Name: Key

Date:

Mr. Richards – Honors Calculus I

FINALS REVIEW #2 (Non-Calculator)

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1. Explain why $\lim_{x\to\infty}\left(\frac{x}{2\cos(x)-3}\right) = -\infty$ but $\lim_{x\to\infty}\left(\frac{x}{3\cos(x)-2}\right) = DNE$.

Because $\cos(x)$ oscillates between $-1$ and $+1$,
$2\cdot\cos(x)$ goes between $-2$ and $2$.
$(-2 \text{ and } 2) - 3 \Rightarrow -5, -1 \Rightarrow$ always negative
$\Rightarrow -\infty$

but

$3[-1,1]-2 \Rightarrow [-3,3]-2 \Rightarrow [-5,1]$ goes to
both $+\infty$ and $-\infty \Rightarrow DNE$

2. Anti-differentiate: $\int \csc^2(x) - e^x + \frac{8}{x} - \frac{8}{x^4}\,dx$

$-\cot(x) - e^x + 8\ln|x| + \frac{8}{3}x^{-3} + C$

3. (On whiteboard/table or separate sheet): Prove that $\lim_{x\to 0}\left(\frac{\sin(x)}{x}\right) = 1$ :)

4. Find the following limits, and discuss the reasoning behind them:

a) $\lim_{x\to -5}\left(\frac{x^2-25}{10x^3+50x^2}\right)$ $= \lim_{x\to -5}\left(\frac{(x-5)(x+5)}{10x^2(x+5)}\right) = \lim_{x\to -5}\left(\frac{x-5}{10x^2}\right)$

$= \frac{-10}{10(25)} = -\frac{1}{25}$

b) $\lim_{x\to\infty}\left(\frac{30x+500{,}000{,}000x^{22}-2x^{23}}{8x^{22}+37{,}000{,}000\cos(x)+x^{23}-4x^2}\right)$

$-2$

c) $\lim_{x\to\infty}\left(\frac{5}{3}x\cdot\tan\left(\frac{1}{x}\right)-\frac{-4x+2}{1-x}\right)$

$\frac{5}{3}\lim_{x\to\infty}\left(x\cdot\tan\left(\frac{1}{x}\right)\right) - \lim_{x\to\infty}\left(\frac{-4x+2}{1-x}\right)$

$u=\frac{1}{x} \to x=\frac{1}{u}$

as $x\to\infty$, $u\to 0$

$\frac{5}{3}\cdot\lim_{x\to\infty}\left(\frac{\tan(u)}{u}\right) - \left(\frac{-4}{-1}\right) = \frac{5}{3}\cdot(1)-4 = \frac{-7}{3}$

d) $\lim_{x\to 4}\left(3\pi\cdot\frac{\sin(-4x+16)}{(2x-8)\cdot\sec(5x-20)}\cdot\frac{x+5}{2x+5}\right)$

$3\pi\cdot\lim_{x\to 4}\left(\frac{\sin(-4x+16)}{2x-8}\right)\cdot\lim_{x\to 4}\left(\frac{1}{\sec(5x-20)}\cdot\frac{x+5}{2x+5}\right)$

$u=-4x+16$

~~$3\pi$~~

~~$-2(2x-8)=-4x+16$~~

$-\frac{1}{2}u=2x-8$

~~$3\pi$~~

as $x\to 4$

$u\to 0$

$3\pi\cdot\lim_{u\to 0}\left(\frac{\sin(u)}{-\frac{1}{2}u}\right)\cdot\frac{1}{1}\cdot\frac{9}{14} = -\frac{27\pi}{7}$