ChE 243: Formula and Data Sheets

Property Relations

 $V = m v (m^{3}); \quad U = m u (kJ); \quad H = m h (kJ); \quad S = m s (kJ/K) \quad [Note: V = volume]$ $H \equiv U + PV (kJ) \implies h = u + Pv (kJ/kg)$ $PE = mgz (kJ); \qquad pe = gz (kJ/kg)$ $KE = \frac{1}{2}m \cdot vel^{2} (kJ); \qquad ke = \frac{1}{2} vel^{2} (kJ/kg) \quad [Note: vel = velocity]$ $c_{v} = \left(\frac{\partial u}{\partial T}\right)_{v} (kJ/kg \cdot K); \quad c_{P} = \left(\frac{\partial h}{\partial T}\right)_{P} (kJ/kg \cdot K)$

Saturated mixtures: $v_{\text{mix}} = v_f + x(v_g - v_f);$ $u_{\text{mix}} = u_f + x(u_g - u_f)$ $h_{\text{mix}} = h_f + x(h_g - h_f);$ $s_{\text{mix}} = s_f + x(s_g - s_f)$

Incompressible substances (i.e. solids and liquids): $c_v \approx c_P \equiv c$; $\Delta u \approx \Delta h \approx c \Delta T$

Ideal gases: Pv = RT; $c_{P0} = c_{v0} + R$ [Note: subscript "0" implies ideal gas]

$$\Delta u = \int_{1}^{2} c_{\nu 0} \, \mathrm{d}T \approx c_{\nu 0} \left(T_{2} - T_{1}\right); \quad \Delta h = \int_{1}^{2} c_{P 0} \, \mathrm{d}T \approx c_{P 0} \left(T_{2} - T_{1}\right)$$

First Law Relations

Closed system (control mass):

$$Q - W = m\left[\left(u_2 + \frac{1}{2}\operatorname{vel}_2^2 + gz_2\right) - \left(u_1 + \frac{1}{2}\operatorname{vel}_1^2 + gz_1\right)\right]; \quad W = \int_1^2 P \, \mathrm{d}V \quad \text{for boundary work}$$

Open system (control volume or CV):

Steady flow

$$\dot{Q}_{\rm CV} - \dot{W}_{\rm CV} = \sum_{\rm exits} \dot{m}_e \left(h_e + \frac{1}{2} \operatorname{vel}_e^2 + gz_e \right) - \sum_{\rm inlets} \dot{m}_i \left(h_i + \frac{1}{2} \operatorname{vel}_i^2 + gz_i \right) \,, \text{ where } \dot{m} = \frac{\operatorname{vel} \cdot A}{v}$$

For single stream steady flow, with $\dot{m}_e = \dot{m}_i \equiv \dot{m}$, define $q \equiv \dot{Q}/\dot{m}$; $w \equiv \dot{W}/\dot{m}$

$$\Rightarrow q - w = \left(h_e + \frac{1}{2} \operatorname{vel}_e^2 + gz_e\right) - \left(h_i + \frac{1}{2} \operatorname{vel}_i^2 + gz_i\right)$$

Transient process (conditions uniform within CV & constant at inlets/exits; neglect changes in PE & KE)

$$Q_{\rm CV} - W_{\rm CV} = \sum_{\rm exits} m_e h_e - \sum_{\rm inlets} m_i h_i + (m_2 u_2 - m_1 u_1)_{\rm CV}$$

Second Law Relations (T in kelvin)

$$dS \equiv \frac{\delta Q_{rev}}{T} \implies Q_{rev} = \int_{1}^{2} T \, dS$$
$$\Delta S_{tot} = \Delta S_{system} + \Delta S_{surr} = S_{gen} \ge 0$$

Incompressible substances (i.e. solids and liquids) with constant specific heats: $\Delta s = c \ln(T_2/T_1)$

Ideal gases (with constant specific heats):

$$\Delta s = \begin{cases} c_{\nu 0} \ln(T_2/T_1) + R \ln(\nu_2/\nu_1) \\ c_{P0} \ln(T_2/T_1) - R \ln(P_2/P_1) \end{cases}$$

$$\Delta s = 0 \quad \Rightarrow \quad P_1 \nu_1^{\ k} = P_2 \nu_2^{\ k} \ ; \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \ ; \quad \frac{T_2}{T_1} = \left(\frac{\nu_1}{\nu_2}\right)^{k-1} \ ; \quad \text{where} \ k = c_{P0}/c_{\nu 0}$$

Closed system (control mass):

$$\Delta S = \sum \frac{Q}{T} + S_{\text{gen}}; \quad S_{\text{gen}} \ge 0$$

Open system (control volume or CV):

Steady flow

$$\frac{\dot{Q}_{\rm CV}}{T} + \dot{S}_{\rm gen} = \sum_{\rm exits} \dot{m}_e \, s_e \, - \, \sum_{\rm inlets} \dot{m}_i \, s_i \; ; \; \; \dot{S}_{\rm gen} \ge 0$$

Transient process (conditions uniform within CV and constant at inlets/exits)

$$\frac{Q_{\rm CV}}{T} + S_{\rm gen} = \sum_{\rm exits} m_e \, s_e \, - \sum_{\rm inlets} m_i \, s_i \, + \, (m_2 \, s_2 - m_1 \, s_1)_{\rm CV} \, ; \, S_{\rm gen} \ge 0$$

Power and refrigeration devices:

$$\eta_{\rm th} \equiv \frac{W_{\rm net,out}}{Q_{\rm H}} = \frac{Q_{\rm H} - Q_{\rm L}}{Q_{\rm H}}; \quad \text{COP}_{\rm Re} \equiv \frac{Q_{\rm L}}{W_{\rm net,in}} = \frac{Q_{\rm L}}{Q_{\rm H} - Q_{\rm L}}; \quad \text{COP}_{\rm HP} \equiv \frac{Q_{\rm H}}{W_{\rm net,in}} = \frac{Q_{\rm H}}{Q_{\rm H} - Q_{\rm L}}$$

$$\eta_{\rm th,Carnot} = \frac{T_{\rm H} - T_{\rm L}}{T_{\rm H}}; \quad \text{COP}_{\rm Re,Carnot} = \frac{T_{\rm L}}{T_{\rm H} - T_{\rm L}}; \quad \text{COP}_{\rm HP,Carnot} = \frac{T_{\rm H}}{T_{\rm H} - T_{\rm L}}$$

$$\eta_{\rm turbine} \equiv \frac{w_{\rm actual}}{w_{s}} = \frac{h_{1} - h_{2,\text{actual}}}{h_{1} - h_{2s}}; \quad \eta_{\rm compressor} \equiv \frac{w_{s}}{w_{\rm actual}} = \frac{h_{2s} - h_{1}}{h_{2,\text{actual}} - h_{1}}$$

Incompressible steady flow: reversible work (neglecting changes in PE & KE) is $w_{CV} = -v \int_{in}^{out} dP$

Generalized Charts

$$T_{R} \equiv \frac{T}{T_{cr}}; \quad P_{R} \equiv \frac{P}{P_{cr}}$$

$$Z \equiv \frac{Pv}{RT}; \quad Z_{h} \equiv \frac{h^{*} - h}{RT_{cr}}; \quad Z_{s} \equiv \frac{s^{*} - s}{R}$$

$$h_{2} - h_{1} = (h_{2} - h_{2}^{*}) + (h_{2}^{*} - h_{1}^{*}) + (h_{1}^{*} - h_{1}); \quad \Delta h^{*} = \int_{1}^{2} c_{P0} \, dT \approx c_{P0}(T_{2} - T_{1})$$

$$s_{2} - s_{1} = (s_{2} - s_{2}^{*}) + (s_{2}^{*} - s_{1}^{*}) + (s_{1}^{*} - s_{1}); \qquad \Delta s^{*} = c_{P0} \ln(T_{2}/T_{1}) - R \ln(P_{2}/P_{1})$$

Dimensions and Units

| Dimension | Unit | Dimension | Unit |
|-------------|---------------|------------|-------------------------------|
| length | metre (m) | force | newton (N) |
| mass | kilogram (kg) | pressure | pascal (Pa) |
| amount | mole (mol) | heat, work | joule (J) |
| temperature | kelvin (K) | power | watt (W) |
| time | second (s) | volume | cubic metre (m ³) |
| | | | |

Constants and Conversion Factors

| $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$ | $1 \text{ J} = 1 \text{ N} \cdot \text{m}$ | 1 W = 1 J/s |
|---|--|-----------------------|
| $1 \text{ Pa} = 1 \text{ N/m}^2$ | $1 \text{ m}^3 = 10^3 \text{ L} = 10^6 \text{ mL}$ | 1 tonne = 1000 kg |
| $1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa}$ | 1 atm = 101.325 kPa | 1 psi = 6.894 757 kPa |
| $R_{\rm u} = 8.314 \ 47 \ {\rm kJ/(kmol \cdot K)}$ | $R = R_{\rm u}/M$ ($M = {\rm molar mass}$) | |
| $g = 9.806\ 65\ \mathrm{m/s^2}$ | 0°C = 273.15 K | |

Linear interpolation

$$y = y_1 + \left(\frac{x - x_1}{x_2 - x_1}\right) \cdot (y_2 - y_1)$$