A space-filling curve derived from the Cantor set.

Here I describe a continuous 2-manifold that fills a 3 dimensional space. This surface is derived from the cantor set and thus shares similar properties with it.

We start by drawing an approximation of the cantor set, as shown in figure 1. Then, every point in the set is replaced with a plane of infinite area. Finally, we draw the connected sum of some of these surfaces.

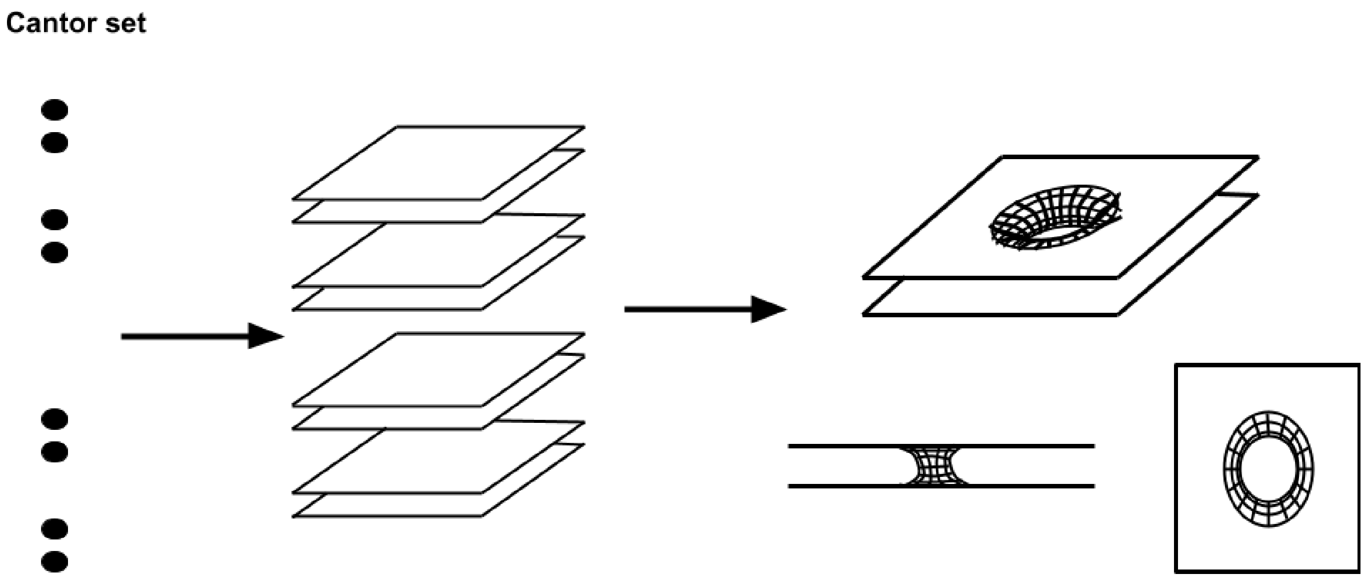

Figure 1 | 3 steps to draw a space-filling curve starting from the cantor set.

For the surface to be continuous there are only a few ways in which the connected sums can be done. Connected sums that punch holes in other surfaces in between, making them discontinuous, are not allowed. To illustrate this, a representation of some of the allowed configuration of the connected sums is shown in fig 2.

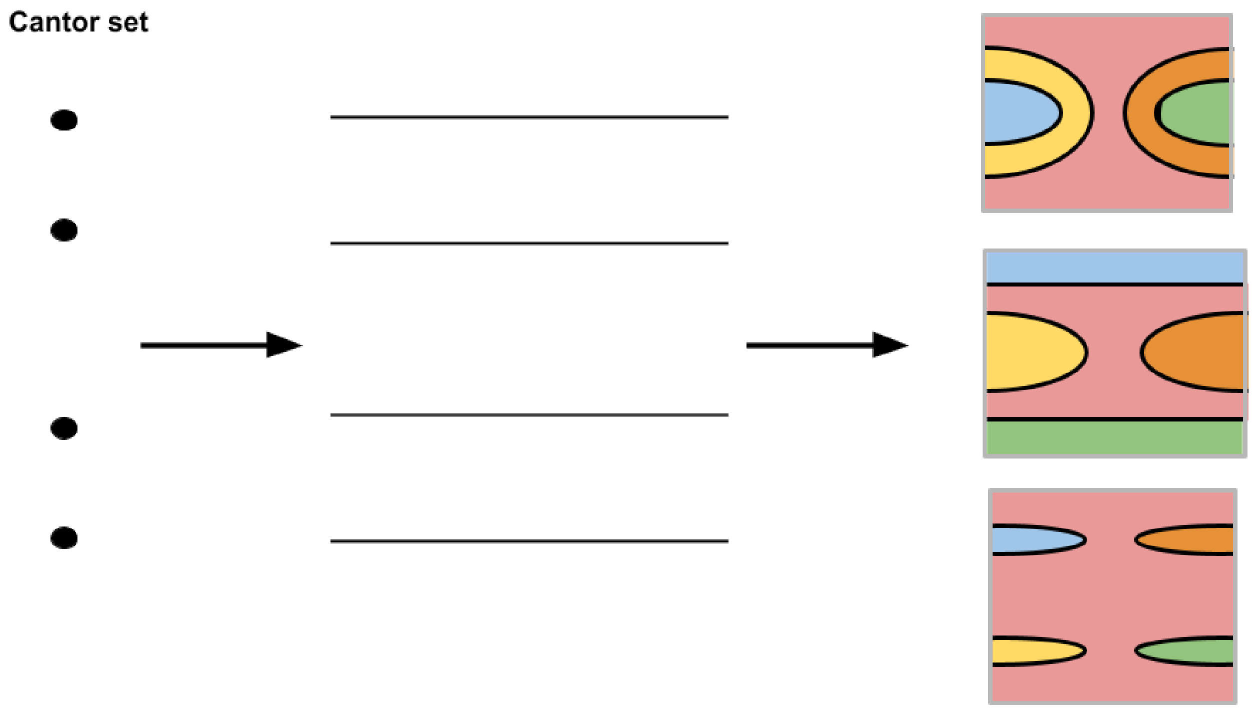

Figure 2 | Only connected sums that result in continuous surfaces are allowed.

Following these steps many different configurations of connected sums of this surface can be made, as shown in figure 3, 4, and 5. These spaces could also be described as topological manifolds with a layered structure given by many foliations.

Given the fact that the surface is derived from the cantor set, we could perhaps declare the following 2 statements about the surface:

1. There is an uncountable amount of connected surfaces.
2. The total sum of all the surfaces' thickness is 0.

It is still not clear to me how to calculate the fractal dimension of this fractal.

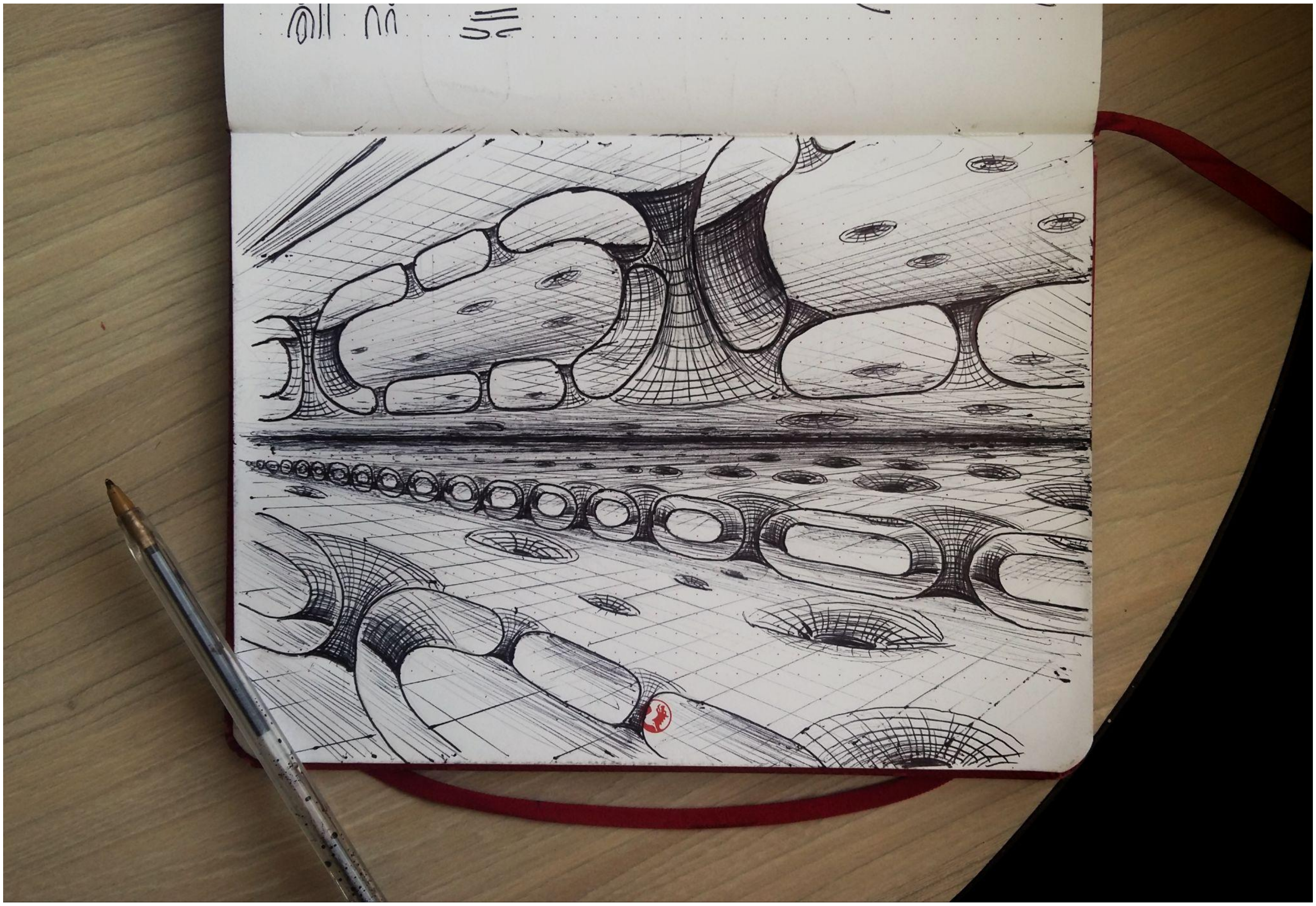

Figure 3 | Perspective drawing of a space-filling curve

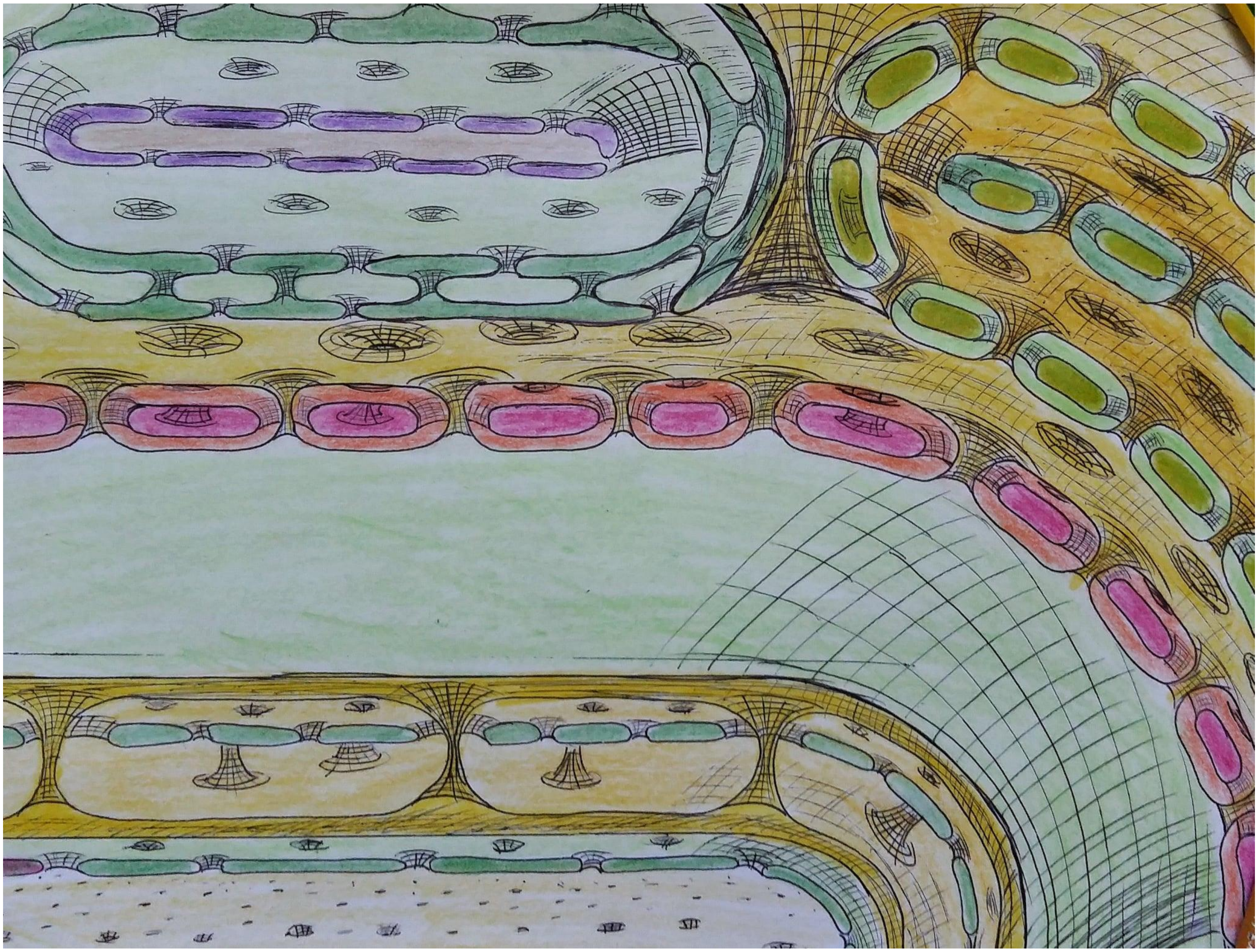

Figure 4 | Colored drawing of a space-filling curve

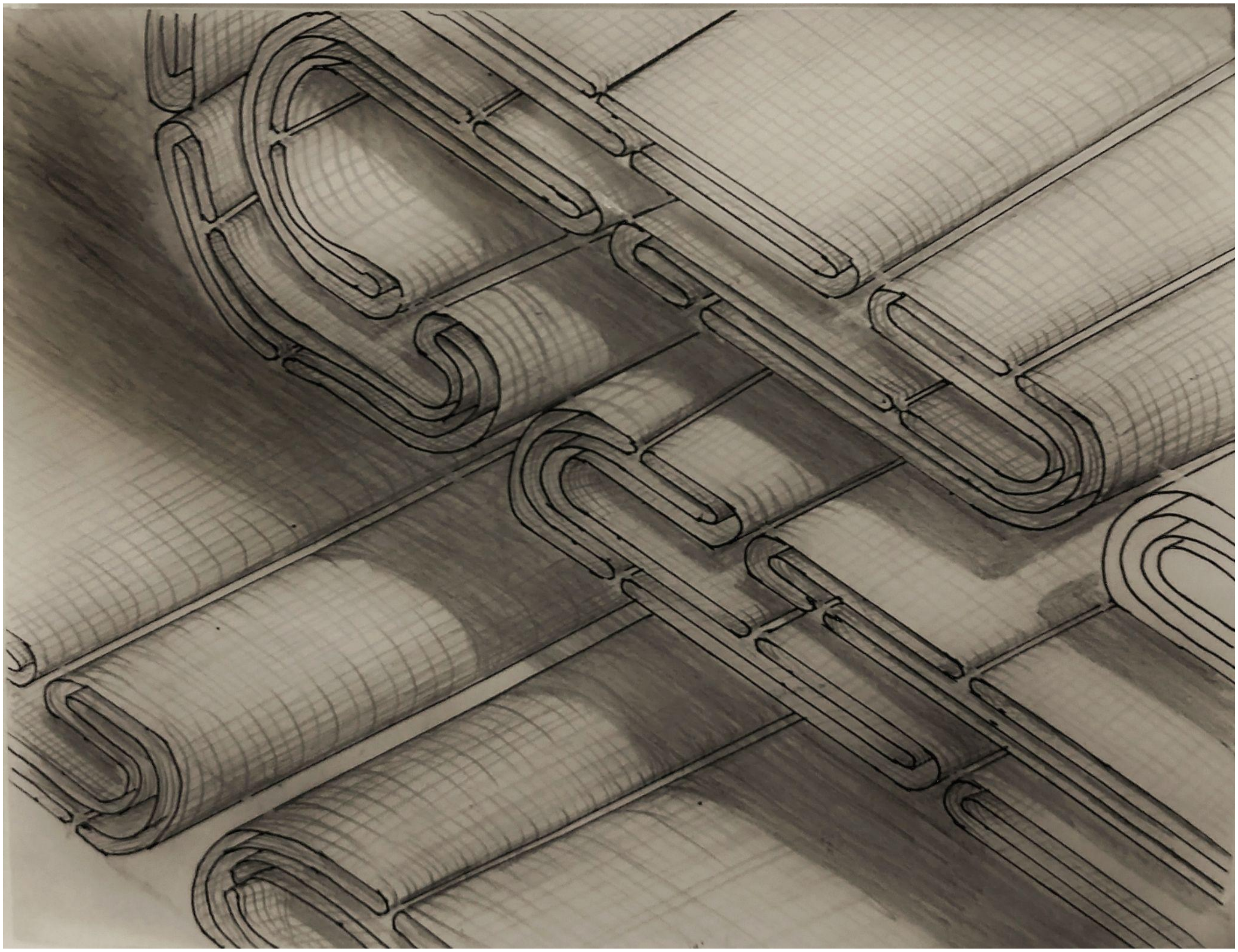

Figure 5 | A space-filling curve in which holes are elongated.