AP PHYSICS C MECHANICS: ROTATIONAL MOTION

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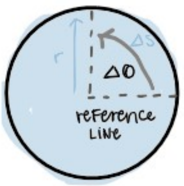
Rotational Kinematics



$\Delta\theta = \Delta s / r$

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ — Angular velocity, Angular acceler; Normal & Calculus formula

Converting from linear to angular values: $\Delta x = r\Delta\theta$, $v = r\omega$, $a = r\alpha$

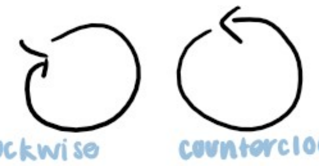
Important → this a represents tangential acceleration (not centripetal acceleration). Tangential acceleration arises from change in speed caused by angular acceleration but centripetal acceleration doesn't cause a change in speed

centripetal acceleration

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$

Kinematic Equations

Requirements for using equations
#1 constant angular acceleration (α)
#2 Rotational motion is considered around a fixed axis



Kinematic Equation	Missing variable
$\Delta\theta = \bar{\omega} t$	α
$\omega = \omega_0 + \alpha t$	$\Delta\theta$
$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	ω
$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$	ω_0
$\omega^2 = \omega_0^2$	t

Rotational Dynamics

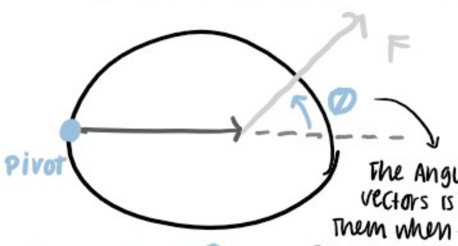
Translational motion Δx — Force = Mass × Acceleration: $\Sigma F = ma$

Rotational motion $\Delta\theta$ — Torque = Rotational inertia × angular a: $\tau = I\alpha$

Torque

How effective a force is in producing an angular acceleration

$$\tau = rF\sin\theta = I\alpha$$



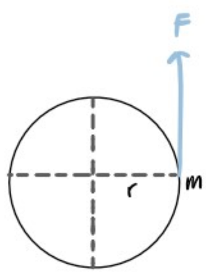
The angle between 2 vectors is angle between them when they start at the same point

$l = r\sin\theta$
lever arm formula

Rotational Inertia

$\Sigma F = ma$ — Newton's second law
$\Sigma F = m(r\alpha)$ — $a = r\alpha$ – linear to angular
$\Sigma F = mr\alpha$
$rF = mr^2\alpha$ — multiply both sides by r to get torque
$\tau = I\alpha$

Rotational Inertia (moment of inertia)



$$I = \int r^2 dm$$

moment of inertia for a point: $I = mr^2$
moment of inertia of entire body via addition: $I = \sum m_i r_i^2$
calculusifying to get integral: $I = \int r^2 dm$

$\int dm = \int \lambda dx$

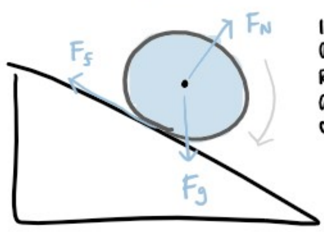
Translational 2 Rotational

Kinetic Energy of Rotation

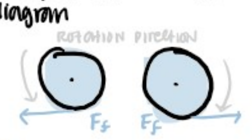
linear KE: $KE = \frac{1}{2}mv^2$
Rotational KE: $KE = \frac{1}{2}I\omega^2$

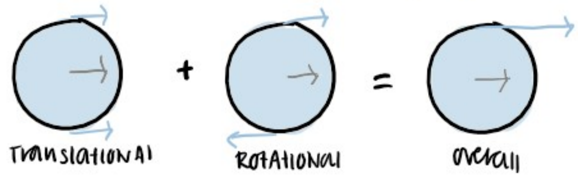
Rolling motion WITHOUT slipping

2018 #3dii



Important - direction of friction is opposite to rotation of wheel - not always towards like in this diagram





as always, use conservation of energy

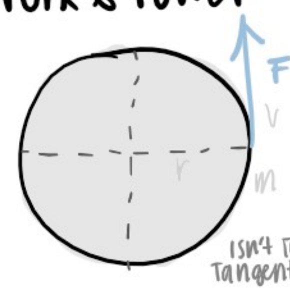
$$\Delta PE = \Delta KE$$

given that there are nonconservative forces (i.e. friction)

$$\Delta KE = \Delta KE_{trans} + \Delta KE_{rot}$$

$v = r\omega$ $\quad = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Work & Power



$W = F\Delta x = F\Delta s$
$W = F(r\Delta\theta)$
$W = (rF)\Delta\theta$
$W = \tau\Delta\theta$

where force is tangential to objects path. If force isn't tangential then only tangential component does work

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

work for a varying F ($\tau = rF$ so varying force means varying τ)

$$W = \int_{F_{x_1}}^{F_{x_2}} F\, dx$$

Notice similarities between linear & rotational forms

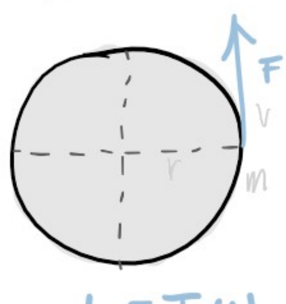
$P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} \Rightarrow P = \tau\omega$

Angular Momentum

$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$

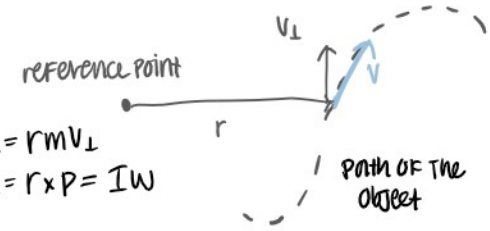
$rF = \frac{\Delta mv}{\Delta t} r$

$\tau = \frac{\Delta(rmv)}{\Delta t}$



$L = rmv$
angular momentum

$$L = I\omega$$



$L = rmv_\perp$
$L = r \times p = I\omega$

Conservation of

Equilibrium

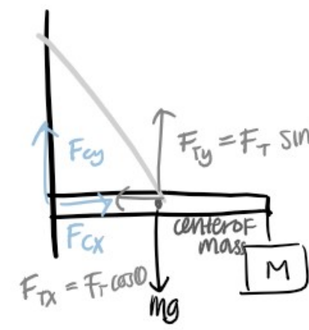
Translational Equilibrium: sum of forces acting on object is zero
Rotational Equilibrium: sum of torques acting on object is 0

Important

$F_{net} = 0 \neq$ velocity is zero
$F_{net} = 0 =$ velocity is constant
$\tau_{net} = 0 \neq$ angular velocity is zero
$\tau_{net} = 0 =$ angular velocity is constant

Static Equilibrium: if an object is at rest then it is in static equilibrium

Uniform Bar Question



The horizontal components of tension require horizontal force exerted by the wall on the bar to maintain equilibrium similarly is weight is to greater for tension to support then the wall would exert upward vertical force to prevent bar from falling

2 Important Equations

$\Sigma \tau = 0$, $\Sigma F = 0$ — True when there is static equilibrium

you can also split $\Sigma F = 0$ into it's horizontal & vertical components

$\Sigma F = 0$: $\Sigma F_x = 0$ horizontal component; $\Sigma F_y = 0$ vertical component

Summary

Linear to Angular Quantities

$s = r\theta \quad v = r\omega \quad a_{tan} = r\alpha$

Linear Equations & Angular Equivalents

Linear	Angular
$\Delta x = \bar{v}t$	$\Delta\theta = \bar{\omega}t$
$v = v_0 + at$	$\omega = \omega + \alpha t$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$\Delta x = vt - \frac{1}{2}at^2$	$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$
$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Basic Rotation Information

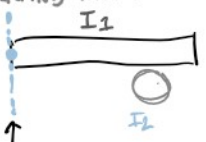
calculusifying to get integral $I = \int r^2 \, dm$

Translational 2 Rotational

$\int dm = \int \lambda \, dx$ ← linear density

Translational Motion	Rotational Motion
force F ⟷	Torque τ
acceleration a ⟷	Rotational Acceleration α
mass M ⟷	Rotational Inertia I
$F_{net} = ma$ ⟷	$\tau_{net} = I\alpha$

Adding multiple moment of inertia's



important - in order to add they MUST have same relative axis with which they get r values

$$I_{tot} = I_1 + I_2$$

To find the total moment of inertia, just add each individual one together & the sum is the total moment of inertia

Conservation of Angular Momentum

$F_{net} = \frac{dP}{dt} \Rightarrow$ If $F_{net} = 0$ the P is constant

$\tau_{net} = \frac{dL}{dt} \Rightarrow$ $\tau_{net} = 0$ the L is constant

Conservation of Angular Momentum

Figure Skater

As a figure skater pulls their arms inward, the move more of their mass closer to the axis of rotation, this means the I (rotational inertia) increases. Due to conservation of angular momentum (no torque), ↓ in I means increase in ω and vise versa

Basic Rotation Information

#1 Rotational Inertia is rotational analog of inertia (a measure of how difficult it is to change objects rotational motion

$I = mr^2$ point of mass

$I = \int r^2 \, dm$ distributed mass

Parallel Axis Theorem

$$I = I_{cm} + mx^2$$

$\tau = r \times F = I\alpha$

Torque Equation — Newton's 2nd Law for Rotation

$KE_r = \frac{1}{2} I \omega^2$ Rotational kinetic energy