12. POTENCIJE I KORJENI

Opci pojmovi i racunske radnje sa potencijama i korjenima

Mnozenje: $x^a \times x^b = x^{a+b}$

Djeljenje: $x^a \div x^b = \frac{x^a}{x^b} = x^{a-b}$

Potenciranje: $\left(x^a\right)^b = x^{a \cdot b}$

Potenciranje produkta: $(x \cdot y)^a = x^a \cdot y^a$

Potenciranje kvocijenta: $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Razlomljeni eksponenti: $x^{\frac{1}{a}} = \sqrt[a]{x},\ \ x^{\frac{b}{a}} = \sqrt[a]{x^b}\ ;\ x \geq 0$

$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m ;\ a \geq 0$

1. $$\frac{\left(x^{-2}y^{-1}\right)^{-3}}{x^4y^2} = \frac{\left(\frac{1}{x^2}\frac{1}{y}\right)^{-3}}{x^4y^2} = \frac{\left(\frac{1}{x^2y}\right)^{-3}}{x^4y^2} = \left(\frac{1}{x^2y}\right)^{-3}\left(\frac{1}{x^2y}\right)^2 = \left(\frac{1}{x^2y}\right)^{-3+2} =$$
$$= \left(\frac{1}{x^2y}\right)^{-1} = x^2y$$

2. $$\frac{c^{-3}d^2 \times c^2d^{-3}}{\left(c^3d^{-2}\right)^{-1}} = \frac{d^2c^2\left(c^3d^{-2}\right)^1}{c^3d^3} = \frac{d^2c^2c^3}{c^3d^3d^2} = \frac{c^2}{d^3}$$

3. $$\frac{1}{x^{-1}}\left(\frac{x^{-1}-y^{-1}}{x^2-y^2}\right) = x\left(\frac{\frac{1}{x}-\frac{1}{y}}{x^2-y^2}\right) = x\left(\frac{\frac{y-x}{xy}}{x^2-y^2}\right) = x\frac{y-x}{x\left(x^2-y^2\right)} = \frac{y-x}{(x-y)(x+y)}$$

4. $$3(x+4)^2(x-3)^{-2} - 2(x-3)^{-3}(x+4)^3 = \frac{3(x+4)^2}{(x-3)^2} - \frac{2(x+4)^3}{(x-3)^3} =$$

$$= \frac{3(x+4)^2(x-3)-2(x+4)^3}{(x-3)^3} = \frac{(x+4)^2\left[5(x-3)-2(x+4)\right]}{(x-3)^3} =$$

$$= \frac{(x+4)^2(3x-9-2x-8)}{(x-3)^3} = \frac{(x+4)^2(x-17)}{(x-3)^3}$$

5. $$\frac{ax^{-2}+a^{-2}x}{a^{-1}+x^{-1}} = \frac{\frac{a}{x^2}+\frac{x}{a^2}}{\frac{1}{a}+\frac{1}{x}} = \frac{\frac{a^3+x^3}{a^2x^2}}{\frac{a+x}{ax}} = \frac{ax(a^3+x^3)}{a^2x^2(a+x)} = \frac{(a+x)(a^2+ax+x^2)}{ax(a+x)} =$$

$$= \frac{(a^2+ax+x^2)}{ax}$$

6. $$4(2x-1)(x+2)^{-1}-(2x-1)^2(x+2)^{-2} = \frac{4(2x-1)}{(x+2)} - \frac{(2x-1)^2}{(x+2)^2} =$$

$$= \frac{4(2x-1)(x+2)-(2x-1)^2}{(x+2)^2} = \frac{4(2x-1)(x+2)-(2x-1)^2}{(x+2)^2} =$$

$$= \frac{(2x-1)\left[(x+2)-(2x-1)\right]}{(x+2)^2} = \frac{(2x-1)(4x+8-2x+1)}{(x+2)^2} = \frac{(2x-1)(2x+9)}{(x+2)^2}$$

7. $$\frac{6x^{-\frac{1}{2}}y^{\frac{2}{3}}}{18x^{-1}}\left(\frac{2y^{\frac{1}{4}}}{x^{\frac{1}{3}}}\right) = \frac{\frac{6y^{\frac{2}{3}}}{x^{\frac{1}{2}}}}{\frac{18}{x}}\left(\frac{2y^{\frac{1}{4}}}{x^{\frac{1}{3}}}\right) = \frac{6xy^{\frac{2}{3}}}{18x^{\frac{1}{2}}}\left(\frac{2y^{\frac{1}{4}}}{x^{\frac{1}{3}}}\right) = \frac{12xy^{\left(\frac{2}{3}+\frac{1}{4}\right)}}{18x^{\left(\frac{1}{2}+\frac{1}{3}\right)}} = \frac{2xy^{\frac{11}{12}}}{3x^{\frac{5}{6}}} = \frac{2y^{\frac{11}{12}}}{3x^{\left(\frac{5}{6}-\frac{6}{6}\right)}} =$$

$$= \frac{2}{3}x^{\frac{1}{2}}y^{\frac{11}{12}}$$

8. $$\left(x^{-1}+2x^{-2}\right)^{-\frac{1}{2}} = \left(\frac{1}{x}+\frac{2}{x^2}\right)^{-\frac{1}{2}} = \left(\frac{x+2}{x^2}\right)^{-\frac{1}{2}} = \left(\frac{x^2}{x+2}\right)^{\frac{1}{2}} = \frac{x}{(x+2)^{\frac{1}{2}}} = x(x+2)^{-\frac{1}{2}}$$

9. $$\left(a^{-2}-a^{-4}\right)^{-\frac{1}{4}} = \left(\frac{1}{a^2}-\frac{1}{a^4}\right)^{-\frac{1}{4}} = \left(\frac{a^2-1}{a^4}\right)^{-\frac{1}{4}} = \left(\frac{a^4}{a^2-1}\right)^{\frac{1}{4}} = \frac{a}{(a^2-1)^{\frac{1}{4}}} = a(a^2-1)^{-\frac{1}{4}}$$

10. $x^2\left(2x-1\right)^{-\frac{1}{2}}+2x\left(2x-1\right)^{\frac{1}{2}}=\dfrac{x^2}{\left(2x-1\right)^{\frac{1}{2}}}+2x\left(2x-1\right)^{\frac{1}{2}}=\dfrac{x^2+2x\left(2x-1\right)^{2\frac{1}{2}}}{\left(2x-1\right)^{\frac{1}{2}}}=$

$=\dfrac{x^2+4x^2-2x}{\left(2x-1\right)^{\frac{1}{2}}}=\dfrac{5x^2-2x}{\left(2x-1\right)^{\frac{1}{2}}}$

11. $\left(3x-1\right)^{-\frac{2}{3}}\left(1-x\right)-\left(3x-1\right)^{\frac{1}{3}}=\dfrac{1-x}{\left(3x-1\right)^{\frac{2}{3}}}-\left(3x-1\right)^{\frac{1}{3}}=\dfrac{\left(1-x\right)-\left(3x-1\right)^{\left(\frac{1}{3}+\frac{2}{3}\right)}}{\left(3x-1\right)^{\frac{2}{3}}}=$

$=\dfrac{1-x-3x+1}{\left(3x-1\right)^{\frac{2}{3}}}=\dfrac{2-4x}{\left(3x-1\right)^{\frac{2}{3}}}$

12. $\sqrt{\dfrac{3a}{4b}-2+\dfrac{4b}{3a}}=\sqrt{\dfrac{9a^2-24ab+16b^2}{12ab}}=\sqrt{\dfrac{\left(3a-4b\right)^2}{4\times 3ab}}=\dfrac{\left(3a-4b\right)}{2\sqrt{3ab}}\times\dfrac{\sqrt{3ab}}{\sqrt{3ab}}=$

$=\dfrac{\sqrt{3ab}\left(3a-4b\right)}{2\times 3ab}=\dfrac{\sqrt{3ab}\left(3a-4b\right)}{6ab}$

13. $\left(\sqrt{\dfrac{2}{a}}+\sqrt{\dfrac{a}{2}}\right)\left(\sqrt{\dfrac{2}{a}}-2\sqrt{\dfrac{a}{2}}\right)=\sqrt{\left(\dfrac{2}{a}\right)^2}-2\sqrt{\dfrac{2}{a}}\sqrt{\dfrac{a}{2}}+\sqrt{\dfrac{a}{2}}\sqrt{\dfrac{2}{a}}-2\sqrt{\left(\dfrac{a}{2}\right)^2}=$

$=\dfrac{2}{a}-2+1-2\dfrac{a}{2}=\dfrac{4-2a-2a^2}{2a}=\dfrac{2-a-a^2}{a}$

14. $\left(\dfrac{2-\sqrt{15}}{2}\right)^2-\left(\dfrac{2-\sqrt{15}}{2}\right)=\dfrac{\left(2-\sqrt{15}\right)^2}{4}-\dfrac{2-\sqrt{15}}{2}=\dfrac{\left(4-4\sqrt{15}+15\right)-2\left(2-\sqrt{15}\right)}{4}=$

$=\dfrac{4-4\sqrt{15}+15-4+2\sqrt{15}}{4}=\dfrac{15-2\sqrt{15}}{4}$

15. $\dfrac{2^{-3}+3^{-1}}{4^{-1}+3^{-2}}=\dfrac{\dfrac{1}{2^2}+\dfrac{1}{3}}{\dfrac{1}{4}+\dfrac{1}{3^2}}=\dfrac{\dfrac{1}{4}+\dfrac{1}{3}}{\dfrac{1}{4}+\dfrac{1}{9}}=\dfrac{\dfrac{3+4}{12}}{\dfrac{9+4}{36}}=\dfrac{\dfrac{7}{12}}{\dfrac{13}{36}}=\dfrac{36\cdot 7}{12\cdot 13}=\dfrac{21}{13}$

16. $\dfrac{\left(5^2+3^5\right)\cdot 2^{-2}}{2^{-3}}=\dfrac{\left(25+27\right)\dfrac{1}{2^2}}{\dfrac{1}{2^3}}=\dfrac{\dfrac{52}{4}}{\dfrac{1}{6}}=\dfrac{52\cdot 8}{4\cdot 1}=104$

17. Prikazi zadani izraz sa pozitivnim eksponentima:

$$\left(p^3q^2\right)^{-1}\cdot p^4q^{-2}=\frac{1}{p^3q^2}p^4\frac{1}{q^2}=\frac{p^4}{p^3q^2q^2}=\frac{p}{q^4}$$

18. $$m^2n\cdot mn^{-2}\cdot mn=m^2n\frac{m}{n^2}mn=m^4$$

19. $$h^2kq^3\cdot h^{-3}k^2q^3=h^2kq^3\frac{k^2q^3}{h^3}=\frac{k^2q^6}{h}$$

20. Rijesi jednakost za vrijednosti od: $m=2,\ n=-3$

$$\frac{m^{-3}m^4}{m^5}=\frac{m^4}{m^5m^3}=\frac{1}{m^4}\Rightarrow\qquad\frac{1}{m^4}=\frac{1}{2^4}=\frac{1}{16}$$

21. $$\frac{m^2n^3\cdot m^{-1}n^{-2}}{\left(m^{-1}n^2\right)^2}=\frac{\frac{m^2n^3}{m^1n^2}}{\frac{n^4}{m^2}}=\frac{m^2n^3m^2}{m^1n^2n^3}=\frac{m^3}{n^3}\Rightarrow\frac{m^3}{n^3}=\frac{2^3}{(-3)^3}=\frac{8}{-27}=-\frac{8}{27}$$

22. $$\frac{\left(m^{-3}n^{-5}\right)\left(m^2n^4\right)}{\left(m^2n\right)^{-2}}=\frac{\frac{m^2n^4}{m^3n^5}}{\frac{1}{m^4n^2}}=\frac{m^2n^4m^4n^2}{m^3n^5}=m^3n\qquad\Rightarrow m^3n=2^3(-3)=8(-3)=-24$$

23. Duzina celicnog profila kome je baza kvadrat stranice a data je izrazom $Va^{-2}n^{-1}$, gdje je V volumen i n- broj komada.
Prikazi izraz sa pozitivnim eksponentima i izracunaj duzinu profila ako je:
$V=162\ \text{cm}^3$ i $n=3$.

$$L=Va^{-2}n^{-1}=\frac{V}{a^2n}\Rightarrow L=\frac{162}{3^3\cdot 3}=\frac{162}{27}=6\ cm$$

24. Fokusna udaljenost lece dana je izrazom $f=\left[(n-1)\left(a^{-1}-b^{-1}\right)\right]^{-1}$
Prikazi izraz sa pozitivnim eksponentima i izracunaj f za: $a=5.9,\ b=7.5\ i\ n=6$.

$$f=\left[(n-1)\left(a^{-1}-b^{-1}\right)\right]^{-1}=\frac{1}{(n-1)\left(a^{-1}-b^{-1}\right)}=\frac{1}{(n-1)\left(\frac{1}{a}-\frac{1}{b}\right)}=\frac{1}{(n-1)\left(\frac{b-a}{ab}\right)}=$$

$$=\frac{ab}{(n-1)(b-a)}\Rightarrow\qquad f=\frac{ab}{(n-1)(b-a)}=\frac{5.9\cdot 7.5}{(6-1)(7.5-5.9)}=5.531$$

25. $\left(\sqrt[4]{p}\right)^{-3}\cdot\sqrt{p^3}=\dfrac{\sqrt{p^3}}{\left(\sqrt[4]{p}\right)^3}=\dfrac{p^{\frac{3}{2}}}{p^{\frac{3}{4}}}=p^{\frac{3}{2}-\frac{3}{4}}=p^{\frac{6-3}{4}}=p^{\frac{3}{4}}$

26. $k^{-2}\left(\sqrt[4]{k}\right)^2\div n^0=\dfrac{k^{\frac{2}{4}}}{k^2}\div 1=k^{\frac{1}{2}}k^{\frac{-4}{2}}=k^{\frac{1-4}{2}}=k^{-\frac{3}{2}}$

27. $\sqrt{\dfrac{y^5}{y^2}}\cdot\left(\sqrt{y}\right)^3=\sqrt{y^{5-3}}\sqrt{y^3}=\sqrt{y^3y^3}=y^3$

28. $\dfrac{\sqrt[3]{a^6}\cdot a^{-3}}{\left(\sqrt{a}\right)^3}=\dfrac{a^{\frac{6}{3}}\dfrac{1}{a^3}}{a^{\frac{3}{2}}}=\dfrac{a^2}{a^{\frac{3}{2}}\cdot a^3}=\dfrac{a^2}{a^{\frac{3}{2}+\frac{6}{2}}}=\dfrac{a^2}{a^{\frac{9}{2}}}=a^{\frac{4}{2}-\frac{9}{2}}=a^{-\frac{5}{2}}$

29. Pojednostavi:

$k^{-\frac{3}{2}}\cdot k^2\cdot k^{\frac{1}{4}}=k^{-\frac{3}{2}+\frac{4}{2}+\frac{1}{4}}=k^{\frac{3}{4}}$

$\left(t^{\frac{1}{2}}\right)^3\div\left(t^{\frac{5}{4}}\right)^2=\dfrac{\left(t^{\frac{1}{2}}\right)^3}{\left(t^{\frac{5}{4}}\right)^2}=\dfrac{t^{3\cdot\frac{1}{2}}}{t^{2\cdot\frac{5}{4}}}=t^{\frac{3}{2}-\frac{5}{2}}=t^{-\frac{2}{2}}=t^{-1}=\dfrac{1}{t}$

$\left(m^5\div m^{1\frac{1}{2}}\right)^{-1}\cdot m^{-\frac{5}{2}}=\dfrac{1}{\dfrac{m^5}{m^{\frac{3}{2}}}}\cdot\dfrac{1}{m^{\frac{5}{2}}}=\dfrac{m^{\frac{3}{2}}}{m^5m^{\frac{3}{2}}}=\dfrac{m^{\frac{3}{2}}}{m^{\frac{15}{2}}}=m^{-\frac{12}{2}}=m^{-6}=\dfrac{1}{m^6}$

30. Radijus kugle dat je izrazom $r=\left(\dfrac{3V}{4\pi}\right)^{\frac{1}{3}}$

a) Izrazi radijus u obliku korjena

b) Izracunaj radijus ako je volumen V=7500 cm^3

a) $r=\left(\dfrac{3V}{4\pi}\right)^{\frac{1}{3}}=\sqrt[3]{\dfrac{3V}{4\pi}}$ *b*) $r=\sqrt[3]{\dfrac{3V}{4\pi}}=\sqrt[3]{\dfrac{3\cdot 7500}{4\pi}}=12.143\ cm$

31. U fizici je poznata jednadzba za plinove u obliku $\left(\sqrt[n-1]{\frac{T_1}{T_2}}\right)^n$

a) Prikazi izraz u eksponencijalnom obliku

b) Izracunaj izraz za $T_1 = 400, T_2 = 300$ i $n = 1.4$

a) $\left(\sqrt[n-1]{\frac{T_1}{T_2}}\right)^n = \left(\frac{T_1}{T_2}\right)^{\frac{n}{n-1}}$ *b*) $\left(\frac{400}{300}\right)^{\frac{1.4}{1.4-1}} = \left(\frac{4}{3}\right)^{3.5} = 2.735$

32. Otpor elektricnog sklopa dan je izrazom $R = \frac{2K^{-\frac{2}{3}} l^{-\frac{1}{3}}}{3D}$

a) Prikazi otpor sa pozitivnim eksponentima

b) Izracunaj R za $D = 5,\ K = 0.008$ *i* $l = 0.125$

c) Za koliko se povecao otpor ako je novo stanje dano sa:

$D = 8,\ K = 0.009$ *i* $l = 0.625$

a) $R = \frac{2K^{-\frac{2}{3}} l^{-\frac{1}{3}}}{3D} = \frac{2}{3D \cdot K^{\frac{2}{3}} \cdot l^{\frac{1}{3}}}$

b) $R_b = \frac{2}{3D \cdot K^{\frac{2}{3}} \cdot l^{\frac{1}{3}}} = \frac{2}{3 \cdot 5 \cdot (0.008)^{\frac{2}{3}} (0.125)^{\frac{1}{3}}} = 6.667\ \Omega$

c) $R_c = \frac{2}{3D \cdot K^{\frac{2}{3}} \cdot l^{\frac{1}{3}}} = \frac{2}{3 \cdot 8 \cdot (0.009)^{\frac{2}{3}} (0.625)^{\frac{1}{3}}} = 2.941\ \Omega$

Razlika u otporu iznosi:

$\Delta R = R_1 - R_2 = 6.667 - 2.267 = 4.4\ \Omega$

33. $3\sqrt{5} - 2\sqrt{3} + 6\sqrt{5} + 4\sqrt{3} = 3\sqrt{5} + 6\sqrt{5} - 2\sqrt{3} + 4\sqrt{3} = 9\sqrt{5} + 2\sqrt{3}$

34. $\sqrt{11} + 3\sqrt{2} - 4\sqrt{11} + \sqrt{2} = 4\sqrt{2} - 3\sqrt{11}$

35. $\left(2\sqrt{3} - \sqrt{5}\right)^2 = \left[\left(2\sqrt{3}\right)^2 + 2\left(2\sqrt{3}\right)\left(\sqrt{5}\right) - \left(\sqrt{5}\right)^2\right] = 4 \cdot 3 + 4\sqrt{15} - 5 = 7 + 4\sqrt{15}$

36. $\left(10\sqrt{6} + 5\sqrt{7}\right)^2 = \left[\left(10\sqrt{6}\right)^2 + 2\left(10\sqrt{6}\right)\left(5\sqrt{7}\right) - \left(5\sqrt{7}\right)^2\right] =$

$= 100 \cdot 6 + 100\sqrt{42} + 25 \cdot 7 = 755 + 100\sqrt{42}$

37. $\left(5\sqrt{10}-2\sqrt{6}\right)\left(3\sqrt{10}-3\sqrt{6}\right)=15\sqrt{10}-15\sqrt{10}\sqrt{6}-6\sqrt{6}\sqrt{10}+6\sqrt{6}\sqrt{6}=$
$=15\cdot 10-15\sqrt{60}-6\sqrt{60}+6\cdot 6=150-21\sqrt{60}+36=186-21\sqrt{4\cdot 15}=$
$=186-42\sqrt{15}$

38. $\left(\sqrt{5}+\sqrt{7}\right)\left(\sqrt{5}-\sqrt{7}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{7}\right)^2=5-7=-2$

39. Vrijeme trajanja jednog punog zamaha klatna (njihala), dano je izrazom $\pi\sqrt{\frac{10l}{2500}}$, gdje je *l*-duzina klatna (cm). Ako je duzina klatna 169 cm, izracunaj vrijeme trajanja jednog njihaja.

$t=\pi\sqrt{\frac{10l}{2500}}=\pi\sqrt{\frac{10\cdot 169}{2500}}=\pi\sqrt{0.0676}=2.53s$

40. Sat ima klatno duzine 81 cm. Za koje vrijeme ucini puni zamah?

$t=\pi\sqrt{\frac{10l}{2500}}=\pi\sqrt{\frac{10\cdot 81}{2500}}=\pi\sqrt{0.324}=1.788s$

41. Klatno za rusenje zgrada, sa celicnom kuglom na kraju, ima duzinu 16m. Izracunaj vrijeme trajanja punog zamaha.

$t=\pi\sqrt{\frac{10l}{2500}}=\pi\sqrt{\frac{10\cdot(16\cdot 100)}{2500}}=\pi\sqrt{6.4}=7.948s$

42. Racionaliziranje nazivnika

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}\cdot\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{\left(\sqrt{5}+\sqrt{3}\right)\sqrt{2}}{2}=\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)$$

43. $\frac{3\sqrt{2}-\sqrt{3}}{2\sqrt{6}}=\frac{3\sqrt{2}-\sqrt{3}}{2\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{3\sqrt{12}-\sqrt{18}}{2\cdot 6}=\frac{1}{12}\left(3\sqrt{4\cdot 3}-\sqrt{2\cdot 9}\right)=$
$=\frac{1}{12}\left(6\sqrt{3}-3\sqrt{2}\right)=\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{4}$

44. $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}}=\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\cdot\left(\sqrt{5}\right)^2+\sqrt{3\cdot 5}}{5}=\frac{15+\sqrt{15}}{5}$

45. $$\frac{5\sqrt{3}+4}{\sqrt{3}+\sqrt{2}} = \frac{5\sqrt{3}-4}{\sqrt{3}-\sqrt{2}} \cdot \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) = \frac{\left(5\sqrt{3}+4\right)\left(\sqrt{3}-\sqrt{2}\right)}{3-2} = 15-5\sqrt{6}+4\sqrt{3}-4\sqrt{2}$$

46. $$\frac{\sqrt{3}-5}{\sqrt{3}+2} = \frac{\sqrt{3}-5}{\sqrt{3}+2} \cdot \left(\frac{\sqrt{3}-2}{\sqrt{3}-2}\right) = \frac{3-2\sqrt{3}-5\sqrt{3}+10}{3-4} = \frac{13-7\sqrt{3}}{-1} = 7\sqrt{3}-13$$

47. $$\frac{2\sqrt{2}+\sqrt{7}}{\sqrt{2}-\sqrt{7}} = \frac{2\sqrt{2}+\sqrt{7}}{\sqrt{2}-\sqrt{7}} \cdot \left(\frac{\sqrt{2}+\sqrt{7}}{\sqrt{2}+\sqrt{7}}\right) = \frac{2\cdot 2+2\sqrt{14}+\sqrt{14}+7}{2-7} =$$
$$= \frac{4+3\sqrt{14}+7}{-5} = \frac{11+3\sqrt{14}}{-5}$$