Trebuchet Calculation:

A trebuchet with a counterweight of 5000kg starts at a height of 8 meters and swings down to a lowest point of 1 meter releasing a 90kg projectile (that started at ground level) at some point in the swing. The counterweight then swings back up to a height of 2m above the ground on the swingthrough. What is the velocity of the projectile if it hits the castle 300m away at 15m above ground level? Assume a perfect trebuchet and no air resistance.(Hint: Use Conservation of Energy).

$$KE = \frac{1}{2}mv^2$$

$$PE = mgh$$

$$KE + PE = Constant$$

Solution:

If the trebuchet's counterweight starts at a height of $8m$ and ends it's swing at a height of $2m$ this means that the counterweight has transferred $6m$ worth of potential energy into kinetic energy for the projectile.

$$PE_{transferred} = (5000kg)(9.80m/s^2)(6m) = 294kJ$$

Since we are assuming perfect energy transfer this means that the projectile now has 294kJ of KE at the start of it's swing. But the projectile hits the castle with a certain amount of PE in it's arc since it is 15m above ground level. This PE has to come from the original KE the projectile started with.

$$PE_{gained} = KE_{lost} = (90kg)(9.80kg/s^2)(15m) = 13.230kJ$$

This energy then has to be subtracted from the original KE of the projectile to find the KE of the projectile upon impact.

$$KE_{impact} = KE_{initial} - KE_{lost} = 294kJ - 13.230kJ = 280.770kJ$$

Now since the equation for KE depends on mass and velocity alone and we know the mass we can figure out the velocity that the projectile has.

$$KE = \frac{1}{2}mv^2$$

$$v^2 = \frac{2KE}{m} = \frac{2(280770J)}{90kg} = 6239.333333$$

$$v = \sqrt{6239.333333} = 78.99m/s$$

The projectile will hit the castle with a final velocity of just under $79m/s$ or $284km/h$ or $178mph$!