## 7.1 - ONLY ONE SOLUTION

$\sin^{-1}(\sin x) \Rightarrow -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\sin(\sin^{-1} x) \Rightarrow -1 \leq x \leq 1$

$\cos^{-1}(\cos x) \Rightarrow 0 \leq x \leq \pi$

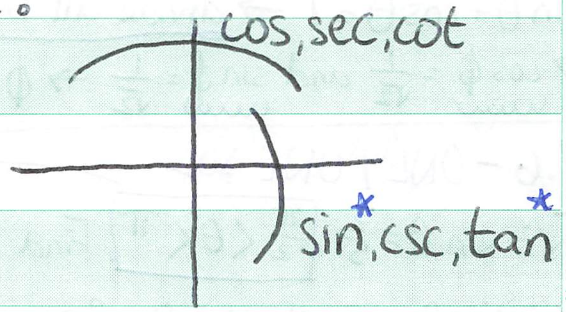
$\cos(\cos^{-1} x) \Rightarrow -1 \leq x \leq 1$

$\tan^{-1}(\tan x) \Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\tan(\tan^{-1} x) \Rightarrow -\infty < x < \infty$

Ex: $\cos^{-1}(1)$, $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$, $\tan^{-1}(0)$

Use:




* Refer to these to see which quadrant they are in

## 7.2 - ONLY ONE SOLUTION

Ex: $\sin(\tan^{-1} \frac{1}{2}) \Rightarrow \tan\theta = \frac{1}{2}\frac{y}{x} \Rightarrow r = \sqrt{5} \Rightarrow \sin\theta = \frac{1}{\sqrt{5}}\frac{y}{r}$

Ex: $\cos(\sin^{-1}(-\frac{1}{3})) \Rightarrow x = 2\sqrt{2} \Rightarrow \cos\theta = \frac{2\sqrt{2}}{3}$

Ex: $\csc^{-1}(2) \Rightarrow \sin^{-1}(\frac{1}{2}) \Rightarrow \theta = \pi/6$

Ex: $\sin(\tan^{-1}(u)) \Rightarrow y = u,\ x = 1,\ r = \sqrt{1+u^2} \Rightarrow \frac{u}{\sqrt{1+u^2}}$

## 7.3 - MANY SOLUTIONS • Adjust the negatives if $0 \leq \theta < 2\pi$

Ex: $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} + 2\pi k$ or $\theta = -\frac{\pi}{3} + 2\pi k$

Ex: $2\sin\theta + 1 = 0 \Rightarrow \theta = -\frac{\pi}{6} + 2\pi k$ or $\theta = \frac{7\pi}{6} + 2\pi k$

Ex: $\tan(\theta - \frac{\pi}{2}) = 1 \Rightarrow \theta - \frac{\pi}{2} = \frac{\pi}{4} + \pi k \Rightarrow \theta =$

Ex: $2\sin^2\theta - 3\sin\theta + 1 = 0 \xRightarrow{\text{factor}} \sin\theta = \frac{1}{2}$ or $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

Ex: $3\cos\theta + 3 = 2\underbrace{\sin^2\theta}_{1-\cos^2\theta} \Rightarrow 3\cos\theta + 3 = 2 - 2\cos^2\theta \xRightarrow{\text{factor}} \cos\theta = -\frac{1}{2}$ or $1 \Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$

Ex: $\underbrace{\cos^2}_{1-\sin^2\theta} + \sin^2 = 2 \Rightarrow \sin^2\theta - \sin\theta + 1 = 0 \Rightarrow$ No solution: $b^2 - 4ac > 0$

## 7.4/7.5 - ONLY ONE SOLUTION

Don't cross multiply.

$\cos A = 1/2 \quad \cos B = 4/5$

Ex: $\sin(\overset{A}{\cos^{-1}\frac{1}{2}} + \overset{B}{\sin^{-1}\frac{3}{5}}) \Rightarrow \sin A = \sqrt{3}/2 \quad \sin B = 3/5$

$\sin(A+B) = \sin A \cos B + \cos A \sin B \rightarrow (4\sqrt{3}+3)/10$

$\sin\theta + \cos\theta = 1 \rightarrow$ divide all by $\sqrt{a^2+b^2}$ $(\sqrt{2}) \rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\rightarrow \cos\phi = \frac{1}{\sqrt{2}}$ and $\sin\phi = \frac{1}{\sqrt{2}} \rightarrow \phi = \pi/4 \rightarrow \sin(\theta + \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \rightarrow \theta = 0, \pi/2$

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## 7.6 - ONLY ONE SOL

Ex: $\sin\theta = \frac{3}{5}$, $\boxed{\frac{\pi}{2} < \theta < \pi}$. Find $\sin(2\theta)$ and $\cos(2\theta)$. $\cos\theta = -\frac{4}{5}$. solve

Ex: $\sin\theta\cos\theta = \frac{-1}{2}, 0 \leq \theta \leq 2\pi \rightarrow \sin(2\theta) = -1$. solve.

Ex: $\cos A = \frac{-3}{5}, \pi < A < 3\pi/2$. Find $\sin\frac{a}{2}, \cos\frac{a}{2}, \tan\frac{a}{2}$. If $\pi < A < \frac{3\pi}{2}$,

$\frac{\pi}{2} < \frac{a}{2} < \frac{3\pi}{4} \therefore \frac{a}{2}$ in Quad 2. $\sin\frac{a}{2}$ is pos, $\cos\frac{a}{2}$ is neg, $\tan\frac{a}{2}$ is neg

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## 7.7 -

$\sin A \sin B = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$

$\sin A \cos B = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

$\cos A \cos B = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$

$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$

$\sin a - \sin b = 2\sin\frac{a-b}{2}\cos\frac{a+b}{2}$

$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$

$\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$

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