
$$V=\frac{kQ}{r}=\frac{k\lambda 2\pi R'}{r}$$

Q = total charge

λ = linear charge density

If the charge is characterized by an area density and the ring by an incremental width dR', then:

$$dV=\frac{k\sigma 2\pi R'dR'}{r}$$



In this form it could be used as a charge element for the determining of the potential of a disc of charge.

Electric Dipole Potential

The potential of an electric dipole can be found by superposing the point charge potentials of the two charges:

$$V=kq\left[\frac{1}{r_+}-\frac{1}{r_-}\right]=kq\left[\frac{r_--r_+}{r_+r_-}\right]$$

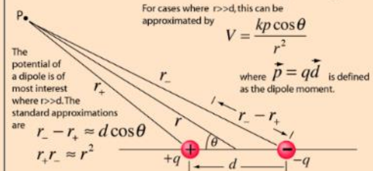
For cases where r>>d, this can be approximated by

$$V=\frac{kp\cos\theta}{r^2}$$

where $\vec{p}=q\vec{d}$ is defined as the dipole moment.

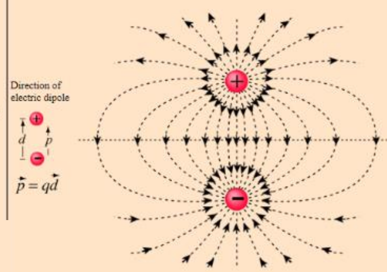
The potential of a dipole is of most interest where r>>d. The standard approximations are

$$r_--r_+\approx d\cos\theta\qquad r_+r_-\approx r^2$$



Electric Dipole Field

The electric field of an electric dipole can be constructed as a vector sum of the point charge fields of the two charges:

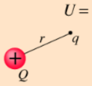


Electric Potential Energy

Potential energy can be defined as the capacity for doing work which arises from position or configuration. In the electrical case, a charge will exert a force on any other charge and potential energy arises from any collection of charges. For example, if a positive charge Q is fixed at some point in space, any other positive charge which is brought close to it will experience a repulsive force and will therefore have potential energy. The potential energy of a test charge q in the vicinity of this source charge will be:

$$U=\frac{kQq}{r}$$

where k is Coulomb's constant.



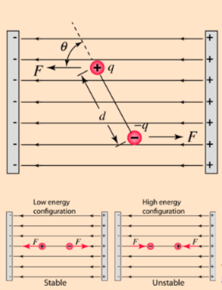
In electricity, it is usually more convenient to use the electric potential energy per unit charge, just called electric potential or voltage.

Application: Coulomb barrier for nuclear fusion

Energy in electron volts

Show

Energy of an Electric Dipole



An electric field produces a torque on a dipole

$$\tau=Ep\sin\theta$$

which tends to take it to its low energy configuration. To rotate it from the low energy state against the field requires work

$$\int_0^\theta\tau d\theta'=\int_0^\theta Ep\sin\theta' d\theta'$$

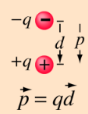
$$=Ep(1-\cos\theta)$$

Choosing $U=0$ at $\theta=90°$ gives a potential energy

$$U=Ep\cos\theta=-\vec{p}\cdot\vec{E}$$

where the shorter form employs the scalar product.

Dipole Moment



$$\vec{p}=q\vec{d}$$

The electric dipole moment for a pair of opposite charges of magnitude q is defined as the magnitude of the charge times the distance between them and the defined direction is toward the positive charge. It is a useful concept in atoms and molecules where the effects of charge separation are measurable, but the distances between the charges are too small to be easily measurable. It is also a useful concept in dielectrics and other applications in solid and liquid materials.

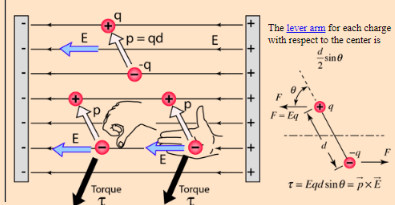
Applications involve the electric field of a dipole and the energy of a dipole when placed in an electric field.

$$E_{dipole}=\frac{1}{4\pi\varepsilon_0}\frac{p}{z^3}\qquad z>>d$$

$$\vec{p}=q\vec{d}$$

Torque on Electric Dipole

The torque produced on an electric dipole by an electric field can be expressed as a vector product with direction given by the right hand rule.



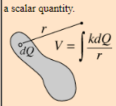
The lever arm for each charge with respect to the center is $\frac{d}{2}\sin\theta$

$$\tau=Eqd\sin\theta=\vec{p}\times\vec{E}$$

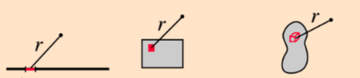
Continuous Charge Distributions

The electric potential (voltage) at any point in space produced by a continuous charge distribution can be calculated from the point charge expression by integration since voltage is a scalar quantity.

$$V=\int\frac{kdQ}{r}$$



The continuous charge distribution requires an infinite number of charge elements to characterize it, and the required infinite sum is exactly what an integral does. To actually carry out the integration, the charge element is expressed in terms of the geometry of the distribution with the use of some charge density.

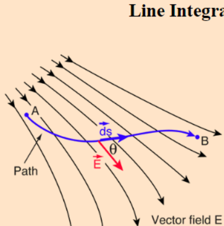


$\lambda dx=dQ$ Linear charge density

$\sigma dA=dQ$ Area charge density

$\rho dV=dQ$ Volume charge density

Line Integral



Vector functions such as electric field and magnetic field occur in physical applications, and scalar products of these vector functions with another vector such as distance or path length appear with regularity. When such a product is summed over a path length where the magnitudes and directions change, that sum becomes an integral called a line integral.

$$\int_A^B\vec{E}\cdot\vec{ds}=\int_A^B E\cos\theta ds$$

A line integral is also used for the general definition of work in mechanics.

Applications

Applications of Line Integrals

The line integral of electric field around a closed loop is equal to the voltage generated in that loop (Faraday's law):

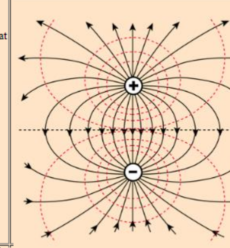
$$\oint\vec{E}\cdot\vec{ds}=-\frac{d\Phi_B}{dt}$$

Such an integral is also used for the calculation of voltage difference since voltage is work per unit charge. Calculating the voltage difference near a point charge is a good example.

The line integral of a force over a path is equal to the work done by that force on the path.

$$W_{ab}=\int_a^b\vec{F}\cdot\vec{ds}$$

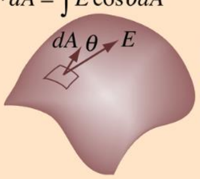
Equipotential lines: dipole



The electric potential of a dipole show mirror symmetry about the center point of the dipole. They are everywhere perpendicular to the electric field lines.

Area Integral

$$\int\vec{E}\cdot\vec{dA}=\int E\cos\theta dA$$

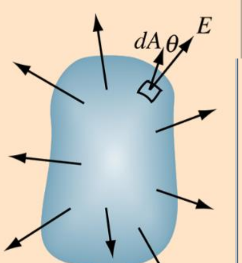


An area integral of a vector function E can be defined as the integral on a surface of the scalar product of E with area element dA. The direction of the area element is defined to be perpendicular to the area at that point on the surface.

The outward directed surface integral over an entire closed surface is denoted

$$\oint\vec{E}\cdot\vec{dA}$$

It is appropriate for such physical applications as Gauss' law.



The Gradient

The gradient is a vector operation which operates on a scalar function to produce a vector whose magnitude is the maximum rate of change of the function at the point of the gradient and which is pointed in the direction of that maximum rate of change. In rectangular coordinates the gradient of function f(x,y,z) is:

$$\nabla f=\left[i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right]f$$

∇ is commonly called "del" and the gradient "del f"

If S is a surface of constant value for the function f(x,y,z) then the gradient on the surface defines a vector which is normal to the surface.

Applications | Conduction heat transfer

The divergence of the gradient is called the LaPlacian. It is widely used in physics.

Gradient, Various Coordinates

Compared to the gradient in rectangular coordinates:

$$\nabla f=\left[i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right]f$$

In cylindrical polar coordinates:

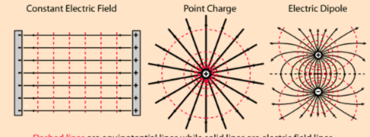
$$\nabla f=\frac{\partial f}{\partial r}1_r+\frac{1}{r}\frac{\partial f}{\partial\theta}1_\theta+\frac{\partial f}{\partial z}k$$

and in spherical polar coordinates:

$$\nabla f=\frac{\partial f}{\partial r}1_r+\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}1_\phi+\frac{1}{r}\frac{\partial f}{\partial\theta}1_\theta$$

Equipotential Lines

Equipotential lines are like contour lines on a map which trace lines of equal altitude. In this case the "altitude" is electric potential or voltage. Equipotential lines are always perpendicular to the electric field. In three dimensions, the lines form equipotential surfaces. Movement along an equipotential surface requires no work because such movement is always perpendicular to the electric field.



Dashed lines are equipotential lines while solid lines are electric field lines. Click on one of the diagrams for further detail.

Vector Calculus Operations

Three vector calculus operations which find many applications in physics are:

1. The divergence of a vector function

$$\nabla\cdot E=\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}$$

2. The curl of a vector function

$$\nabla\times E=\left(\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right)i+\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)j+\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)k$$

3. The Gradient of a scalar function

$$\nabla f=\frac{\partial f}{\partial x}i+\frac{\partial f}{\partial y}j+\frac{\partial f}{\partial z}k$$

These examples of vector calculus operations are expressed in Cartesian coordinates, but they can be expressed in terms of any orthogonal coordinate system, aiding in the solution of physical problems which have other than rectangular symmetries.

Current	I	ampere	A
Charge	Q, q, e	coulomb	C=A*s
Current density	j	-	A/m^2
Volume charge density	ρ	-	C/m^3
Surface charge density	σ	-	C/m^2
Linear charge density	λ	-	C/m
Electric potential Voltage emf	φ V ε	volt	V=J/C
Electric field	E	-	N/C, V/m
Electric flux	Φ	-	V*m
Electric moment	p_e	-	C*m
Resistance	R, r	ohm	Ω=V/A
Specific resistance	ρ	-	Ω*m
Capacitance	C	farad	F=C/V
Specific conductivity	σ	-	(Ω*m)$^{-1}$
Magnetic field	B	tesla	T=N/(A*m)
Magnetic flux	Φ	weber	Wb=T*m^2=V*s
Inductance Mutual-inductance	L M	henri	H=Wb/A
Magnetic moment	p_m	-	A*m^2
Polarization	P	-	C/m^2
Magnetization	I	-	A/m