Tangent Plane, normal line

* surface has equation $F(x,y,z)=0$

* **normal vector to T. Plane**

$$= \langle F_x(x_1,y_1,z_1), F_y(x_1,y_1,z_1), F_z(x_1,y_1,z_1) \rangle$$

* **equation of Tangent Plane to surface $F(x,y,z)=0$ at $P(x_1,y_1,z_1)$**

$$F_x(x-x_1)+F_y(y-y_1)+F_z(z-z_1)=0$$

* **equation of normal line:**

Parametric equation
$$\left\{ \begin{array}{l} x = x_1 + F_x t \\ y = y_1 + F_y t \\ z = z_1 + F_z t \end{array} \right\} t \in R$$

* equation of normal line:

symmetric equation
$$\frac{X-X_1}{F_X} = \frac{y-y_1}{F_y} = \frac{Z-Z_1}{F_z}$$

• Find equation for The tangent Plane and the normal line at $P(2,-3,1)$. (162)

$4X^2 - y^2 + 3Z^2 = 10$

$f = 4X^2 - y^2 + 3Z^2 - 10 = 0$

$f_x(2,-3,1) = 8X = 16$

$f_y(2,-3,1) = -2y = 6$

$f_z(2,-3,1) = 6Z = 6$

normal vector $\langle 16, 6, 6 \rangle$

$\langle f_x, f_y, f_z \rangle$

equation of (T.P):

$F_x(X - X_1) + F_y(y - y_1) + F_z(z - z_1) = 0$

$16(X - 2) + 6(y + 3) + 6(z - 1) = 0$

$16X + 6y + 6z - 6 - 32 + 18 = 0$

$16X + 6y + 6z - 20 = 0$

equation of normal line:

$$\left.\begin{aligned} X &= 2 + 16t \\ y &= -3 + 6t \\ z &= 1 + 6t \end{aligned}\right\} \quad t \in R$$

Find equations for The tangent plane (163)
and normal line to the surface :
$X^2 - 2X + 2y^2 + 3Z^2 = 5$ at Point $P(0,1,1)$

$f = X^2 - 2X + 2y^2 + 3Z^2 - 5 = 0$

$$\left.\begin{aligned} f_x(0,1,1) &= 2X - 2 = -2 \\ f_y(0,1,1) &= 4y = 4 \\ f_z(0,1,1) &= 6z = 6 \end{aligned}\right\} \text{normal vector } \langle -2, 4, 6 \rangle$$

equation of Tangent Plane:

$$F_x(X - X_1) + F_y(y - y_1) + F_z(z - z_1) = 0$$

$$-2(X - 0) + 4(y - 1) + 6(z - 1) = 0$$

$$-2X + 4y - 4 + 6z - 6 = 0$$

$$-2X + 4y + 6z - 10 = 0 \to X - 2y - 3z = -5$$

equation of the normal line:

$$\left.\begin{aligned} X &= 0 - 2t \\ y &= 1 + 4t \\ z &= 1 + 6t \end{aligned}\right\} t \in R$$

find equation of the tangent Plane and normal line to the surface: (168)

$Z = e^{2y} \cos 2X$ at Point $(\frac{\pi}{2}, 0, -1)$

$f(X, y, z) = Z - e^{2y} \cos 2X = 0$

$f_x(X, y, z) = 2 e^{2y} \sin 2X \to f_x(\frac{\pi}{2}, 0, -1) = 0$

$f_y(X, y, z) = -2 e^{2y} \cos 2X \to f_y(\frac{\pi}{2}, 0, -1) = +2$

$f_z(X, y, z) = 1 \qquad \to f_z(\frac{\pi}{2}, 0, -1) = 1$

equation of The Tangent Plane:

$F_x(X - X_1) + F_y(y - y_1) + F_z(Z - Z_1) = 0$

$0 \quad +2(y - 0) + 1(Z + 1) = 0$

$2y + z + 1 = 0$

equation of The normal line:

$$\left.\begin{aligned} X &= \frac{\pi}{2} \\ y &= 2t \\ z &= -1 + t \end{aligned}\right\} \quad t \in R$$

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The surface S is given by:

$$x^2 - xy = z^2 \tan^{-1} xy - \pi$$

a. find the Parameteric equations of the normal line to S at Point $(1,1,-2)$

b. find The intersection of this normal line with (YZ-Plane).

a. $f(x,y,z) = x^2 - xy - z^2 \tan^{-1} xy + \pi = 0$

$f_x(x,y,z) = 2x - y - z^2 \cdot \frac{y}{1+(xy)^2} \rightarrow f_x(1,1,-2) = -1$

$f_y(x,y,z) = -x - z^2 \frac{x}{1+(xy)^2} \rightarrow f_y(1,1,-2) = -3$

$f_z(x,y,z) = -2z \tan^{-1} xy \rightarrow f_z(1,1,-2) = \pi$

equation of The normal line:

$$\left.\begin{aligned} x &= 1 - t \\ y &= 1 - 3t \\ z &= -2 + \pi t \end{aligned}\right\} t \in R$$

normal vector $\langle -1, -3, \pi \rangle$

b. The intersection Point is:

YZ-Plane: $(x = 0) \rightarrow x = 1 - t$

$0 = 1 - t \rightarrow (t = 1)$

$y = 1 - 3t = 1 - 3 = -2$

$z = -2 + \pi t = -2 + \pi \rightarrow$ (intersection Point) $(0, -2, -2+\pi)$

At what Point does the normal line to the surface : $z = x^2 + y^2$ at Point $(1, -1, 2)$ intersect the xy-Plane. (170)

$f(x, y, z) = x^2 + y^2 - z = 0$

$f_x = 2x \rightarrow f_x(1, -1, 2) = 2$

$f_y = 2y \rightarrow f_y(1, -1, 2) = -2$

$f_z = -1 \rightarrow f_z(1, -1, 2) = -1$

normal vector : $\langle 2, -2, -1 \rangle$

equation of The normal line :

$x = 1 + 2t \qquad y = -1 - 2t \qquad z = 2 - t$

intersect xy-Plane $\rightarrow (z = 0)$

$z = 2 - t \rightarrow 0 = 2 - t \rightarrow t = 2$

$$\left.\begin{array}{l} x = 1 + 2(2) \rightarrow x = 5 \\ y = -1 - 2(2) \rightarrow y = -5 \\ z = 2 - 2 \rightarrow z = 0 \end{array}\right\} (5, -5, 0)$$

find The Points on : $X^2-2y^2+3z^2=11$ (surface) at which the normal line to the surface is Perpendicular to the Plane : 171

$\pi : X-2y+6z=1$

$F = X^2-2y^2+3z^2-11=0$

$f_x = 2X \rightarrow f_x(X,y,z) = 2X$

$f_y = -4y \rightarrow f_y(X,y,z) = -4y$

$f_z = 6z \rightarrow f_z(X,y,z) = 6z$

normal vector for T. Plane :

$\langle 2X, -4y, 6z \rangle$

$\vec{n} = \langle 1, -2, 6 \rangle$ for vector Plane

$\langle 2X, -4y, 6z \rangle // \langle 1, -2, 6 \rangle$ (normal line // $\vec{n}$)

$$\left.\begin{aligned} 2X &= c \\ -4y &= -2c \\ 6z &= 6c \end{aligned}\right\} \quad \left.\begin{aligned} X &= \tfrac{1}{2}c \\ y &= \tfrac{1}{2}c \\ z &= c \end{aligned}\right\}$$ (in surface equation)

$X^2-2y^2+3z^2=11$

$(\tfrac{1}{2}c)^2-2(\tfrac{1}{2}c)^2+3c^2=11$

$\tfrac{1}{4}c^2-\tfrac{1}{2}c^2+3c^2=11$

$c^2-2c^2+12c^2=44$

$11c^2=44 \rightarrow c^2=4$

$(c=\pm 2)$

$c=2 : \rightarrow (X=1, y=1, z=2)$

$c=-2 : \rightarrow (X=-1, y=-1, z=-2)$

Find The Points on the surface:

$Z = 8 - 3X^2 - 2y^2$ at which The Tangent Plane is Perpendicular to the line:

$X = 2 - 6t \quad , \quad y = 7 + 8t \quad , \quad Z = 5 - t$

$$f(x, y, z) = -3X^2 - 2y^2 + 8 - Z = 0$$

$f_x = -6X \rightarrow f_x(X, y, z) = -6X$

$f_y = -4y \rightarrow f_y(X, y, z) = -4y$

$f_z = -1 \rightarrow f_z(X, y, z) = -1$

normal vector to T. Plane:

$$\left.\begin{aligned} X &= 2 - 6t \\ y &= 7 + 8t \\ z &= 5 - t \end{aligned}\right\}$$

$$\langle -6X, -4y, -1 \rangle \parallel \langle -6, 8, -1 \rangle$$

$-6X = -6C \rightarrow C = X$

$-4y = 8C \rightarrow C = -\frac{1}{2}y$

$-1 = -C \rightarrow C = 1$

$$\left.\begin{aligned} X &= 1 \\ y &= -2 \\ Z &= 8 - 3X^2 - 2y^2 \\ Z &= 8 - 3 - 8 \end{aligned}\right\}$$

Point $= (1, -2, -3) \leftarrow z = -3$

Find The Points on the hyperboloid of Two sheets : $X^2 - 2y^2 - 4z^2 = 16$ at which The Tangent Plane is Parallel To The Plane : $4X - 2y + 4z = 5$

$F(x, y, z) = X^2 - 2y^2 - 4z^2 - 16 = 0$

$F_x(X, y, z) = 2X \rightarrow F_x(X, y, z) = 2X$

$F_y(x, y, z) = -4y \rightarrow F_y(X, y, z) = -4y$

$F_z(X, y, z) = -8z \rightarrow F_z(X, y, z) = -8z$

normal vector $= \langle 2X, -4y, -8z \rangle$

- Tangent Plane Parallel to Plane :

$4X - 2y + 4z = 5$

(normal vector) Parallel (normal vector)

$\langle 2X, -4y, -8z \rangle$ Parallel $\langle 4, -2, 4 \rangle$

$2X = 4C \rightarrow x = 2C$

$-4y = -2C \rightarrow y = \frac{1}{2}C$

$-8z = 4C \rightarrow z = -\frac{1}{2}C$

Points on The surface :

$$X^2 - 2y^2 - 4z^2 = 16$$

$$4c^2 - \frac{1}{2}c^2 - c^2 = 16$$

$$\frac{5}{2}c^2 = 16 \rightarrow c^2 = \frac{32}{5} \rightarrow c = \pm 4\sqrt{\frac{2}{5}}$$

$c = 4\sqrt{\frac{2}{5}}$

$X = 8\sqrt{\frac{2}{5}}$

$y = 2\sqrt{\frac{2}{5}}$

$z = -2\sqrt{\frac{2}{5}}$

$P_1\left(8\sqrt{\frac{2}{5}}, 2\sqrt{\frac{2}{5}}, -2\sqrt{\frac{2}{5}}\right)$

$c = -4\sqrt{\frac{2}{5}}$

$X = -8\sqrt{\frac{2}{5}}$

$y = -2\sqrt{\frac{2}{5}}$

$z = 2\sqrt{\frac{2}{5}}$

$P_2\left(-8\sqrt{\frac{2}{5}}, -2\sqrt{\frac{2}{5}}, 2\sqrt{\frac{2}{5}}\right)$

Find the Points on the surface:

$$xy + yz + zx - x - z^2 = 0$$

where The Tangent Plane is Parallel to the (xy-Plane.)

$$f(x,y,z) = xy + yz + zx - x - z^2 = 0$$

$f_x(x,y,z) = y + z - 1$

$f_y(x,y,z) = x + z$

$f_z(x,y,z) = y + x - 2z$

normal vector

$$\underbrace{\langle y+z-1, x+z, y+x-2z \rangle}_{n_1}$$

xy-Plane: $\rightarrow$ $(z = 0)$ $\downarrow$ Plane

$$\langle f_x, f_y, f_z \rangle \parallel \underbrace{\langle 0, 0, 1 \rangle}_{n_2}$$

$$\langle y + z - 1, x + z, y + x - 2z \rangle \parallel \langle 0, 0, 1 \rangle$$

① $y + z - 1 = 0$
② $x + z = 0$
③ $y + x - 2z = c$
④ $xy + yz + zx - x - z^2 = 0$ $\rightarrow$ المعادلة الأساسية

$\rightarrow y = 1 - z$
$\rightarrow x = -z$

نعوضها in ④

$$xy + yz + zx - x - z^2 = 0$$

$$(-z)(1-z) + (-z+1)z + z(-z) + z - z^2 = 0$$

$$-\cancel{z} + \cancel{z^2} - \cancel{z^2} + \cancel{z} - z^2 + z - z^2 = 0$$

$$-2z^2 + z = 0$$

$$2z^2 - z = 0$$

$$z(2z-1) = 0 \rightarrow$$

$$z = 0 \qquad\qquad z = \frac{1}{2}$$

$$\downarrow \qquad\qquad \downarrow$$

$$x = 0 \qquad\qquad x = -\frac{1}{2}$$

$$\downarrow \qquad\qquad \downarrow$$

$$y = 1 \qquad\qquad y = \frac{1}{2}$$

$$(0, 1, 0) \qquad\qquad \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

The tangent line to the curve:

$r(t) = t^{-1}\,\vec{i} + (t-1)\,\vec{j} + t^2\,\vec{k}$ at the Point

$(1, 0, 1)$ contains the Point.

(A) $(-1, 2, 5)$ (B) $(-2, 3, 5)$ (C) $(3, -3, -2)$ (D) $(\frac{1}{2}, 1, 4)$ (E) $(1, 3, 2)$

$$r(t) = t^{-1}\,\vec{i} + (t-1)\,\vec{j} + t^2\,\vec{k}$$

$$r(t) = \langle t^{-1},\ t-1,\ t^2 \rangle$$

$$r'(t) = \langle -t^{-2},\ 1,\ 2t \rangle$$

Let : $r(t) = \langle t^{-1}, t-1, t^2 \rangle$

Point $(1, 0, 1) \rightarrow t - 1 = 0 \rightarrow (t = 1)$

$$r'(t) = \langle -t^{-2},\ 1,\ 2t \rangle$$

$$t = 1 \rightarrow r'(1) = \langle \underset{a}{-1},\ \underset{b}{1},\ \underset{c}{2} \rangle$$

Point $(1, 0, 1)$
$\quad x_0, y_0, z_0$

$$\left.\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}\right\} \quad \left.\begin{aligned} x &= 1 - t \\ y &= t \\ z &= 1 + 2t \end{aligned}\right\}$$

بالتعويض بالنقط المعطاة نجد أن الـ (A) هي المطلوبة

$$A(-1, 2, 5) \;\rightarrow\; \left.\begin{aligned} -1 &= 1 - t \rightarrow t = 2 \\ 2 &= t \qquad\quad\ \ t = 2 \\ 5 &= 1 + 2t \qquad t = 2 \end{aligned}\right\} \text{محققة}$$

Tangent Planes to level surface: (14.6)

$$F(x,y,z)=K \qquad (K) \text{ Constant}$$

Equation of the tangent plane:

$$F_x(x_0,y_0,z_0)\cdot(x-x_0)+F_y(x_0,y_0,z_0)\cdot(y-y_0)+F_z(x_0,y_0,z_0)\cdot$$
$$\cdot(z-z_0)=0$$

The normal line (symmetric equation):

$$\frac{x-x_0}{F_x(x_0,y_0,z_0)}=\frac{y-y_0}{F_y(x_0,y_0,z_0)}=\frac{z-z_0}{F_z(x_0,y_0,z_0)}$$

Find the equation of the tangent plane and normal line at $P(-2,1,-3)$ to the ellipsoid:

$$\frac{x^2}{4}+y^2+\frac{z^2}{9}=3$$

$$K=3 \rightarrow F(x,y,z)=\frac{x^2}{4}+y^2+\frac{z^2}{9}$$

$$\left.\begin{aligned} F_x(x,y,z)&=\frac{x}{2}\\ F_y(x,y,z)&=2y\\ F_z(x,y,z)&=\frac{2z}{9}\end{aligned}\right\} \begin{aligned} F_x(-2,1,-3)&=-1\\ F_y(-2,1,-3)&=2\\ F_z(-2,1,-3)&=-\frac{2}{3}\end{aligned}$$

The equation of the tangent line at $(-2, 1, -3)$:

$$F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+F_z(x_0,y_0,z_0)(z-z_0)=0$$

$$-1(x+2)+2(y-1)-\frac{2}{3}(z+3)=0$$

$$-x-\cancel{2}+2y-\cancel{2}-\frac{2}{3}z-\cancel{\frac{6}{3}}=0$$

$$-x+2y-\frac{2}{3}z-6=0$$

$$3x-6y+2z+18=0$$

symmetric equation of the normal line:

$$\frac{x-x_0}{F_x(x_0,y_0,z_0)}=\frac{y-y_0}{F_y(x_0,y_0,z_0)}=\frac{z-z_0}{F_z(x_0,y_0,z_0)}$$

$$\frac{x+2}{-1}=\frac{y-1}{2}=\frac{z+3}{-\frac{2}{3}}$$

Equation of the tangent plane:

(f) continuous function (14.4)

$Z = f(x, y)$; Point $P(X_0, y_0, z_0)$

$$Z - Z_0 = f_x(X_0, y_0)(X - X_0) + f_y(X_0, y_0)(y - y_0).$$

Find the tangent Plane to the elliptic Parboloid $Z = 2X^2 + y^2$ at $P(1, 1, 3)$.

$$Z = f(x, y) = 2X^2 + y^2$$

$$f_x(x, y) = 4X \to f_x(1, 1) = 4$$

$$f_y(x, y) = 2y \to f_y(1, 1) = 2$$

$$Z - Z_0 = f_x(X_0, y_0)(X - X_0) + f_y(X_0, y_0)(y - y_0)$$

$$(X_0, y_0, z_0) = (1, 1, 3)$$

$$Z - 3 = \underbrace{f_x(1, 1)}(X - 1) + \underbrace{f_y(1, 1)}(y - 1)$$

$$Z - 3 = 4(X - 1) + 2(y - 1)$$

$$Z - 3 = 4X - 4 + 2y - 2$$

$$Z = 4X + 2y - 6 + 3$$

$$Z = 4X + 2y - 3$$

Differential to Approximate:

(or)

Linear Approximation:

- $f(x,y,z) \simeq f(a,b,c) + df$
- $df = f_x\,(x-a) + f_y\,(y-b) + f_z\,(z-c)$
- $\Delta x = x - a \;;\; \Delta y = y - b \;;\; \Delta z = z - c$
- **(x, y, z):** هي القيم التي تحتوي على فاصلة أو هي الزاوية غير المشهورة
- **(a, b, c):** هي القيم الصحيحة القريبة من (x,y,z) أو هي الزاوية المشهورة.

Change in (y):

$$\Delta f \cong df = f_x\,(x-a) + f_y\,(y-b) + f_z\,(z-c)$$

Use differentials to approximate:

$$\frac{2}{\sqrt{(3.98)}} - 5\sqrt[5]{31.84}$$

$x = 3.98 \to a = 4$

$y = 31.84 \to b = 32$

$$f(x,y) = \frac{2}{\sqrt{x}} - 5\sqrt[5]{y} = 2x^{-\frac{1}{2}} - 5y^{\frac{1}{5}}$$

$$f(4,32) = \frac{2}{2} - 5(2) = 1 - 10 = -9$$

$$f_x(x,y) = 2\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} = -\frac{1}{\sqrt{x^3}}$$

$$* \; f_x(4,32) = -\frac{1}{8}$$

$$f_y(x,y) = -5\left(\frac{1}{5}\right)y^{-\frac{4}{5}} = -\frac{1}{\sqrt[5]{y^4}}$$

$$* \; f_y(4,32) = -\frac{1}{16}$$

$$df = f_x(x-a) + f_y(y-b)$$

$$= -\frac{1}{8}\left(-\frac{2}{100}\right) - \frac{1}{16}\left(-\frac{16}{100}\right)$$

$$= \frac{1}{4} \cdot \frac{1}{100} + \frac{1}{100}$$

$$df = \frac{1+4}{400} = \frac{5}{400} = \frac{1}{80}$$

$\Delta x = x - a$

$= 3.98 - 4$

$\Delta x = -0.02$

$\Delta y = y - b$

$= 31.84 - 32$

$\Delta y = -0.16$

$$f(3.98, 31.84) \simeq L(3.98, 31.84) = f(4,32) + df$$

$$= -9 + \frac{5}{400} = -9 + \frac{1}{80} = \frac{-720+1}{80} = -\frac{719}{80}$$

$$= -8.9875$$

Use differential to approximate the change in f. (149)

from $(-2,3)$ to $(-2.02, 3.01)$

$f(x,y) = x^2 - 3x^3y^2 + 4x - 3y^3 + 6$

$f(-2,3) = 4 - 3(-8)(9) + 4(-2) - 3(27) + 6$
$= 137$

$f_x(x,y) = 2x - 9x^2y^2 + 4$

$f_x(-2,3) = -4 - 324 + 4 = -324$

$f_y(x,y) = -6x^3y - 9y^2$

$f_y(-2,3) = 144 - 81 = 63$

$\Delta x = -2.02 + 2$ ($x - a$)

$\Delta x = -0.02$

$\Delta y = 3.01 - 3$ ($y - b$)

$\Delta y = 0.01$

$$df = f_x \cdot (x-a) + f_y(y-b)$$

$$= (-324)(-0.02) + (63)(0.01)$$

$$= 6.48 + 0.63$$

$$df = 7.11$$

$$f(-2.02, 3.01) \simeq L(-2.02, 3.01)$$

$$= f(-2,3) + df$$

$$= 137 + 7.11 = 144.11$$

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find linear approximate:

$$u(s,t) = (s+t)^2 + 3\cos(s-2) - 3t + 4$$

at $P(2,1)$ and use it to approximate

The value $u(2.01, 0.97)$

- $u(\overset{a}{2}, \overset{b}{1}) = 9 + 3\cos(0) - 3 + 4 = 13$

* $u_s(s,t) = 2(s+t) - 3\sin(s-2)$

  $u_s(2,1) = 6 - 3(0) = 6$

* $u_t(s,t) = 2(s+t) - 3$

  $u_t(2,1) = 2(3) - 3 = 6 - 3 = 3$

* $u(s,t) \simeq u(a,b) + u_s(s-a) + u_t(t-b)$

  $\simeq 13 + 6(s-2) + 3(t-1)$

  $\simeq 13 + 6s - 12 + 3t - 3$

$u(s,t) \simeq 6s + 3t - 2$

المطلب الثاني من السؤال :

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$$u(2.01, 0.97) \simeq L(2.01, 0.97)$$

$$= 6s + 3t - 2$$

$$= 6\left(\frac{201}{100}\right) + 3\left(\frac{97}{100}\right) - 2$$

$$= \frac{1206 + 291 - 200}{100} = \frac{1297}{100} = 12.97$$

find (L-A) : $f(x,y) = \sqrt{x^2+y^2}$ at $(3,4)$ and $f(3.02, 3.99)$

$f(3,4) = 5$; $f_x(x,y) = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}} = \frac{3}{5}$; $f_y(x,y) = \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}} = \frac{4}{5}$

(L-A) : $W + dW$; $w = f(a,b) = f(3,4) = 5$

$dw = f_x(x-3) + f_y(y-4) = \frac{3}{5}(x-3) + \frac{4}{5}(y-4) \rightarrow$ (L-A) : $W + dW$

The linear approximate of (f) : $\left(\frac{3}{5}x + \frac{4}{5}y - 5\right)$

$$f(3.02, 3.99) \simeq L(3.02, 3.99)$$

$$= \frac{3}{5}x + \frac{4}{5}y$$

$$= \frac{3}{5}\left(\frac{302}{100}\right) + \frac{4}{5}\left(\frac{399}{100}\right) = \frac{906 + 1596}{500}$$

$$= 5.004$$

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$f(x,y,z) = e^{xyz} + \frac{x}{y}$

a. (show that) f is differentiable at (2,1,0)

b. find (L-A)(f) at (2,1,0)

c. use (b) to find f(1.99, 1.01, -0.01)

a. $f_x = yze^{xyz} + \frac{1}{y} \Rightarrow f_x(2,1,0) = 1$

$f_y = xze^{xyz} - \frac{x}{y^2} \Rightarrow f_y(2,1,0) = -2$

$f_z = xye^{xyz} \Rightarrow f_z(2,1,0) = 2$

f_x, f_y, f_z (exist) and (f) continuous at (2,1,0) $\Rightarrow$ f differentiable at (2,1,0)

b. (L-A): (W+dW) ; $W = f(2,1,0) = 3$

$dw = f_x(x-a) + f_y(y-b) + f_z(z-c) = 1(x-2) - 2(y-1) + 2(z-0)$

(L-A): $3 + 1(x-2) - 2(y-1) + 2(z-0)$

$3 + x - 2 - 2y + 2 + 2z = x - 2y + 2z + 3$

The linear approximate (f): $x - 2y + 2z + 3$

c. $f(1.99, 1.01, -0.01) \simeq L(1.99, 1.01, -0.01)$

$= \frac{199}{100} - 2\frac{101}{100} - 2\frac{1}{100} + 3 = \frac{199 - 202 - 2}{100} + 3$

$= \frac{199 - 202 - 2 + 300}{100} = \frac{295}{100} = 2.95$

Let $f(x,y,z) = (y+z)e^{x+z}$

(a) Find an equation for the tangent Plane at the Point $(-1,2,1)$ to the surface $f(x,y,z) = 3$.

(b) use a linear approximation to estimate $f(-1.03, 2.01, 0.98)$.

(a)

$$f(x,y,z) = (y+z)e^{x+z}$$

$$f_x(x,y,z) = (y+z)e^{x+z}$$

$$f_y(x,y,z) = e^{x+z}$$

$$f_z(x,y,z) = e^{x+z} + (y+z)e^{x+z}$$

$$f_x(-1,2,1) = 3e^{0} = 3$$

$$f_y(-1,2,1) = e^{-1+1} = e^{0} = 1$$

$$f_z(-1,2,1) = e^{-1+1} + (2+1)e^{-1+1} = e^{0} + (3)e^{0}$$

$$= 1 + 3 = 4$$

equation of the tangent plane:

$$f_x(x-x_1) + f_y(y-y_1) + f_z(z-z_1) = 0$$

$$3(x+1) + 1(y-2) + 4(z-1) = 0$$

(b) $L(x,y,z) \approx f(x,y,z) + f_x(x-x_1)$
$+ f_y(y-y_1) + f_z(z-z_1)$

$$= 3 + 3(x+1) + (y-2) + 4(z-1)$$

From $(\overset{a}{-1}, \overset{b}{2}, \overset{c}{1})$ to $(\overset{x}{-1.03}, \overset{y}{2.01}, \overset{z}{0.98})$

$\Delta x = x - a \rightarrow \Delta x = -1.03 + 1 = -0.03$
$\Delta y = y - b \rightarrow \Delta y = 2.01 - 2 = 0.01$
$\Delta z = z - c \rightarrow \Delta z = 0.98 - 1 = -0.02$

$$L(x,y,z) \approx 3 + 3(-0.03) + 0.01 + 4(-0.02)$$
$$\approx 3 - 0.09 + 0.01 - 0.08$$
$$\approx 3 - 0.08 - 0.08$$
$$\approx 3 - 0.16 = 2.84$$

Let: $f(x,y,z) = x^2\sqrt{y} + \sqrt{9+z^2}$ (152)

(a): find linear approximate of (f) at Point $(1,4,4)$

(b): use Part (a) to find approximate Value of $(0.9)^2\sqrt{4.1} + \sqrt{9+(3.9)^2}$

$$f(\overset{a}{1},\overset{b}{4},\overset{c}{4}) = 7$$

$$\left.\begin{aligned} f_x(x,y,z) &= 2x\sqrt{y} \\ f_y(x,y,z) &= x^2\frac{1}{2\sqrt{y}} \\ f_z(x,y,z) &= \frac{2z}{2\sqrt{9+z^2}} \end{aligned}\right\} \begin{aligned} f_x(1,4,4) &= 4 \\ f_y(1,4,4) &= \frac{1}{4} \\ f_z(1,4,4) &= \frac{8}{10} = \frac{4}{5} \end{aligned}$$

- $f(x,y,z) \simeq f(a,b,c) + df$
- $df = f_x(x-a) + f_y(y-b) + f_z(z-c)$

$$df = 4(x-1) + \frac{1}{4}(y-4) + \frac{4}{5}(z-4)$$

$$df = 4x - 4 + \frac{1}{4}y - 1 + \frac{4}{5}z - \frac{16}{5}$$

$$df = 4x + \frac{1}{4}y + \frac{4}{5}z - \frac{41}{5}$$

- $f(x,y,z) \simeq \boxed{7} + 4x + \frac{1}{4}y + \frac{4}{5}z - \frac{41}{5} = 4x + \frac{1}{4}y + \frac{4}{5}z - \frac{6}{5}$

الطلب (b):

(b):

$$f(0.9, 4.1, 3.9) \simeq L(0.9, 4.1, 3.9)$$

$$= 4x + \frac{1}{4}y + \frac{4}{5}z - \frac{6}{5}$$

$$= 4\left(\frac{9}{10}\right) + \frac{1}{4}\left(\frac{41}{10}\right) + \frac{4}{5}\left(\frac{39}{10}\right) - \frac{6}{5}$$

$$= \frac{36}{10} + \frac{41}{40} + \frac{156}{50} - \frac{6}{5}$$

$$= \frac{720 + 205 + 624 - 240}{200}$$

$$= \frac{1309}{200} = 6.545$$

find linear Approximate of:

$f(x,y,z) = y\sin x + 2yz + 6$ at $P(0,1,0)$ and use it to find approximate Value for $f(0.01, 0.98, -0.02)$

- $f(x,y,z) \simeq f(a,b,c) + df$

$*$ $f(\overset{a}{0},\overset{b}{1},\overset{c}{0}) = (1)\sin(0) + 2(1)(0) + 6 = \boxed{6}$

$f_x(x,y,z) = y\cos x$ — $f_x(0,1,0) = 1$

$f_y(x,y,z) = \sin x + 2z$ — $f_y(0,1,0) = 0$

$f_z(x,y,z) = 2y$ — $f_z(0,1,0) = 2$

$*$ $df = f_x(x-a) + f_y(y-b) + f_z(z-c)$

$df = 1(x-0) + 0(y-1) + 2(z-0)$

$df = \boxed{x + 2z}$

- $f(x,y,z) \simeq 6 + x + 2z$

$f(0.01, 0.98, -0.02) \simeq 6 + \frac{1}{100} + 2\left(-\frac{2}{100}\right)$

$\simeq \frac{597}{100}$

$f(x,y,z) = xy^2 + x^2\sqrt{1+z}$ at $(2,-1,3)$

and use it to approximate $f(2.1,-0.98,3.04)$

- $f(\overset{a}{2}, \overset{b}{-1}, \overset{c}{3}) = 2 + 8 = 10$

$$\left.\begin{aligned} f_x(x,y,z) &= y^2 + 2x\sqrt{1+z} \\ f_y(x,y,z) &= 2xy \\ f_z(x,y,z) &= x^2 \frac{1}{2\sqrt{1+z}} \end{aligned}\right\} \begin{aligned} f_x(2,-1,3) &= 1+8=9 \\ f_y(2,-1,3) &= -4 \\ f_z(2,-1,3) &= 1 \end{aligned}$$

- $f(x,y,z) \simeq f(a,b,c) + f_x(x-a) + f_y(y-b) + f_z(z-c)$

$f(x,y,z) \simeq 10 + 9(x-2) - 4(y+1) + 1(z-3)$

$\simeq 10 + 9x - 18 - 4y - 4 + z - 3$

$f(x,y,z) \simeq 9x - 4y + z - 15$

$df = f_x(x-a) + f_y(y-b) + f_z(z-c)$

$df = 9(x-2) - 4(y+1) + 1(z-3)$

$df = 9x - 18 - 4y - 4 + z - 3$

$df = 9x - 4y + z - 25$

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الطلب (b) :

(155)

(b) :

$$f(2.1,-0.98,3.04) \approx L(2.1,-0.98,3.04)$$

$$= 9X - 4y + z - 15$$

$$= 9\left(\frac{21}{10}\right) - 4\left(\frac{-98}{100}\right) + \frac{304}{100} - 15$$

$$= \frac{189}{10} + \frac{392}{100} + \frac{304}{100} - 15$$

$$= \frac{1890 + 392 + 304 - 1500}{100}$$

$$= \frac{1086}{100} = 10.86$$

Find The linearization of (f):

$$f(x,y) = \frac{x^2}{2} + \frac{y^2}{4} + xy + 3\cos(x-2) - 3y + 4$$

at $P(2,1)$

- $f(2,1) = 2 + \frac{1}{4} + 2 + \cancel{3(1)} \cancel{-3} + 4$

  $f(2,1) = 8 + \frac{1}{4} = 8\frac{1}{4} = \frac{33}{4}$

- $f_x(x,y) = x + y - 3\sin(x-2)$

  $f_x(2,1) = 2 + 1 - 3(0) = 3$

- $f_y(x,y) = \frac{y}{2} + x - 3$

  $f_y(2,1) = \frac{1}{2} + 2 - 3 = \frac{5}{2} - 3 = -\frac{1}{2}$

$$f(x,y) \approx L(x,y)$$

$$= f(2,1) + f_x(2,1)\underbrace{(x-2)}_{\Delta x} + f_y(2,1)\underbrace{(y-1)}_{\Delta y}$$

$$f(x,y) \approx \frac{33}{4} + 3(x-2) - \frac{1}{2}(y-1).$$