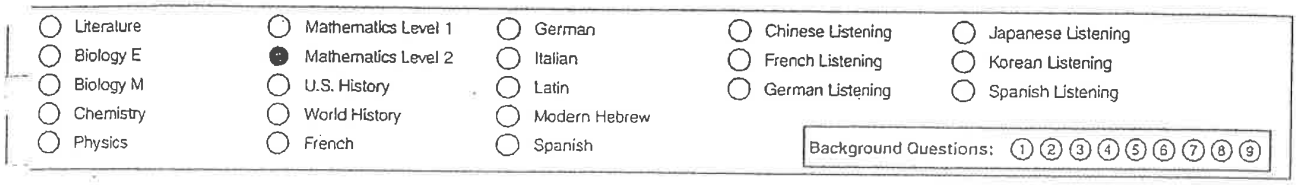
MATHEMATICS LEVEL 2 TEST

The top portion of the page of the answer sheet that you will use to take the Mathematics Level 2 Test must be filled in exactly as illustrated below. When your supervisor tells you to fill in the circle next to the name of the test you are about to take, mark your answer sheet as shown.



After filling in the circle next to the name of the test you are taking, locate the Background Questions section, which also appears at the top of your answer sheet (as shown above). This is where you will answer the following Background Questions on your answer sheet.

BACKGROUND QUESTIONS

Please answer Part I and Part II below by filling in the appropriate circle in the Background Questions box on your answer sheet. The information you provide is for statistical purposes only and will not affect your test score.

Part I. Which of the following describes a mathematics course you have taken or are currently taking? (FILL IN ALL CIRCLES THAT APPLY.)

- Algebra I or Elementary Algebra OR Course I of a college preparatory mathematics sequence —Fill in circle 1.
- Geometry OR Course II of a college preparatory mathematics sequence —Fill in circle 2.
- Algebra II or Intermediate Algebra OR Course III of a college preparatory mathematics sequence —Fill in circle 3.
- Elementary Functions (Precalculus) and/or Trigonometry OR beyond Course III of a college preparatory mathematics sequence —Fill in circle 4.
- Advanced Placement Mathematics (Calculus AB or Calculus BC) —Fill in circle 5.

Part II. What type of calculator did you bring to use for this test? (FILL IN THE ONE CIRCLE THAT APPLIES. If you did not bring a scientific or graphing calculator, do not fill in any of circles 6-9.)

- Scientific —Fill in circle 6.
- Graphing (Fill in the circle corresponding to the model you used.)

  Casio 9700, Casio 9750, Casio 9800, Casio 9850, Casio 9860, Casio FX 1.0, Casio CG-10, Sharp 9200, Sharp 9300, Sharp 9600, Sharp 9900, TI-82, TI-83, TI-83 Plus, TI-83 Plus Silver, TI-84 Plus, TI-84 Plus Silver, TI-85, TI-86, TI-Nspire, or TI-Nspire CX —Fill in circle 7.

  Casio 9970, Casio Algebra FX 2.0, HP 38G, HP 39 series, HP 40 series, HP 48 series, HP 49 series, HP 50 series, TI-89, TI-89 Titanium, TI-Nspire CAS, or TI-Nspire CX CAS —Fill in circle 8.

  Some other graphing calculator —Fill in circle 9.

When the supervisor gives the signal, turn the page and begin the Mathematics Level 2 Test. There are 100 numbered circles on the answer sheet and 50 questions in the Mathematics Level 2 Test. Therefore, use only circles 1 to 50 for recording your answers.

MATHEMATICS LEVEL 2 TEST

REFERENCE INFORMATION

THE FOLLOWING INFORMATION IS FOR YOUR REFERENCE IN ANSWERING SOME OF THE QUESTIONS IN THIS TEST.

Volume of a right circular cone with radius r and height h: $V = \frac{1}{3}\pi r^2 h$

Volume of a sphere with radius r: $V = \frac{4}{3}\pi r^3$

Volume of a pyramid with base area B and height h: $V = \frac{1}{3}Bh$

Surface area of a sphere with radius r: $S = 4\pi r^2$

DO NOT DETACH FROM BOOK.

GO ON TO THE NEXT PAGE

MATHEMATICS LEVEL 2 TEST

For each of the following problems, decide which is the BEST of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding circle on the answer sheet.

Notes: (1) A scientific or graphing calculator will be necessary for answering some (but not all) of the questions in this test. For each question you will have to decide whether or not you should use a calculator.

(2) For some questions in this test you may have to decide whether your calculator should be in the radian mode or the degree mode.

(3) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(4) Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. The range of f is assumed to be the set of all real numbers $f(x)$, where x is in the domain of f.

(5) Reference information that may be useful in answering the questions in this test can be found on the page preceding Question 1.

USE THIS SPACE FOR SCRATCH WORK.

1. What are the distinct zeros of the function defined by $f(x) = (x^2 - 4)(x + 2)$?

(A) -2 only
(B) 2 only
(C) -2 and 2 only
(D) -4, -2, and 2
(E) -4, 2, and 4

GO ON TO THE NEXT PAGE

USE THIS SPACE FOR SCRATCH WORK.

2. If $x > 0$ and $y < 0$, then $x + y$ can be which of the following?

I. Greater than 0
II. Equal to 0
III. Less than 0

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

3. If $n^3 = 12.346$, then $n^6 =$

(A) 24.69
(B) 152.42
(C) 156.25
(D) 4,857.69
(E) 11,290.94

4. In the xy-plane, if $(2, 3)$ is the midpoint of $\overline{MN}$ and N has coordinates $(8, 12)$, then M has coordinates

(A) $(4, 6)$
(B) $(4, -6)$
(C) $(-4, -6)$
(D) $(-4, 6)$
(E) $(-6, -9)$

GO ON TO THE NEXT PAGE

USE THIS SPACE FOR SCRATCH WORK.

If $f(x) = \dfrac{x-2}{(x+1)(x-3)}$, what is the domain of f?

(A) All real numbers except $x = 0$
(B) All real numbers except $x = 1$ and $x = -3$
(C) All real numbers except $x = -1$ and $x = 3$
(D) All real numbers except $x = -1$, $x = 3$, and $x = 2$
(E) All real numbers

If the points in the xy-plane are reflected about the line $y = 2x + 1$, which of the following points remains fixed?

(A) $(8, 17)$
(B) $(2, -3)$
(C) $(1, 0)$
(D) $(0, 0)$
(E) $(-5, 8)$

For which of the following values of x is $\cos x = \sin x$?

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{2}$

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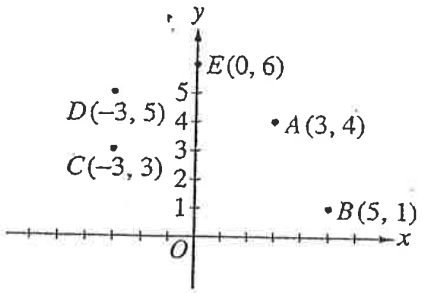
USE THIS SPACE FOR SCRATCH WOR

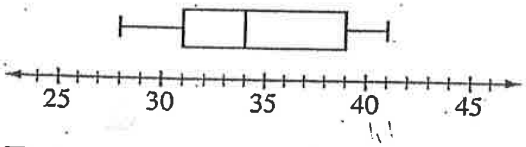
8. If $f(x) = x^2 - 4$, for what positive value of x is $f(x) = 3.2$?

(A) 0.89
(B) 1.10
(C) 1.79
(D) 2.68
(E) 6.24



9. Through which of the lettered points in the figure above will the curve $x^2 + y^2 = 25$ pass?

(A) A (B) B (C) C (D) D (E) E



10. The boxplot above represents the number of students from schools participating in a community service event. What is the range of this data set?

(A) 8 (B) 13 (C) 28 (D) 34 (E) 41

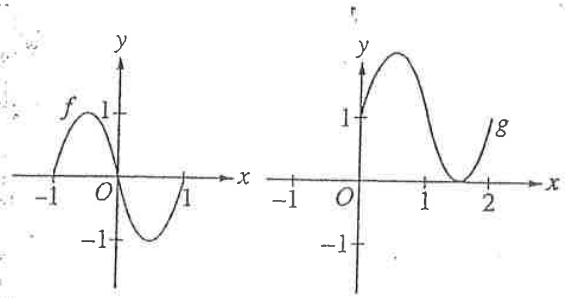
GO ON TO THE NEXT PAGE

MATHEMATICS LEVEL 2 TEST—*Continued*

USE THIS SPACE FOR SCRATCH WORK.

1. Let T be the temperature, in degrees Fahrenheit, of liquid in a certain pitcher, and let N be the number of ice cubes put into the pitcher. If $T = -\frac{9}{5}N + 77$, what is the least number of ice cubes that must be put into the pitcher to cool the liquid to a temperature of 32 degrees Fahrenheit?

(A) 23 (B) 24 (C) 25 (D) 26 (E) 27



12. In the figure above, the graphs of $y = f(x)$ and $y = g(x)$ are shown. Which of the following statements is true?

(A) $g(x) = f(x-1) + 1$
(B) $g(x) = f(x+1) + 1$
(C) $g(x) = f(x) + 1$
(D) $g(x) = f(x) + f(1)$
(E) $g(x) = f(x) + f(1) + 1$

GO ON TO THE NEXT PAGE

USE THIS SPACE FOR SCRATCH WORK.

13. All of the following expressions are equivalent to each other EXCEPT

(A) $-3(t-2)(t-4)$
(B) $-3(2-t)(4-t)$
(C) $3(t-2)(4-t)$
(D) $3(2-t)(t-4)$
(E) $3(t-2)(t-4)$

$$t = 17.4m^{\frac{1}{4}}$$

14. The equation above is a model for the time t, in seconds, required for the blood to circulate once completely through the body of a mammal of mass m kilograms. According to this model, how long does it take for the blood of a 5,200-kilogram elephant to circulate completely through its body?

(A) 0.3 minutes
(B) 2.5 minutes
(C) 5.0 minutes
(D) 20.9 minutes
(E) 6.3 hours

15. A number n is decreased by 4, and then the result is divided by 2. If the cube of the quotient is $-3\frac{3}{8}$, what is the value of n?

(A) $\frac{1}{2}$ (B) 1 (C) $3\frac{1}{3}$ (D) 5 (E) 7

GO ON TO THE NEXT PAGE

USE THIS SPACE FOR SCRATCH WORK.

6. If $(r + 2) + (s + 4)i = 8 + 6i$ for real numbers r and s, what is the value of r?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

x	$f(x)$
0	−1
1	0
2	1
3	2
4	3

x	$g(x)$
−1	1
0	4
1	3
2	0
3	2

7. The tables above show values for two functions f and g. What is the value of $g(f(3))$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

In the xy-plane, what is the greater value of the x-coordinates of the points at which the graphs of $y = -x^2 + 100$ and $y = 6$ intersect?

(A) 47 (B) 10.3 (C) 9.7 (D) −9.7 (E) −10.3

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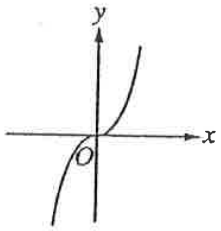
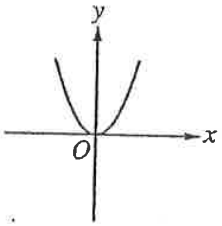
USE THIS SPACE FOR SCRATCH WORK.

19. Which of the following functions has an inverse function?

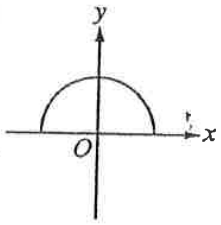
(A)



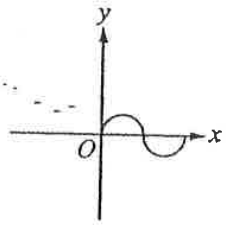
(B)



(C)



(D)



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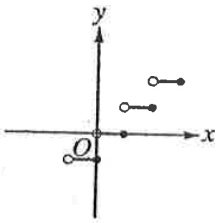

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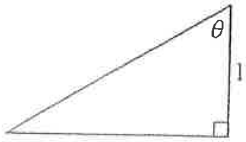

20. In the figure above, if $\tan\theta = k$, then $\sin\theta =$

(A) $\frac{1}{k}$

(B) $\frac{1}{\sqrt{1+k^2}}$

(C) $\frac{k}{\sqrt{1+k^2}}$

(D) $\frac{\sqrt{1+k^2}}{k}$

(E) $\sqrt{1+k^2}$

Score	0	1	2	3	4	5	6	7	8	9	10
Number of Students	0	0	1	0	1	1	2	3	5	3	4

. The distribution of scores for 20 students on a ten-question quiz is shown in the table above. What is the difference between the median score and the mean score for the 20 students?

(A) 0.45
(B) 1.51
(C) 2
(D) 7.55
(E) 8

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22. If $\log_a 25 = 0.7$, then $a =$

(A) 2.01
(B) 5.01
(C) 9.52
(D) 35.71
(E) 99.32

23. If a function f is defined by

$$f(x) = \begin{cases} 2 & \text{for } x > 3 \\ 2 - x^2 & \text{for } x \leq 3, \end{cases}$$

then $f(4) - f(-2) =$

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

24. Which of the following is equal to $\cos x$ for all values of x?

(A) $\cos(\pi - x)$

(B) $\cos(\pi + x)$

(C) $\cos\left(\frac{\pi}{2} + x\right)$

(D) $\cos\left(\frac{\pi}{2} - x\right)$

(E) $\cos(2\pi - x)$

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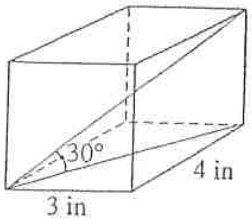
USE THIS SPACE FOR SCRATCH WORK.



25. The diagonal of a rectangular box whose base measures 3 inches by 4 inches forms a 30° angle with the base of the box, as shown in the figure above. What is the volume of the box, in cubic inches?

(A) 30
(B) 34.64
(C) 48.50
(D) 51.96
(E) 60

26. The functions f and g are defined so that $f(g(x)) = x$. If $g(x) = 7 - 8x$, which of the following is equal to $f(x)$?

(A) $8x - 7$

(B) $8x + 7$

(C) $\frac{-x-7}{8}$

(D) $\frac{x-7}{8}$

(E) $\frac{-x+7}{-8}$

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27. Which of the following functions are increasing for all values of x?

I. $f(x) = \left(\frac{1}{2}\right)^x$

II. $f(x) = 2^x$

III. $f(x) = 2x + 6$

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

28. In the xy-plane, at what point does the graph of $y = \log_4(4 - x)$ intersect the x-axis?

(A) $(-5, 0)$
(B) $(0, 0)$
(C) $(1, 0)$
(D) $(2, 0)$
(E) $(3, 0)$

29. In the xy-plane, the vertices of a triangle have coordinates $(0, 0)$, $(3, 2)$, and $(-3, 2)$. What is the measure of the largest angle of this triangle?

(A) 56.3°
(B) 67.4°
(C) 83.6°
(D) 112.6°
(E) 140.3°

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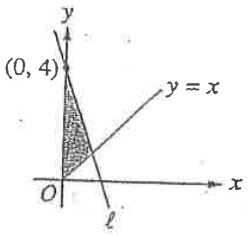
USE THIS SPACE FOR SCRATCH WORK.

0. The value, in dollars, of a commemorative coin is given by $V = \frac{1}{2}e^{0.1t}$, where t represents the time, in years, since the coin was minted in 1835. In what year was the coin worth \$1,220 ?

(A) 1843
(B) 1869
(C) 1899
(D) 1913
(E) 1977

. If two spherical surfaces with unequal diameters ntersect, the intersection is a

(A) circle only
(B) circle or sphere only
(C) circle or point only
(D) sphere or point only
(E) circle, sphere, or point



n the figure above, if the area of the shaded triangular region is 2, what is the slope of line ℓ?

(A) −4
(B) −3
(C) −2
(D) −1
(E) 0

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USE THIS SPACE FOR SCRATCH WORK.

33. In the xy-plane, which of the following equations has a graph that is the same as the graph defined by the parametric equations $x = 3\cos t$ and $y = 5\sin t$?

(A) $3x - 5y = 0$

(B) $3x + 5y = 1$

(C) $9x^2 + 25y^2 = 1$

(D) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(E) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

34. If $3^x \cdot 3^y = 9^3$, then $x + y =$

(A) 3 (B) 4 (C) 5 (D) 6 (E) 9

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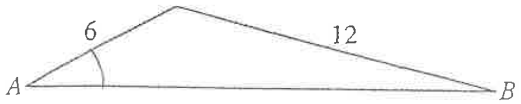
USE THIS SPACE FOR SCRATCH WORK.



35. In the figure above, if $\sin A = \frac{1}{2}$ and if the lengths of two sides are as shown, what is $\sin B$?

(A) $\frac{1}{4}$

(B) $\frac{\sqrt{3}}{3}$

(C) $\frac{\sqrt{5}}{5}$

(D) $\frac{1}{2}$

(E) It cannot be determined from the information given.

36. A drawer contains 5 black gloves, 6 brown gloves, and 7 blue gloves. If two gloves are randomly taken from the drawer without replacement, one after the other, what is the probability that they both will be blue?

(A) $\frac{7}{54}$

(B) $\frac{7}{51}$

(C) $\frac{7}{11}$

(D) $\frac{13}{18}$

(E) $\frac{227}{300}$

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MATHEMATICS LEVEL 2 TEST—*Continued*

USE THIS SPACE FOR SCRATCH WORK.

37. Let $a_1, a_2, \ldots$ be an arithmetic sequence with $a_1 = 5$ and $a_2 = 9$. What is the value of the first term of the sequence to exceed 1,000 ?

(A) 1,001
(B) 1,002
(C) 1,003
(D) 1,004
(E) 1,005

38. If $\pi \leq x \leq \frac{3\pi}{2}$ and $\tan x = 1.246$, then $\sin x =$

(A) 0.8945
(B) 0.7799
(C) −0.1709
(D) −0.7799
(E) −0.8945

39. A pizzeria is offering a special in which a large pizza has 1, 2, or 3 different toppings chosen from a list of 8 toppings. How many different combinations of toppings are possible for one large pizza?

(A) 6 (B) 8 (C) 24 (D) 56 (E) 92

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USE THIS SPACE FOR SCRATCH WORK.

40. Which of the following contains only values of x such that $3^x < x + 3$?

(A) $x < -2.961$
(B) $-2.961 < x < 1.335$
(C) $0.039 < x < 4.335$
(D) $x < 0.039$ or $x > 4.335$
(E) $x > 1.335$

41. If $F(x) = (x-1)F(x-1)$ for all positive numbers x, and if $F(0.4) = 2.7$, then $F(1.4) =$

(A) 1.08
(B) 1.96
(C) 3.70
(D) 4.78
(E) 7.29

42. The intensity of light varies inversely with the square of the distance from the light. If the intensity of a light is 150 units when the light is 3 meters away, what is the intensity when the same light is 12 meters away?

(A) 0.2
(B) 2.3
(C) 8.6
(D) 9.4
(E) 37.5

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USE THIS SPACE FOR SCRATCH WORK.

43. For positive numbers a and b, what is the period of $y = a\tan(bx)$?

(A) b (B) $\frac{b}{a}$ (C) $\frac{\pi}{b}$ (D) $\frac{2\pi}{b}$ (E) $b\pi$

44. A surveyor determines that the measure of the angle of elevation to the top of a vertical cliff is 30°. Moving 30 feet toward the cliff, the surveyor determines that the measure of the angle of elevation is 45°. Assuming that the ground is level, what is the height of the cliff to the nearest foot?

(A) 17 feet
(B) 21 feet
(C) 30 feet
(D) 41 feet
(E) 71 feet

45. If $x - y > 0$, then $|y - x| - (y - x) =$

(A) $2x - 2y$
(B) $2y - 2x$
(C) $-2y$
(D) $2x$
(E) 0

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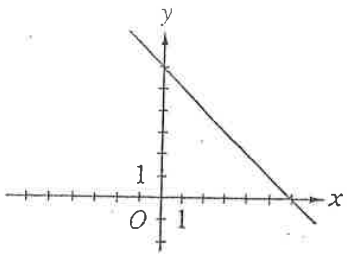
USE THIS SPACE FOR SCRATCH WORK.

16. If $f(0) = f(6)$, which of the following graphs could represent $f(x)$?

(A)



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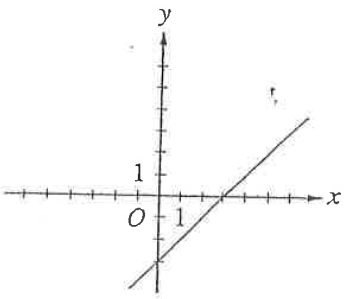

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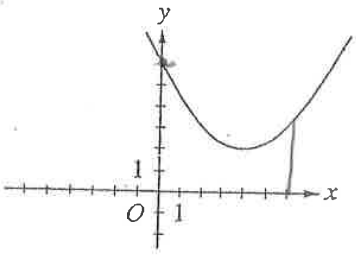

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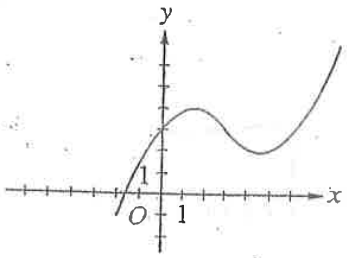
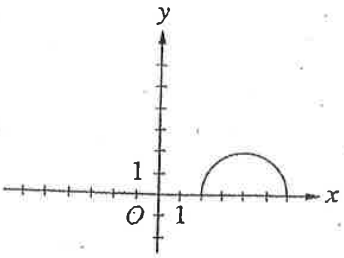


(E)



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USE THIS SPACE FOR SCRATCH WORK

47. A data set consists of 99 numerical observations. One more observation is added to bring the total to 100 observations. Which of the following statistics of the data set CANNOT decrease when the 100th observation is added?

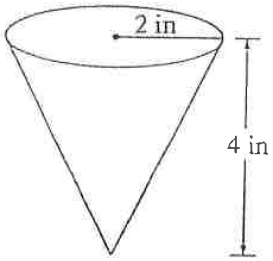
(A) Mean
(B) Median
(C) Minimum
(D) Range
(E) Standard deviation

48. If $P(x) = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$ is a polynomial function with rational coefficients, which of the following is the least number of real zeros that P can have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

GO ON TO THE NEXT PAGE

USE THIS SPACE FOR SCRATCH WORK.



9. The figure above shows a conical drinking cup with a base of radius 2 inches and a depth of 4 inches. What is the depth d, in inches, of a conical drinking cup that is similar in shape to the cup in the figure but has half the volume?

(A) 1.0
(B) 1.6
(C) 2.0
(D) 2.5
(E) 3.2

10. A, B, and C are matrices. If A has dimensions 2×5, C has dimensions 2×4, and $A \cdot B = C$, what are the dimensions of B?

(A) 2×4
(B) 4×2
(C) 4×5
(D) 5×2
(E) 5×4

STOP

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST ONLY.
DO NOT TURN TO ANY OTHER TEST IN THIS BOOK.

NO TEST MATERIAL ON THIS PAGE