

General Mathematics Sample Resources



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Mathematics and Health – HSC FOCUS TOPIC

Body Measurements – Scatter plots

REMEMBER:

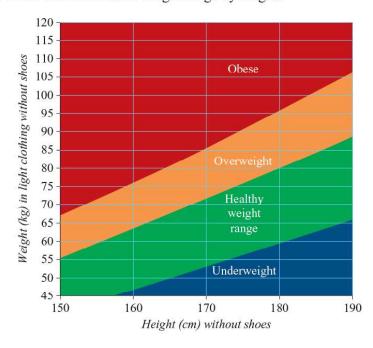
Scatterplot

- A scatterplot is a graph of ordered pairs of numbers.
- Each ordered pair is a dot on the scatterplot.

Interpreting a scatterplot:

- 1 Look for a clear pattern.
- 2 Linear relationships approximate a straight line.
- 3 Non-linear relationships approximate a curve.

The chart below shows the adult weight range by height.



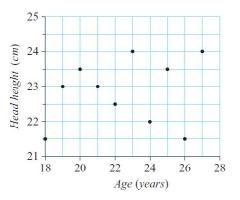
- **a** Use the chart to decide if the following measurements are underweight, healthy, overweight or obese.
 - i 170 cm tall and weighs 80 kg
- ii 185 cm tall and weighs 60 kg
- iii 155 cm tall and weighs 55 kg
- iv 167 cm tall and weighs 100 kg
- **b** How much should these adults lose or gain to be in the middle of the healthy weight range?
 - i Rose is 160 cm and 85 kg.
- ii Nate is 175 cm and 80 kg.
- iii Jade is 165 cm and 45 kg.
- iv Sam is 190 cm and 115 kg.

[Mathematics and Health]

Property of Prestige Tuition

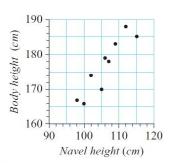
The scatterplot shows head height versus age for 10 people.

- a What is the head height of a person who is 21 years old?
- **b** What is the age of the person who has a head height of 22 cm?
- e How many people have a head height greater than 23 cm?
- **d** Is there a clear relationship between these two variables?



The scatterplot shows the navel height versus the body height for nine students.

- a What is the body height for the student with a navel height of 112 cm?
- **b** What is the navel height for the student with a body height of 166 cm?
- c Use the scatterplot to predict the body height of a student with a navel height of 110 cm.
- **d** Is there a clear relationship between these two variables?



The table below shows forearm length and hand length.

| Forearm length (cm) | 27 | 28 | 26 | 28 | 29 | 27 | 25 | 26 | 27 |
|---------------------|----|------|----|----|------|----|------|----|------|
| Hand length (cm) | 19 | 18.5 | 18 | 19 | 19.5 | 18 | 17.5 | 18 | 18.5 |

- a Draw a scatterplot of the data in the table.
- **b** Describe the relationship between forearm length and hand length.



Correlation coefficient

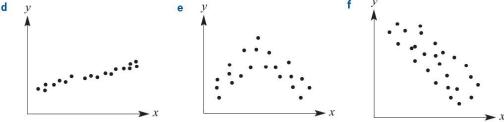
Correlation coefficient

Correlation coefficient (r) measures the strength of a linear relationship ($-1 \le r \le +1$).

- Positive correlation (0 to +1) Both quantities increase or decrease at the same time.
- Zero or no correlation (0) No relationship between the quantities.
- Negative correlation (-1 to 0) One quantity increases, the other quantity decreases.

High correlation between two variables does not imply causation.

State whether the correlation coefficient is positive, negative or zero.



Describe the strength of the linear relationships with the following correlation coefficients.

- a r = 0.3
- **b** r = -1

r = 0.9

- d r = -0.9
- r = -0.3
- f = r 1

Calculate the correlation coefficient to three decimal places for each table.

| a | x | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|----|----|----|----|----|
| | y | 2 | 5 | 7 | 10 | 12 | 16 | 19 | 20 |

| b | | | | | | | | | | | | |
|---|---|----|----|----|----|----|----|----|----|----|----|----|
| | x | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| | y | 35 | 31 | 29 | 26 | 20 | 19 | 16 | 14 | 10 | 6 | 3 |

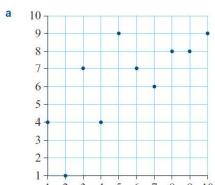
Draw a scatterplot that shows:

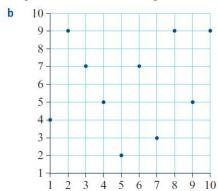
- a high positive correlation
- c zero correlation

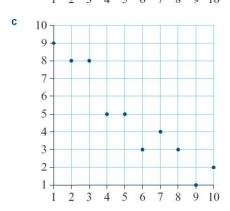
- **b** low positive correlation
- d low negative correlation

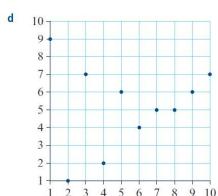


Calculate the correlation coefficient to three decimal places for each scatterplot.









A developing nation compared body mass to income. The correlation coefficient for these quantities was -0.2. What is the meaning of this correlation?

The table below shows the heart rate (beats per minute) and the score out of 20 in a test.

| Heart rate (h) | 62 | 63 | 66 | 68 | 70 | 72 | 75 | 78 | 79 | 80 |
|-------------------|----|----|----|----|----|----|----|----|----|----|
| Score on test (s) | 19 | 18 | 16 | 18 | 15 | 14 | 11 | 10 | 9 | 8 |

- a Draw a scatterplot using the data in the table.
- **b** State whether the relationship is positive or negative.
- c Describe the strength of the relationship as perfect, high or low.
- d Calculate the value of the correlation coefficient. Answer correct to four decimal places.

There is a high positive correlation between health and income.

- a This correlation is commonly thought to reflect a causal link running from income to health. Explain how higher income can cause better health.
- **b** Some experts believe the correlation is a causal link running from health to income. Explain how better health can cause higher income.



Line of Best Fit

REMEMBER:

Equation of least-squares line of best fit

The equation is given by y = mx + b where

Gradient (or slope) is $m = r \frac{s_y}{s}$ and y-intercept is $b = \overline{y} - m\overline{x}$

r – Correlation coefficient

 s_x and s_y – Standard deviation of x and y

 \overline{x} and \overline{y} – Mean of x and y

Draw a scatterplot and a line of best fit by eye for the following points.

- **a** (0, 0) (10, 30) (20, 67) (30, 93) (40, 126) (50, 158) (60, 178)
- **b** (5, 20) (10, 42) (15, 73) (20, 94) (25, 122) (30, 150) (35, 165)
- **c** (0, 6) (2, 24) (3, 39) (4, 44) (5, 59) (6, 64) (7, 79) (8, 84)
- **d** (10, 55) (12, 45) (14, 20) (16, 40) (18, 30) (20, 28) (22, 25)

The equation relating a person's weight (in kg) to shoe size is given below: weight = $2.2 \times$ shoe size + 48.1 (only for shoe sizes between 6 and 12).

- **a** What is the weight of a person with a shoe size of 7? Is this interpolation or extrapolation?
- **b** What is the weight of a person with a shoe size of 5? Is this interpolation or extrapolation?
- **c** What is the weight of a person with a shoe size of 11? Is this interpolation or extrapolation?

Find the equation of the least-squares line of best fit to two decimal places for each data set.

- **a** $\overline{x} = 11.38$, $s_x = 1.87$, $\overline{y} = 230.7$, $s_y = 97.87$ and r = 0.94
- **b** $\overline{x} = 35.54$, $s_x = 5.41$, $\overline{y} = 56.12$, $s_y = 9.58$ and r = -0.81
- $\overline{x} = 5.631$, $s_x = 3.598$, $\overline{y} = 78.135$, $s_y = 40.134$ and r = 0.946
- **d** $\overline{x} = 100.79$, $s_x = 6.43$, $\overline{y} = 59.18$, $s_y = 5.71$ and r = -0.76

The heights and masses of young children are measured and recorded below.

| Height h (cm) | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|------|------|
| Mass m (kg) | 1.5 | 3.1 | 3.6 | 5.5 | 6.0 | 6.9 | 7.6 | 8.6 | 10.0 | 11.2 |

- a Determine the equation of the least-squares line of best fit.
- **b** What is the expected mass of a child given their height is 73 cm?
- c What is the expected height of a child given their mass is 4.8 kg?
- **d** What is the expected mass of a child given their height is 48 cm?
- What is the expected height of a child given their mass is 9.0 kg?

Name



[HSC/Prelim] [General Mathematics]

[Measurement & Mathematics and Resources] 2017

General Instructions

- Reading time [5] minutes
- Working time [60] minutes
- Write using blue or black pen
- Draw diagrams in pencil
- [Board approved calculators may be used
- Write your name at the top of this page

Total marks - [80] marks

Part A - [20] marks

Attempt questions [1] – [20] Allow about [35] minutes for this part

Part B - [60] marks

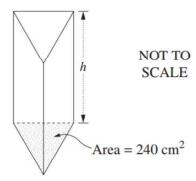
Attempt questions [21] – [42] Allow about [1] hour and [55] minutes for this part





1. (2016) 1 mark

A container is in the shape of a triangular prism which has a capacity of 12 litres. The area of the base is 240 cm².

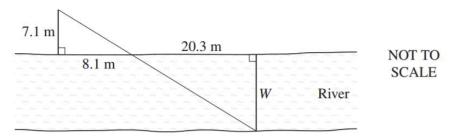


What is the distance, h, between the two triangular ends of the container?

- (A) 5 cm
- (B) 20 cm
- (C) 25 cm
- (D) 50 cm

2. (2016) 1 mark

The width (W) of a river can be calculated using two similar triangles, as shown in the diagram.



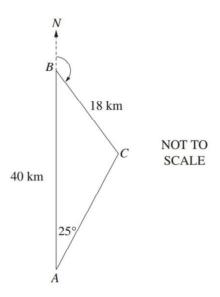
What is the approximate width of the river?

- (A) 17.8 m
- (B) 19.3 m
- (C) 23.2 m
- (D) 24.9 m



3. (2016) 1 mark

The diagram shows towns A, B and C. Town B is 40 km due north of town A. The distance from B to C is 18 km and the bearing of C from A is 025° . It is known that $\angle BCA$ is obtuse.

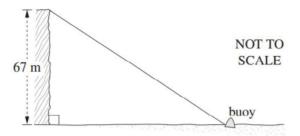


What is the bearing of C from B?

- (A) 070°
- (B) 095°
- (C) 110°
- (D) 135°

4. (2015) 1 mark

From the top of a cliff 67 metres above sea level, the angle of depression of a buoy is 42°.



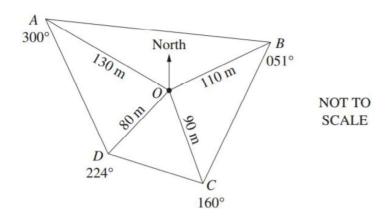
How far is the buoy from the base of the cliff, to the nearest metre?

- (A) 60 m
- (B) 74 m
- (C) 90 m
- (D) 100 m



5. (2015) 1 mark

The diagram shows a radial survey of a field ABCD.

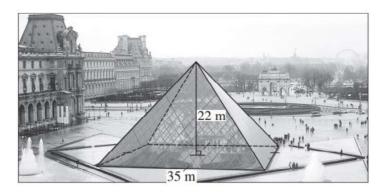


In triangle AOB, what is the size of $\angle AOB$?

- (A) 51°
- (B) 111°
- (C) 125°
- (D) 249°

6. (2015) 1 mark

The Louvre Pyramid in Paris has a square base with side length 35 m and a perpendicular height of 22 m.



What is the volume of this pyramid, to the nearest m³?

- (A) 257 m^3
- (B) 1027 m^3
- (C) 8983 m³
- (D) $26\,950\,\mathrm{m}^3$