AP PHYSICS REVIEW SHEET

By some dumbass not qualified to write about physics

Notes from class notes, Flipping Physics, Khan Academy, and Michael Richmond's Lectures (HIGHLY RECOMMENDED)

This doc should cover everything needed for Physics 1 and both Physics Cs.

*I wrote this in the span of like 1 day so I might be missing stuff. Also I got kinda tired around SHM and linear momentum, and got a bit lazy with my work

Physics C Mech
Monday 5/11 11am Central

Physics C E+M
Monday 5/11 1pm Central

Physics 1
Thursday 5/14 3pm Central

AP Physics is a high school equivalent of introductory level college physics.

Document Notes

Physics 1 does not have Electric Charge, Electric Force, DC Circuits, or Mechanical Waves and Sound

Physics C Mech does not have Oscillations or Gravitation

Physics C E+~~M~~ does not have Magnetism

There's also apparently a lot less math this year

If you do want to use this on the AP Exam (don't know why you would), either print it out or save your own copy of it. Don't access the google doc during the exam! CB doesn't want people doing that

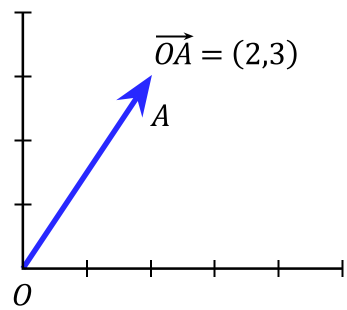
Doc is based off of my gov doc

Basics

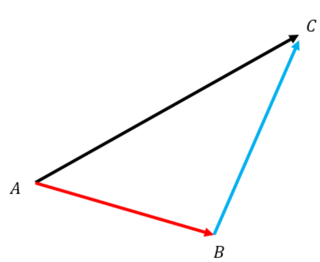
Physics is the study of the motion of the universe and seeks to understand how the universe works.

- **MATH NEEDED TO UNDERSTAND THIS DOCUMENT**
  - Arithmetic (add, subtract, multiply, divide)
  - Exponents and powers
  - SI Notation
  - Algebra
  - Trig
    - Includes ***small angle approximation*** – which states that for small angles (less than 11 degrees)

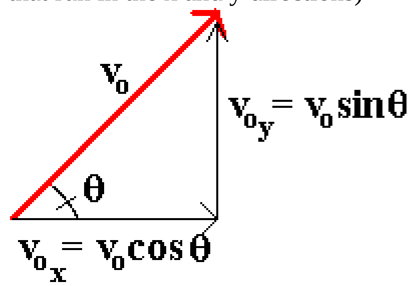
      $$\sin\theta \approx \tan\theta \approx \theta$$
  - Calculus (for Physics C, mostly just basic derivatives and integrals)
- **VECTORS**
  - ***Scalar Quantity*** – Has a magnitude (size) only
    - 14 students
    - 10 degrees
    - 10 m tall
  - ***Vector Quantity*** – Has a magnitude and direction
    - 10 mph North
    - 15 mph south
  - Vectors can be represented on a graph



    - The length of the vector represents it's size
    - The direction of the arrow represents the vector direction
  - The above vector can be described as:
    - Words : "Three units +x, Two units +y"
    - Parentheses: (3, 2)
    - Components: A(x) = 3, A(y) = 2
    - Unit Vectors: 3î, 2ĵ
  - Vectors can be added graphically, by putting them head to tail



  - Vectors can also be broken up into components (the parts of the vector that run in the x and y directions)



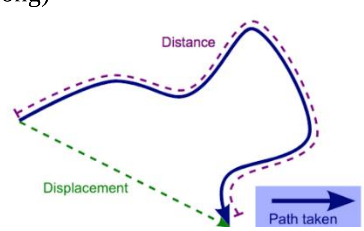
    - Components are written as

      $$\vec{A} = (A_x, A_y)$$
    - A_x is the vector's x component
    - A_y is the vector's y component
    - Vectors can be easily added by adding components separately

      $$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$$
  - Vector Trig
    - $A_x = A\cos\theta$
    - $A_y = A\sin\theta$
    - $A = \sqrt{A_x^{\,2} + A_y^{\,2}}$
- **SI UNITS**
  - ***Kilogram (kg)***: Mass
  - ***Meter (m)***: Length
  - ***Second (s)***: Time
  - Angles are either measured in ***Degrees or Radians***
  - Metric Prefixes – May appear before units and affect their scale
    - Giga = 10^9
    - Mega = 10^6
    - Kilo = 10^3
    - Milli = 10^-3
    - Micro = 10^-6
    - Nano = 10^-9

Kinematics

- ***Kinematics*** – The description of motion
- **Displacement and Velocity**
  - ***Displacement (x)*** – The distance between the start and ending position. Vector (ex. Path is 10 miles North)
  - ***Distance*** – The length of the path traveled. Scalar (ex. Path is 15 miles long)



  - ***Velocity (v)*** – The rate of change of displacement
    - ***Average Velocity*** – The mean value of velocity over some period of time

$$Velocity = \frac{Displacement}{Time}$$

- ***Instantaneous Velocity*** – The exact velocity at a certain point in time

$$Velocity = \frac{dv}{dt}$$

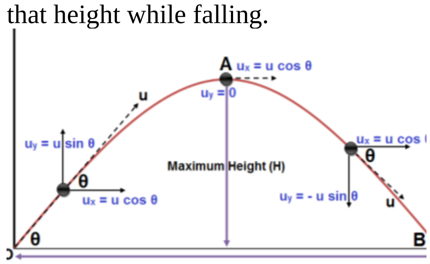
- ***Speed*** – The rate of change of *distance*. Measures how fast something is going, and the direction of movement doesn't matter.
- ***Acceleration (a)*** – The rate of change of an object's velocity
  - ***Average Acceleration*** – Total change in velocity over a period of time

$$Acceleration = \frac{v - v_0}{t - t_0}$$

  - ***Instantaneous Acceleration*** – Acceleration at a certain point

$$Acceleration = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

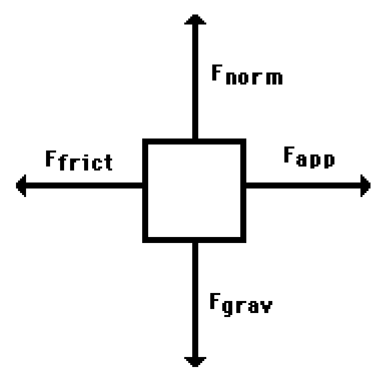
  - If something is moving in a circle at a constant speed, it's still has acceleration because while the magnitude of velocity stays the same, the *direction* is changing
- **Kinematics Equations (Assuming constant acceleration)**
  - $x = x_0 + vt$
  - $v = v_0 + at$
  - $x = x_0 + v_0t + \frac{1}{2}at^2$
  - $v^2 = {v_0}^2 + 2a(x - x_0)$
- ***Freefall*** – Motion when the only acceleration is from gravity
  - Objects in freefall will undergo a constant acceleration of 9.8 m/s^2 <u>down</u>
  - This acceleration is represented by a letter "g": $g \equiv 9.8\frac{m}{s^2}$
  - *In most cases, +y is moving upwards and +x is moving to the right. If this is the case, $g = -9.8\frac{m}{s^2}$ because gravity accelerates <u>down</u>
- ***2D Kinematics*** – Motion in two directions. Solve by splitting it up and determining what happens in the x and y directions independently.
  - In most cases **the motion of an object in two different directions are independent** (horizontal motion doesn't affect vertical motion)
  - It usually helps to **break a 2D problem into two 1D problems**.
    - There may be some common features (ex. The time it takes or the starting position)
  - Equations – You can break the motion into two pieces, each of which follows normal kinematics equations
    - $v_x = v_{0x} + a_xt$
    - $x = x_0 + v_{0x} + 0.5a_xt^2$
    - ${v_{0x}}^2 = {v_0}^2 + 2a_x(x - x_0)$
    - $temp$
    - $v_y = v_{0y} + a_yt$
    - $y = y_0 + v_{0y}t + 0.5a_yt^2$
    - ${v_{0y}}^2 = {v_0}^2 + 2a_y(y - y_0)$
  - **General Steps to Solving**
    - Read the problem
    - Draw a picture of what's happening
    - Write down known values
    - Determine what you're looking for
    - Check the Kinematics equations and see if one turns what you know into what you want.
- ***Projectile Motion*** – A special case of 2D Kinematics where $a_x = 0$ and $a_y = -g$
  - Horizontal speed is constant
  - Vertical acceleration is constant (usually its gravity)
  - A projectile motion usually involves something being thrown into the air
  - Treat like a normal 2D kinematics question
  - Note: At the peak of the object's trajectory, $v_y = 0$
  - Note: If an object is thrown from one eight, and then falls back to that height, it takes the same amount of time to rise to the peak as it takes to fall back down
  - Note: The speed of the object at a certain height while rising is the same as the speed of the object at that height while falling.



  - Remember Vector Components!
    - $v_{0x} = V cos\ \theta$
    - $v_{0y} = V sin\ \theta$

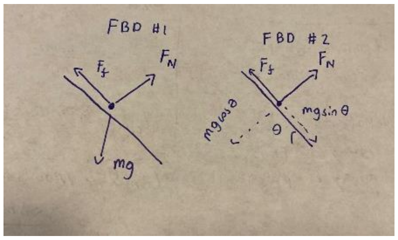
Dynamics (Forces)

- ***Forces*** – Things that causes acceleration or change in shape to an object
- ***Newton's Second Law:*** $F = ma$
  - Force = Mass * Acceleration
  - Units of kg*m/s^2 (Newton)
  - When there is no net force, there is no acceleration
    - $F = ma$
    - $0 = ma$
    - $a = 0$
    - No acceleration = constant velocity
    - (See First Law)
- **<u>Mass vs. Weight</u>**
  - Mass is how much stuff something has, while weight measures how much gravity pulls on an object. That's why your weight would be greater on Jupiter because Jupiter pulls down on your harder, but your mass wouldn't change.
- ***Newton's First Law***: Objects at rest will stay at rest, objects in motion will stay in motion (w/ constant velocity) UNLESS there's external force applied to it
- **Solving Dynamics Problems**
  - If you know the mass and all the forces acting on an object, you can now describe it's motion.
  - If there are multiple forces acting on a body, <u>be sure to add them properly as vectors.</u> (See FBDs)
- **List of Common Forces**
  - ***Gravity (mg or F_g)*** – Pulls objects down towards a center of mass
  - ***Normal (F_N)*** – Pushes surfaces away from each other
  - ***Tension (F_T)*** – Force applied by stretched ropes (or similar objects)
  - ***Friction (F_f or just f)*** – Opposes motion along 2 surfaces in contact
  - ***Electric Force (F_E)*** – Force between charged objects
  - ***Spring Force (F_S)*** – Force applied by a spring
  - ***Applied Force (F_A)*** – Generic name for force applied by something
- ***Free Body Diagrams*** – A picture showing all forces acting on an object. This helps add all the forces up properly. Ex.



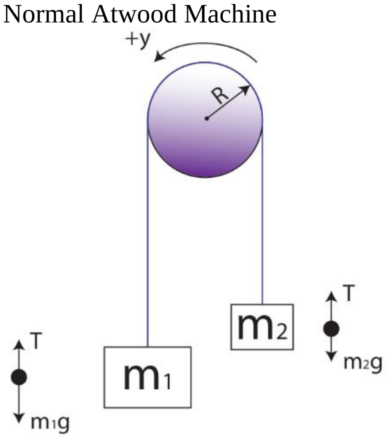
  - $\sum F_y = F_N - F_G$
  - $\sum F_x = F_{app} - F_f$
  - Forces that are not explicitly in the x or y direction should be broken up into *components*.
    - *If a problem requires you do draw a FBD, **draw the**

components in a SEPARATE FBD

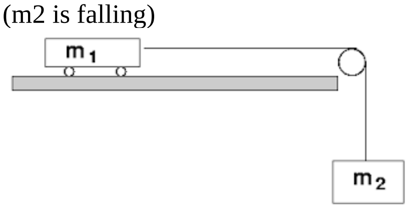
- 

(Sorry for shitty drawing)

- ***Newton's Third Law:*** If a body exerts a force on another body, then the second body exerts an equal and opposite force on the first
    - $F_{a\ on\ b} = -F_{b\ on\ a}$
    - *Just because the forces are the same, the result on each body may not be the same.
    - Ex. Bob (m=200kg) runs into Joe (400kg). They both exert a 800N force on each other
        - $a_{joe} = \frac{F_{Bob\ on\ Joe}}{m_{Joe}} = 800/40 = 20m/s^2$
        - $a_{bob} = \frac{F_{Joe\ on\ Bob}}{m_{Bob}} = 800/200 = 4m/s^2$
- ***Atwood machine*** – A pair of masses hung by a string going over a pulley. Common physics problem
    - Normal Atwood Machine



        - $\sum F_{m1} = m_1 a = m_1 g - T$
        - $\sum F_{m2} = m_2 a = T - m_2 g$
        - Usually this becomes a system of equations problem, so just solve for the missing value(s)
    - Modified Atwood Machine (m2 is falling)

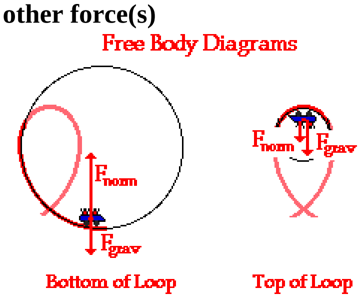


        - $\sum F_{m2} = m_2 g - T$
        - $\sum F_{m1} = T - F_f$**
        - **Sometimes the table is frictionless. There is no net force in the y direction on m_1 because Normal – Gravity = 0
- **<u>Friction:</u>** Force opposing motion of two objects moving against each other.
    - ***Static Friction*** – Friction between two objects <u>not moving</u>. Opposes the start of motion.

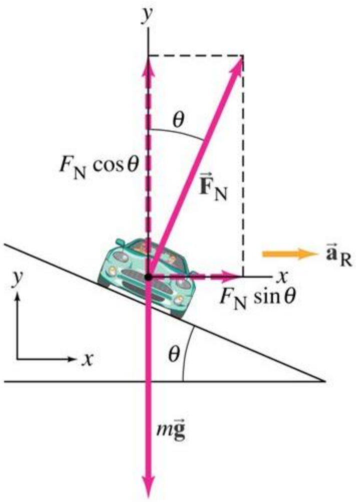
$$F_s = F_N * f_s$$

        - f_s is the coefficient of static friction, which depends on the surface
    - ***Kinetic Friction*** – Friction between moving surfaces

$$F_k = F_N * f_k$$

        - f_k is the coefficient of kinetic friction, which depends on the surface
        - A rough surface (ex. sandpaper) would have a high coefficient of kinetic friction, while a smooth surface (ex. ice) would have a low one
    - Friction acts opposite to <u>motion</u>. If acceleration points to the left and the object is moving right, the friction will point left <u>because it opposes the direction of velocity</u>.
    - **<u>Air Resistance</u>**: Resistance against a falling object by the air
      *I don't think this will be on the AP, but just in case
        - $F_{air} = k * A * v$
        - k = constant (varies based on air)
        - A = Area perpendicular to motion
- **<u>Solving Differential Equations</u>**
    - Also don't think this is needed, but just in case I'll just give an example
    - Person swimming, F(water) = -kv. What's dude's velocity at some time t after push off?
        - $F_{water} = -kv$
        - $ma = -kv$
        - $a = -\frac{k}{m} * v$
        - $\frac{dv}{dt} = -\frac{k}{m} * v$
        - $\frac{1}{v} dv = -\frac{k}{m} dt$
        - $\int \frac{1}{v} dv = \int -\frac{k}{m} dt$
        - $ln|v| = \int -\frac{k}{m} dt$
        - $ln|v| = -\frac{kt}{m} + C$
        - $v = e^{-\frac{bt}{m}} * e^c$
        - At $t = 0, v = e^c, so\ v_0 = e^c$
        - $v = v_0 e^{\frac{-bt}{m}}$
- **<u>Uniform Circular Motion</u>** – When something moves in a circle at a constant speed
    - *The velocity is constantly changing because direction is always changing. The object is also accelerating because direction is changing, and acceleration is a vector.
    - The magnitude of the acceleration is the same, but the direction is changing.
    - Normal kinematic equations don't work
    - An object in UCM will always be accelerating ***towards the center of the circle***. This is ***centripetal acceleration***.
        - $a = \frac{v^2}{R}$
        - R is the radius of the circle
        - a is centripetal acceleration
- **<u>Centripetal Force</u>** – In order to make something move in a circle, a force must be constantly applied to the object. This force is centripetal force.
    - $F_c = ma_c = \frac{mv^2}{r}$
    - Centripetal force is caused by something pulling towards the center of the circle, or something pushing from the exterior (or both).
    - <u>The sum of all forces pointing towards the center of the circle equals the centripetal force.</u>
    - **Centripetal Force is not a force itself, it must be caused by some other force(s)**
    - 

        - $\sum F_{bottom} = \frac{mv^2}{R} = F_N - mg$
        - $\sum F_{top} = \frac{mv^2}{R} = F_N + mg$
- **<u>Banked Curves</u>** – When a car moves on a circular track and the track is angled inwards



*Note – Don't do what they did here! Draw one FBD with just F_N and a separate FBD where you split up F_N into F(N) cos theta and F(N) sin theta.

- Here, $F_C = \frac{mv^2}{R} = F_N sin\,\theta$ because $F_N sin\theta$ is the force pointing towards the center of the circle

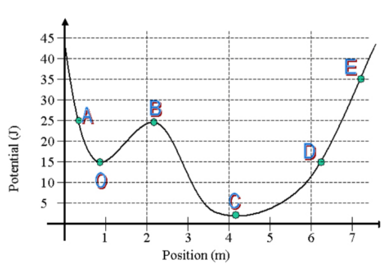
- **<u>Gravity</u>**
  - Gravity is usually simplified as F=mg
  - But the actual Law of Gravitation states that <u>any</u> two objects attract each other with a force
  $$F = G\frac{m_1 * m_2}{r^2}$$
    - G is a constant.
    $$G = 6.67 * 10^{-11} N\frac{kg^2}{m^2}$$
    - R is the distance between the (center of) two objects
  - Relatively speaking, gravity is a <u>very weak</u> force. It's near impossible to measure the force between ordinary objects (ex. between yourself and a nearby car).
    - However, on large scales (planets+), gravity is very important
  - Your <u>weight</u> could change based on where you are on Earth. You weight slightly more at sea level than on top of Mt. Everest because at Mt. Everest, the distance between you and the center of the Earth (r) is greater than at sea level.

Energy and Springs

- **<u>Kinetic Energy and Work</u>**
  - Kinetic Energy (KE) is the energy of motion
  $$KE = \frac{1}{2}mv^2$$
    - Units of kg*m^2/s^2 (Joule)
  - Work (W) measures the change in kinetic energy, and directly relates Force to energy
  $$W = KE_f - KE_i = F \cdot \mathrm{d}$$
    - Note that F in the work equation is <u>only Force in the direction of displacement</u>
    $$W = Fdcos\,\theta$$
    Where theta is the angle between the Force and the direction of displacement
  - Ex. A dude jumps off a building. At the 25th floor, he's going at 30m/s, and at the 20th floor, he's going at 70m/s. What's the difference in height between the floors?
    - $W = \triangle KE = F_g \cdot d$
    - $W = 0.5mv_2^2 - 0.5mv_1^2 = mgd$
    *v2 is speed at 20th floor, v1 is speed at 25th floor
    - $0.5 * 70^2 - 0.5 * 30^2 = g * d$
    - $2000 = 10d$
    - $d = 20m$
  - If the force applied is variable (changing), then the equation becomes
  $$W = \int F(x)dx$$
  (Integral bounds are from initial to final displacement)
  - Sometimes you might calculate work to be negative work. If you perform positive work, then you're spending energy, while if you perform negative work, then you're storing energy. Examples of negative work/storing energy:
    - Springs being compressed
    - Elevators rising
- **<u>Springs</u>**
  - Springs are important because the force they exert depends on how much they're stretched.
  - Springs have an equilibrium point, a point where they exert no net force. The further away from equilibrium you pull/push the spring, the more force the spring will exert to try to bring itself back to equilibrium.
  - Spring force is also sometimes called ***restoring force***, because it's trying to restore the spring back to it's equilibrium point.
  $$F_s = -kx$$
    - Restoring force acts opposite of displacement (direction you push/pull the spring away from equilibrium)
    - k is the ***spring constant***, which varies based on the spring. Tighter springs have a higher spring constant
    - x is the distance the spring is stretched.
- ***Power*** – The rate at which work is done
$$P = \frac{W}{t}$$
(Units of Watts)
  - Ex. If you exert 784 N to go up 2 meters of stairs over a period of 1.8 seconds, then your power in that period of time is
  $$Power = \frac{W}{t} = \frac{784 \cdot 2}{1.8}$$
  $$Power = 871\, Watts$$
  - Power can also be rewritten as
  $$P = Force * Velocity$$
  (Since P=W/t=Fd/t, and d/t is velocity)
- ***Potential Energy*** – An object's potential to do work. Often represented by **U**
  - Ex. A hanging box has the <u>potential</u> to do work if you release it.
  - To find the PE of something, ask "how much work did it take to arrange it into it's present configuration"
    - Ex. Bob lifts a box of mass m a height h.
    To do so, he must exert a force F=mg upwards to lift it, so the work done is
    $$W = F \cdot H = FH = mgH$$
    - This is the Gravitational PE of the box at that height.
    $$GPE = mgH$$
  - List of Potential Energies
    - $Gravitational\ PE = mgh$
    h is distance from the ground/where PE=0
    - $Spring\ PE = \frac{1}{2}kx^2$
    x is distance from equilibrium point, or where PE=0
    - $ElectricPE = k\frac{q_1 * q_2}{r}$
    - (More on EPE later)
  - Note that usually in a problem involving Potential Energy, you want to define the point where the PE = 0. If it involves simple gravity, PE=0 is usually at ground level.
- ***Mechanical Energy*** – Sum of all Kinetic and Potential energies of a system
$$ME = \sum KE + \sum PE$$
- In some cases, energy is **conserved**, meaning that
$$E_{initial} = E_{final}$$
- If something like friction is present, then
$$E_{initial} = E_{final} + W_{external}$$

(For friction, W(external) would be work done by friction)

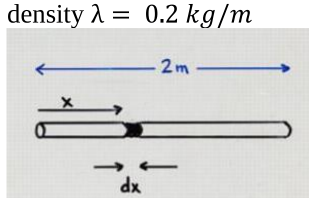
- **<u>Conservative vs. Nonconservative Forces</u>**
  - An object that does a round trip under the influence of a conservative force will not see any changes in energy.
    - Ex. Gravity
    - If a ball is thrown up, it has some initial KE. It slows due to gravity and falls back down. It's final KE when it returns is <u>the same</u> as the initial KE.
    - Each state had 1 vote, regardless of population
  - An object that does a round trip under the influence of a nonconservative force will have a different KE than when it started.
    - Ex. friction
    - Say you want to make a box move in a circle, so you give it a push, giving the box an initial KE. When the box returns back around, friction has slowed it down, and so it's final KE is less than its initial KE.
  - "The net work done on an object moving through a closed path is <u>zero</u> for conservative forces and <u>nonzero</u> for nonconservative forces"
  - *All conservative forces have an associated Potential Energy (GPE, SPE, etc) and conservative forces follow the law of conservation of energy (E = KE + PE), which makes problems easy to solve.
- **<u>Force and Potential Energy</u>**
  - $U(B) - U(A) = -\int_a^b F(x)dx$
    - U(x) is PE at location "x"
    - There is a <u>negative sign</u> in front of the integral.
  - Example: $F_g = -mg$
    - $U(B) - U(A) = -\int_a^b F(x)dx$
    - $= -\int_b^a -mgdx$
    - $= mg\int_b^a dx$
    - $= mg(B - A)$
    - $= mgh$
  - Sometimes Potential Energy is represented on a graph



    - To move to spots of higher PE, you need to spend energy to get there.
- **<u>Equilibrium and Potential Energy</u>**
  - $F = -\frac{dU}{dx}$
    (Found by reversing the integral earlier)
    - If PE increases as you move right, the Force pushes left
    - If PE increases as you move left, then Force pushes right
  - ***Equilibrium*** is the point where a spring, or any object in SHM, exerts no force.

Momentum

- ***Center of Mass*** – Pretty self explanatory
  - $x_{cm} = \frac{x_1m_1+x_2m_2+\cdots}{m_1+m_2+\cdots}$
  - $x_{cm} = \frac{\sum x_i m_i}{\sum m_i}$
    - (Also applies to other dimensions)
    - $y_{cm} = \frac{\sum y_i m_i}{m_i}$
  - If you're trying to find the center of mass of a continuous object (the center of mass of a blob) then
    - $x_{cm} = \frac{\int x dm}{M}$
    - This is a volume integral
- ***Solving Volume Integrals***
  - General Steps
    - Draw a picture
    - Find a way to divide the object into convenient pieces (cubes, circles, slices, etc)
    - Write an expression for the mass of each piece
    - Write the integral (with limits)
    - Integrate
  - Example
    - A rod has length L=2m and linear density $\lambda = 0.2\ kg/m$



    - Break the rod into small segments with length dx. Each segment has
      Density: λ
      Mass: $\lambda dx = dm$
    - The mass is the integral of all the small segments from one end to the other

$$Mass = \int_{x=0}^{x=2} dm = \int_{x=0}^{x=2} \lambda dx$$
$$= \lambda \int_{x=0}^{x=2} dx = \lambda(2-0)$$
$$= 0.2\frac{kg}{m} * 2m = 0.4kg$$

    - $x_{cm} = \frac{\int_{x=0}^{x=2} x dm}{M} = \frac{1}{0.4}\int_{x=0}^{x=2} x\lambda dx$
      $= \frac{1}{0.4} = \left[\lambda\frac{1}{2}x^2\right]_0^2$
      $\frac{1}{0.4} = 0.2 * 2 = 1m$
    - *Sometimes density will be a function, so be sure to factor that in.

$$dm = dx(\lambda(x))$$

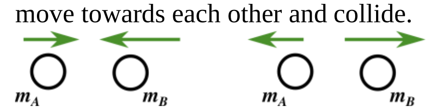
- ***Momentum***

$$p \equiv mv$$

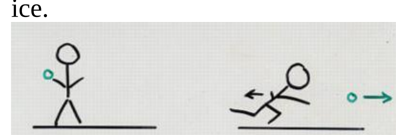
  - Momentum is <u>conserved</u> if there are no external forces acting on the object/system

$$p_{final} = p_{initial}$$

  - Momentum is a vector (in direction of velocity)
  - Both momentum and energy are important because they make problems easier to solve when they are conserved.
    - Momentum is most useful in collisions and when there's a system of multiple objects
  - Ex. 2 balls in an isolated system move towards each other and collide.

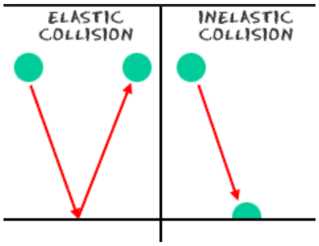


    $p_{final} = p_{initial}$
    $m_Av_A + m_Bv_B = m_Av_{Af} + m_Bv_{Bf}$
    If you know some of the values, you can solve for the others.
  - Ex. A pitcher throws a ball while on ice.



    $p_i = p_f$
    $m_pv_p + m_bv_b = m_pv_{pf} + m_bv_{bf}$
    (p = pitcher, b= ball)
    - Momentum tells us that the pitcher will end up sliding <u>backwards</u> on the ice.
  - As long as there are no external forces on a system of objects, the motion of the <u>center of mass</u> of those objects WILL NOT CHANGE, even if they collide or split apart.
- **<u>Types of Collisions</u>**
  - ***(Perfectly) Elastic Collision*** – A collision where no KE is lost.

- *Inelastic Collision* – Collision where KE is lost



- There are no elastic collisions in real life. If the collision is assumed to be elastic, the problem will tell you so.
- Momentum is conserved in both collisions

- **Force, Momentum, and Impluse**
  - Conservation of momentum > conservation of energy in collisions because objects tend to crumple or stick together, making energy hard to find.
  - The amount by which momentum changes is proportional to ***Impulse***
    - ***Impulse = Force * Time***
    - $p_f - p_i = Ft$
  - If force changes over time

$$p_f - p_i = \int_{t_i}^{t_f} F(t)dt$$

    - Given a graph of Force as a function of time, the area under the curve is impulse.
  - Ex. Bob pushes a car with mass 1500 kg with 300N. How long until the car reaches 4 m/s?

$p_i = 0$
$p_f = mv = 1500 * 4 = 6000$
$\triangle p = p_f - p_i = 6000 = Ft$

Plug in values and solve

- **2D Collisions**
  - Remember, momentum is a vector, and can be broken down into components (momentum in x and y directions)
  - Momentum is still conserved in each dimension

$$\sum mv_{xi} = \sum mv_{xf}$$
$$\sum mv_{yi} = \sum mv_{yf}$$

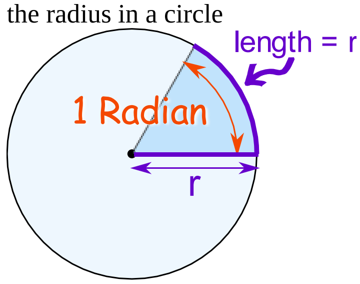
  - Kinetic energy is conserved in elastic collisions (but not in each dimension)

$$KE_i = KE_f$$

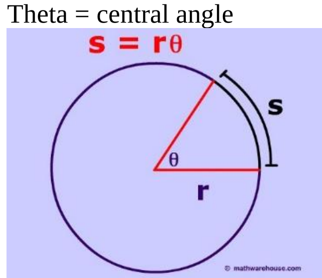
  - Split up the momentum of each object into x and y components, then apply conservation of momentum to each component respectively.

Rotational Kinematics

- Rotational Kinematics refers to the motion of rotating objects
- **Radians**
  - A radian is a notation for the angle or something, kinda like degrees.
  - 1 radian is when the arclength equals the radius in a circle



  - 1 radian = 180 degrees/ pi (approx. 57.3 degrees)
  - $\boldsymbol{s = r\,\theta}$
    - S = arclength
    - R = radius
    - Theta = central angle



  - When writing an angle, ALWAYS indicate degrees or radians
- ***Angular Velocity (w)*** – The rate at which the angle changes with respect to time. Represented by the Greek letter omega (though I'm just typing "w")
  - ***Average Angular Velocity***

$$\boldsymbol{\bar{w} = \frac{\theta_2 - \theta_1}{t_2 - t_1}}$$

  - ***Instantaneous Angular Velocity***

$$\boldsymbol{w = \frac{d\theta}{dt}}$$

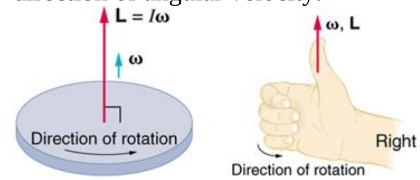
  - Units of rad/s
- ***Angular Acceleration*** (α) – The change in angular velocity with respect to time
  - ***Average Angular Acceleration***

$$\boldsymbol{\alpha = \frac{w_2 - w_1}{t_2 - t_1}}$$

  - ***Instantaneous Acceleration***

$$\boldsymbol{\alpha = \frac{dw}{dt}}$$

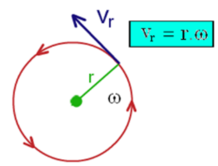
  - Units of rad/s^2
- The direction of angular velocity is along the axis of rotation.
  - Right hand rule – Curl the fingers of your right hand in the direction of motion. Stick your thumb out. The direction of your thumb is the direction of angular velocity.



- **Converting Linear to Rotational**
  - The speed of a point on a rotating body will have a *linear* (aka tangential) speed

$$\boldsymbol{v = wr}$$

    - Where r is the distance from the point to the axis of rotation
    - The direction of this motion is constantly changing. It is always perpendicular to the line joining it to the axis of rotation.



  - The tangential acceleration (rate of change of linear/tangential velocity) is

$$\boldsymbol{a_t = \alpha * r}$$

    - The t subscript denotes tangential acceleration.
- **Rotational Kinematic Equations** (Assuming constant rotational acceleration)
  - $\boldsymbol{w = w_0 + at}$
  - $\boldsymbol{\theta = \theta_0 + w_0 t + \alpha t}$
  - $\boldsymbol{w^2 = {w_0}^2 + 2a(\theta - \theta_0)}$
  - Basically kinematics equations, but replace x with theta, v with w, and a with alpha.
- **Rotational Kinetic Energy and Inertia**
  - Rotating objects also have kinetic energy, it just takes a bit more work.
  - Ex. An object is tied to a rope and is being swung in a circle. (If there are multiple objects in the system, add up the KE of each object).

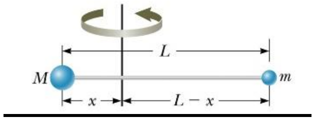
$\boldsymbol{KE = 0.5mv^2}$
$\boldsymbol{v = wr}$
$\boldsymbol{KE = 0.5m(wr)^2 = 0.5mw^2r^2}$

  - The general formula for KE is the sum of the KE for each discrete piece of mass in a system.

$$\boldsymbol{KE(total) = \sum_i 0.5m_i v_i^2}$$
$$\boldsymbol{= 0.5\left(\sum_i m_i r_i^2\right) w^2}$$

  - The terms with "i" subscripts can be separated into a term called ***moment of inertia (I).***
    - $\boldsymbol{I \equiv \sum_i m_i r_i^2}$

- $KE = 0.5 * I_{total} * w^2$
- If a body is composed of a small number of discrete masses, use the above equation(s) (ex. if there are 2 masses attached to ends of a rotating massless rod)



- If the body contains extended items, or is a continuous distribution of matter, it becomes an integral

$$I = \int r^2 dm$$

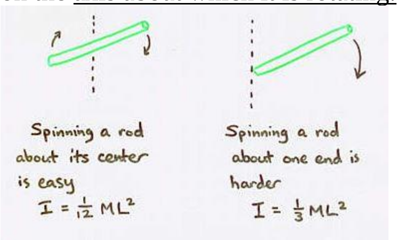
- It helps to break down dm into a product of density and d(Volume)

$$I = \int r^2 dm$$
$$dm = \rho dV$$
$$I = \int r^2 \rho DV = \int \rho r^2 DV$$

- Ex. Say 2 balls are attached to ends of a rod with mass w/ uniform density. In this case, break it up into three components

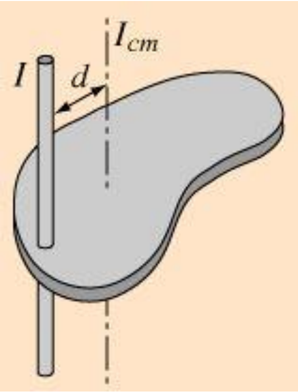
$$I_{total} = I_{ball1} + I_{ball2} + I_{rod}$$
$$KE = 0.5 * I_{total} * w^2$$

- When solving for moment of inertia of an object,
  - Try breaking into simple pieces
  - Draw a careful picture
  - If needed, integrate from one end to the other.
- The moment of inertia also depends on the <u>axis about which it is rotating.</u>



  - In the second case, there's more mass far away from the axis, so it takes more work to move
- ***Parallel Axis Theorem***: If you move the axis of rotation a parallel distance away from the center of mass (creating a parallel axis), then

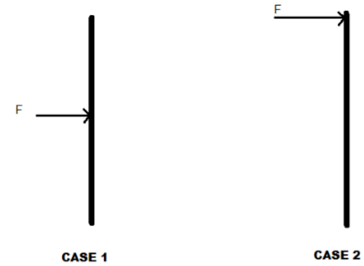
$$I_{parallel\ axis} = I_{cm} + Md^2$$



- ***Torque*** – Measure of how hard one pushes an object to make it rotate. Represented by greet letter τ

$$\tau = \alpha * I$$

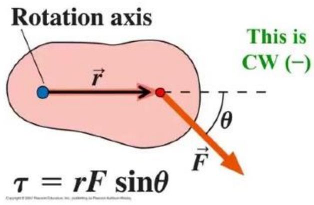
  - *Note the last part: "to make it rotate". The same force can give rise to different torques, depending on where the force is applied.



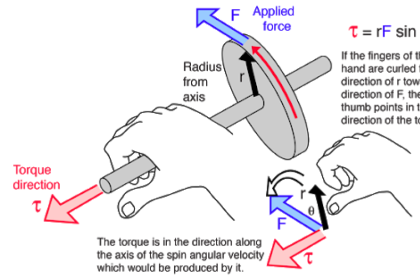
  -
    - In case 1, the force will just push the rod to the right
    - In case 2, the force will case the rod to <u>rotate</u> and experience angular acceleration.
  - Torque can also be expressed as

$$\tau = rF\sin\theta$$

    - r is distance from axis of rotation, theta is the angle between F and r.
    - Torque is max at an angle of 90 degrees



  - Right hand rule (for torque)
    - Curl fingers in the direction of force
    - Stick out thumb. Thumb points in direction of torque.



    - *text in above picture isn't that important
  - $\textbf{Net Toruqe} = \sum_i T_i$ (Net torque is the sum of all torques on an object)
  - Torque is basically Newton's Second Law's rotational counterpart
    - $F = ma$
    - $\tau = I\alpha$
- Work and Torque
  - If torque is constant, $W = \tau\theta$
  - If torque isn't constant, $W = \int \tau d\theta$
    - Also $dW = \tau d\theta$
    - $\textbf{Power} = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$
  - Also

$$dw = \tau d\theta$$
$$\tau = I\alpha$$
$$dW = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\frac{d\theta}{dt}$$
$$dW = I\omega\frac{d\omega}{d\theta}d\theta$$
$$dW = I\omega d\omega$$
$$W = \int dW = \int_{w_1}^{w_2} I\omega d\omega$$

  - Work done by torque = Change in rotational KE

$$W = KE_{rot\ final} - KE_{rot\ initial}$$

- **Rolling without slipping**
  - Objects that roll without slipping have a fixed relation between their linear and angular velocities.
  - If radius is fixed
    - $v = r\omega$
    - $a = r\alpha$
  - The KE of an object rolling without slipping (ex. a wheel rolling down a ramp) has KE from both sliding and spinning

$$KE_{tot} = KE_{sliding} + KE_{spinning}$$
$$KE_{tot} = 0.5mv^2 + 0.5I\omega^2$$

    - I is moment of inertia about center of mass
- Common moments of inertia
  - Single point mass

$$I = Mr^2$$

    - r is distance from axis of rotation
    - M is mass of point mass
  - Multiple point masses connected by a line

$$I = \frac{m_1 r_1^2 + m_2 r_2^2 + \cdots}{m_1 + m_2 + \cdots}$$

- r is distance mass is away from axis of rotation

- Rod rotating about its center

$$I = \frac{1}{12} mL^2$$

  - L is length of the rod
  - M is mass of the rod
- Rod rotating about its endpoint.

$$I = \frac{1}{3} mL^2$$

- Hoop

$$I = MR^2$$

  - R is radius of the hoop
- Solid cylinder/disk

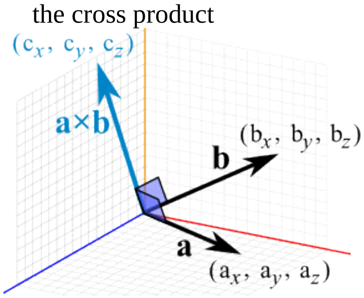
$$I = \frac{1}{2} MR^2$$

  - R is radius of the disk
- Solid Sphere

$$I = \frac{2}{5} MR^2$$

- Hollow Sphere (Shell)

$$I = \frac{2}{3} MR^2$$

- A few notes on inertia
  - A solid disk would have a higher moment of inertia than a ring of the same mass because all the mass is concentrated further away from the center of mass. Higher inertia means that if placed on a ramp, it will roll down slower than the disk.
  - Same applies to a hollow ball and a solid ball of the same mass.
- **Vector Cross Product (Vector Product)** Given 2 vectors A and B, the cross product (A x B) has
  - Magnitude $|AxB|$ or $ABsin\theta$
  - Direction is given by the right hand rule
    - Orient right hand so fingers point in direction of A
    - Curl fingers from A to B
    - Thumb now points in direction of the cross product
  - 

  - *The order of vectors matters
  - Parallel vectors = cross product of 0
  - Perpendicular vectors = cross product of AB.
- **Angular Momentum (L)**

$$L = I\omega$$

$$L = r \times p$$

  - (A body of momentum p located at position r has angular momentum L = r x p)
  - L points along the axis of rotation
  - Angular momentum depends on the choice of origin. An object may have positive angular momentum relative to one point, and negative relative to another.
  - Angular momentum is also conserved
  - $\frac{dL}{dt} = \tau_{net}$

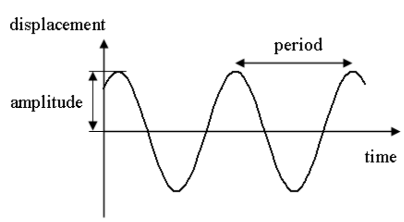
Simple Harmonic Motion

- SHM is ***oscillatory*** motion – involving a back and forth or up and down sequence where no net movement occurs
  - Pendulums
  - Springs pulled from rest
- SHM is a special case of oscillatory motion where
  - It has a rest position (equilibrium)
  - There is a restoring force which tries to bring objects to the rest position
  - Restoring Force is proportional to displacement from the rest position
  - $F = -kx$
    - K is a constant
    - X is distance from equilibrium
- SHM always displays the same sort of behavior with time

$$x = Acos(\omega t)$$

$$x = Asin\left(\omega t + \frac{\pi}{2}\right)$$

  - X is the oscillating quantity (position, height, etc.)
  - A is the amplitude, or the maximum variation form rest
  - Omega is the angular frequency, or the number of radians/sec for oscillation
  - SHM on a graph of x verses time usually looks like



  - You can differentiate the SHM functions (w/ respect to time) to find velocity

$$v(t) = \frac{dx}{dt} = -\omega Asin(\omega t)$$

  - You can differentiate again to find acceleration

$$a(t) = \frac{d^2x}{dt^2} = -\omega^2 Acos(\omega t)$$

  - You can further combine this with Force to find

$$-kx = m * a(t)$$

$$a(t) = -\omega^2 x(t) = -\frac{k}{m} x(t)$$

    - There is a relationship

$$\frac{k}{m} = \omega^2$$

    - Between mass, k (constant), and angular frequency.
- SHM in Springs
  - Springs will always exert a force to try and reach equilibrium, or a point where there's no net force

$$F = -kx$$

    - K is the spring constant
    - X is displacement from equilibrium.
  - When a spring is stretched out, it pulls back
  - When a spring is pushed inwards, it exerts a force back to original position
  - ***Frequency (f)*** is a measure of the # of cycles/per second (Hz)

$$Angular\ Frequency: \omega = \frac{k}{m}$$

$$Frequency: f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

  - ***Period (T)*** is 1/f, and is the time for one oscillation

$$Period: T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

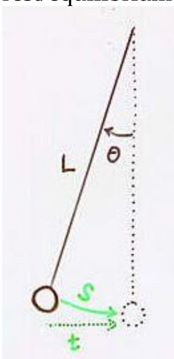
    - M is mass of the oscillating object on the spring, k is the spring constant
  - Springs also have a Potential Energy of

$$SPE = 0.5kx^2$$

    - X is the distance from equilibrium
    - As the spring moves, it converts SPE into KE, and energy is conserved.
- SHM in Pendulums
  - A pendulum is where a mass is suspended by a string and swings with (ideally) air resistance.
    - In an ideal pendulum, the string is massless and there's no air resistance. The object will always return to the same height and SHM occurs.
  - Pendulum must act like SHM

$$s(t) = Acos(\omega t)$$

- S is the distance from rest/equilibrium



$$S = L\sin\theta$$

- You can use the small angle approximation here: If theta is small then

$$t \approx L\sin\theta$$
$$s \approx t$$

- In a pendulum, $k = \frac{mg}{L}$
- So, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\frac{mg}{L}}{m}} = \sqrt{\frac{g}{L}}$
- $T = 2\pi\sqrt{\frac{L}{g}}$
- $f = \frac{1}{2\pi\sqrt{\frac{L}{g}}}$

Electricity

- Types of Charges
  - Positive
  - Negative
  - Opposite charges attract
  - Similar Charges Repel
- Charge is measured in multiples of a unit called e.
  - Proton: +1e
  - Electron: -1e
  - You can't have charges like 0.5e or pi e.
  - $\boldsymbol{e = 1.6x10^{-19}C}$
    - C is coulombs
- Charged is conserved 0 the total amount of electric charge in a closed system is constant.
  - Ex. a box has 3 electrons (-3e) and 9 protons (+9e). If nothing happens to the box, the charge inside will always be +6e
- Conductors and Insulators
  - Conductors are materials that allow charge to move freely within them (metals)
  - Insulators keep charge "suck" in place (glass, wood, rubber, etc.)
  - No material is a perfect insulator, even things like air will conduct charge very slowly. A charged object will eventually lose it's charges to the surrounding environment.
  - The Earth is basically an infinite charge sink. A ***grounded*** object is one that conducts charge into the Earth.
- **Coulomb's Law**
  - The electric force on particle B due to particle A is

$$\boldsymbol{F = k\frac{q_A a_B}{r^2}\hat{r}}$$

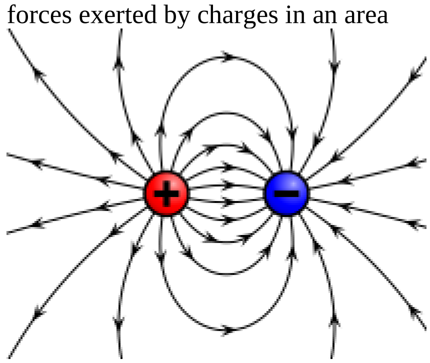
    - k is Coulomb's constant, which is

$$\boldsymbol{k = 8.99x10^9\frac{Nm^2}{C^2}}$$

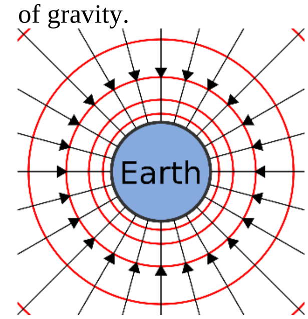
    - q(a) is charge on particle A (Coulombs)
    - q(b) is charge on particle B (Coulombs)
    - r is the distance from A to B
    - (r hat is the unit vector from A to B)
    - *To find the force on particle B, employ the direction from A to B
    - Newton's Second Law still applies:

$$\boldsymbol{F_{a\,on\,b} = -F_{b\,on\,a}}$$

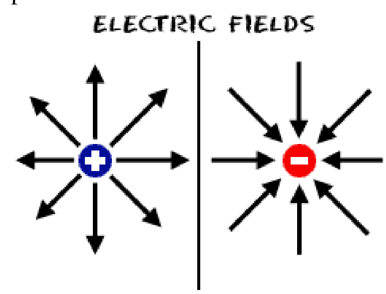
    - Finally check to make sure it makes sense. Opposites attract, similar charges should repel.
  - Electric Force is much stronger than Gravitational Force
- **Electric Field**
  - "If we place a tiny charge q at this spot, what would be the net force on it due to all the other charges?"
  - An electric field expresses this – "What is the net electric field at this position, due to all the charges?"
  - The electric field is created by electric charges, and represents the forces exerted by charges in an area



    - Fields aren't just electric, you can have gravitational fields as well, where lines point in the direction of gravity.



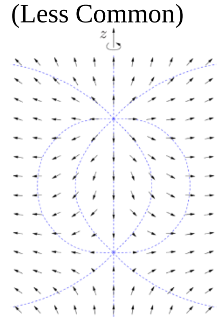
    - Electric field convention is that field arrows point towards negative charges and away from positive ones.



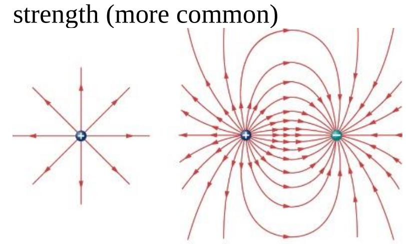
  - E field created by a single point charge Q is

$$\boldsymbol{E = k\frac{Q}{r^2}}$$

  - E fields are represented in 2 common ways
    - Using arrows, where the length and direction of each arrow represents strength and direction (Less Common)



    - Using continuous lines. The density of lines show field strength (more common)



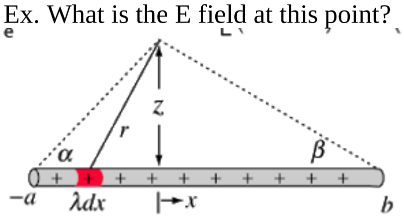
  - ***Superposition***: The net electric field of all charges is equal to the (vector) sum of the fields created by each charge.

$$\boldsymbol{E_{tot} = \sum_{i=1}^{N} E_i}$$

- **E Field due to a body of charge (line/disk)**
  - If a body is filled with a continuous charge distribution, then you integral over all the charge.

$$E = \int \frac{k}{r^2} dq$$

  - To integrate
    - Draw a picture
    - Break the distribution into "nice" pieces (beads on a line, or rings on a disk)
    - Pick one small piece and write the equation for that piece alone.
    - Integrate (one dimension at a time)
    - Check to see if the result makes sense
  - Ex. What is the E field at this point?



    - Consider the tiny red chunk

$$dE = \frac{kdq}{r^2} = \frac{k\lambda dx}{x^2 + z^2}$$

$$dE_y = \frac{k\lambda dx}{x^2 + z^2} * \frac{l}{\sqrt{x^2 + z^2}}$$

    - *dE_y is the E-field in the y-direction, which we want to find because the point is in the y direction. X refers to the distance from the red chunk to z. X is negative here, but since it's squared, it doesn't matter. You multiply by $\frac{l}{\sqrt{x^2+z^2}}$ because that's sin theta, and that gives you the y component.
    - To find the total y component, integrate

$$Total\ E_y = \int_{-a}^{a} dE_y$$

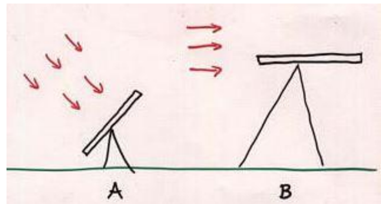
$$= \int_{-a}^{b} k\lambda z \frac{dx}{(x^2 + z^2)\text{^\{3/2\}}}$$

    - Do some algebra and calc, im too lazy
- **Particles in an Electric Field**
  - A particle with charge q experiences a force when placed in an E-Field

$$F = qE$$

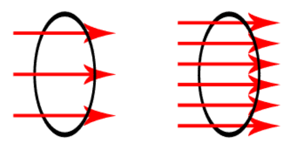
$$a = \frac{F}{m} = \frac{qE}{m}$$

    - The particle undergoes constant acceleration, so you can use 1D kin
- **Gauss' Law**
  Describes the interactions between electric forces and E-fields.
  - ***Flux*** is "How much stuff passes through an area?"
  - ***Electric Flux*** is "How much charge passes through an area"
    - *Flux depends on the angle between the E-field and the surface.

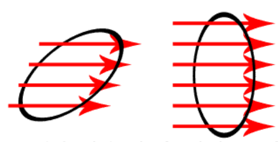


More flux in A, less flux (no flux) in B

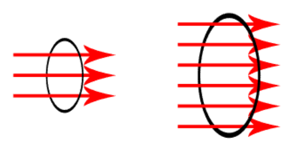
  - Flux can be modified in 3 ways
    - Changing the field strength (stronger field = more flux)
    - Changing the surface area (larger surface area = more flux)
    - Changing the orientation (flux is greatest when the E field is perpendicular to the surface).



Flux is proportional to the density of flow.



Flux varies by how the boundary faces the direction of flow.



Flux is proportional to the area within the boundary.

  - Electric Flux through some surface is given by the equation

$$\phi = E \cdot A = EAcos\theta$$

    - E is the E-field at the surface
    - A is the area of the surface
    - Theta is the angle between the E-field and the area
  - *If the surface isn't flat

$$\phi = \int E \cdot dA$$

    - Divide up the surface into tiny flat surfaces (with area dA), and add all them together.

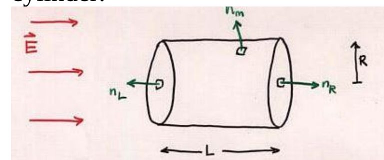
$$d\phi = E \cdot dA$$

$$\phi = \int d\phi = \int EdA$$

  - Flux passing through a closed surface

The net flux passing through any (empty) closed surface is 0.

  - Ex. An E-field passes through a cylinder.



  - Divide the surface into pieces
    - Cap on the left
    - Cap on the right
    - Band Around the Middle
  - Find the flux through each piece

$$\phi_{left} = E \cdot A = \pi R^2 * E$$

$$\phi_{right} = E \cdot A = \pi R^2 * -E$$

$$\phi_{band} = E \cdot A = 0$$

  Note that the flux out of the right is negative because it is leaving the surface.
  - Add up the flux through each piece

$$\phi_{left} + \phi_{right} + \phi_{band} = 0$$

  - ***Gauss' Law***
  - net electric flux $= \frac{\text{Electric Charge Inside}}{\epsilon_0}$

$$\phi = \frac{q}{\epsilon_0}$$

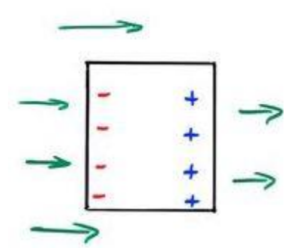
$$\oint E \cdot dA = \frac{q}{\epsilon_0}$$

$$E \cdot A = \frac{q}{\epsilon_0}$$

    - q is the enclosed charge in a surface
    - ϵ_0 is a constant known as the permittivity of free space

$$\epsilon_0 = 8.854 * 10^{-12}$$

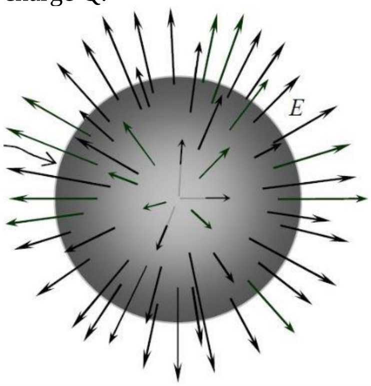
    - Gauss' Law hold true for any closed surface
- **Charge in a Conductor**
  - If you pass an E-field through a metal block, the charges in the block will split.
    - Positive charges will move away from the E-field. Negative charges will move towards the E-field.



    - This spilt of charges will create an E-field within the block that cancels out the external E-field.
  - The charge on the outer surface of a conductor is always spread out niformly. The E-field just outside a

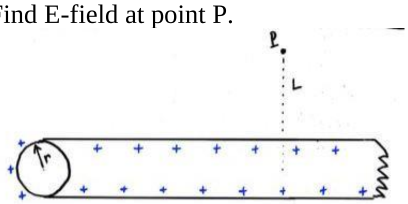
conductor is uniform, and is perpendicular to the surface.

- **Gauss' Law Example(s)**
  - Find the E-field due to a uniform ball of charge of radius R and total charge Q.



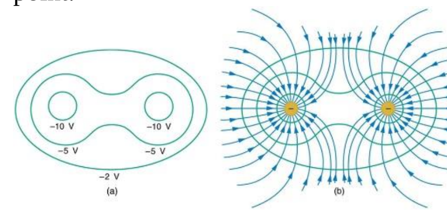
$$\oint \boldsymbol{EdA} = \frac{\boldsymbol{Q_{enc}}}{\boldsymbol{\epsilon_0}}$$
$$\boldsymbol{E}\oint \boldsymbol{dA} = \frac{\boldsymbol{Q_{enc}}}{\boldsymbol{\epsilon_0}}$$
$$\boldsymbol{E4\pi r^2} = \frac{\boldsymbol{Q_{enc}}}{\boldsymbol{\epsilon_0}}$$

    - Factored out E because the E field is the same at all points on the surface
    - $\oint \boldsymbol{dA}$ is just the sum of all the tiny areas making up the Gaussian surface (all of the dAs), which is 4*pi*r^2
    - If you don't know the charge enclosed (for example, if the Gaussian surface is <u>inside</u> the sphere), then you can do
$$\frac{\boldsymbol{q_{enc}}}{\boldsymbol{q}} = \frac{\boldsymbol{V_{surface}}}{\boldsymbol{V_{entire\ ball}}}$$
      (Assuming uniform charge density)
$$\boldsymbol{q_{enc}} = \boldsymbol{\rho(v_{surface})}$$
  - Find E-field at point P.

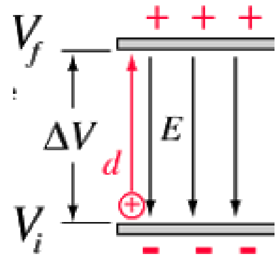


$$E \cdot A = \frac{q_{enc}}{\epsilon_0}$$
$$\boldsymbol{E(2\pi Ld)} = \frac{\boldsymbol{q_e nc}}{\boldsymbol{\epsilon_0}}$$

  - General Steps for using Gauss' Law
    - Draw a simple closed figure
    - Find the charge inside
    - Find the flux through the surface
    - Use Gauss' Law to find E
- **Electric Potential Energy**
  - Electric Potential Energy is the potential energy from electric charges.
    Ex. Imagine there's one positive charge and one negative charge, separated by a distance r(1). The negative charge is pulled away from the positive charge to a new distance r(2), which requires work.
$$W = \int F dx$$
$$W = \int qE dx$$
$$W = -q\int_{r1}^{r2} \frac{kq}{r^2} dr$$
$$W = -kq^2\left(-\frac{1}{r_2} + \frac{1}{r_1}\right)$$
    - *q was changed to negative in step 3 because it's a negative charge being pulled
  - Electric Potential Energy is given by the equation
$$\triangle U = -W_E$$
$$\triangle U = -q\int E dx$$
    - In the above example,
$$\triangle U = -\left[-kq^2\left(-\frac{1}{r_2} + \frac{1}{r_1}\right)\right]$$
    - This is the amount of energy needed for some force to move the negative particle away from the positive one.
- **Electric Potential (Voltage)**
  - $V = \frac{U_e}{q}$
  - Electric potential takes out the effect of charge.
  - Describes electric potential energy per unit charge at some point in space
  - "How much would the EPE of an imaginary positive charge change if we moved it to this spot?"
- **Electric Potential and E-Field**
  - $E = -\frac{dV}{ds}$
$$\boldsymbol{E_x} = \frac{\boldsymbol{dV}}{\boldsymbol{dx}}$$
$$\boldsymbol{E_y} = \frac{\boldsymbol{dV}}{\boldsymbol{dy}}$$
    - E field is the derivative of voltage with respect to distance from the source of charge.
  - $V_B - V_A = -\int_A^B E dx$
    - The potential difference between points A and B is given above.
    - Dx is a tiny section of a path from A to B
  - Sometimes, Electric Potential can be graphed sorta like a topographic map. The lines on these graphs are called equipotential lines, and represent the electric potential at that point.



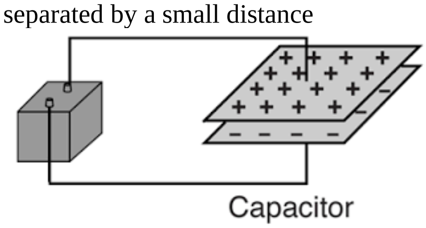
  - In a constant E-field, $\triangle V = -Ed$



  - Electric Potential due to a single point charge is given by
$$V = \frac{kQ}{r}$$
    - Q is the charge of the point charge
    - R is the distance from the point charge
    - If there are multiple point charges, add up all the voltages
$$\boldsymbol{V_{net}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \frac{\boldsymbol{kQ_i}}{\boldsymbol{r_i}}$$
  - Electric Potential a continuous distribution of charge (ex. a line of charge or a disk of charge) is given by
$$V = \int \frac{kdQ}{r}$$
    - As with all integrals, draw it out and find a way to break the shape into small segments

Capacitors and Circuits

- <u>**Capacitor**</u>: Something that stores charge.
  - Capacitors usually maintain a bunch of positive and negative charges separated by a small distance



  - Capacitors maintain a charge +Q on one plate and -Q on the other plate when you apply a voltage to it. Capacitance (F) is measured by
$$\boldsymbol{C} \equiv \frac{\boldsymbol{Q}}{\boldsymbol{V}}$$
    - Q = charge held within
    - V = Voltage across the device
  - The E-field on each plate is given by

$$E = \frac{Q/A}{\epsilon_0}$$

- The E-field between plates is uniform, and so the voltage between the plates is

$$\triangle V = -Ed$$

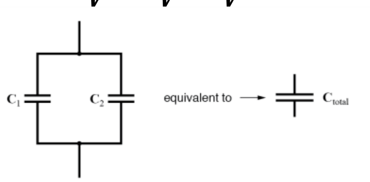
$$\triangle V = \frac{Q}{A\epsilon_0}d$$

- So given this, capacitance can also be rewritten as

$$C = \frac{A\epsilon_0}{d}$$

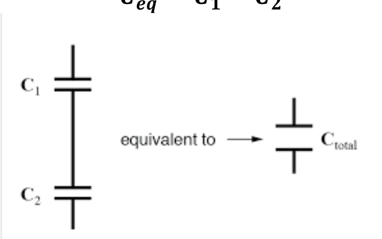
- Capacitors in Circuits
  - If capacitors are connected in parallel, then

$$C_{eq} = \frac{Q_{eq}}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$$



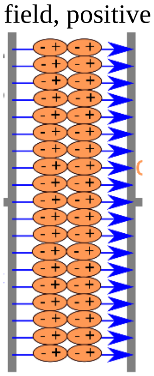
  - If capacitors are connected in series, then

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



  - Energy stored in a capacitor is

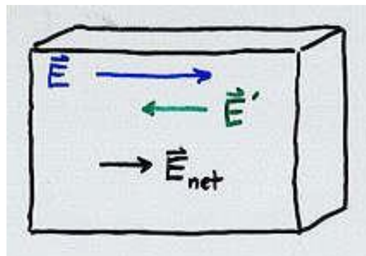
$$Energy = 0.5CV^2$$

- **Dielectrics**
  - Dielectric material is material that's made up of molecules which have one end slightly more positively charged than the other.
  - Normally these molecules orient randomly, but if you pass an E field through the, they'll line up with the E-field and create their own E-field.
    - (Negative ends point towards E-field, positive ends point away)
    - 
  - As a result, the material emits an E field of it's own, which opposes the outside E-field.

    - The strength of this E field is given by

$$E_{dielectric} = -\left(\frac{k-1}{k}\right)E_{external}$$

    - Where k is the dielectric constant of the material (k is always >1)
    - The induced E field is always weaker than the external E-field.
  - Inside a block of material, the net E field becomes



$$E_{net} = E_{ext} + E_{dielectric}$$

$$E_{net} = \frac{1}{k}E_{ext}$$

  - If a dielectric is placed between the plates of a capacitor with e-field E_0, then

$$E = \frac{1}{k}E_0$$

$$V = \frac{1}{k}V_0$$

  - The dielectric decreases voltage between plates, which makes adding charge easier. Capacitance increases when a dielectric is added

$$C = \frac{Q}{v_0/k}$$

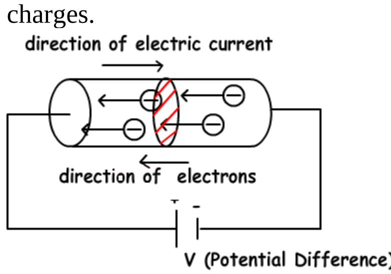
$$C = kC_0$$

- **Current (I)** is the net flow of electric charge in some direction

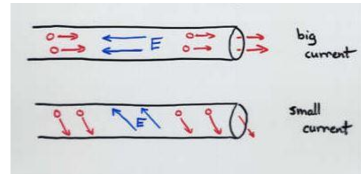
$$I = \frac{dQ}{dt}$$

$$I = \frac{Q}{t}$$

  - *Convention is that current flows in the direction of motion of <u>positive</u> charges.



  - Electrons move because an E-field within the wire is pushing them
    - Stronger E field = Bigger current
    - *In a big current, charges will basically fly down the wire. In a small current, charges may slow because they're bouncing around the wire more.



  - For some materials (Ohmic materials), current is proportional to electric flux

$$I = \sigma\phi = \sigma EA\cos\theta$$

    - σ = conductivity
- **Resistance** is a measure of opposition to current flow in an electrical circuit.
  - $R \equiv \frac{V}{I}$ (Ohm's Law)
  - $R = \rho\frac{L}{A}$
    - Where ρ is resistivity of a material

$$\rho = \frac{1}{\sigma}$$

    - L is length of the resistor
    - A is cross-sectional area of the resistor
  - ***Ohm's Law*** gives a relationship between R, V, and I

$$V = IR$$

- **Power (P) (Watts)**
  - Electric current can be converted into something useful (ex. heat or light) in a resistor.
    - In a resistor, the moving current bumps into the material of the resistor and heats it up.
    - $\triangle (ThermalE\} = \triangle EPE = -q\triangle V$
    - If a total of n charges pass through the resistor

$$Q = Nq$$

$$\triangle(ThermalE) = Q\triangle V$$

    - And if these charges pass through the resistor over a period of time $\triangle t$, the rate at which energy is transferred is

$$\frac{\triangle TE}{\triangle T} = \frac{Q}{\triangle t}\triangle V = I\triangle V$$

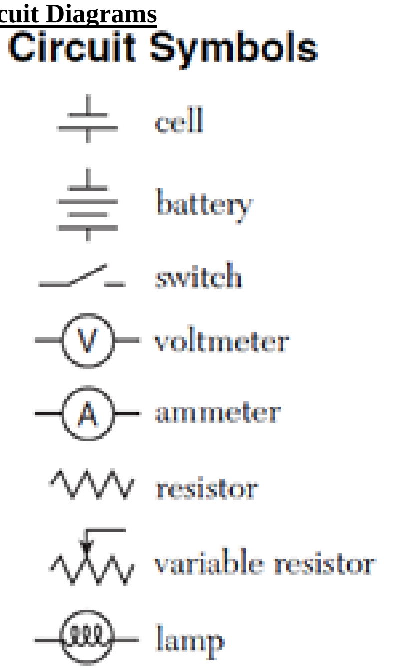
    - The ***power*** dissipated in a resistor as current runs through it is

$$P = I\triangle V$$

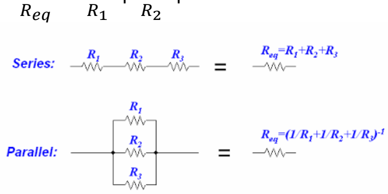
    - Can also be rewritten

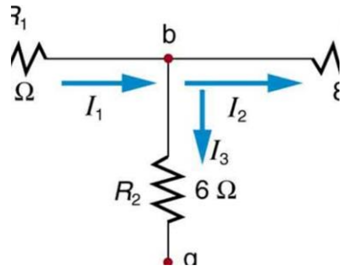
$$P = I^2R = \frac{(\triangle V)^2}{R}$$

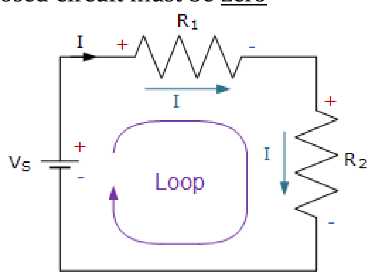
- **Circuits**
  - Circuits are primarily composed of batteries, resistors, and capacitors
  - Circuits can be compared to waterworks
    - Battery = pump

    - Voltage = Water level
    - Current = Flowing water
    - Resistor = Vertical drop
  - ***Electromotive Force (emf)*** is the same thing as voltage.
    - The difference between the emf of a battery and its terminal voltage is that a real battery has an internal resistance
    - $V = \varepsilon - IR$
- <u>**Circuit Diagrams**</u>
  - 

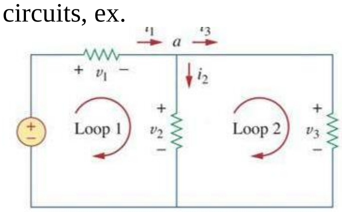
    - The lines in a circuit diagram are wires
    - A cell is basically the same thing as a battery
- Resistor Combinations
  - Resistors in series
    - Current is the same through both
    - Voltage different across each
    - $R_{eq} = R_1 + R_2 + \cdots$
  - Resistors in parallel
    - Current is different through each
    - Voltage is the same across each
    - $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$



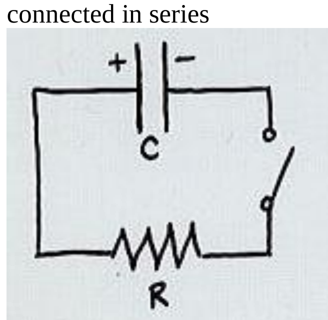
  -
- <u>**Kirchhoff's Rules**</u>
  - (Junction Rule) The sum of currents <u>into</u> a junction equals the sum of currents <u>out</u> of the junction
    - A junction is a point where wires meet/converge
    - 

    - $I_1 = I_2 + I_3$
  - (Loop Rule) The sum of potential charges across all elements of a closed circuit must be <u>zero</u>
    - 

    - $V - IR_1 - IR_2 = 0$
    - *There can be multiple closed circuits, ex.



In this case, the sum of the things in loop 1 = 0, and the sum of the things in loop 2 = 0.
  - To solve
    - It'll take way too long to write out all the steps, so just watch this [video]
- <u>**RC Circuits**</u>
  - A circuit where a capacitor and resistor are connected
  - Ex. A capacitor and resistor are connected in series



If the capacitor is charged, then there's a voltage

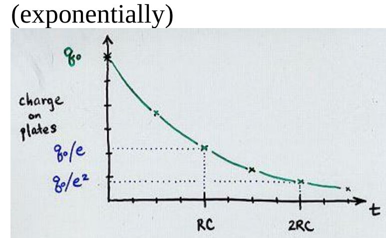
$$V = Q/C$$

across the wire, which sets up an E-field in the wire. If the switch is closed, the current is strong at first, then decreases as the charge on the capacitor goes down.

$$V_{cap} - IR = 0$$

$$\frac{q}{C} - IR = 0$$
$$q = IRC$$
$$q = -\frac{dq}{dt}RC$$
$$\frac{dq}{dt} = q\left(-\frac{1}{RC}\right)$$
$$q(t) = q_0 e^{-\frac{1}{RC}t}$$

- Charge on capacitor decreases (exponentially)



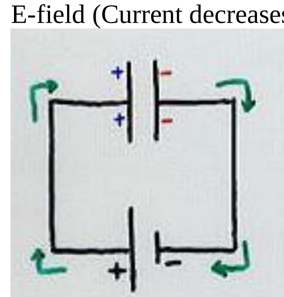
- The combination of Resistance and Capacitance

$$\tau = RC$$

- Is also known as the time constant of the circuit

$$q(t) = q_0 e^{-\frac{t}{\tau}}$$
$$I(t) = -\frac{dq}{dt} = -\left(-\frac{1}{\tau} q_0 e^{\frac{-t}{\tau}}\right)$$
$$I_0 = \frac{q_0}{RC}$$

- <u>**RC Circuits (Charging a capacitor)**</u>
  - Ex. an uncharged capacitor is placed into a circuit with a battery.
    - The battery creates an e-field on the wire which pushes positive charge onto one plate and negative charge onto another (Current Flows)
    - As charge builds up on the plates, the capacitor sets up an opposing E-field (Current decreases)



    - Eventually the charge on the capacitor is so large that the E-field of the capacitor negates the E-field of the battery (Current Stops)

$$V_{cap} = -V_{battery}$$
$$q_{final} = CV_{battery}$$

    - Charge on the capacitor follows a general equation

$$q(t) = CV\left[1 - e^{-t/RC}\right]$$

<u>**FRQ WRITING STRATEGIES**</u>

- **ANSWER THE STUPID QUESTION**

- The hardest part about physics is not knowing how to do the math of the problem but knowing what equation you’re supposed to use in a situation.
    - Write down quantities you know, and what you’re looking for.
    - Determine what kind of question this is (ex. a RC circuit question, or a simple kinematics question)
    - It might help to draw everything out
    - Look on your equation sheet (if you need to) and try to find something that matches up
    - Geometry Integrals can be somewhat tricky. Draw out the problem, and separate the volume/area/length into tiny pieces (dx or whatever), and write equations for this value before subbing it back in to the integral (ex. $dm = \rho dV$)
- Print a copy of the equation sheet, and add whatever necessary equations you need to it
- Write FAST – you don’t have as much time as you think you do
    - Writing neatly is a nice bonus, but don’t prioritize it. My handwriting looks like s***, and I can still get 5s.
- Write “scientifically” – Unlike English, words actually mean what they’re supposed to mean in science. So, don’t try to use fancy vocab which confuses your writing and forces your grader to analyze your stuff.
- If you have a “based on part a” kinda question, make sure to incorporate stuff from part an into your answer
- <u>FRQs are different in that you’re not only trying to give the right answer, but you’re also trying to convince the grader that you understand what you’re talking about. So, don’t go around making up BS.</u>
- Show your work! A lot of the times, showing work is necessary for credit.
- Apparently on the 2020 exams, there is a lot less math, so keep that in mind
- Make sure to use the right units! In this doc, I got kinda lazy and didn’t do that, but indicate units.
- Don’t be afraid to skip any parts of the question you don’t know how to do.
- Look over the scoring guidelines of previous exams
- Watch Flipping Physics’ last minute AP review videos

Formatting inspired by **Imbesi’s AP Psych Review Sheet**

I spent 9 hours on this I wanna die