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n-simplex numbers

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written by Adi Cox 9th June 2015

What is an n-simplex number?

An n-simplex has a visual representation with n vertices and all these vertices are connected to every other vertex. So if we number the vertices from zero to n we get a number if we follow the path of all the connections. Sometimes the connections are used twice. This is when n is even as in the case for 4-simplices.

where 2 and 1 connections are used twice:
01230213 base 4 = 6951 base 10

where 2 and 3 connections are used twice:
01230231 base 4 = 6957 base 10

All even n-simplex numbers have odd order vertices and as such all even n-simplex numbers use one connection path twice, unless n = 2.

All odd n-simplex numbers have even order vertices and as such all odd n-simplex numbers use all connection paths once only and as such this makes odd n-simplices Eulerian, unless n = 0 where there are no connection paths.

0-simplex number

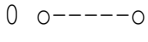
therefore set 0-SN{NULL} is the set of 0-simplex numbers.

1-simplex number

0 o

therefore set 1-SN{0} is the set of 1-simplex numbers.

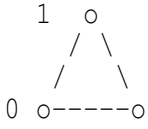
2-simplex numbers



01 base 2 = 1 base ten
10 base 2 = 2 base ten

therefore set 2-SN{1,2} is the set of 2-simplex numbers.

3-simplex numbers



0120 base 3 = 15 base 10
0210 base 3 = 21 base 10

1021 base 3 = 34 base 10
1201 base 3 = 46 base 10

2012 base 3 = 59 base 10
2102 base 3 = 65 base 10

therefore set 3-SN{15,21,34,46,59,65} is the set of 3-simplex numbers.

the inverse	the converse
0120 and 2102	0120 and 0210
1021 and 1201	1021 and 1201
2012 and 0210	2012 and 2102

The group of '3-simplex numbers and *equal digit numbers' is an order 9 cyclic group. The 'equal digit numbers' make the order 3 cyclic subgroup:

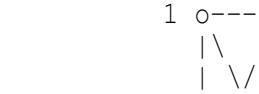
0 = 0000*
a = 0120
b = 0210
c = 1111*
d = 1021
e = 1201
f = 2222*
g = 2012
h = 2102

+3	0	a	b	c	d	e	f	g	h
0	0	a	b	c	d	e	f	g	h
a	a	b	0	e	c	d	g	h	f
b	b	0	a	d	e	c	h	f	g
c	c	e	d	f	h	g	0	a	b
d	d	c	e	h	g	f	b	0	a
e	e	d	c	g	f	h	a	b	0
f	f	g	h	0	b	a	c	e	d
g	g	h	f	a	0	b	e	d	c
h	h	f	g	b	a	0	d	c	e

rearranging we get:

+3	0*	c*	f*	a	b	d	e	g	h
0*	0*	c*	f*	a	b	d	e	g	h
c*	c*	f*	0*	e	d	h	g	a	b
f*	f*	0*	c*	g	h	b	a	e	d
a	a	e	g	b	0	g	d	h	f
b	b	d	h	0	a	f	c	f	g
d	d	h	b	g	f	g	f	0	a
e	e	g	a	d	c	f	h	b	0
g	g	a	e	h	f	0	b	d	c
h	h	b	d	f	g	a	0	c	e

4-simplex numbers



Using the programs that I have written below we find that 4-simplex numbers have eight digits and that the 4-simplex number set 4-SN{} has order 192.

Below is the full list of all 192, 4-simplex numbers

001 01203123 base 4 = 6363 base 10
002 01203132 base 4 = 6366 base 10
003 01203213 base 4 = 6375 base 10
004 01203231 base 4 = base 10
005 01230123 base 4 = base 10
006 01230132 base 4 = base 10
007 01230213 base 4 = base 10
008 01230231 base 4 = base 10
009 01302123 base 4 = base 10
010 01302132 base 4 = base 10
011 01302312 base 4 = base 10
012 01302321 base 4 = base 10
013 01320123 base 4 = base 10
014 01320132 base 4 = base 10
015 01320312 base 4 = base 10
016 01320321 base 4 = base 10
017 02103123 base 4 = base 10
018 02103132 base 4 = base 10
019 02103213 base 4 = base 10
020 02103231 base 4 = base 10
021 02130123 base 4 = base 10
022 02130132 base 4 = base 10
023 02130213 base 4 = base 10
024 02130231 base 4 = base 10
025 02301213 base 4 = base 10
026 02301231 base 4 = base 10
027 02301312 base 4 = base 10
028 02301321 base 4 = base 10
029 02310213 base 4 = base 10
030 02310231 base 4 = base 10
031 02310312 base 4 = base 10
032 02310321 base 4 = base 10
033 03102123 base 4 = base 10
034 03102132 base 4 = base 10
035 03102312 base 4 = base 10
036 03102321 base 4 = base 10
037 03120123 base 4 = base 10
038 03120132 base 4 = base 10
039 03120312 base 4 = base 10
040 03120321 base 4 = base 10
041 03201213 base 4 = base 10
042 03201231 base 4 = base 10
043 03201312 base 4 = base 10
044 03201321 base 4 = base 10
045 03210213 base 4 = base 10
046 03210231 base 4 = base 10
047 03210312 base 4 = base 10

048 03210321 base 4 = base 10
049 10213023 base 4 = base 10
050 10213032 base 4 = base 10
051 10213203 base 4 = base 10
052 10213230 base 4 = base 10
053 10231023 base 4 = base 10
054 10231032 base 4 = base 10
055 10231203 base 4 = base 10
056 10231230 base 4 = base 10
057 10312023 base 4 = base 10
058 10312032 base 4 = base 10
059 10312302 base 4 = base 10
060 10312320 base 4 = base 10
061 10321023 base 4 = base 10
062 10321032 base 4 = base 10
063 10321302 base 4 = base 10
064 10321320 base 4 = base 10
065 12013023 base 4 = base 10
066 12013032 base 4 = base 10
067 12013203 base 4 = base 10
068 12013230 base 4 = base 10
069 12031023 base 4 = base 10
070 12031032 base 4 = base 10
071 12031203 base 4 = base 10
072 12031230 base 4 = base 10
073 12301203 base 4 = base 10
074 12301230 base 4 = base 10
075 12301302 base 4 = base 10
076 12301320 base 4 = base 10
077 12310203 base 4 = base 10
078 12310230 base 4 = base 10
079 12310302 base 4 = base 10
080 12310320 base 4 = base 10
081 13012023 base 4 = base 10
082 13012032 base 4 = base 10
083 13012302 base 4 = base 10
084 13012320 base 4 = base 10
085 13021023 base 4 = base 10
086 13021032 base 4 = base 10
087 13021302 base 4 = base 10
088 13021320 base 4 = base 10
089 13201203 base 4 = base 10
090 13201230 base 4 = base 10
091 13201302 base 4 = base 10
092 13201320 base 4 = base 10
093 13210203 base 4 = base 10
094 13210230 base 4 = base 10
095 13210302 base 4 = 31026 base 10
096 13210320 base 4 = 31032 base 10
097 20123013 base 4 = 34503 base 10
098 20123031 base 4 = 34509 base 10
099 20123103 base 4 = 34515 base 10
100 20123130 base 4 = 34524 base 10
101 20132013 base 4 = 34695 base 10
102 20132031 base 4 = base 10
103 20132103 base 4 = base 10
104 20132130 base 4 = base 10
105 20312013 base 4 = base 10
106 20312031 base 4 = base 10
107 20312301 base 4 = base 10

108 20312310 base 4 = base 10
109 20321013 base 4 = base 10
110 20321031 base 4 = base 10
111 20321301 base 4 = base 10
112 20321310 base 4 = base 10
113 21023013 base 4 = base 10
114 21023031 base 4 = base 10
115 21023103 base 4 = base 10
116 21023130 base 4 = base 10
117 21032013 base 4 = base 10
118 21032031 base 4 = base 10
119 21032103 base 4 = base 10
120 21032130 base 4 = base 10
121 21302103 base 4 = base 10
122 21302130 base 4 = base 10
123 21302301 base 4 = base 10
124 21302310 base 4 = base 10
125 21320103 base 4 = base 10
126 21320130 base 4 = base 10
127 21320301 base 4 = base 10
128 21320310 base 4 = base 10
129 23012013 base 4 = base 10
130 23012031 base 4 = base 10
131 23012301 base 4 = base 10
132 23012310 base 4 = base 10
133 23021013 base 4 = base 10
134 23021031 base 4 = base 10
135 23021301 base 4 = base 10
136 23021310 base 4 = base 10
137 23102103 base 4 = base 10
138 23102130 base 4 = base 10
139 23102301 base 4 = base 10
140 23102310 base 4 = base 10
141 23120103 base 4 = base 10
142 23120130 base 4 = base 10
143 23120301 base 4 = base 10
144 23120310 base 4 = base 10
145 30123012 base 4 = base 10
146 30123021 base 4 = base 10
147 30123102 base 4 = base 10
148 30123120 base 4 = base 10
149 30132012 base 4 = base 10
150 30132021 base 4 = base 10
151 30132102 base 4 = base 10
152 30132120 base 4 = base 10
153 30213012 base 4 = base 10
154 30213021 base 4 = base 10
155 30213201 base 4 = base 10
156 30213210 base 4 = base 10
157 30231012 base 4 = base 10
158 30231021 base 4 = base 10
159 30231201 base 4 = base 10
160 30231210 base 4 = base 10
161 31023012 base 4 = base 10
162 31023021 base 4 = base 10
163 31023102 base 4 = base 10
164 31023120 base 4 = base 10
165 31032012 base 4 = base 10
166 31032021 base 4 = base 10
167 31032102 base 4 = base 10

```
168   31032120 base 4 =  base 10
169   31203102 base 4 =  base 10
170   31203120 base 4 =  base 10
171   31203201 base 4 =  base 10
172   31203210 base 4 =  base 10
173   31230102 base 4 =  base 10
174   31230120 base 4 =  base 10
175   31230201 base 4 =  base 10
176   31230210 base 4 =  base 10
177   32013012 base 4 =  base 10
178   32013021 base 4 =  base 10
179   32013201 base 4 =  base 10
180   32013210 base 4 =  base 10
181   32031012 base 4 =  base 10
182   32031021 base 4 =  base 10
183   32031201 base 4 =  base 10
184   32031210 base 4 =  base 10
185   32103102 base 4 =  base 10
186   32103120 base 4 =  base 10
187   32103201 base 4 =  base 10
188   32103210 base 4 =  base 10
189   32130102 base 4 =  base 10
190   32130120 base 4 =  base 10
191   32130201 base 4 = 59169 base 10
192   32130210 base 4 = 59172 base 10
```

The program below converts the 8 digit base 4, 4-simplex number to a base 10, 4-simplex number.

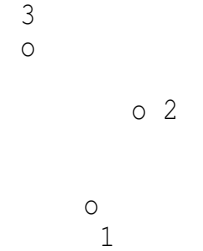
```
<!DOCTYPE HTML PUBLIC " - //W3C//DTD XHTML 1.0 Transitional//EN"
"http://www.w3.org/TR/xhtml1/DTD/xhtml1-transitional.dtd
">
<html xmlns="http://www.w3.org/1999/xhtml">
<head>
<title>A for loop</title>
</head>
<body>
<script type="text/javascript" language="javascript">
document.write("<h3>n base b = x base 10<h3>")

n=1203123;
b=4;


m1=n-10*Math.floor(n/10);

m2=(((n-100*Math.floor(n/100))
-(n-10*Math.floor(n/10)))/10);

m3=(((n-1000*Math.floor(n/1000))
-(n-100*Math.floor(n/100)))/100);

m4=(((n-10000*Math.floor(n/10000))
-(n-1000*Math.floor(n/1000)))/1000);

m5=(((n-100000*Math.floor(n/100000))
-(n-10000*Math.floor(n/10000)))/10000);
```

```
m6=(((n-1000000*Math.floor(n/1000000))
-(n-100000*Math.floor(n/100000)))/100000);

m7=(((n-10000000*Math.floor(n/10000000))
-(n-1000000*Math.floor(n/1000000)))/1000000);

m8=(((n-100000000*Math.floor(n/100000000))
-(n-10000000*Math.floor(n/10000000)))/10000000);


x1=m1;
x2=m2*b;
x3=m3*b*b;
x4=m4*b*b*b;
x5=m5*b*b*b*b;
x6=m6*b*b*b*b*b;
x7=m7*b*b*b*b*b*b;
x8=m8*b*b*b*b*b*b*b;

x=x1+x2+x3+x4+x5+x6+x7+x8;

document.write(n," base ",b," = ",x," base 10","<br />");


</script>
</body>
</html>
```

We get the output below

```
n base b = x base 10

1203123 base 4 = 6363 base 10
```

5-simplex numbers



This is where it gets a little more difficult. I have struggled to write a program to work out all the 5-simplex numbers, so I have done what I can by hand.

It does appear that in 5-simplex numbers they alway start and end with the same number, as with the other odd number that we have looked at the 3-simplex numbers.

Without any proof I conjecture that as 3-simplex numbers in base 3

are one number twice, which is always the first and the last number and one of each of the other two numbers. This will give a number like 1021.

So a 5-simplex number in base 5 are one number three times, which is always the first, the last and one in the middle somewhere and then two of the other four numbers. This will give an 11 digit number like 01203423140.

I believe that there will be 2340, 5-simplex numbers and this is how I have worked it out:

01203142340
01203143240
01203241340
01203234130
01203413240
01203423140
01204132430
01204134230
01204231430
01204234130
01204314230
01204324130

01230241340
01230243140
01230413420
01230431420
01231402430
01231403420
01231420340
01231420430
01231430240
01231430420
01234024130
01234031420
01234130240
01234130420

01240231430
01240234130
01240341320
01241320340
01241320430
01241340230
01241340320
01243023140
01243041320
01243140230
01243140320
01243203140
01243204130

Above are the 39 numbers that are 012xxxxxxx0. We can multiply 39 by 12 for all the 5-simplex numbers that begin and end in zero:

012xxxxxxx0
013xxxxxxx0
014xxxxxxx0

021xxxxxxx0
023xxxxxxx0
024xxxxxxx0
031xxxxxxx0
032xxxxxxx0
034xxxxxxx0
041xxxxxxx0
042xxxxxxx0
043xxxxxxx0

And then finally we multiply 12 times 39 by 5 for each of the different begining and end numbers 0,1,2,3,4. So we get:

12 x 39 x 5 = 2340

therefore the order of the set for 5-simplex numbers, set 5-SN{} has order 2340.

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n-simplex numbers

The number of digits of an n-simplex number in its n base is:

for even n numbers n(n/2)
for odd n numbers (n(n-1)/2)+1

even example where n = 4:
(n/2)n ---> ([4]/2)[4] = 8

e.g. 01230213 base 4

odd example where n = 5:
(((n-1)/2)n)+1 ---> ((([5]-1)/2)[5])+1 = 11

e.g. 01234031420 base 5

n	number of digits in base n	number of n-simplex numbers in set n-SN{}
00	000	000
01	001	001
02	002	002
03	004	006
04	008	192
05	011	2340 ?
06	018	????
07	022	
08	032	
09	037	
10	050	
11	056	
12	072	
13	079	
14	098	

15		106
16		128
17		137
18		162
19		172
20		200

Below is the program for finding 3-simplex numbers

```
<!DOCTYPE HTML PUBLIC " - //W3C//DTD XHTML 1.0 Transitional//EN"
"http://www.w3.org/TR/xhtml1/DTD/xhtml1-transitional.dtd
">
<html xmlns="http://www.w3.org/1999/xhtml">
<head>
<title>A for loop</title>
</head>
<body>
<script type="text/javascript" language="javascript">
document.write("<h3>x1, x2, x3, x4<h3>")
//B=28;
//D=-160;
u=0;
n=3;
for (x1 = u; x1 < n; ++x1)
{
for (x2 = u; x2 < n; ++x2)
{
for (x3 = u; x3 < n; ++x3)
{
for (x4 = u; x4 < n; ++x4)
{
if
((x1+x2+x3+x4)<(6)&&(x1!=x2)&&(x2!=x3)&&(x3!=x4)&&(x1!=x3)&&(x2!=x4))
{
document.write(x1,", ", x2,", ", x3,", ",x4," ",  "<br />");
}
}
}
}
}
</script>
</body>
</html>
```

Below is the output of the program above

```
x1, x2, x3, x4
0, 1, 2, 0
0, 2, 1, 0
1, 0, 2, 1
1, 2, 0, 1
2, 0, 1, 2
```

2, 1, 0, 2

Below is the program for finding 4-simplex numbers

```
<!DOCTYPE HTML PUBLIC " - //W3C//DTD XHTML 1.0 Transitional//EN"
"http://www.w3.org/TR/xhtml1/DTD/xhtml1-transitional.dtd
">
<html xmlns="http://www.w3.org/1999/xhtml">
<head>
<title>A for loop</title>
</head>
<body>
<script type="text/javascript" language="javascript">
document.write("<h3>x1, x2, x3, x4, x5, x6, x7, x8  <h3>")

u=0;
n=4;
for (x1 = u; x1 < n; ++x1)
{
for (x2 = u; x2 < n; ++x2)
{
for (x3 = u; x3 < n; ++x3)
{
for (x4 = u; x4 < n; ++x4)
{
for (x5 = u; x5 < n; ++x5)
{
for (x6 = u; x6 < n; ++x6)
{
for (x7 = u; x7 < n; ++x7)
{
for (x8 = u; x8 < n; ++x8)
{

if((x1+x2+x3+x4+x5+x6+x7+x8)==(12)&&(x2!=x1)&&(x3!=x1)&&(x3!=x2)&&
(x4!=x2)&&(x4!=x3)&&(x5!=x3)&&(x5!=x4)
&&(x5!=x2)&&(x6!=x4)&&(x6!=x5)&&(x7!=x1)&&(x7!=x6)&&(x8!=x7))

{
document.write(x1, x2, x3, x4, x5,  x6,  x7, x8,   "<br />");
}

}
}
}
}
}
}
}
}
</script>
</body>
</html>
```

Below is the output of the program above

```
x1, x2, x3, x4, x5, x6, x7, x8
01203123
01203132
01203213
01203231
01230123
01230132
01230213
01230231
01302123
01302132
01302312
01302321
01320123
01320132
01320312
01320321
02103123
02103132
02103213
02103231
02130123
02130132
02130213
02130231
02301213
02301231
02301312
02301321
02310213
02310231
02310312
02310321
03102123
03102132
03102312
03102321
03120123
03120132
03120312
03120321
03201213
03201231
03201312
03201321
03210213
03210231
03210312
03210321
10213023
10213032
10213203
10213230
10231023
```

10231032
10231203
10231230
10312023
10312032
10312302
10312320
10321023
10321032
10321302
10321320
12013023
12013032
12013203
12013230
12031023
12031032
12031203
12031230
12301203
12301230
12301302
12301320
12310203
12310230
12310302
12310320
13012023
13012032
13012302
13012320
13021023
13021032
13021302
13021320
13201203
13201230
13201302
13201320
13210203
13210230
13210302
13210320
20123013
20123031
20123103
20123130
20132013
20132031
20132103
20132130
20312013
20312031
20312301
20312310
20321013
20321031
20321301
20321310
21023013

21023031
21023103
21023130
21032013
21032031
21032103
21032130
21302103
21302130
21302301
21302310
21320103
21320130
21320301
21320310
23012013
23012031
23012301
23012310
23021013
23021031
23021301
23021310
23102103
23102130
23102301
23102310
23120103
23120130
23120301
23120310
30123012
30123021
30123102
30123120
30132012
30132021
30132102
30132120
30213012
30213021
30213201
30213210
30231012
30231021
30231201
30231210
31023012
31023021
31023102
31023120
31032012
31032021
31032102
31032120
31203102
31203120
31203201
31203210
31230102

31230120
31230201
31230210
32013012
32013021
32013201
32013210
32031012
32031021
32031201
32031210
32103102
32103120
32103201
32103210
32130102
32130120
32130201
32130210