Experiment 5: Hydraulic Jump

Key Concepts: Validity of the Equation, Hydraulic Jump Phenomenon

Refer to: Roberson, Crowe & Elger, 7th ed., Chapter 15, pp. 675-697

I. Introduction

A hydraulic jump in an open channel of small slope is shown in Figure 14. In engineering practice the hydraulic jump frequently appears downstream from overflow structures (spillways) or underflow structures (sluice gates) where velocities are high. It may be used to effectively dissipate kinetic energy and thus prevent scour of the channel bottom, or to mix chemicals in a water or sewage treatment plant. In design calculations the engineer is concerned mainly with prediction of existence, size, and location of the jump.

A hydraulic jump is formed when liquid at high velocity discharges into a zone of lower velocity, creating a rather abrupt rise in the liquid surface (a standing wave) accompanied by violent turbulence, eddying, air entrainment, and surface undulations.

A flow is **supercritical** when:

$$F_r = \frac{V}{\sqrt{gy}} > 1 \quad (1)$$

where F_r is the Froude number (see "Dimensionless Fluid Parameters" on page 8), V is the fluid velocity, g is the gravitational constant, and y is fluid depth.

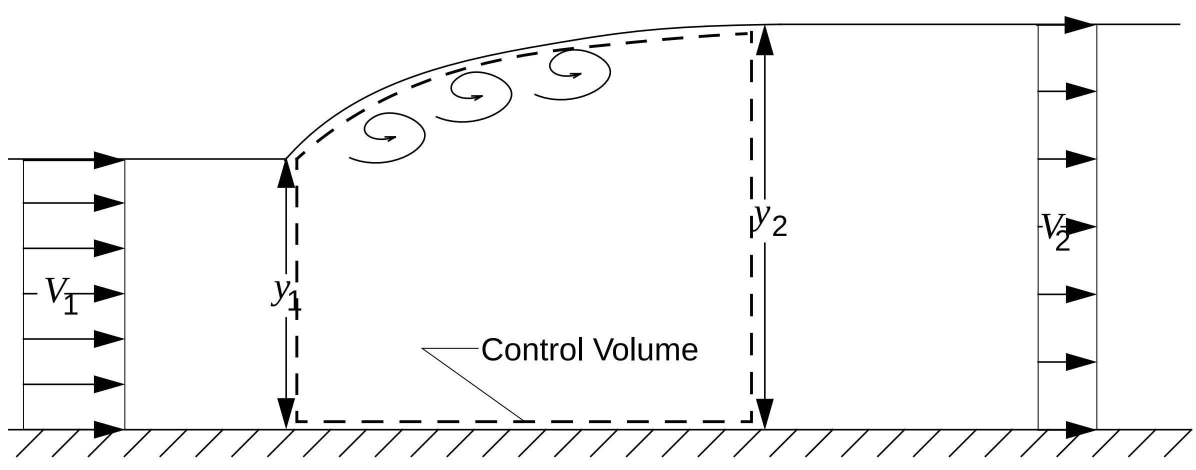

Figure 14. Hydraulic jump

For a channel of rectangular cross-section and constant width, b:

$$F_r = \frac{q}{y}\frac{1}{\sqrt{gy}} \tag{2}$$

where $q = Q/b$, the flowrate per unit width of the channel. In supercritical flow, disturbances travel downstream, and upstream water levels are unaffected by downstream control. Supercritical flows are characterized by high velocity and small flow depth and are also known as *shooting flows*.

A flow is subcritical when:

$$F_r = \frac{V}{\sqrt{gy}} < 1 \tag{3}$$

In subcritical flow, disturbances travel upstream *and* downstream, and upstream water levels are affected by downstream control. Subcritical flows are characterized by low velocity and large flow depth and are also known as *tranquil* flows. In a hydraulic jump, supercritical flow changes to subcritical flow over a short horizontal distance.

Specific energy (*E*) in a channel section is the sum of the elevation head and velocity head, measured with respect to the channel bottom:

$$E = y + \frac{V^2}{2g} \tag{4}$$

For a rectangular channel of constant width b and constant discharge Q,

$$E = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2 \tag{5}$$

Consider a plot of depth, y, vs. specific energy, E, for a given flow rate (Figure 16). This plot is known as a specific energy diagram. As the depth increases from a small value, the specific energy decreases to a minimum value, E_c. The depth associated with this minimum value of specific energy is called **critical depth**, y_c, and the associated Froude number is unity. As the depth continues to increase, the specific energy increases, eventually approaching the $y = E$ line. For each value of specific energy greater than the minimum specific energy, there are two associated depths of flow. One, y_1, is less than the critical depth (supercritical), and one, y_2, is greater than the critical depth (subcritical).

Using equation (5), the energy loss through the jump may be determined:

$$\Delta E = E_1 - E_2 = \left[y_1 + \frac{1}{2g}\left(\frac{q}{y_1}\right)^2\right] - \left[y_2 + \frac{1}{2g}\left(\frac{q}{y_2}\right)^2\right] \tag{6}$$

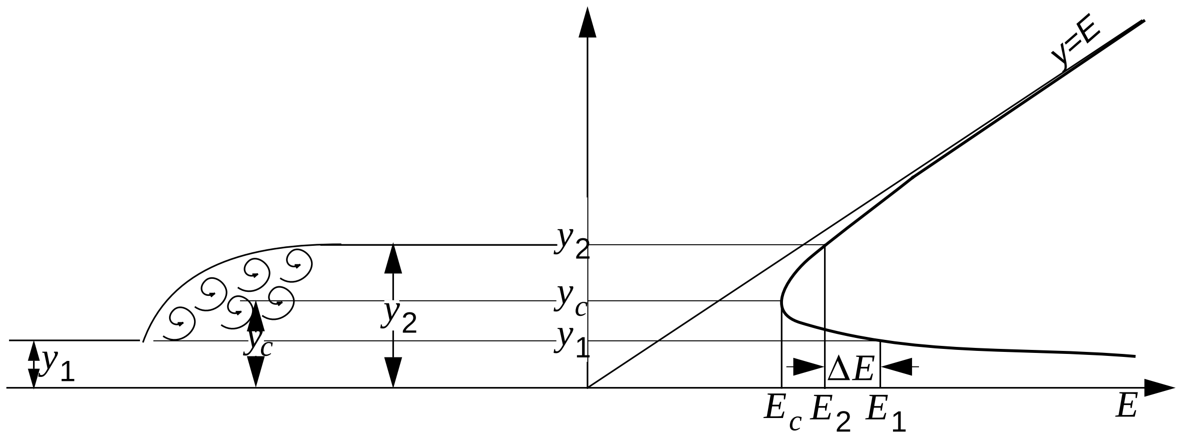

Figure 15. Hydraulic jump profile and associated specific energy diagram.

Consider the integral momentum equation:

$$\sum \mathbf{F} = \int_{\text{c.s.}} \mathbf{V}\rho\mathbf{V} \bullet \mathbf{dA} \tag{7}$$

where $\mathbf{F}$ is an external force, $\mathbf{V}$ is the velocity vector, positive to the left, ρ is the density of the fluid, and $\mathbf{dA}$ is the area vector, positive outward from the stationary control volume. The control volume contains the jump, as shown in Figure 14. Assuming a uniform velocity distribution across the area $\mathbf{A}$ yields:

$$\sum \mathbf{F} = \mathbf{V}\rho\mathbf{V} \bullet \mathbf{A} \tag{8}$$

Ignoring boundary friction and for small channel slopes,

$$F_1 - F_2 = \rho Q(V_2 - V_1) \tag{9}$$

where $F_1 = \gamma y_1^2/2$ and $F_2 = \gamma y_2^2/2$ are the hydrostatic forces upstream of the jump and downstream of the jump, respectively. After substitution for F_1 and F_2, the momentum equation becomes:

$$\frac{q^2}{gy_1} + \frac{y_1^2}{2} = \frac{q^2}{gy_2} + \frac{y_2^2}{2} \tag{10}$$

Equation (10) is a quadratic in y_2/y_1, the solution of which may be written (the derivation is non-trivial!):

$$y_2 = \frac{y_1}{2}\left(\sqrt{1+8F_{r_1}^2} - 1\right) \tag{11}$$

or

$$y_1 = \frac{y_2}{2}\left(\sqrt{1+8F_{r_2}^2} - 1\right) \tag{12}$$

These equations show that $y_2/y_1 > 1$ only when $F_{r_1} > 1$ and $F_{r_2} < 1$, thus proving the necessity of supercritical flow for hydraulic jump formation. Another way of visualizing this is by defining a **momentum function**, also known as specific momentum or specific force, *M*:

$$M = \frac{q^2}{gy} + \frac{y^2}{2} \tag{13}$$

where the term q^2/gy is the momentum of the flow passing through the channel section per unit time per unit weight of water, and the term $y^2/2$ is the force per unit weight of water.

Plotting M as a function of y for a constant flowrate (Figure 16), the solution of equation (10) occurs when $M_1 = M_2$. The depths y_1 and y_2 at which $M_1 = M_2$ are called **sequent depths**.

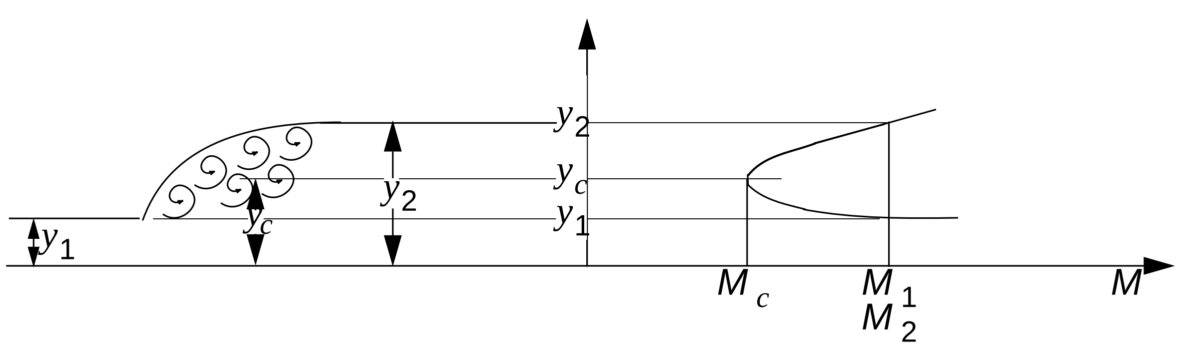

Figure 16. Hydraulic jump profile and associated M - curve.

A stable hydraulic jump will form only if the three independent variables (y_1, y_2, F_{r_1}) of the hydraulic jump equation conform to the relationship of equations (11) and (12).[1] The upstream depth, y_1, and the Froude number, F_{r_1}, are controlled by an upstream headgate for a given discharge. In this experiment, the downstream depth, y_2, is controlled by a downstream tailgate and *not by the hydraulic jump*. Denoting the actual *measured* downstream depth as y_2 and the sequent depth as y_2', the following observations may be made:

- if $y_2 = y_2'$, a stable jump forms;
- if $y_2 > y_2'$ the downstream M is greater than the upstream M, and the jump moves upstream;
- if $y_2 < y_2'$ the downstream M is less than the upstream M and the jump moves downstream.

II. Objective

Determine the validity of the integral momentum and specific energy equations for the hydraulic jump phenomenon. Determine the stability and characteristics of the jump obtained in the lab using the impulse-momentum and specific energy equations.

III. Anticipated Results

At a minimum, you should be able to anticipate:

1) What happens to the energy curves as the flowrate increases?
2) What happens to the M curves as the flowrate increases?
3) Will the jump be stable?

IV. Apparatus

1) Horizontal, glass-walled flume with headgate and tailgate controls.
2) Metered water supply.
3) Point gages.
4) Scale.

V. Procedure

1) Check the reservoir water level.
2) Level the flume to horizontal if necessary. Measure the flume width.
3) Zero the gages to the bottom of the channel.
4) Lower the tailgate to approximately level. Start the pump and open the discharge control valve. Use flowrates between 85 gallons per minute and fully open (about 115 gpm)
5) Position the headgate so that the upstream water surface is near the top of the headgate. Do not overflow the headgate or channel.
6) Once steady-state conditions have been reached, record the discharge rate.
7) Position the tailgate (raise it) to create a hydraulic jump in the center of the flume. Very small adjustments are required once the jump is near the center of

the channel. Give the system a few moments to equilibrate before making more adjustments. Verify the jump is stable and not moving upstream or downstream. The jump will not remain perfectly steady but should not move during a period of several minutes.

8) Using the point gages, determine the water surface levels upstream and downstream of the jump.
9) Raise the headgate a small amount (only small amounts are required; give the system a few moments to equilibrate) and repeat steps 7 through 9 for at least five headgate positions.
10) Repeat steps 5 through 10 for three flowrates.

VI. Data Control

Data control consists of plotting water surface depth (both upstream and downstream of the jump) versus E and versus M (on separate plots) for one set of flow data.

VII. Results

1) Plot the specific energy curves for the three laboratory flowrates on the same graph. Denote on each curve the critical depth, and the loss of energy in one jump. Hint: critical depth occurs when E is a minimum and $F_r = 1$. Hint: plot the y=E line. What happens as the flowrate increases?
2) Plot the M - curves for the three flowrates on the same graph. Denote the sequent depths and the critical depth for one jump (use a y_1 value from your lab data and equation (11)). What is the relationship between your calculated sequent depth and the measured depth? What is the relationship between the sequent depths and the critical depth? What happens as the flowrate increases?
3) Is the jump stable? Why or why not?

VIII. Suggested Data Sheet Headings ([ ] indicate the units of measurement)

Run #	Discharge Q		q	Point Gage 1			Point Gage 2		
	[gpm]	[cfs]	[ ]	Bed	W.S.	y_1	Bed	W.S.	y_2

Flume width (b) ______________ [ ]