MATHS 8

MATHEMATICS LEVEL 2 TEST

REFERENCE INFORMATION

THE FOLLOWING INFORMATION IS FOR YOUR REFERENCE IN ANSWERING SO
THE QUESTIONS IN THIS TEST.

Volume of a right circular cone with radius r and height h: $V = \frac{1}{3}\pi r^2 h$

Volume of a sphere with radius r: $V = \frac{4}{3}\pi r^3$

Volume of a pyramid with base area B and height h: $V = \frac{1}{3}Bh$

Surface Area of a sphere with radius r: $S = 4\pi r^2$

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MATHEMATICS LEVEL 2 TEST

For each of the following problems, decide which is the BEST of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding circle on the answer sheet.

Notes: (1) A scientific or graphing calculator will be necessary for answering some (but not all) of the questions in this test. For each question you will have to decide whether or not you should use a calculator.

(2) For some questions in this test you may have to decide whether your calculator should be in the radian mode or the degree mode.

(3) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(4) Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. The range of f is assumed to be the set of all real numbers $f(x)$, where x is in the domain of f.

(5) Reference information that may be useful in answering the questions in this test can be found on the page preceding Question 1.

USE THIS SPACE FOR SCRATCHWORK.

1. For what values of x is the expression $\frac{3x^2 - 3}{x - 1}$ undefined?

(A) -1 only
(B) 0 only
(C) 1 only
(D) -3 and 3
(E) -1 and 1

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2. In the xy-plane, if $y = mx + 7$ and $y = kx - 7$ are equations of parallel lines, which of the following must be true?

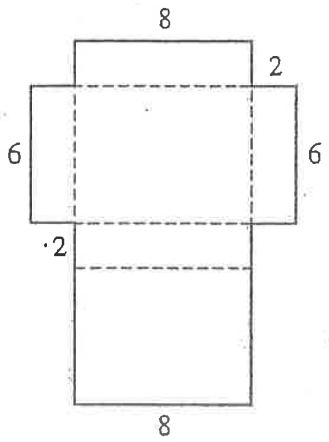
   (A) $m = k$

   (B) $m = -k$

   (C) $m = -\frac{1}{k}$

   (D) $m + k = -1$

   (E) $m - k = -1$



3. The net for a rectangular solid is shown in the figure above. When the solid is formed, what is its volume?

   (A) 32 (B) 96 (C) 144 (D) 152 (E) 192

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MATHEMATICS LEVEL 2 TEST—*Continued*

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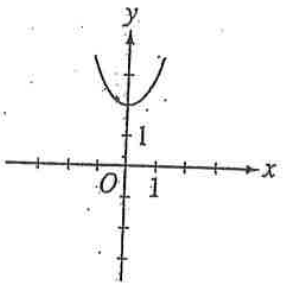
If p is a prime number, then $2p + 1$ is a prime number.

4. Which of the following prime numbers could be a value of p that yields a COUNTEREXAMPLE to the statement above?

(A) 19 (B) 11 (C) 5 (D) 3 (E) 2



5. If the parabola in the figure above is translated 3 units to the right, which of the following could represent the equation of this translated parabola?

(A) $y = (x - 3)^2 + 2$
(B) $y = (x + 3)^2 + 2$
(C) $y = (x - 3)^2 - 2$
(D) $y = (x + 3)^2 - 2$
(E) $y = (x - 3)^2 + 5$

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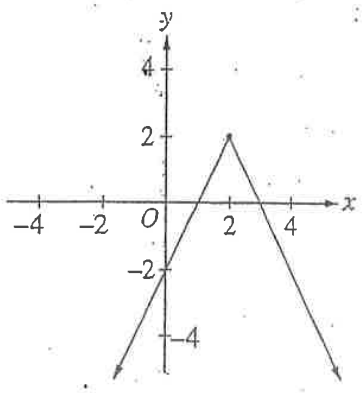
6. The graph of the linear function f has slope -3. If the graph of f passes through the point $(2, 6)$, which of the following could be an expression for $f(x)$?

(A) $-3x$
(B) $-3x + 12$
(C) $-3x + 20$
(D) $3x$
(E) $3x - 16$



7. What is the range of the function whose graph is shown above?

(A) The set of all real numbers less than or equal to 2
(B) The set of all real numbers greater than or equal to -2
(C) The set of all real numbers greater than or equal to 2
(D) The set of all real numbers between 1 and 3
(E) The set of all real numbers

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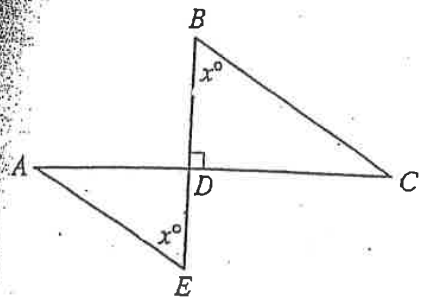

8. In the figure above, $x = 58$ and $BD = 4$. What is the length of $\overline{CD}$?

(A) 2.50
(B) 4.72
(C) 6.40
(D) 7.55
(E) 8

9. If $|x| = |y|$, which of the following must be true?

(A) $\sqrt{x} = \sqrt{y}$
(B) $x = y$
(C) $x = -y$
(D) $x^2 = y^2$
(E) $x^3 = y^3$

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10. A certain radioactive substance decomposes at a rate such that the amount of mass remaining at the end of each day is $\frac{1}{2}$ the amount left at the end of the previous day. If the mass at the beginning of the first day is M_0, which of the following could express the mass remaining at the end of t days?

(A) $2M_0t$

(B) $2(M_0)t$

(C) $\frac{2M_0}{t}$

(D) $\frac{M_0t}{2}$

(E) $\frac{M_0}{2^t}$

11. If $\sin x = 0.90$ and $0 < x < \frac{\pi}{2}$, then $\cos \frac{x}{2} =$

(A) 0.22
(B) 0.45
(C) 0.56
(D) 0.85
(E) 0.97

12. If $f(x) = 10^x$ and $x > 0$, for what value of x is $f(x) = 11.2$?

(A) 0.89
(B) 0.95
(C) 1.05
(D) 1.07
(E) 1.12

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MATHEMATICS LEVEL 2 TEST—*Continued*

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13. If $y = \dfrac{1}{(2x-3)^2}$, what value does y approach as x approaches $\dfrac{3}{2}$?

(A) $\dfrac{1}{36}$

(B) 0

(C) $\dfrac{1}{36}$

(D) 1

(E) The value of y increases without bound.

14. If $(1 - \cos x)(1 + \cos x) = 0.64$, what is the value of $\sin^2 x$?

(A) 0.36
(B) 0.41
(C) 0.60
(D) 0.64
(E) 0.80

15. In the xy-plane, the points $(0, 4)$, $(4, 0)$, $(8, 4)$, and $(4, 8)$ are vertices of a square. If the x- and y-coordinates of each vertex are doubled, a new square is formed. The area of the new square is how many times the area of the original square?

(A) 8

(B) 4

(C) 2

(D) $\dfrac{1}{2}$

(E) $\dfrac{1}{4}$

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16. If $y = 4x - 12$, what is the value of y when it is twice the value of x?

(A) 4 (B) 6 (C) 12 (D) 24 (E) 30

17. In the xy-plane, the endpoints of $\overline{AB}$ have coordinates $(-1, 0)$ and $(0, 1)$. If O is the origin and M is the midpoint of $\overline{AB}$, what is the slope of $\overline{OM}$?

(A) -2

(B) -1

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

(E) 1

Score	Frequency
10	3
15	4
20	7
25	2
30	8
Total	24

18. What is the mean of the scores in the frequency distribution shown above?

(A) 20
(B) 20.42
(C) 21.67
(D) 22.92
(E) 30

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$7^6 = 10r + s$

19. In the equation above, if r and s are positive integers and s is less than 10, what is the value of s?

(A) 1
(B) 2
(C) 3
(D) 7
(E) 9

20. If $\tan^2 x = 1.0913$, which of the following could cot x equal?

(A) −1.0447
(B) 0.3022
(C) 0.9573
(D) 1.0447
(E) 1.4461

21. Let f, g, and h be linear functions satisfying $f(g(x)) = h(x)$ for all x. If $f(x) = x + 1$ and $h(x) = 2x - 1$, then $g(x) =$

(A) $2x + 1$
(B) $2x - 2$
(C) $x - 1$
(D) $x + 1$
(E) $x + 2$

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22. If b and c are real numbers, what is the greatest possible number of real solutions of $x = x^2 + bx + c$?

(A) None
(B) One
(C) Two
(D) Three
(E) Infinitely many

23. The graph of $x^2 - 6x + y^2 + 4y - 12 = 0$ in the xy-plane is a circle with radius 5, centered at

(A) $(-9, 4)$
(B) $(-3, -2)$
(C) $(-3, 2)$
(D) $(3, -2)$
(E) $(9, -4)$

24. A sphere and a cone each have the same radius and volume. If the volume of the sphere is 1,000 cubic centimeters, what is the height of the cone?

(A) 4.0 cm
(B) 5.4 cm
(C) 8.9 cm
(D) 15.8 cm
(E) 24.8 cm

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MATHEMATICS LEVEL 2 TEST—*Continued*

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25. Let f be a polynomial function such that the graph of $y = f(x)$ contains the points $(-2, -2)$, $(-1, 4)$, and $(2, 1)$. Which of the following must be true about the zeros of f?

(A) f has a zero between $x = -2$ and $x = -1$.
(B) f has a zero between $x = -1$ and $x = 2$.
(C) f has no zeros.
(D) f has exactly one zero.
(E) f has exactly two zeros.

$$(6 \triangle 4) \oplus (5 \triangle 3) = 2\frac{1}{2}$$

26. In the equation above, $\oplus$ and $\triangle$ represent two distinct operations chosen from the set $\{+, -, \times, \div\}$. What is the value of $(6 \oplus 2) \triangle 4$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

27. If m and n are integers between -4 and 4, inclusive, then for how many ordered pairs (m, n) does $2^m = 4^n$?

(A) Five
(B) Four
(C) Three
(D) One
(E) None

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28. If f is defined for all numbers x by $f(x) = 1 - x^2$, for which of the following values of x does $f(x) = \frac{f(x)}{3}$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 6

29. The depth of the water D, in feet, at the end of a fishing pier is related to the number of hours h after midnight by $D(h) = 8.5 - 4.2\cos\left(\frac{\pi}{6}(h - 3)\right)$.

How much does the depth change from its lowest point to its highest point?

(A) 4.2 ft
(B) 4.3 ft
(C) 8.4 ft
(D) 8.5 ft
(E) 12.7 ft

30. What is the probability that the sum of two numbers, chosen at random with replacement from the integers from 1 to 10 inclusive, is equal to 7?

(A) 0.01
(B) 0.06
(C) 0.1
(D) 0.5
(E) 0.9

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31. If $\log_2 x = -1.7$, what is the value of x?

(A) −3.40
(B) −0.85
(C) 0.31
(D) 2.89
(E) 3.25

32. Which of the following gives only values of x for which $\frac{1}{2}x^3 + 2x^2 - 2 > 0$?

(A) $-1.2 < x < -1.0$
(B) $-3.7 < x < 1.0$
(C) $x < -3.7$ and $x > 1.0$
(D) $x < -3.7$ and $-1.2 < x < 1.0$
(E) $-3.7 < x < -1.2$ and $x > 1.0$

33. The rate R of decomposition, in moles per liter per second, of a certain chemical is given by the formula $R = kC^m$, where C is the chemical's concentration, in moles per liter, and k and m are constants. If $R = 1.0$ when $C = 0.2$ and $R = 16.0$ when $C = 0.8$, what is the value of m?

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

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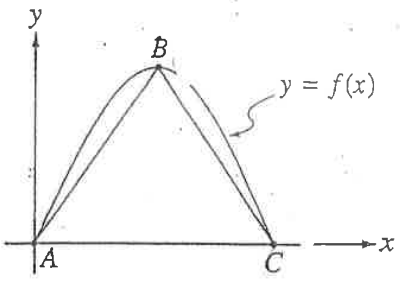

34. The figure above shows a portion of the graph of a function f, where $f(x) = 2\sin(2x)$. One of the vertices of $\triangle ABC$ is a maximum point of the graph of f, and the other two vertices are consecutive x-intercepts of this graph. What is the area of $\triangle ABC$?

(A) $\frac{\pi}{2}$
(B) π
(C) 2π
(D) 3π
(E) 4π

35. A piece of paper is 0.0[illegible] inch thick. Each time the paper is folded in half, the thickness is doubled. Someone claims that the paper can be folded in half 12 times. If this were true, how thick, to the nearest foot, would the folded paper be?

(A) 1,475 ft
(B) 1,229 ft
(C) 123 ft
(D) 10 ft
(E) 5 ft

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MATHEMATICS LEVEL 2 TEST—*Continued*

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$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$$

36. For function f defined above, what is the value of $f(4.5) - f(-4.5)$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) 9

3.1 3.1 3.3 3.3

1.1, 1.1, 1.3, 1.3

37. The weights, in pounds, of four books are shown above. The standard deviation of these weights is 0.1 pound. Each book is packaged into a separate shipping box. Each empty box weighs 2 pounds. What is the standard deviation of the weights of the four packages?

(A) 0.1 pound
(B) 0.2 pound
(C) 2.0 pounds
(D) 2.1 pounds
(E) 8.1 pounds

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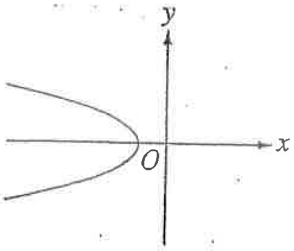

38. The graph in the figure above could be a portion of the graph of which of the following pairs of parametric equations?

(A) $x = t$, $y = t^2 + 1$

(B) $x = t$, $y = t^2 - 1$

(C) $x = t^2 - 1$, $y = t$

(D) $x = -t^2 + 1$, $y = t$

(E) $x = -t^2 - 1$, $y = t$

39. If x, y, and z are consecutive positive integers, the product xyz must be divisible by which of the following numbers?

I. 2
II. 3
III. 4

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

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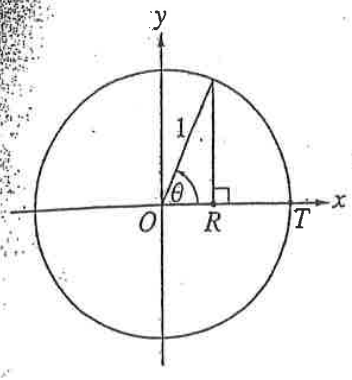
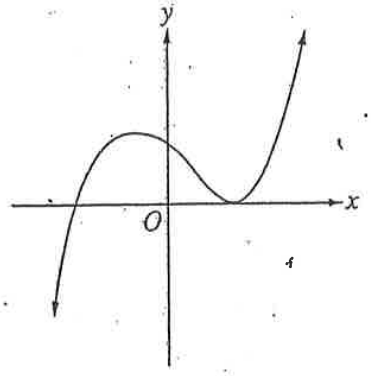


0. In the figure above, what is the length of $\overline{RT}$ in terms of θ?

(A) $\sin\theta - \cos\theta$
(B) $\tan\theta - 1$
(C) $1 - \sin\theta$
(D) $1 - \cos\theta$
(E) $2\sin\theta\cos\theta$



1. The graph of the function f, given by $f(x) = a(x+b)(x+c)(x+d)$, where a, b, c, and d are constants, is shown in the figure above. Which of the following could be true?

(A) $a = 0$, $b \neq c$, $b \neq d$, and $c \neq d$.
(B) $a < 0$, $b = c$, and $c \neq d$.
(C) $a < 0$ and $b = c = d$.
(D) $a > 0$, $b = c$, and $c \neq d$.
(E) $a > 0$ and $b = c = d$.

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42. Which of the following equations has exactly two distinct real roots?

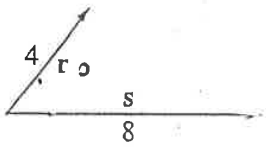
(A) $(x^2 + 8)(x^2 + 4) = 0$

(B) $(x^3 - 8)(x^2 + 5) = 0$

(C) $(x^2 - 1)(x^2 + 1) = 0$

(D) $(x^4 - 1)(x + 2) = 0$

(E) $(x^2 - 4)(x^2 - 9) = 0$



43. In the figure above, r and s are vectors of magnitude 4 and 8, respectively. If the angle between the two vectors is 50°, what is the magnitude of vector r + s ?

(A) 5.5
(B) 6.2
(C) 8.9
(D) 11.0
(E) 12.0

44. A person invests $3,500 at 5.2 percent annual interest compounded quarterly. How much will this investment be worth at the end of 1 year?

(A) $3,545.50
(B) $3,682.00
(C) $3,685.57
(D) $4,286.77
(E) $5,706.65

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MATHEMATICS LEVEL 2 TEST—*Continued*

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45. Which of the following functions is the same as its inverse function?

I. $f(x) = \frac{1}{x}$

II. $g(x) = \frac{1}{x} + 1$

III. $h(x) = \frac{1}{x-1} + 1$

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

46. What is the period of the function f, where $f(x) = \left|\cos\left(\frac{1}{2}x\right)\right| + 3$?

(A) $\frac{\pi}{2}$
(B) π
(C) $\frac{3\pi}{2}$
(D) 2π
(E) 4π

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47. In the xyz-coordinate system, what is an equation of the set of all points in space equidistant from points $A(1, 1, 3)$ and $B(1, 2, 3)$?

(A) $x = 1$

(B) $y = \frac{1}{2}$

(C) $y = 1$

(D) $y = \frac{3}{2}$

(E) $z = 3$

48. A laboratory receives 16 samples from an experiment. A group of 4 of these samples is selected for testing. One sample from this group of 4 is then selected for a special second test. In how many different ways could the samples for this two-stage testing procedure be selected?

(A) 24
(B) 64
(C) 1,820
(D) 7,280
(E) 3,312,400

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49. The probability that an incoming phone call is long-distance is $\frac{1}{3}$, and that it is local is $\frac{2}{3}$. If two incoming phone calls are independent events, what is the probability that one will be local and the other long-distance?

(A) $\frac{2}{9}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{5}{9}$ (E) 1

50. If $x = \arctan y$, then $\sqrt{1 + y^2} =$

(A) $|\sin x|$
(B) $|\cos x|$
(C) $|\sec x|$
(D) $|\cot x|$
(E) $|\csc x|$

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