STRIKE NTSE

CBSE Test Paper 01

Chapter 05 Arithmetic Progression

1. In an AP, if a = 4, n = 7 and a_n = 4, then the value of 'd' is **(1)**
   a. 0
   b. 1
   c. 3
   d. 2
2. The next two terms of the AP : k, 2k + 1, 3k + 2, 4k + 3, ... are **(1)**
   a. 5k + 4 and 6k + 5
   b. 4k + 4 and 4k + 5
   c. 5k + 5 and 6k + 6
   d. 5k and 6k
3. The common difference of the A.P. can be **(1)**
   a. only negative
   b. only zero
   c. positive, negative or zero
   d. only positive
4. The 7th term from the end of the A.P. – 11, – 8, – 5, ..., 49 is **(1)**
   a. 28
   b. 31
   c. -11
   d. -8
5. The common difference of the A.P whose $a_n = -3n + 7$ is **(1)**
   a. 3
   b. 1
   c. -3
   d. 2

6. If 5 times the 5th term of an AP is equal to 10 times the 10th term, show that its 15th term is zero. **(1)**

7. Write the first term a and the common difference d of A.P. -1.1, - 3.1, -5.1, - 7.1,... **(1)**

8. For what value of n are the n^{th} term of the following two AP's are same 13, 19, 25, …. and 69, 68, 67 …. **(1)**

9. Find k, if the given value of x is the k^{th} term of the given AP $5\frac{1}{2}$, 11, $16\frac{1}{2}$, 22, …, x = 550. **(1)**

10. Find the 6^{th} term from the end of the A.P. 17,14,11,…, - 40 **(1)**

11. How many terms of the AP 17,15,13,11,… must be added to get the sum 72? **(2)**

12. Find n. Given a = first term = -18.9, d = common difference = 2.5, a_n = the nth term = 3.6, n = ? **(2)**

13. Find the number of terms in each of the following APs. 18, $15\frac{1}{2}$, 13, ….., – 47. **(2)**

14. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find the common difference and the number of terms. **(3)**

15. The 14th term of an A.P. is twice its 8th term. If the 6th term is -8, then find the sum of its first 20 terms. **(3)**

16. Find the 6th term from end of the AP 17, 14, 11, …, –40. **(3)**

17. The houses of a row in a colony are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x. **(3)**

18. The sum of first n terms of an A.P. is $3n^2 + 4n$. Find the 25^{th} term of this A.P. **(4)**

19. If sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of the first 10 terms. **(4)**

20. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP. **(4)**

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Answers

1. a. 0

   Explanation: Given: a = 4, n = 7 and a_n = 4, then

   $a_n = a + (n-1)d$

   $\Rightarrow 4 = 4 + (7-1)d$

   ⇒4 - 4 = 6d

   $\Rightarrow 6d = 0$

   $\Rightarrow d = 0$

2. a. 5k + 4 and 6k + 5

   Explanation: Given: $k, 2k+1, 3k+2, 4k+3, \ldots\ldots\ldots\ldots\ldots$

   Here $d = 2k + 1 - k = k + 1$

   Therefore, the next two terms are

   $4k + 3 + k + 1 = 5k + 4$ and $5k + 4 + k + 1 = 6k + 5$

3. c. positive, negative or zero

   Explanation: The common difference of the A.P. can be positive, e.g. 1, 2, 3, 4 ..... d is +ve and series is increasing negative e.g 4, 3, 2, 1 ...... d is - ve and series is decreasing

   or zero also and the AP becomes constant e.g 4, 4, 4, 4 ....

4. b. 31

   Explanation: Reversing the given A.P., we have

   $49, 46, 43, \ldots\ldots\ldots\ldots\ldots, -11$

   Here, $a = 49, d = 46 - 49 = -3$ and $n = 7$

   $\therefore a_n = a + (n-1)d$

   $\Rightarrow a_7 = 49 + (7-1) \times (-3)$

   = $49 + 6 \times (-3)$

   $\Rightarrow a_7 = 49 - 18 = 31$

5. c. – 3

   Explanation: Given: $a_n = -3n + 7$

Putting $n = 1, 2, 3$, we get

$a = -3 \times 1 + 7 = -3 + 7 = 4$

$a_2 = -3 \times 2 + 7 = -6 + 7 = 1$

$a_3 = -3 \times 3 + 7 = -9 + 7 = -2$

$\therefore$ Common difference $(d) = a_2 - a = 1 - 4 = -3$

6. Let 1^{st} term = a and common difference = d.

   $a_5 = a + 4d$, $a_{10} = a + 9d$

   According to the question, $5 \times a_5 = 10 \times a_{10} \Rightarrow 5(a + 4d) = 10(a + 9d) \Rightarrow 5a + 20d = 10a + 90d \Rightarrow a = -14d$

   Now, $a_{15} = a + 14d \Rightarrow a_{15} = -14d + 14d = 0$.

7. -1.1, - 3.1, -5.1, - 7.1,...

   First term (a) = -1.1

   We know that common difference is difference between any two consecutive terms of an A.P.

   So, common difference(d) = (-3.1) - (-1.1)

   = -3.1 + 1.1

   = -2

8. n^{th} term of 13, 19, 25, ... = nth term of 69, 68, 67, ...

   13 + (n - 1) 6 = 69 + (n - 1) (-1)

   13 + 6n - 6 = 69 - n + 1

   n + 6n = 70 - 7

   7n = 63

   n = 9

   Therefore, n = 9

9. $a = 5\frac{1}{2} = \frac{11}{2}$, $d = a_2 - a_1 = 11 - \frac{11}{2} = \frac{11}{2}$ and x = 550

   A.T.Q., $a_k = x$

   $\Rightarrow a + (k-1)d = 550$

   $\Rightarrow \frac{11}{2} + (k-1)\frac{11}{2} = 550$

   $\Rightarrow \frac{11}{2} + \frac{11}{2}k - \frac{11}{2} = 550$

   $\Rightarrow \frac{11}{2}k = 550$

$\Rightarrow$ k = $\frac{550\times2}{11} = 100$

10. A.P. is 17,14,11,..., - 40

    We have,

    l = Last term = -40 , a = 17 and, d = Common difference= 14 - 17 = - 3

    $\therefore$ 6th term from the end = l - (n -1)d

    = l - (6-1) d

    = -40 - 5 $\times$ (-3)

    = -40 + 15

    = -25

    So, 6th term of given A.P. is -25.

11. Given A.P. is 17, 15, 13, 11........

    Here, 1st term (a) = 17 and common difference (d) = (15 - 17) = -2

    Let the sum of n terms be 72. Then,

    S_n = 72

    $\Rightarrow$ $\frac{n}{2}$ · {2a + (n - 1)d} = 72

    $\Rightarrow$ n· {2 × 17 + (n - 1)(-2)} = 144

    $\Rightarrow$ n(36 - 2n)=144

    $\Rightarrow$2n^2- 36n + 144 = 0

    $\Rightarrow$ n^2 - 18n + 72 = 0

    $\Rightarrow$$n^2$ - 12n - 6n + 72 = 0

    $\Rightarrow$ n(n - 12) - 6(n - 12) = 0

    $\Rightarrow$(n - 12)(n - 6) = 0

    $\Rightarrow$ n = 6 or n = 12.

    $\therefore$ sum of first 6 terms = sum of first 12 terms = 72.

    This means that the sum of all terms from 7th to 12th is zero.

12. a_n = a + (n - 1)d

    $\Rightarrow$ 3.6 = - 18.9 + (n - 1) (2.5)

    $\Rightarrow$ 3.6 + 18.9 = (n - 1) (2.5)

    $\Rightarrow$ 22.5 = (n - 1) (2.5)

    $\Rightarrow n-1=\frac{22.5}{2.5}$

    $\Rightarrow$ n - 1 = 9

$\Rightarrow$ n = 10

13. $18, 15\frac{1}{2}, 13, \ldots, -47$

Here, a = 18

$d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = -\frac{5}{2}$

a_n = -47

Let the number of terms be n.

Then,

a_n = -47

$\Rightarrow$ a + (n - 1)d = -47

$\Rightarrow 18 + (n-1)\left(-\frac{5}{2}\right) = -47$

$\Rightarrow -\frac{5}{2}(n-1) = -47 - 18$

$\Rightarrow -\frac{5}{2}(n-1) = -65$

$\Rightarrow \frac{5}{2}(n-1) = 65$

$\Rightarrow n - 1 = \frac{65 \times 2}{5}$

$\Rightarrow$ n - 1 = 26

$\Rightarrow$ n = 26 + 1

$\Rightarrow$ n = 27

Hence, the number of terms of the given AP is 27.

14. Let the given AP contains n terms.

First term, a = 5

Last term, l = 45

S_n = 400

$\Rightarrow \frac{n}{2}$ [a + l] = 400

$\Rightarrow \frac{n}{2}$ [5 + 45] = 400

$\Rightarrow$n $\times$ 50 = 800

$\Rightarrow$n = 16

Thus, the given AP contains 16 terms.

Let d be the common difference of the given AP.

then,

T_{16} = 45

$\Rightarrow$a + 15d = 45

$\Rightarrow 5 + 15d = 45$

$\Rightarrow 15d = 40$

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$.

Therefore, common difference of the given AP is $\frac{8}{3}$.

15. Let first term be a and common difference be d.

Here, $a_{14} = 2a_8$

a + 13d = 2(a + 7d)

a + 13d = 2a + 14d

a = - d...(i)

$a_6 = -8$

a + 5d = -8 ... (ii)

Putting the value of a from (i) in (ii), we get

-d + 5d = -8

4d = -8

d = -2

Put d = -2 in (i)

a = -(-2)

a = 2

So ,a = 2, d = - 2

$S_{20} = \frac{20}{2}[2 \times 2 + (20-1)(-2)]$

$= 10[4 + 19 \times (-2)]$

= 10(4 - 38)

= 10 $\times$ (-34)

= - 340. Which is the required sum of first 20 terms.

16. The given AP is 17, 14, 11, ..., – 40

Here, a = 17

d = 14 - 17 = -3

l = -40

Let there be n terms between in the given AP

Then, nth term = -40

$\Rightarrow$ a + (n - 1)d = -40 $\because a_n = a + (n-1)d$

$\Rightarrow$ 17 + (n - 1) (-3) = -40

$\Rightarrow$ (n - 1) (-3) = -40 - 17

$\Rightarrow$ (n - 1) (-3) = -57

$\Rightarrow n-1=\frac{-57}{-3}$

$\Rightarrow$ n - 1= 19

$\Rightarrow$ n = 19 + 1

$\Rightarrow$ n = 20

Hence, there are 20 terms in the given AP.

Now, 6th term from the end

= (20 - 6 + 1)th term from the beginning

= 15th term from the beginning

= a + (15 - 1)d $\because a_n=a+(n-1)d$

= 17 + 14 (-3)

= 17 - 42

= - 25

Hence, the 6th term from the end of the given AP is -25.

17. According to the question, we have to find the value of x.

We are given an AP, namely 1,2,3,..., (x -1), x, (x +1),..., 49

such that 1 + 2 + 3 +... + (x -1) = (x +1) + (x + 2) +... + 49.

Thus, we have S_{x-1} = S_{49}- S_x ... (i)

Using the formula, $S_n=\frac{n}{2}$ (a + l) in (i), we have,

$\frac{(x-1)}{2}\cdot\{1+(x-1)\}=\frac{49}{2}\cdot(1+49)-\frac{x}{2}\cdot(1+x)$

$\Rightarrow\frac{x(x-1)}{2}+\frac{x(x+1)}{2}=1225$

$\Rightarrow 2x^2$ = 2450 $\Rightarrow$ x^2 = 1225 $\Rightarrow$ x = $\sqrt{1225}$ = 35

Hence, x = 35.

18. According to the question,

Sum of n terms of the A.P. $S_n = 3n^2 + 4n$

$S_1 = 3 \times 1^2 + 4 \times 1 = 7 = t_1$ ....(i)

$S_2 = 3 \times 2^2 + 4 \times 2 = 20 = t_1 + t_2$ ....(i)

$S_3 = 3 \times 3^2 + 4 \times 3 = 39 = t_1 + t_2 + t_3$ .... (iii)

From (i), (ii), (iii)

$t_1 = 7$, $t_2 = 13$, $t_3 = 19$

Common difference, d = 13 - 7 = 6

25^{th} of the term of this A.P., t_{25} = 7 + (25 - 1)6

= 7 + 144 = 151

$\therefore$ The 25^{th} term of the A.P. is 151.

19. Consider the A.P. whose first term and common difference are ‘a’ and ‘d’ respectively.

If sum of first 6 terms of an A.P. is 36.

$S_6 = 36$

$\therefore \frac{6}{2}$ [2a + (6 – 1)d] = 36 $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow$ 3[2a + 5d] = 36

$\Rightarrow$ 2a + 5d = $\frac{36}{3}$

$\Rightarrow$ 2a + 5d = 12 ...(i)

If sum of first 16 terms is 256,

So, $S_{16} = 256$

$\Rightarrow \frac{16}{2}$ [2a + (16 – 1)d] = 256

$\Rightarrow$ 8[2a + 15d] = 256

$\Rightarrow$ 2a + 15d = $\frac{256}{8}$

$\Rightarrow$ 2a + 15d = 32 ...(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{l} 2a + 15d = 32 \quad ...(ii) \\ 2a + \ \ 5d = 12 \quad [\text{From (i)}] \\ - \quad - \qquad - \\ \hline \qquad 10d = 20 \end{array}$$

$$\begin{array}{l} 2a + 15d = 32 \\ 2a + \ \ 5d = 12 \\ - \quad - \qquad - \\ \hline \qquad 10d = 20 \end{array}$$

$\Rightarrow$ d = 2

Now, 2a + 5d = 12 [From (i)]

$\Rightarrow$ 2a + 5(2) = 12

$\Rightarrow$ 2a + 10 = 12

$\Rightarrow$ 2a = 12 – 10

$\Rightarrow$ a = $\frac{2}{2}$

$\Rightarrow$ a = 1

Hence, a = 1 and d = 2

So, S_{10} = $\frac{10}{2}$ [2a + (10 – 1)d]

= 5[2(1) + 9(2)]

= 5[2 + 18]

= 5[20]

= 100

$\Rightarrow$ S_{10} = 100

Hence, the sum of first 10 terms is 100.

20. Let the first term and the common difference of the AP be a and d respectively.

According to the question,

Third term + seventh term = 6

$\Rightarrow$ [a + (3 - 1)d] + [a + (7 - 1)d] = 6 = a + (n - 1)d

$\Rightarrow$ (a + 2d) + (a + 6d) = 6 $\Rightarrow$ 2a + 8d = 6

$\Rightarrow$ a + 4d = 3 ..... (1)

Dividing throughout by 2 &

(third term) (seventh term) = 8

$\Rightarrow$ (a + 2d) (a + 6d) = 8

$\Rightarrow$ (a + 4d - 2d) (a + 4d + 2d) = 8

$\Rightarrow$ (3 - 2d) (3 + 2d) = 8

$\Rightarrow$ 9 - 4d^2 = 8

$\Rightarrow 4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d + \pm\frac{1}{2}$

Case I, when $d = \frac{1}{2}$

Then from (1), $a + 4\left(\frac{1}{2}\right) = 3$

$\Rightarrow$ a + 2 = 3 $\Rightarrow$ a = 3 - 2 $\Rightarrow$ a = 1

$\therefore$ Sum of first sixteen terms of the AP = S_{16}

$= \frac{16}{2}[2a + (16 - 1)d] \because S_n = \frac{n}{2}[2a + (n - 1)d]$

= 8[2a + 15d]

$= 8[2(1) + 15(\frac{1}{2})]$

$= 8[12 + \frac{15}{2}]$

$= 8[\frac{19}{2}]$

$= 4 \times 19 = 76$

Case II. When $d = -\frac{1}{2}$

Then from (1),

$a + 4\left(-\frac{1}{2}\right) = 3$

$\Rightarrow$ a - 2 = 3 $\Rightarrow$ a = 3 + 2 $\Rightarrow$ a = 5

$\therefore$ Sum of first sixteen terms of the AP = S_{16}

$= \frac{16}{2}[2a + (16-1)d] \because S_n = \frac{n}{2}[2a + (n-1)d]$

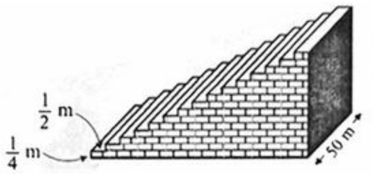
$= 8[2a + 15d] = 8\left[2(5) + 15\left(-\frac{1}{2}\right)\right] = 8\left[10 - \frac{15}{2}\right] = 8\left[\frac{5}{2}\right] = 20$

CBSE Test Paper 02

Chapter 05 Arithmetic Progression

1. Find the sum of first‘n’ terms of an A.P.? **(1)**
   a. 7n – 8
   b. S = $\frac{n}{2}$[2a + (n - 1)d]
   c. 2n + 3
   d. $n^2 + 2$
2. The sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$ is **(1)**
   a. 680
   b. 672
   c. 640
   d. 600
3. The 17th term of an AP exceeds its 10th term by 7, then the common difference is **(1)**
   a. -1
   b. 1
   c. 2
   d. 0
4. The first term of an AP is 5, the last term is 45 and the sum is 400. The number of terms is **(1)**
   a. 16
   b. 20
   c. 17
   d. 18
5. If a, b and c are in A. P., then the value of $\frac{a-b}{b-c}$ is **(1)**
   a. $\frac{a}{b}$
   b. 1
   c. $\frac{c}{a}$
   d. $\frac{b}{c}$
6. Find the common difference of the A.P. and write the next two terms of A.P. 119,136,153,170,..... **(1)**

7. lf 2x, x + 10, 3x + 2 are in A.P., find the value of x. **(1)**
8. Find 7^{th} term from the end of the AP : 7, 10 , 13,....,184. **(1)**
9. Find k, if the given value of x is the k^{th} term of the given AP 25,50, 75,100,..., x = 1000. **(1)**
10. Find the 11th term from the end of the AP 10,7,4,....,-62. **(1)**
11. Find the common difference d and write three more terms. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ **(2)**
12. The house of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the following it. Find this value of x. **(2)**
13. Divide 24 in three parts such that they are in AP and their product is 440. **(2)**
14. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8th term, we get 6. **(3)**
15. Find the second term and n^{th} term of an A.P. whose 6^{th} term is 12 and the 8^{th} term is 22. **(3)**
16. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace. **(3)**



17. If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term. **(3)**
18. If the sum of Rs 1890 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 50 less than its preceding prize. Then find the value of each of the prizes. **(4)**
19. Let a sequence be defined by $a_1 = 1$, $a_2 = 1$ and, $a_n = a_{n-1} + a_{n-2}$ for all n > 2. Find $\frac{a_{n+1}}{a_n}$ for n = 1,2,3,4. **(4)**
20. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find. n and S_n, if a = 5, d = 3 and a_n = 50. **(4)**

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Solution

1. b. S = $\frac{n}{2}$[2a + (n - 1)d]

   Explanation: let a = 1^{st} term, d = common difference,

   S_n = sum of 1st n terms of an AP

   then S_n = (a) + (a + d) + (a + 2d) + ...{a + (n - 3) d} + { a + (n - 2)d} + {a + (n - 1)d} .... (i)

   Now Rewrite S_n as follows

   S_n = {a +(n-1)d} + {a +(n-2)d }+ {a + (n - 3)d} ....(a + 3d) + (a + 2d) + (a + d) + a .... (ii)

   adding the terms i and ii vertically

   adding 1st term of both we get (a) + {a + (n-1)d} = 2a + (n-1)d

   adding 2nd term of both (a + d) + {a + (n - 2)d} = 2a + (n-1)d

   adding 3rd terms of both (a + 2d) + {a + (n-3)d} = 2a + (n-1)d

   since there are n terms in each of the equations i and ii , adding both the equations we get

   $2S_n$ = n{2a + (n-1)d}

   S_n = $\frac{n}{2}$ {2a + (n-1)d}.

2. b. 672

   Explanation: Given: $a_n = 3 + 2n$

   $\therefore a_1 = 3 + 2 \times 1 = 3 + 2 = 5$

   $a_2 = 3 + 2 \times 2 = 3 + 4 = 7$

   $\therefore d = a_2 - a_1 = 7 - 5 = 2$

   Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

   $\Rightarrow S_{24} = \frac{24}{2}[2 \times 5 + (24-1)2]$

   $\Rightarrow S_{24} = 12[10 + 23 \times 2]$ = 12 [10 + 46] = 672

3. b. 1

   Explanation: According to question,

   Given that the 17th term of an A.P exceeds its 10th term by 7 .

d = ?

$\Rightarrow$ a + 16d = a + 9d + 7

$\Rightarrow$16d - 9d = 7

$\Rightarrow$7d = 7

$\Rightarrow$ d = $\frac{7}{7}$ = 1

Therefore , common difference = 1.

4. a. 16

Explanation: Given: $a = 5, l = 45, \mathrm{S}_n = 400$

$\therefore \mathrm{S}_n = \frac{n}{2}(a + l)$

$\Rightarrow 400 = \frac{n}{2}(5 + 45)$

$\Rightarrow 800 = n \times 50$

$\Rightarrow n = 16$

5. b. 1

Explanation: If a, b and c are in A.P.,

$b - a = c - b$

$-(a - b) = -(b - c)$

$a - b = b - c$

dividing both sides by $b - c$

$\frac{a-b}{b-c}$ = $\frac{b-c}{b-c}$

$\frac{a-b}{b-c}$ =1

6. Given A.P is

119, 136, 153, 170........

We know that common differnce is difference between any consecutive terms of an A.P.

So, common difference = 136 - 119 = 17

5th term = 170 + 17 = 187 (a_5 = a + 4d)

6th term = 187 + 17 = 204. (a_6 = a + 5d)

7. lf 2x, x + 10, 3x + 2 are in A.P.,we have to find the value of x.

Since, 2x, x + 10, 3x + 2 are in A.P.therefore 2 (x + 10) = 2x + 3x + 2

$\Rightarrow$ 2x + 20 = 5x + 2

$\Rightarrow$ 3x = 18 $\Rightarrow$ x = 6

8. Given, AP is 7, 10 , 13,....,184.

   we have to find 7th term from the end

   reversing the AP , 184,.....,13,10,7.

   now, d = common difference = 7-10 = -3

   $\therefore$ 7^{th} term from the beginning of AP =$a+(7-1)d = a+6d$

   $= 184 + (6 \times (-3))$

   = 184 - 18 = 166

9. $a = 25,\ d = 50 - 25 = 25,\ \ x = 1000$

   A.T.Q. , $a_k = x$

   $\Rightarrow$ a+$(k-1)d = 1000$

   $\Rightarrow$ 25 + (k - 1)25 = 1000

   $\Rightarrow$ (k - 1)25 = 975 $\Rightarrow$ k - 1 = $\frac{975}{25}$

   $\Rightarrow$ k - 1 = 39 $\Rightarrow$ k = 40

10. We have

    a = 10, d = (7-10) = -3, l = -62 and n = 11.

    $\therefore$11th term from the end = [l - (n -1) $\times$ d]

    = {-62 - (11 -1) $\times$ (-3)}

    = (-62 + 30) = -32.

    Hence, the 11th term from the end of the given AP is -32.

11. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

    $a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

    $a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

    $a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

    i.e. a_{k+1} - a_k is the same every time.

    So, the given list of numbers forms an AP with the common difference d = $\sqrt{2}$.

    The next three terms are:

    $\sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

    $5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

    and $6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

12. The consecutive numbers on the houses of a row are 1, 2, 3, ..., 49

    Clearly this list of number forming an AP.

Here, a = 1

d = 2 - 1 = 1

$S_{x-1} = S_{49} - S_x$

$\Rightarrow \frac{x-1}{2}[2a+(x-1-1)d] - \frac{x}{2}[2a+(x-1)d]$

$\because S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \frac{x-1}{2}[2(1)+(x-2)(1)] = \frac{49}{2}[2(1)+(48)(1)] - \frac{x}{2}[2(1)+(x-1)(1)]$

$\Rightarrow \frac{x-1}{2}[x] = 1225 - \frac{x(x+1)}{2}$

$\Rightarrow \frac{(x-1)(x)}{2} + \frac{x(x+1)}{2} = 1225$

$\Rightarrow \frac{x}{2}(x-1+x+1) = 1225$

$\Rightarrow x^2 = 1225$

$\Rightarrow x = \sqrt{1225} \Rightarrow x = 35$

Hence, the required value of x is 35.

13. Let the required numbers in A.P.are (a - d), a and (a + d).

Sum of these numbers = (a - d) + a + (a + d) = 3a

Product of these numbers = $(a - d)\times a\times(a + d)=a(a^2 - d^2)$

But given, sum = 24 and product = 440

$\therefore$ 3a = 24 $\Rightarrow$ a = 8

and $a(a^2 - d^2) = 8(64 - d^2) = 440$ [$\because$ a = 8]

Or, $64 - d^2 = 55$

Or, $d^2 = 64 - 55$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

When a = 8 and d = 3

The required numbers are (5, 8, 11).

When a = 8 and d = -3

The required numbers are (11, 8, 5).

14. Let the first term be a and the common difference be d.

$a_n = a + (n - 1)d$

Here given, $a_3 = 9$

or, a + 2d = 9 ....(i)

a_8 - a_5 = 6

or, (a + 7d) - (a + 4d) = 6

a + 7d - a - 4d = 6

or, 3d = 6

or, d = 2 ....(ii)

Substituting this value of d from (ii) in (i), we get

or, a + 2(2) = 9

or, a + 4 = 9

or a = 9 - 4

or, a = 5

a = 5 and d = 2

So, A.P. is 5,7,9,11,....

15. Given a_6 = 12

$\Rightarrow$ a + (6 - 1)d = 12

$\Rightarrow$ a + 5d = 12 ....(i)

and, a_8 = 22

$\Rightarrow$ a + (8 - 1)d = 22

$\Rightarrow$ a + 7d = 22 ....(ii)

Subtracting equation (i) from (ii), we get

(a + 7d) - (a + 5d) = 22 - 12

$\Rightarrow$ a + 7d - a - 5d = 10

$\Rightarrow$ 2d - 10

$\Rightarrow d = \frac{10}{2} = 5$

Using value of d in equation (i), we get

a + 5 $\times$ 5 = 12

$\Rightarrow$ a = 12 - 25 = -13

Second term(a_2) = a + (2 - 1)d

= -13 + 1(5)

= -13 + 5 = -8

nth term(a_n) = a + (n - 1)a

= -13 + (n - 1)(5)

= 5n - 18

16. Volume of concrete required to build the first step, second step, third step, ... (in m^2) are

$\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50$

$\Rightarrow \frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \ldots\ldots$

$\therefore$ Total volume of concrete required = $\frac{50}{8} + 2 \times \frac{50}{8} + 3 \times \frac{50}{8} + \ldots.$

$= \frac{50}{8}[1 + 2 + 3 + \ldots\ldots.]$

$S_n = \frac{n}{2}$ [(2a + (n - 1)d]

$S_{15} = \frac{50}{8} \times \frac{15}{2}[2 \times 1 + (15 - 1) \times 1]\ [\because n = 15]$

$= \frac{50}{8} \times \frac{15}{2} \times 16$ = 750 m^3

17. We have,

$a_9 = 0$

$\Rightarrow$ a + (9 - 1)d = 0

$\Rightarrow$ a + 8d = 0

$\Rightarrow$ a = -8d

To prove: $a_{29} = 2a_{19}$

Proof: LHS = a_{29}

= a + (29 - 1)d

= a + 28d

= -8d + 28d = 20d

RHS = $2a_{19}$

= 2 a + (19 - 1)d]

= 2[-8d + 18d]

= 2 $\times$10d

= 20d

$\therefore$ LHS = RHS

Hence, 29^{th} term is double the 19^{th} term.

18. Let 1^{st} prize be Rs x.

The series in A.P. is x, x - 50, x - 100, x - 150,........

Where a = x, d = - 50, S_n = 1890, n = 7.

As we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow \frac{7}{2}[2x + (6)(-50)] = 1890$
$\Rightarrow \frac{7}{2}[2x - 300] = 1890$
$\Rightarrow 2x - 300 = 1890(2/7)$
$\Rightarrow 2x = 540 + 300$
$\Rightarrow x = \frac{840}{2} = 420$
The prizes are: Rs 420, Rs 370, Rs 320, Rs 270, Rs 220, Rs 170, Rs 120.

19. Given a_1 =1 and a_2 = 1 and $a_n = a_{n-1} + a_{n-2}$

So $a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

Now putting n = 1,2,3 and 4 in a_{n+1}/a_n we get
$\frac{a_2}{a_1} = \frac{1}{1} = 1$
$\frac{a_3}{a_2} = \frac{2}{1} = 2$
$\frac{a_4}{a_3} = \frac{3}{2} = 1.5$
$\frac{a_5}{a_4} = \frac{5}{3} = 1.67$

20. Given,
First term(a) = 5
Common difference(d) = 3
and, nth term (a_n) = 50

$\Rightarrow a + (n - 1)d = 50$

$\Rightarrow 5 + (n - 1)(3) = 50$

$\Rightarrow 5 + 3n - 3 = 50$

$\Rightarrow 3n = 50 - 5 + 3$

$\Rightarrow 3n = 48$

$\Rightarrow \quad n = \frac{48}{3} = 16$
Therefore, $S_n = \frac{n}{2}[a + a_n]$

$= \frac{16}{2}[5 + 50]$
$= 8 \times 55 = 440$

CBSE Test Paper 03

Chapter 5 Arithmetic Progression

1. 200 logs are stacked in a such a way that 20 logs in the bottom row, 19 logs in the next row, 18 logs in the row next to it and so on. The total number of rows is **(1)**
   a. 16
   b. 12
   c. 15
   d. 10
2. Sum of n terms of the series, $\sqrt{2}+\ \sqrt{8}+\ \sqrt{18}+\sqrt{32}+\ldots\ldots\ldots\ldots$is **(1)**
   a. $\frac{n(n-1)}{2}$
   b. $\frac{n(n+1)}{\sqrt{2}}$
   c. $\frac{n(n+1)}{2}$
   d. $\frac{n(n-1)}{\sqrt{2}}$
3. A sum of Rs.700 is to be used to award 7 prizes. If each prize is Rs.20 less than its preceding prize, then the value of the first prize is **(1)**
   a. Rs.160
   b. Rs.100
   c. Rs.180
   d. Rs.200
4. The sum of odd numbers between 0 and 50 is **(1)**
   a. 625
   b. 600
   c. 500
   d. 2500
5. The common difference of an A.P. in which $a_{18}-a_{14}=32$ is **(1)**
   a. – 8
   b. 6
   c. 8
   d. – 6

6. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$. **(1)**

7. For an AP, if $a_{18} - a_{14} = 32$ then find the common difference d. **(1)**
8. Find the first four terms of an A.P. whose first term is - 2 and common difference is - 2. **(1)**
9. If the sum of n terms of an A.P. is $2n^2 + 5n$, then find the 4th term. **(1)**
10. What is the common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} \ldots$? **(1)**
11. In the following AP's find the missing terms: 2, __, 26. **(2)**
12. Is 68 a term of the AP : 7, 10 , 13 ,....? **(2)**
13. Find 51 is a term of given A.P.or not where the A.P. is 5,8,11,14, ... **(2)**
14. In the following situation, does the list of numbers involved make an arithmetic progression, and why? The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum. **(3)**
15. If 7^{th} term of an A.P. is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term. **(3)**
16. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149. **(3)**
17. In an A.P. the first term is 8, n^{th} term is 33 and the sum to first n terms is 123. Find n and the common difference(d). **(3)**
18. Each year, a tree grows 5 cm less than it did the preceding year. If it grew by 1 m in the first year, then in how many years will it have ceased growing? **(4)**
19. If the sum of the first m terms of an AP be n and the sum of its first n terms be m then show that the sum of its first (m + n) terms is -(m + n). **(4)**
20. Find the sum of all integers between 100 and 550, which are divisible by 9. **(4)**

Solution

1. a. 16

   Explanation: The number of logs in the row from bottom to the top are 20, 19, 18, ... which form an AP with first term 20 and common difference
   $19 - 20 = -1.$
   Let the 200 logs be arranged in n rows.
   Then $\mathrm{S}_n = 200$
   $\Rightarrow 200 = \frac{n}{2}[2a + (n-1)d]$
   $\Rightarrow 400 = n[2 \times 20 + (n-1)(-1)]$
   $\Rightarrow n^2 - 41n + 400 = 0$
   $\Rightarrow n^2 - 16n - 25n + 400 = 0$
   $\Rightarrow n(n-16) - 25(n-16) = 0$
   $\Rightarrow (n-25)(n-16) = 0$
   $\Rightarrow n - 25 = 0$ or $n - 16 = 0$
   $\Rightarrow n = 25$ or $n = 16$
   $n = 25$ is not poaaible as on calculating number of logs in 25th row, there is negative number of logs, which is not possible.
   Therefore, number of rows are 16.

2. b. $\frac{n(n+1)}{\sqrt{2}}$

   Explanation: Given: $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots\ldots\ldots\ldots$
   $\Rightarrow \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots\ldots\ldots\ldots\ldots\ldots$
   Here, $a = \sqrt{2}, d = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$
   $\therefore \mathrm{S}_n = \frac{n}{2}[2a + (n-1)d]$
   $\Rightarrow \mathrm{S}_n = \frac{n}{2}\left[2 \times \sqrt{2} + (n-1) \times \sqrt{2}\right]$
   $\Rightarrow \mathrm{S}_n = \frac{n}{2}\left[2\sqrt{2} + n\sqrt{2} - \sqrt{2}\right]$
   $\Rightarrow \mathrm{S}_n = \frac{n}{2}\left[\sqrt{2} + n\sqrt{2}\right]$
   $\Rightarrow \mathrm{S}_n = \frac{n\sqrt{2}}{2}(1+n) = \frac{n(n+1)}{\sqrt{2}}$

3. a. Rs.160

Explanation: Let the first prize be a.

The seven prizes form an AP with first term a and common difference,

$d=-20$ n=7

Now the sum of all seven prizes = Rs. 700

$\therefore \mathrm{S}_n=700$

$\Rightarrow \frac{n}{2}[2a+(n-1)d]=700$

$\Rightarrow \frac{7}{2}[2a+(7-1)(-20)]=700$

$\Rightarrow 2a-120=200$

$\Rightarrow 2a=320$

$\Rightarrow a=160$

Therefore, the value of first prize is Rs. 160.

4. a. 625

Explanation: Odd numbers between 0 and 50 are 1, 3, 5, 7, ..., 49 Here

$a=1, d=3-1=2$ and

= $\frac{25}{2}\times 50$

S_{25} = 25×25

= 625

5. c. 8

Explanation: Given: $a_{18}-a_{14}=32$

$\Rightarrow a+(18-1)d-[a+(14-1)d]=32$

$\Rightarrow a+17d-a-13d=32$

$\Rightarrow 4d=32$

$\Rightarrow d=8$

6. Let a be the first term and d be the common difference of the given AP. Then,

S_n= $\frac{n}{2}\cdot$[2a+(n-l)d],

$\therefore 3(S_8-S_4)=3[\frac{8}{2}(2a+7d)-\frac{4}{2}(2a+3d)]$

= $3[4(2a+7d)-2(2a+3d)]=6(2a+11d)$

$=\frac{12}{2}\cdot(2a+11d)=S_{12}$.

Hence, S_{12}= $3(S_8 - S_4)$.

7. We know, n^{th} term of an AP is given by $a_n=a+(n-1)d$, where a is the first term and d is the common difference

Given, $a_{18}-a_{14}=32$

$\Rightarrow (a+(18-1)d)-(a+(14-1)d)=32$
$\Rightarrow (a+17d)-(a+13d)=32$
$\Rightarrow 4d=32$
$\Rightarrow d=8$

8. given a_1= -2, common difference d= -2

   a_1 = -2 ,

   $a_2 = a_1 + d = -2 + (-2) = -4$

   $a_3 = a_2 + d = -4 + (-2) = -6$

   $a_4 = a_3 + d = =-6 + (-2) = -8$

   ∴ First four terms are - 2, - 4, - 6, - 8

9. Let the sum of n terms of A.P. = S_n.

   Given, $S_n = 2n^2 + 5n$

   Now, n^{th} term of A.P, = $S_n - S_{n-1}$

   or, $a_n = (2n^2 + 5n) - [2(n - 1)^2 + 5 (n - 1)]$

   $a_n= (2n^2 + 5n) - [2(n^2 -2n + 1) + 5 (n - 1)]$

   $a_n = 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$

   $a_n = 2n^2 + 5n - [2n^2+ n - 3]$

   $a_n = 2n^2 + 5n - 2n^2 - n + 3$

   $a_n = 4n + 3$

   4^{th} term a_4= $4\times 4+3=16+3=19$

10. Common difference(d) = $a_2 - a_1$ = $\frac{(1-p-1)}{p}=\frac{-p}{p}=-1$

    therefore, d = -1

11. 2, __, 26
    We know that the difference between consecutive terms is equal in any A.P.
    Let the missing term be x .
    x - 2 = 26 - x ⇒ 2x = 28 ⇒ x = 14
    Therefore, missing term is 14.

12. Given AP 7,10,13...
    Here, first term $a=7$ and common difference $d=a_2-a_1=10-7=3$,

Assume that 68 is n^{th} term of given AP

We know that n^{th} term is given by $a_n = a + (n-1)d$

$\Rightarrow 68 = 7 + (n-1) \times 3$

$\Rightarrow 61 = (n-1) \times 3$

$\Rightarrow \frac{61}{3} + 1 = n$

$\Rightarrow n = \frac{64}{3} = 31\frac{1}{3}$

Since, n cannot be fraction, but a whole number. Therefore, 68 is not a term of given AP.

13. Given, A.P. is 5,8,11,14, ...

Here a = 5 and d = (8 - 5) = 3.

Let the nth term of the given AP be 51. Then,

$T_n = 51$

We know that $T_n = a + (n - 1)d$.

$\Rightarrow$ a + (n -1)d = 51

$\Rightarrow$ 5 + (n- 1) $\times$ 3 = 51 [Because, a = 5 and d = 3]

$\Rightarrow$5 + 3n - 3 = 51

$\Rightarrow$ 3n = 51 - 2

$\Rightarrow$ 3n = 49

$\Rightarrow$ n = $16\frac{1}{3}$.

But, the number of terms cannot be a fraction.

Therefore, 51 is not a term of the given Arithmetic progression.

14. Amount of money after 1 year $= Rs10000\left(1 + \frac{8}{100}\right) = a_1$

Amount of money after 2 year $= Rs10000\left(1 + \frac{8}{100}\right)^2 = a_2$

Amount of money after 3 year $= Rs10000\left(1 + \frac{8}{100}\right)^3 = a_3$

Amount of money after 4 year $= Rs10000\left(1 + \frac{8}{100}\right)^4 = a_4$

$a_2 - a_1 = Rs10000\left(1 + \frac{8}{100}\right)^2 - Rs10000\left(1 + \frac{8}{100}\right)$

$= Rs10000\left(1 + \frac{8}{100}\right)\left(1 + \frac{8}{100} - 1\right)$

$= 10000(1 + \frac{8}{100})(\frac{8}{100})$

$a_3 - a_2$

$= 10000\left(1+\frac{8}{100}\right)^2 - 10000\left(1+\frac{8}{100}\right)$

$= 10000\left(1+\frac{8}{100}\right)\left(1+\frac{8}{100}-1\right)$

$= 10000\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$

Since.$a_3 - a_2 \neq a_2 - a_1$.It does not form AP.

15. Let the first term be a and the common difference be d.

$t_n = a + (n-1)d$

Given, $t_7 = \frac{1}{9}$

$t_9 = \frac{1}{7}$

$a + 6d = \frac{1}{9}$ .....(i)

and $a + 8d = \frac{1}{7}$ ....(ii)

On subtracting eqn.(i) from (ii)

$a + 8d - a - 6d = \frac{1}{7} - \frac{1}{9}$

or, $2d = \frac{2}{63}$

or, $d = \frac{1}{63}$

Substituting the value of d in (ii) we get

$a + 8 \times \frac{1}{63} = \frac{1}{7}$

or, a = $\frac{1}{7} - \frac{8}{63}$

or, a = $\frac{9-8}{63} = \frac{1}{63}$

$t_{63} = a + 62d$

$\therefore \quad t_{63} = \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$

or, $t_{63} = \frac{63}{63} = 1$

Hence, t_{63}= 1.

16. Here, d = 7

a_{22} = 149

Let the first term of the AP be a.

We know that a_n = a + (n - 1)d

$\Rightarrow a_{22}$ = a + (22 - 1)d

$\Rightarrow a_{22}$ = a + 21d

$\Rightarrow$ 149 = a + (21) (7)

$\Rightarrow$ 149 = a + 147 $\Rightarrow$ a = 2

Again, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_{22} = \frac{22}{2}[2(2) + (22-1)7]$

$\Rightarrow$ S_{22} = (11) [4 + 147]

$\Rightarrow$ S_{22} = (11) (151) $\Rightarrow$ S_{22} = 1661

Hence, the sum of the first 22 terms of the AP is 1661.

17. Given First term (a) = 8

and, n^{th} term (a_n) = 33

$\Rightarrow$ a + (n - 1)d = 33

$\Rightarrow$ 8 + (n - 1)d = 33

$\Rightarrow$ (n - 1)d = 33 - 8

$\Rightarrow$ (n - 1)d = 25 .....(i)

and, Sum of first n terms = 123
$\Rightarrow \frac{n}{2}[a + a_n] = 123$
$\Rightarrow \frac{n}{2}[8 + 33] = 123$
$\Rightarrow \frac{n}{2} \times 41 = 123$
$\Rightarrow n = \frac{123 \times 2}{41}$ $\Rightarrow$ n = 6
Put value of n in equation (i)

(6 - 1)d = 25

$\Rightarrow$ 5d = 25

$\Rightarrow d = \frac{25}{5} = 5$

18. Given that , tree grows 5 cm less than preceding year, and grew by 1m (100 cm) in the first year.
means, 95cm in the 2nd year, 90 in the 3rd year , 85 in the fourth year and so on.
growth in the year in which it will stop growing will be 0cm
Therefore, The following sequence can be formed.
i.e, $100, 95, 90, \dots, 0$ which is an AP.
Here, $a = 100, d = 95 - 100 = -5$ and $l = 0$
Let $l = a_n = a + (n-1)d$
Then, $0 = 100 + (n-1)(-5)$
$0 = 100 - 5n + 5$
$0 = 105 - 5n$
$5n = 105 \Rightarrow n = 21$

Hence, Tree will ceased growing in 21 years.

19. Let a be the first term and d be the common difference of the given AP. Then,

S_m = n $\Rightarrow \frac{m}{2}$ [2a + (m-1)d] = n

$\Rightarrow$ 2am + m(m- 1)d - 2n ...... (i)

And, S_n = m $\Rightarrow \frac{n}{2}$ [2a + (n - 1)d] = m

$\Rightarrow$ 2an + n(n - 1)d = 2m ...... (ii)

On subtracting (ii) from (i), we get

2a(m-n) + [(m^2 - n^2) - (m - n)]d = 2(n - m)

$\Rightarrow$ (m - n)[2a + (m + n - 1)d] = 2(n - m)

$\Rightarrow$ 2a + (m + n- 1)d = -2 ..... (iii)

Sum of the first (m + n) terms of the given AP

= $\frac{(m+n)}{2}$ · {2a + (m + n - 1)d}

$= \frac{(m+n)}{2} \cdot (-2) = -(m+n)$ [using (iii)].

Hence, the sum of first (m + n) terms of the given AP is -(m + n).

20. According to the question,

All integers between 100 and 550, which are divisible by 9

= 108, 117, 126,.........., 549

First term (a) = 108

Common difference(d) = 117 - 108 = 9

Last term(a_n) = 549

$\Rightarrow$ a + (n - 1)d = 549

$\Rightarrow$ 108 + (n - 1)(9) = 549

$\Rightarrow$ 108 + 9n - 9 = 549

$\Rightarrow$ 9n = 549 + 9 - 108

$\Rightarrow$ 9n = 450

$\Rightarrow \quad n = \frac{450}{9} = 50$

Sum of 50 terms = $\frac{n}{2}[a + a_n]$

$= \frac{50}{2}[108 + 549]$

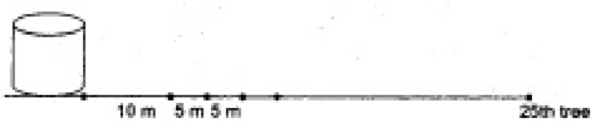
$= 25 \times 657$ = 16425

CBSE Test Paper 04
Chapter 5 Arithmetic Progression

1. Which term of the A.P. 121, 117, 113, ... is its first negative term? **(1)**
   a. 32
   b. 33
   c. 30
   d. 31
2. The A.P. whose third term is 16 and the difference of 5th term from the 7th term is 12, then the A.P. is **(1)**
   a. 4, 11, 18, 25, ...
   b. 4, 14, 24, 34, ...
   c. 4, 10, 16, 22, ...
   d. 4, 6, 8, 10, ...
3. If $a_1 = 4 \; and \; a_n = 4a_{n-1} + 3, \; n > 1,$ then the value of a_4 is **(1)**
   a. 320
   b. 329
   c. 319
   d. 300
4. In an A.P. it is given that a = 5, d = 3 and a_n = 50, then the value of ‘n’ is **(1)**
   a. 16
   b. 18
   c. 20
   d. 15
5. If the second term of an AP is 13 and its fifth term is 25, then its 7th term is **(1)**
   a. 37
   b. 33
   c. 39
   d. 35
6. In the A.P. 2, x, 26 find the value of x. **(1)**
7. Find 9^{th} term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$ **(1)**
8. If they form an AP, find the common difference d and write three more terms. 0.2,

0.22, 0.222, 0.2222, ... **(1)**

9. If sum of first n terms of an AP is $2n^2 + 5n$. Then find S_{20}. **(1)**
10. Find the sum of each of the following APs: 0.6, 1.7, 2.8,... to 100 terms. **(1)**
11. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. **(2)**
12. In a certain A.P. 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term. **(2)**
13. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P. **(2)**
14. Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $a_n = 7 - 3n$. **(3)**
15. The 12th term of an AP is -13 and the sum of its first four terms is 24. Find the sum of its first 10 terms. **(3)**
16. Prove that the 11^{th} term of an A.P. cannot be $n^2 + 1$. Justify your answer. **(3)**
17. There are 25 trees at equal distances of 5m in a line with a water tank, the distance of the water tank from the nearest tree being 10m. A gardener waters all the trees separately, starting from the water tank and returning back to the water tank after watering each tree to get water for the next. Find the total distance covered by the gardener in order to water all the trees. **(3)**



18. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find n and a_n, if a = 2, d = 8 and $S_n = 90$. **(4)**
19. Find the sum of all natural numbers between 200 and 300 which are divisible by 4. **(4)**
20. In a garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row. How many rows are there of rose plants? Also find the total number of rose plants in the garden. **(4)**

Solution

1. a. 32

Explanation: Here $a = 121, d = 117 - 121 = -4$

For the first negative term, $a_n < 0$

$\Rightarrow 121 + (n-1)(-4) < 0$

$\Rightarrow 121 - 4n + 4 < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow n > \frac{125}{4}$

$\Rightarrow n > 31\frac{1}{4}$

Therefore 32nd term is the first negative term.

2. c. 4, 10, 16, 22, ...

Explanation: Given: $a_3 = 16 \Rightarrow a + 2d = 16$ ....(i)

And $a_7 - a_5 = 12 \Rightarrow a + 6d - a - 4d = 12$

$\Rightarrow 2d = 12 \Rightarrow d = 6$

Putting value of d in eq. (i), we get

$a + 2 \times 6 = 16$

$\Rightarrow a = 4$

Therefore, A.P. is 4, 10, 16, 22, ...

3. c. 319

Explanation: Given: $a_1 = 4$ and $a_n = 4a_{n-1} + 3,\ n > 1,$

$\therefore a_2 = 4a_{2-1} + 3 = 4a_1 + 3 = 4 \times 4 + 3 = 16 + 3 = 19$

and$a_3 = 4a_{3-1} + 3 = 4a_2 + 3 = 4 \times 19 + 3 = 76 + 3 = 79$

and $a_4 = 4a_{4-1} + 3 = 4a_3 + 3 = 4 \times 79 + 3 = 316 + 3 = 319$

4. a. 16

Explanation: Given: a = 5, d = 3 and a_n = 50

$\therefore a_n = a + (n-1)d$

$\Rightarrow 50 = 5 + (n-1) \times 3$

$\Rightarrow 45 = (n-1) \times 3$

$\Rightarrow \frac{45}{3}$ = n-1

$\Rightarrow n - 1 = 15$

$\Rightarrow n = 16$

5. b. 33

Explanation: Given: $a_2 = 13$

$\Rightarrow a + (2 - 1)\,d = 13$

$\Rightarrow a + d = 13$ ....(i)

And $a_5 = 25$

$\Rightarrow a + (5 - 1)\,d = 25$

$\Rightarrow a + 4d = 25$ ....(ii)

Solving eq. (i) and (ii),

we get $a = 9$ and $d = 4$

$\therefore a_7 = a + (7 - 1)\,d$

= $9 + (7 - 1) \times 4$

= 9 + 6 $\times$ 4

= 9 + 24 = 33

6. $2, x$ and 26 are in A.P.

$\therefore x - 2 = 26 - x$

or, $x + x = 26 + 2$

or, $2x = 28$

or, $x = \frac{28}{2}$ = 14

7. Given, A.P $= \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$

First term(a) = $\frac{3}{4}$

Common difference (d) = $\frac{5}{4} - \frac{3}{4} = \frac{2}{4}$

As we know,

$a_n = a + (n - 1)d$

$\Rightarrow a_9 = \frac{3}{4} + (9 - 1) \times \frac{2}{4}$

$= \frac{3}{4} + 8 \times \frac{2}{4}$

$= \frac{3}{4} + \frac{16}{4} = \frac{19}{4}$

8. 0.2, 0.22, 0.222, 0.2222, 000

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

As $a_2 - a_1 \neq a_3 - a_2$, the given list of numbers does not form an AP.

9. $S_n = 2n^2 + 5n$
   $S_{20} = 2(20)^2 + 5(20)$
   $= 2(400) + 100 = 900.$

10. Here, $a = 0.6,\ d = 1.7 - 0.6 = 1.1\ and\ n = 100$
    Now we know that, $S_n = \frac{n}{2}[2a + (n - 1)d]$
    Therefore, $S_n = \frac{100}{2}[2 \times 0.6 + (100 - 1)(1.1)]$
    $= 50[1.2 + 108.9]$
    $= 50 \times 110.1$
    $= 5505$

11. We have,
    $a_4 = 0$
    $a + 3d = 0$
    $3d = -a$
    or $-3d = a$..........(i)
    Now,
    $a_{25} = a + 24d$
    $= -3d + 24d$ [Putting value of a from eq(i)]
    $= 21d$...........(ii)
    $a_{11} = a + 10d$
    $= -3d + 10d$
    $= 7d$.........(iii)
    From eq(ii) and (iii), we get
    $a_{25} = 21d$
    $a_{25} = 3(7d)$
    $a_{25} = 3a_{11}$
    Hence Proved

12. Let the 1st term be a and common difference be d.

$a_n = a + (n-1)d$

According to the question, $a_{32} = 2a_{12}$

a +31d = 2 (a + 11d)

a + 31d = 2a + 22d

a - 2a = 22d - 31d

a = 9d

a_{70} = a + 69d = 9d + 69d = 78d

a_{31} = a + 30d = 9d + 30d = 39d

a_{70} = 78d

a_{70} = 2(39d)

$a_{70} = 2a_{31}$

$a_{70} = 2a_{31}$ **Hence Proved**.

13. Let a be the first term and d be the common difference of the given A.P. Then,

$\therefore . S_6 = 42$

$\frac{6}{2}(2a + 5d) = 42$

$2a + 5d = 14 \ldots\ldots\ldots(i)$

It is given that

$a_{10} : a_{30}$ = 1: 3

$\Rightarrow \quad \frac{a+9d}{a+29d} = \frac{1}{3}$

$3a + 27d = a + 29d$

$2a = 2d$

$a = d \ldots\ldots\ldots (ii)$

From (i) and(ii) we get

a = d = 2

Hence $a_{13} = 2 + 12 \times 2 = 26$ and $a_1 = 2$

14. Given, $a_n = 7 - 3n$

Put n = 1, $a_1 = 7 - 3 \times 1 = 7 - 3 = 4$

Put n = 2, $a_2 = 7 - 3 \times 2 = 7 - 6 = 1$

Common difference(d) = 1 - 4 = -3

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{25} = \frac{25}{2}[2 \times 4+(25-1)(-3)]$

$= \frac{25}{2}[8-72]$

$= \frac{25}{2} \times -64$

= -800

15. 12^{th} term = T_{12} = -13

$\Rightarrow$a + 11d = -13...(i)

S_4 = 24

$\Rightarrow \frac{4}{2}$ [2a + 3d] = 24

$\Rightarrow$2[2a + 3d] = 24

$\Rightarrow$2a + 3d = 12...(ii)

Multiplying equation (i) by 2, we get

2a + 22d = -26...(iii)

Subtracting (ii) from (i), we get

19d = -38

$\Rightarrow$d = -2

$\Rightarrow$a + 11(-2) = -13...[from (i)]

$\Rightarrow$a = 9

$\therefore$ Sum fo first 10 terms, $S_{10}=\frac{10}{2}$ [2(9) + 9(-2)] = 5[18 - 18] = 5 $\times$ 0 = 0

16. Let n^{th} term of A.P.

$a_n = n^2+1$

Putting the value of n = 1, 2 ,3 ,.....we get

$a_1 = 1^2 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

$a_3 = 3^2 + 1 = 10$

The obtained sequence = $2, 5, 10, 17\ldots\ldots$

Its common difference

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

or, 5 - 2 $\neq$ 10 - 5 $\neq$ 17 - 10

$\therefore$ 3 $\neq$ 5 $\neq$ 7

Since the sequence has no. common difference

Hence, n^2+1 is not a form of n^{th} term of an A.P.

17. Assume that gardener is standing near the well initially, and he did not return to the well after watering the last tree.

distance covered by gardener to water 1^{st} tree and return to the initial position=10m + 10m = 20m.

Distance covered by gardener to water 2^{nd} tree and return to the initial position =15m + 15m =3 0m.

Distance covered by gardener to water 3^{rd} tree and return to the initial position =20m + 20m = 40m.

$\therefore$ Distance covered by the gardener to water the plants are in AP.

Here a = 20, d = 10

Total distance covered by the gardener is given by Sn, where n = 25.

$\Rightarrow$Sn = $\frac{25}{2}$[2(20) + (25 - 1)10] = 3500.

Thus, the total distance covered by the gardener is 3500m.

18. Given that, a = 2, d = 8 and S_n = 90.

As, $S_n = \frac{n}{2}[2a+(n-1)d]$

$90=\frac{n}{2}[4+(n-1)8]$

90 = n[2 + (n - 1)4]

90 = n[2 + 4n - 4]

90 = n(4n - 2) = $4n^2$ - 2n

$4n^2$ - 2n - 90 = 0

$4n^2$ - 20n + 18n - 90= 0

4n(n - 5) + 18(n - 5) = 0

(n - 5)(4n + 18) = 0

Either n = 5 or $n=-\frac{18}{4}=\frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, n = 5

a_n = a + (n - 1)d

a_5 = 2 + (5 - 1)8

= 2 + 4(8)

= 2 + 32 = 34

19. All the numbers between 200 and 300 which are divisible by 4 are 204, 208, 212, 216, ..., 296

Here, a_1 = 204

a_2 = 208

a_3 = 212

a_4 = 216

$\therefore$ a_2 - a_1 = 208 - 204 = 4

a_3 - a_2 = 212 - 208 = 4

a_4 - a_3 = 216 - 212 = 4

$\because$ a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... (= 4 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, a = 204

d = 4

l = 296

Let the number of terms be n. Then,

l = a + (n - 1)d

$\Rightarrow$ 296 = 204 + (n - 1)4

$\Rightarrow$ 92 = (n - 1)4

$\Rightarrow$ n - 1 = 23

$\Rightarrow$ n = 23 + 1

$\Rightarrow$ n = 24

$\therefore S_n = \frac{n}{2}(a + l)$

$= \left(\frac{24}{2}\right)(204 + 296)$

=(12) (500)

= 6000.

Hence, the sum of all the natural numbers between 200 and 300 which are divisible by 4 is 6000.

20. The number of rose plants in the 1^{st}, 2^{nd}, ... are 23,21,19,...........5

$a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

$\therefore a_n = a + (n - 1)d$

or, $5 = 23 + (n - 1)(-2)$

or, $5 = 23 - 2n + 2$

or, $5 = 25 - 2n$

or, $2n = 20$

or, $n = 10$

Total number of rose plants in the flower bed,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$$
$$= 5[46 - 20 + 2]$$

$S_{10} = 5(46 - 18)$

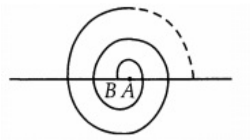
$= 5(28)$

$S_{10} = 140$

CBSE Test Paper 05
Chapter 5 Arithmetic Progression

1. The sum of the first 15 multiples of 8 is **(1)**
   a. 900
   b. 960
   c. 1000
   d. 870
2. If the angles of a right angled triangle are in A.P. then the angles of that triangle will be **(1)**
   a. $45^\circ,\ 45^\circ,\ 90^\circ$
   b. $30^\circ,\ 60^\circ,\ 90^\circ$
   c. $40^\circ,\ 50^\circ,\ 90^\circ$
   d. $20^\circ,\ 70^\circ,\ 90^\circ$
3. In an A.P., if $S_n = 3n^2 + 2n$, then the value of 'a_n' is **(1)**
   a. 7n – 2
   b. 9n – 4
   c. 8n – 3
   d. 6n – 1
4. The sum of (a + b), (a – b), (a – 3b), ... to 22nd term is **(1)**
   a. 22a + 440b
   b. 22a – 440b
   c. 20a + 440b
   d. 22a – 400b
5. The first and last terms of an A.P. are 1 and 11. If their sum is 36, then the number of terms will be **(1)**
   a. 7
   b. 5
   c. 8
   d. 6
6. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots\ldots$ an A.P.? Give reason. **(1)**
7. The sum of three numbers in AP is 21 and their product is 231. Find the numbers. **(1)**

8. Find a and b such that the numbers a, 9, b, 25 form an AP. **(1)**
9. For an A.P., if $a_{25} - a_{20} = 45$, then find the value of d. **(1)**
10. Find the common difference of the AP : $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ **(1)**
11. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the A.P. **(2)**
12. Write the expression $a_n - a_k$ for the AP: a, a + d, a + 2d, ... and find the common difference of the AP for which 20^{th} term is 10 more than the 18^{th} term. **(2)**
13. The sum of the first three terms of an A.P. is 33. If the product of first and the third term exceeds the second term by 29, find the AP. **(2)**
14. If the m^{th} term of an AP be $\frac{1}{n}$ and its nth term be $\frac{1}{m}$, then show that its (mn)th term is 1. **(3)**
15. Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7^{th} term is two more than thrice of its 3rd term. **(3)**
16. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is (2m - 1):(2n -1). **(3)**
17. A spiral is made up of successive semi-circles with centres alternately at A and B starting with A, of radii 1 cm, 2 cm, 3 cm,.... as shown in the figure. What is the total length of spiral made up of eleven consecutive semi-circles? **(3)**



18. In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{13}{2}n$. Find its 25^{th} term. **(4)**
19. Find the sum of all integers between 100 and 550 which are not divisible by 9. **(4)**
20. If the sum of the first n terms of an A.P. is $4n - n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly, find the third, the tenth and the nth terms. **(4)**

Solution

1. b. 960

   Explanation: Multiples of 8 are 8, 16, 24, ...

   Here $a = 8, d = 16 - 8 = 8$ and $n = 15$

   Now, $\mathrm{S}_n = \frac{n}{2}[2a + (n-1)d]$

   $\Rightarrow \mathrm{S}_{15} = \frac{15}{2}[2 \times 8 + (15-1)8]$

   $\Rightarrow \mathrm{S}_{15} = \frac{15}{2}[16 + 14 \times 8]$

   = $\frac{15}{2} \times 128$

   = 15 $\times$ 64

   = 960

2. b. $30^\circ,\ 60^\circ,\ 90^\circ$

   Explanation: Let the three angles of a triangle be $a - d, a$ and $a + d$.

   $\therefore a - d + a + a + d = 180^\circ$

   $\Rightarrow 3a = 180^\circ$

   $\Rightarrow a = 60^\circ$

   Therefore, one angle is of 60° and other is 90° (given).

   Let third angle be x°, then

   $60^\circ + 90^\circ + x^\circ = 180^\circ$

   $\Rightarrow 150^\circ + x^\circ = 180^\circ$

   $\Rightarrow x^\circ = 180^\circ - 150^\circ = 30^\circ$

   Therefore, the angles of the right angled triangle are $30^\circ, 60^\circ, 90^\circ$.

3. d. 6n – 1

   Explanation: Given: $S_n = 3n^2 + 2n$

   $\mathrm{S}_1 = 3(1)^2 + 2 \times 1 = 3 + 2 = 5$

   $\Rightarrow a = 5$

   S_2 = $3(2)^2$ + 2 $\times$ 2

   S_2 = 3 $\times$ 4 + 4

   S_2 = 12+4

$S_2 = 16$

$\Rightarrow a_1 + a_2 = 16$

$\Rightarrow a_1 = 5$

$\Rightarrow a_2 = 11$

$\therefore d = a_2 - a_1 = 11 - 5 = 6$

$\therefore a_n = a + (n-1)\,d$

$= 5 + (n-1)\,6 = 5 + 6n - 6 = 6n - 1$

4. b. 22a – 440b

Explanation: Given: $a = a + b, d = a - b - a - b = -2b$

$\therefore S_{22} = \frac{22}{2}\left[2\,(a+b) + (22-1)\,(-2b)\right]$

$= 11\left[2a + 2b + (21)\,(-2b)\right]$

$\Rightarrow S_{22} = 11\left[2a + 2b - 42b\right]$

$= 11\left[2a - 40b\right]$

$= 22a - 440b$

5. d. 6

Explanation: Given: $a = 1, l = 11$ and $S_n = 36$

$\therefore S_n = \frac{n}{2}\,(a + l)$

$\Rightarrow 36 = \frac{n}{2}(1 + 11)$

$\Rightarrow 72 = n \times 12$

$\Rightarrow n = 6$

6. Common difference,

$d_1 = \sqrt{6} - \sqrt{3}$

$= \sqrt{3}(\sqrt{2} - 1)$

$d_2 = \sqrt{9} - \sqrt{6}$

$= \sqrt{3 \times 3} - \sqrt{2 \times 3}$

$= 3 - \sqrt{6}$

$d_3 = \sqrt{12} - \sqrt{9}$

$= \sqrt{4 \times 3} - \sqrt{9}$

$= 2\sqrt{3} - 3$

As common difference does not equal.

Hence, The given series is not in A.P.

7. Let the required numbers be (a-d), a and (a + d)............(1)

   Then, according to question, (a - d) + a + (a + d) = 21

   $\Rightarrow$ 3a = 21

   $\Rightarrow$ a = 7.

   And, (a - d)· a· (a + d) = 231 $\Rightarrow$ a (a^2 - d^2) = 231

   $\Rightarrow$7(49 - d^2) = 231 [$\because$a = 7]

   $\Rightarrow$7d^2 = 343 - 231 = 112

   $\Rightarrow$$d^2$ = 16

   $\Rightarrow$d = $\pm$ 4.

   Thus, a = 7 and d = $\pm$4. Now substitute these values of a and d in above equation (1).

   Therefore, the required numbers are(3,7,11) or (11,7,3).

8. The numbers $a, 9, b, 25$ form an AP,

   we have

   $9 - a = b - 9 = 25 - b.$

   Now, b - 9 = 25 - b $\Rightarrow$ 2b = 34 $\Rightarrow$ b = 17.

   And, 9 - a = b - 9 $\Rightarrow$ a + b = 18 $\Rightarrow$ a + 17 = 18 $\Rightarrow$ a = 1.

   Hence, $a = 1 \; and \; b = 17.$

9. Let the first term of an A.P be a and common difference d.

   $a_n = a + (n-1)d$

   $\therefore a_{25}$- a_{20} =$[a + (25-1)d] - [a + (20-1)d]$

   or, 45 = $a + 24d - a - 19d$

   or, 45 = $5d$

   or, $d = \frac{45}{5}$ = 9

10. Common difference(d) = $n^{th} term - (n-1)^{th} term$

    $\therefore d = a_2 - a_1$

    $d = (\frac{1-p}{p}) - (\frac{1}{p}) = \frac{(1-p)-(1)}{p} = \frac{-p}{p} = -1$

    $d = -1$

11. Let the middle most terms of the A.P. be $(a-d), a, (a+d)$

    Given $a - d + a + a + d = 225$

    $3a = 225$

or, $a = 75$

and the middle term = $\frac{37+1}{2}$ = 19th term

$\therefore$ A.P. is

(a - 18d),....(a - 2d), {a - d), a, (a + d), (a + 2d),..........$(a + 18d)$

Sum of last three terms

$(a + 18d) + (a + 17d) + (a + 16d) = 429$

or,$3a + 51d = 429$

or, $225 + 51d = 429$

or, d = 4

First term $a_1 = a - 18d = 75 - 18 \times 4 = 3$

$a_2 = 3 + 4 = 7$

Hence, A.P. = 3, 7, 11 ,.........., 147

12. $a_n = a + (n - 1)d$; $a_k = a + (k - 1)d$

Now, $a_n - a_k = [a + (n - 1)d] - [a + (k - 1)d] = (n - 1)d - (k - 1)d = (n - 1 - k + 1)d$

$a_n - a_k = (n - k)d$ ....(1)

Let $a_{18} = x$.

$a_{20} = x + 10$

Taking n = 20 and k = 18, equation (1) becomes

$a_{20} - a_{18} = (20 - 18)d \Rightarrow (x + 10) - x = 2d \Rightarrow d = 5$

13. Let the first three terms in A.P. be a - d, a, a + d. It is given that the sum of these terms is 33.

$\therefore$ a - d + a + a + d = 33

$\Rightarrow$ 3a = 33

$\Rightarrow$ a = 11

It is given that

$a_1 \times a_3 = a_2 + 29$

(a - d)(a + d) = a + 29

$a^2 - d^2 = a + 29$

$121 - d^2 = 11 + 29$

$d^2 = 121 - 40 = 81$

$d = \pm 9$

If d = 9 then the series is 2,11,20,29

If d = -9 then the series is 20,11,2,-7,-16

14. Let a be the first term and d be the common difference of the given AP. Now, we know that in general mth and nth terms of the given A.P can be written as

T_m = a + (m-l)d and T_n = a + (n-1)d respectively.

Now, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$ (given).

∴ a + (m-1)d = $\frac{1}{n}$ ....(i)

and a + (n-1)d = $\frac{1}{m}$ .... (ii)

On subtracting (ii) from (i), we get

(m-n)d = $(\frac{1}{n} - \frac{1}{m}) = (\frac{m-n}{mn}) \Rightarrow d = \frac{1}{mn}$

Putting d = $\frac{1}{mn}$ in (i), we get

a + $\frac{(m-1)}{mn} \Rightarrow a = \{\frac{1}{n} - \frac{(m-1)}{mn}\} = \frac{1}{mn}$

Thus, a= $\frac{1}{mn}$ and d= $\frac{1}{mn}$

∴Now, in general (mn)th term can be written as T_{mn} = a +(mn-1)d

= $\{\frac{1}{mn} + \frac{(mn-1)}{mn}\}$ [∵a= $\frac{1}{mn}$]

= 1.

Hence, the (mn)th term of the given AP is 1.

15. Here, we have the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7^{th} term is two more than thrice of its 3rd term.

Let "a" be the first term and" d"be the common difference of the given A.P. Therefore, $a_3 = 7$ and $a_7 = 3a_3 + 2$ [Given]

$\Rightarrow$a + 2d = 7 and a + 6d = 3(a + 2d) + 2

$\Rightarrow$ a + 2d = 7 and a + 6d = 3a + 6d + 2

$\Rightarrow$ a + 2d = 7 and a -3a = 6d - 6d + 2

$\Rightarrow$a + 2d = 7 and -2a = 2

$\Rightarrow$a + 2d = 7 and a = -1

$\Rightarrow$-1 + 2d = 7

$\Rightarrow$2d = 7 + 1 = 8

$\Rightarrow$d = 4

$\Rightarrow$ a = -1 and d = 4

Putting n = 20, a = - 1 and d = 4 in $S_n = \frac{n}{2}\{2a + (n-1)d\}$,we get

$S_{20} = \frac{20}{2}\{2 \times -1 + (20-1) \times 4\} = \frac{20}{2}(-2 + 76) = 740$

16. Let first term of given A.P. be a and common difference be d also sum of first m and first n terms be S_m and S_n respectively

$\therefore \quad \frac{S_m}{S_n} = \frac{m^2}{n^2}$

or, $\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2} \times \frac{n}{m}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

or, $m(2a + (n-1)d) = n[2a + (m-1)d]$

Now, $\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$

$= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$

or, $= \frac{a+2ma-2a}{a+2na-2a}$

or, $= \frac{2ma-a}{2na-a}$

or, $= \frac{a(2m-1)}{a(2n-1)}$

or, $= \frac{(2m-1)}{(2n-1)}$

= $2m - 1 : 2n - 1$

The ratio of its m^{th} and n^{th} terms is $2m - 1 : 2n - 1$.

Hence proved

17. Let r_1, r_2,.... be the radii of semicircles and l_1, l_2 .....be the lengths of circumferences of semi-circles, then

l_1 = $\pi r_1 = \pi(1) = \pi$cm

$l_2 = \pi r_2 = \pi(2) = 2\pi$cm

$l_3 = \pi r_3 = \pi(3) = 3\pi$cm

....

$l_{11} = \pi r_{11} = \pi(11) = 11\pi$cm

$\therefore$ Total length of spiral

= $l_1 + l_2 + \ldots + l_{11}$

= $\pi + 2\pi + 3\pi + \ldots + 11\pi$

$= \pi(1 + 2 + 3 + 4 .... + 11)$
$= \pi \times \frac{11 \times 12}{2}$
$= 66 \times 3.14$
$= 207.24cm$

18. According to the question,
Given Sum of n terms $(S_n) = \frac{3n^2}{2} + \frac{13}{2}n$
Put n = 24, $S_{24} = \frac{3 \times 24 \times 24}{2} + \frac{13 \times 24}{2}$
= 864 + 156
= 1020
Put n = 25, $S_{25} = \frac{3 \times 25 \times 25}{2} + \frac{13 \times 25}{2}$
$= \frac{1875}{2} + \frac{325}{2}$
$= \frac{2200}{2} = 1100$
$\therefore$ 25th term $(a_{25}) = S_{25} - S_{24}$
= 1100 - 1020
= 80

19. All integers between 100 and 550, which are divisible by 9
= 108, 117, 126,.........., 549
First term (a) = 108
Common difference(d) = 117 - 108 = 9
Last term(a_n) = 549
$\Rightarrow$a + (n - 1)d = 549
$\Rightarrow$108 + (n - 1)(9) = 549
$\Rightarrow$108 + 9n - 9 = 549
$\Rightarrow$9n = 549 + 9 - 108
$\Rightarrow$ 9n = 450
$\Rightarrow \quad n = \frac{450}{9} = 50$
Sum of 50 terms = $\frac{n}{2}[a + a_n]$
$= \frac{50}{2}[108 + 549]$
$= 25 \times 657$
= 16425
Now, sum of all integers between 100 and 550 which are not divisible by 9
= Sum of all integers between 100 and 550 - Sum of all integers between 100 and 550

which are divisible by 9

= [101 + 102 + 130 +......+ 549] - 16425

= $\frac{549 \times 550}{2} - \frac{100 \times 101}{2}$ - 16425

= 150975 - 5050 - 16425

= 129500

20. Given that,

$S_n = 4n - n^2$

First term, a = S1 = $4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms = S_2

= $4(2) - (2)^2 = 8 - 4 = 4$

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$

= 3 + (n - 1)(-2)

= 3 - 2n + 2

= 5 - 2n

Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1. 3^{rd}, 10^{th} and n^{th} terms are -1, -15, and 5 - 2n respectivey.