

**CFA<sup>®</sup>**

# Fundamentals

2<sup>nd</sup> Edition



**KAPLAN**  **SCHWESER**

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# CFA<sup>®</sup> FUNDAMENTALS, 2ND EDITION

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CFA FUNDAMENTALS, 2ND EDITION

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# FOREWORD

## Who Should Use this Book?

This book's primary function is to aid those who are interested in becoming CFA Charterholders but feel they might need a refresher before they start studying for the Level I exam. This text will help you gain (or regain) the background necessary to begin your studies in the CFA Program.

A requirement for entry into the CFA Program™ is a bachelor's degree from a four-year college or university. (Graduating seniors may also enter the program; see the Prospective CFA Candidates link at [www.cfainstitute.org](http://www.cfainstitute.org) for additional information). A business degree, however, is not specifically required. This means CFA candidates come from different educational backgrounds and many walks of life.

## What is the Structure of the CFA Program?

CFA Institute, the governing body that creates and administers the CFA exams, is headquartered in Charlottesville, Virginia. In July of each year, CFA Institute releases the curriculum for the following year's exams. The curriculum changes every year to reflect changes in the field of investment management and innovations in financial markets.

The CFA Program is a series of three examinations. Level I is administered twice per year, on the first Saturday of June and the first Saturday of December. Levels II and III are administered only once per year on the first Saturday of June. Each exam is a 6-hour experience (three hours in the morning and three hours in the afternoon), and candidates must pass the exams in order.

- *Level I-Investment Tools* (100% multiple choice): The Level I curriculum focuses on the tools of investment finance. The main topic areas are: Ethical and Professional Standards; Quantitative Methods; Economics; Financial Reporting and Analysis; Corporate Finance; Equity Investments; Fixed Income; Derivatives; Alternative Investments; and Portfolio Management. The multiple choice questions will be in sections according to these headings. The exam consists of 240 multiple-choice questions, 120 in the morning session and 120 in the afternoon session. All topic areas are tested in each session.

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- *Level II-Analysis and Valuation* (100% selected response item set): In an item set, you are given a vignette (typically one to three pages long) and a set of six multiple choice questions that relate to that vignette. The Level II exam consists of 20 item sets (120 questions), ten item sets in the morning session and ten in the afternoon session. The topic areas are the same as Level I, but not all topics are tested in each session of the Level II exam.

The purpose of the Level II exam is to apply and expand on the tools that were introduced at Level I. For example, at Level I, you learn the basics of derivative securities. The Level II derivatives curriculum is much more in-depth, introducing additional tools and instruments and applying those tools to investment valuation. In general, questions focus on a deeper treatment of the material than the Level I exam.

- *Level III-Synthesis and Portfolio Management* (combination constructed response essay and selected response item-set): The morning session of the Level III exam is in constructed response essay format. You are given a scenario and asked to answer several questions relating to it. The answer can range from one or two words to a paragraph or a calculation. There are usually ten to fifteen multi-part constructed response essay questions in the morning session of the exam. The questions vary from length and value and total 180 points. The afternoon session has ten 18-point (six question) selected response item sets, similar to the format for Level II.

The main focus of the Level III exam is portfolio management. You will use the tools and analysis from the previous two levels to develop investment policies and appropriate portfolios for both individual and institutional investors (pension funds, endowments, life insurance companies, etc.). Also, you will learn to protect existing investment positions from the effects of market volatility (risk) through the use of derivative instruments (hedging). Perhaps because of its coverage or simply because it is the final level, many CFA candidates consider Level III the most enjoyable of the three levels. That doesn't mean it is easy, though. Like Levels I and II, the Level III exam is not to be taken lightly.

In addition to passing the three exams, candidates must also have four years of relevant work experience to be awarded a Charter.

## How Do I Register for the CFA Program?

To register for the CFA Program you need to complete an application form that can be obtained from the CFA Institute Web site, [www.cfainstitute.org](http://www.cfainstitute.org).

## How Can Kaplan Schweser Help Me Achieve My Goals?

Kaplan Schweser is the premier provider of study tools for the CFA examination. Our core product is our SchweserNotes, but we also offer live seminars around the world and a world-class online educational program; our extensive SchweserPro question bank; two printed volumes of Practice Exams for each level; and our Live Mock Exam at locations around the world, two weeks before each CFA exam. Please visit our Web site at [www.schweser.com](http://www.schweser.com) for more information.

# QUANTITATIVE METHODS

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## AN INTRODUCTION TO ALGEBRA

The best place to develop an understanding of quantitative methods is with basic algebra. *Webster Collegiate Dictionary*<sup>1</sup> defines algebra as “any of various systems or branches of mathematics or logic concerned with the properties and relationships of abstract entities manipulated in symbolic form under operations often analogous to those of arithmetic.” This simply means we can use a letter in the place of a number to symbolize an unknown value in an equation.<sup>2</sup>

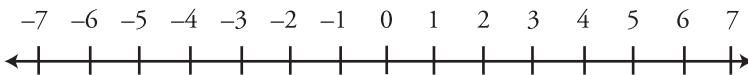
An equation shows the relationship between two or more variables.<sup>3</sup> It is analogous to a perfectly balanced lever with an equal sign acting as the fulcrum. Whatever is on the left of the equal sign is balanced with (i.e., equals) whatever is on the right (e.g.,  $a = 6$ ).

Before we actually solve several sample equations, let’s look at some mathematical properties that will make algebra easier for us. After stating the rules, we’ll use those rules to solve algebraic equations. We will begin with a brief review of the properties of negative numbers.

**Negative Numbers.** Think of all possible numbers as sitting on a continuous line with zero at the exact center.

Figure 1: Number Line

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If you think of numbers this way, you can see there is a negative number for every possible positive number. And each number and its counterpart are spaced exactly the same distance from zero, just on opposite sides. Thus adding a number and its exact opposite (i.e., its negative counterpart) yields a value of zero. Adding three and negative three together yields zero. Adding five and negative five together yields zero, too. Even adding 1,999,999 and  $-1,999,999$  together yields zero.

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1 *Merriam Webster’s Collegiate Dictionary*, 10th ed., 2000.

2 In algebra, an unknown value is symbolized by a letter whose value we calculate by solving an equation.

3 A variable is a mathematical symbol (letter) in an equation that represents something (e.g., cost, size, length).

Here are some basic properties for working with negative numbers:

**Property 1:** *When adding a positive and a negative number, the sign of the sum is the same as the sign of the number with the largest absolute value.*<sup>4</sup>

Consider the equation  $x = (-4) + (+3)$ . Since the absolute value of negative four is greater than the absolute value of positive three, the sum has a negative sign. The answer to the equation is  $x = (-4) + (3) = -1$ .

**Property 2:** *When subtracting one number from another, the sign of the second number is changed, and the two numbers are added together.*

Thus,  $3 - (+4) = -1$  can be rewritten as  $3 + (-4) = -1$ . Similarly,  $3 - (-4) = 7$  can be rewritten as  $3 + 4 = 7$ .

**Property 3:** *Multiplying an even amount of negative numbers yields a product with a positive sign. Multiplying an odd number of negative numbers yields a product with a negative sign.*

Look at some examples:

	# of negative signs
$(-1) \times (-1) = +1$	2
$(-1) \times (-1) \times (-1) = -1$	3
$(-1) \times (-1) \times (-1) \times (-1) = +1$	4
$(-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$	5

Notice the number of negative signs in each equation. You will see that when there is an even number, the product has a positive sign, and when there is an odd number, the product has a negative sign.

**Algebraic Rules.** Now that we have reviewed the properties of negative numbers, let's focus our attention on the properties and rules governing algebraic equations. The intent of this section is to refresh your memory on algebraic manipulation that will be necessary to solve complex problems on the CFA exam.

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<sup>4</sup> Absolute value is the value of the number, ignoring the sign (e.g., the absolute value of +3 is 3, and the absolute value of -3 is also 3).



## Quantitative Methods

**Rule 1:** We can multiply or divide all terms<sup>5</sup> on both sides of an equation by the same letter or number without changing the relationship expressed by the equation or the value of the unknown variables.

Let's multiply both sides of Equation 1 by the number 5.

$$a = 6 \tag{1}$$

$$5 \times a = 5 \times 6 \Rightarrow 5a = 30 \Rightarrow a = 6$$

By multiplying each side of Equation 1 by five, we did not alter the basic relationship between the left-hand and right-hand side of the equation.<sup>6</sup> In fact, we could have multiplied every term in the equation by any number or any letter without changing the relationship expressed by the equation. In every case, the unknown value, signified by the letter *a* in Equation 1, would still equal 6.

We can also *divide* both sides of the equation by a number or letter without altering the relationship expressed by the equation. Recall Equation 1:

$$a = 6 \tag{1}$$

This time let's divide both sides by the number *one*.

$$\frac{a}{1} = \frac{6}{1} \Rightarrow a = 6$$

Dividing by *one* does not change the relationship expressed in Equation 1 since the value of *a* remains unchanged. To make the example easy, we relied on a very basic rule. Any term (number or letter) multiplied *or* divided by *one* equals the original number or letter. So:

$$6 \times 1 = 6 \text{ and } \frac{6}{1} = 6$$

Stated differently:

$$a \times 1 = a \text{ and } \frac{a}{1} = a$$

---

5 *Term* is the mathematical expression for an entry in an equation. Equation 1 only has two terms: the letter *a* on the left side of the equation and the number 6 on the right.

6 A number in combination with (multiplied by) a letter is referred to as the *coefficient* of the letter (the variable the letter represents). In this case, 5 is the coefficient of *a*.

Even though we used the number one in the example, we could have divided every term in Equation 1 by any number or letter and the result would have been the same. In every case, the letter  $a$  would have the same value (i.e., 6 in this case).

**Rule 2:** *We can add (or subtract) the same letter or number from both sides of an equation without changing the relationship expressed by the equation or the value of the unknown.*

Let's look again at Equation 1, and this time we will add and subtract a number from both sides.

$$a = 6 \tag{1}$$

Adding and subtracting *one*,

$$a + 1 = 6 + 1$$

or

$$a - 1 = 6 - 1$$

The best way to solve an algebraic equation with one unknown is to collect all terms with the unknown on the left of the equal sign (the left side of the equation) and to collect all known values on the right side of the equation.

$$a + 1 = 6 + 1 \Rightarrow a = 6 + 1 - 1 \Rightarrow a = 6$$

$$a - 1 = 6 - 1 \Rightarrow a = 6 - 1 + 1 \Rightarrow a = 6$$

Every time you move a variable or number from one side of the equation to the other, its sign changes (i.e., a negative becomes positive or a positive becomes negative).

Notice that in both previous cases, the “1” on the left-hand side of the equation changed signs when we moved the number to the right side of the equation. When we added one to the left side of the equation and then moved it to the right, it went from positive to negative. When we subtracted one from the left side and then moved it to the right, it went from negative to positive. In both cases the positive and negative ones on the right side of the equation canceled each other out (sum to

## Quantitative Methods

zero).<sup>7</sup> In fact, if we add any number or letter to, or subtract any number or letter from both sides of the equation, we observe the same outcome. The additional numbers or letters will cancel each other out on the right side of the equation, and the value of the unknown will be left unchanged.

Now that we have a few rules to make algebra easier for us, we'll proceed to a few examples. Consider the following: If apples cost \$0.25 each and you have \$1.00, how many apples can you buy?

Since each apple costs \$0.25, if you multiply that price by the number of apples you buy, the total cannot be greater than \$1.00. We will let the letter A represent the number of apples you buy. Assume you buy one apple. That means  $A = 1$  and you spend  $1 \times \$0.25 = \$0.25$ . If you buy two apples,  $A = 2$  and you spend  $2 \times \$0.25 = \$0.50$ . If  $A = 3$  you spend \$0.75, and if  $A = 4$  you spend exactly \$1.00. Thus the answer is 4. You can buy a maximum of 4 apples with your \$1.00, as long as they cost \$0.25 each.

We can formalize the relationship between the amount of money available to purchase apples, the cost of apples, and the number of apples purchased in the following algebraic equation:

$$A = \frac{M}{C} = \frac{\$1.00}{\$0.25} = 4 \quad (2)$$

where:

A = the number of apples you can buy (the unknown)

M = the maximum number of money you can spend (\$1.00)

C = the cost per apple (\$0.25)

We could state the problem another way. How much would four apples cost if each apple costs \$0.25. If apples cost \$0.25 each, four apples will cost \$1.00. This modification can be shown by rearranging Equation 2 into Equation 2a:

$$A = \frac{M}{C} \Rightarrow \quad (2)$$

$$M = AC \Rightarrow M = 4 \times 0.25 = \$1.00 \quad (2a)$$

---

<sup>7</sup> The term *sum* simply means to add numbers together. In this case, we summed (added) one and negative one, which equals zero.

In transforming Equation 2 into Equation 2a, we have utilized Rule 1 by multiplying each side of Equation 2 by the letter  $C$ :

$$CA = \frac{CM}{C} \Rightarrow CA = M$$

We rewrite this equation as follows:

$$M = AC$$

The letter  $M$  is the unknown, and by convention, we put the unknown variable on the left side of the equation. Of course, the result is exactly the same either way.

The equation is equally valid in either direction, so it really doesn't matter which way we write it. That also holds for the direction we multiply the variables. Notice that we rearranged the combination  $CA$  to  $AC$ . This is another simple trick, which makes absolutely no difference to the value of  $M$ . You may confirm this relationship by multiplying three times two and then multiplying two times three. You'll get six in either case. This property holds, regardless of the numbers or variables used, *as long as you're multiplying*.

Now, let's look at the same problem from another perspective. Let's assume you already purchased eight apples for \$2.40.

What was the cost per apple? To solve this problem, let's set up Equation 2 exactly as before:

$$A = \frac{M}{C} \tag{2}$$

where:

$A$  = the number of apples you bought (8)

$M$  = the amount of money you spent (\$2.40)

$C$  = the cost per apple (the unknown)

This time, however, we know how much money you spent (\$2.40) and how many apples you bought (8). Again, we substitute values for letters wherever we can. The result is:

$$8 = \frac{\$2.40}{C}$$

We face a new challenge. How do you solve for  $C$  when it's in the denominator (i.e., bottom) on the right side of the equation and you would like it in the

## Quantitative Methods

numerator (i.e., top) on the left side? To get the  $C$  out of the denominator on the right side of the equation, we'll utilize Rule 1 and multiply each side of the equation by  $C$ :

$$C \times 8 = C \times \frac{\$2.40}{C}$$

The  $C$ s on the right side of the equation cancel each other out. Remember, any number or algebraic letter divided by itself equals *one*, so  $\frac{C}{C} = 1$ . The equation now reads:

$$8C = \$2.40$$

Now we again must utilize Rule 1 and divide each side of the equation by the number eight in order to isolate the unknown,  $C$ , on the left side of the equation:

$$\frac{8 \times C}{8} = \frac{\$2.40}{8} \Rightarrow C = \frac{\$2.40}{8} = \$0.30$$

You might have noticed we have designated “multiplied by” in two different ways in our example. First we used  $AC$  to mean  $A$  times  $C$ . Then we used  $C \times 8$  to mean  $C$  times eight. Each of these ways is entirely appropriate. In fact, we could also have written  $A(C)$  or  $(A)(C)$  to indicate  $A$  times  $C$ . So there are at least four ways to write exactly the same thing.

Here's another algebra example related to apples. You have a total of \$5.00 to spend. If the apples cost \$0.79 per pound, how many pounds can you buy? Letting:

- $P$  = the number of pounds you can buy (unknown)
- $C$  = the cost per pound (\$0.79)
- $M$  = the total amount of money you can spend (\$5.00)

The amount of money you can spend,  $M$ , divided by the price per pound,  $C$ , will yield the number of pounds you can purchase,  $P$ . Let's set up the equation.

$$P = \frac{M}{C} \Rightarrow P = \frac{\$5.00}{\$0.79} = 6.33 \tag{3}$$

Now let's alter this problem to determine the maximum amount of apples you can buy for \$5.00. We know you cannot spend more than \$5.00, but you can certainly spend less if you prefer. We also know that the number of pounds you buy,  $P$ ,

multiplied by the cost per pound,  $C$ , is the total amount you spend. Let's set up that equation:

$$P \times C \leq M \quad (3a)$$

Equation 3a is interpreted as follows: "The number of pounds you buy times the cost per pound must be less than or equal to the total money you can spend." In Equation 3a, notice we have replaced the equal sign with an inequality. In this case the sign does not say the left side of the equation *must* equal the right side. Instead it says the left side must be *equal to or less than* the right side.<sup>8</sup> An inequality sign has some properties that are the same as an equal sign and other properties that are different. The following section demonstrates the similarities and dissimilarities.

Let's put some numbers in Equation 3a:

$$P \times C \leq M \Rightarrow P \times \$0.79 \leq \$5.00$$

$$P \leq \frac{\$5.00}{\$0.79} \Rightarrow P \leq 6.33$$

Notice we get exactly the same *numerical* answer as before. When we solved for  $P$  the first time we got exactly 6.33 pounds. Now, however, the answer to the equation tells us the amount purchased,  $P$ , can be a range of values. It can be any value *equal to or less than* 6.33 pounds. In other words, with just \$5.00 to spend, you cannot buy more than 6.33 pounds, but you can certainly buy less than 6.33 pounds if you wish.

A word of caution is appropriate here. Even though the  $\leq$  sign didn't really change the answer to our problem, it is interpreted quite differently from the  $=$  sign. For instance, we used  $a = 6$  in the very beginning as an example of an equation. The equal sign is interpreted as saying  $a$  is equal to six and no other value. That is a strict equality.

If we substitute  $\leq$  for  $=$ , the meaning of the equation changes dramatically because the two signs actually ask two different questions. The  $=$  asks for a specific value. The  $\leq$  sign asks for a *range* of values, including the maximum value (or upper limit).

---

<sup>8</sup> It is helpful to think of the point of the inequality sign as pointing to the side of the equation that has the smaller value.

## Quantitative Methods

The equation  $a \leq 6$  is interpreted as saying the letter  $a$  can have any value as long as it is equal to or less than 6. The unknown,  $a$ , could have a value that would include all negative values. We could change the  $=$  sign in our example to a  $\leq$  sign because we wanted to know the absolute maximum we could buy, but we could buy a smaller amount if we wanted.

**Rule 3:** *Multiplying or dividing an inequality by a negative number changes the direction of the inequality.*

Remember that an inequality is denoted by the less than or equal to  $\leq$  sign or greater than or equal to  $\geq$  sign. Multiplying or dividing the inequality by a negative number changes the direction of the inequality. For example:

$$-3x \leq 9 \Rightarrow \frac{(-3x)}{-3} \leq \frac{(9)}{-3} \Rightarrow x \geq -3$$

Observe that in the preceding equation, the inequality sign changes direction after dividing both sides of the equation by negative three. Now let's take the previous result and multiply by negative three:

$$x \geq -3 \Rightarrow -3(x) \geq -3(-3) \Rightarrow -3x \leq 9$$

Notice that after multiplying by negative three, the equation is right back where it began.

**Parentheses.** Parentheses are used to group together variables and numbers that should be considered together in the equation. Consider the following equation:

$$3(x + 4) = 15 \tag{4}$$

We interpret this equation as *three* times the quantity<sup>9</sup>  $(x + 4)$  equals 15. We can treat the terms in the parentheses as if they are a single term. That means we can divide both sides of the equation by *three* (using Rule 1) and solve for the unknown.

$$\frac{3(x + 4)}{3} = \frac{15}{3} \Rightarrow (x + 4) = 5$$

---

<sup>9</sup> Quantity is simply the mathematical expression used to denote the parentheses and everything inside them.

Once the *three* is no longer outside the parentheses on the left side of the equation, we can simply eliminate the parentheses and solve for the unknown.

$$x + 4 = 5 \Rightarrow x = 5 - 4 = 1$$

Let's look at a more complex equation:

$$3 = \frac{15}{(x + 4)} \quad (4a)$$

At first this may appear difficult, but it's actually just another way of writing Equation 4. We can see this by multiplying both sides of Equation 4a by the quantity  $(x + 4)$  as follows:

$$(x + 4) \times 3 = \frac{(x + 4) \times 15}{(x + 4)} \Rightarrow 3(x + 4) = 15 \quad (4)$$

To get the quantity  $(x + 4)$  out of the denominator on the right side of the equation, we multiplied both sides of the equation by  $(x + 4)$  as if it were a single term. That leaves  $3(x + 4)$  on the left side of the equation, and since  $\frac{x + 4}{x + 4}$  on the right cancels to 1, we are left with only the 15 on the right side of the equation. The result is Equation 4, which we solved earlier.

There is another way to solve Equation 4. As an alternative, we can multiply everything inside the parentheses by *three* (multiply through by *three*<sup>10</sup>) instead of dividing both sides of the equation by *three*. Note that we get exactly the same answer. This is because we are solving the equation in two different ways without changing the relationship of the equation in any way.

$$3(x + 4) = 15 \Rightarrow \quad (4)$$

$$3x + 12 = 15 \Rightarrow 3x = 15 - 12 \quad (4b)$$

$$3x = 3 \Rightarrow x = 1$$

---

<sup>10</sup> This is also known as distributing. The three, in this case, gets distributed across every number or variable inside the parentheses.



## Quantitative Methods

Now let's assume you begin with the form in Equation 4b. How would you go about solving it? Let's *factor out* a number common to all the terms on the left side of the equation.<sup>11</sup> To factor the left side of the equation we'll find the largest number that divides evenly into both  $3x$  and  $12$ . The largest number is *three*, so we'll factor *three* out of both terms.

$$3x + 12 = 15 \Rightarrow 3(x + 4) = 15$$

Dividing both sides of the equation by *three*,

$$\frac{3}{3}(x + 4) = \frac{15}{3} \Rightarrow (x + 4) = 5$$

$$x + 4 = 5 \Rightarrow x = 5 - 4 \Rightarrow x = 1$$

It should be clear from the solution that factoring the left side of the equation did not alter the value of the unknown.

**Exponents.** Algebraic equations become slightly more complicated when an exponent affects one or more of the variables as in Equation 5:

$$x^2 = 9 \tag{5}$$

Equation 5 is interpreted as “x squared<sup>12</sup> equals nine.” The solution for the value of  $x$  is that number which when multiplied by itself (squared) equals *nine*. You may be able to determine that the answer is *three*, since  $3 \times 3 = 9$ .

Just in case you're confronted with more difficult equations involving exponents, let's see how to solve Equation 5. In order to solve for a variable, the exponent associated with that variable must equal *one*.<sup>13</sup> In this example, in order to change the value of the exponent to one, we can take the square root of both sides of the equation. Since we used easy numbers, the equation is easy to solve as follows:

$$\sqrt{x^2} = \sqrt{9} \Rightarrow x^1 = 3^1 \Rightarrow x = 3$$

---

11 Factors are simply numbers that when multiplied together yield the original number. The numbers 3 and 5 are factors of 15, since  $3 \times 5 = 15$ . Likewise, 2 and 4 are the factors of 8. A *prime number* is divisible only by itself and 1. The number 3 has no factors other than 1 and 3, so it is considered a prime number.

12 Squaring a number simply means multiplying it by itself. For instance 2 squared is  $2 \times 2 = 4$ .

13 Any number, variable, or quantity raised to the power of one is equal to itself. In other words,  $5^1 = 5$ . Five raised to the power of one is equal to five.

Another way to designate square root is with the exponent  $\frac{1}{2}$ . Let's try solving the equation using that method. Instead of using the square root sign, or radical, we'll use  $\frac{1}{2}$  as an exponent.

$$(x^2)^{1/2} = (9)^{1/2}$$

We're trying to find the square root of  $x^2$  and the square root of *nine*. When you see a term with more than one exponent, simply multiply the exponents together. In this example, when you multiply the exponents associated with the  $x$  variable, you get an exponent of  $2 \times \frac{1}{2} = 1$  and  $x^1$  or simply  $x$  on the left of the equation. Since there is an implied exponent of *one* associated with any number or letter, we can say we actually have  $(9^1)^{1/2}$  on the right side of the equation. Again we multiply the exponents and get  $1 \times \frac{1}{2} = \frac{1}{2}$ , leaving us with  $9^{1/2}$  on the right side of the equation:

$$x = 9^{1/2} = 3$$

Since an exponent of  $\frac{1}{2}$  indicates square root, we're back to our original statement;  $x$  equals the square root of *nine*.

**Parentheses and Exponents.** The next logical step is to solve an equation in which we have a set of parentheses with an exponent. Remember that quantities within parentheses can be treated like a single term. Consider the following equation:

$$(x + 5)^3 = 8 \tag{6}$$

Keep in mind that in order to solve for a variable, its exponent must equal *one*. In this example we can accomplish this by taking the cube root (third root) of both sides.<sup>14</sup> Remember, we're going to treat the quantity inside the parentheses just like a single term and multiply the exponents, as we did before.

$$[(x + 5)^3]^{1/3} = 8^{1/3} \Rightarrow x + 5 = 8^{1/3}$$

$$x + 5 = 2 \Rightarrow x = -3$$

---

<sup>14</sup> The cube, or third root of a number, is the number that, when multiplied by itself three times, equals the original number. For example, the cube root of 1,000 is 10, since  $10 \times 10 \times 10 = 1,000$ .

## SYSTEMS OF EQUATIONS

A somewhat more advanced use for algebra is finding the value of two unknown variable terms. When dealing with two unknowns, we will use two equations,<sup>15</sup> and we will solve them “simultaneously.” Let’s say you are faced with the following two equations:

$$3x + 4y = 20 \text{ and } x + 3y = -4$$

Even though it might seem like quite a daunting task, solving for these two unknowns,  $x$  and  $y$ , is actually just a matter of following a very logical series of steps while utilizing the rules of algebra.

*Step 1:* Start by setting up the equations as if you were going to add them together like two numbers.

$$\begin{array}{r} 3x + 4y = 20 \\ + x + 3y = -4 \\ \hline = ? \end{array}$$

*Step 2:* Using Rule 1, we multiply all the terms in one or both of the equations so that adding or subtracting them will eliminate one of the variables. In our case we multiply the bottom equation by minus three:

$$\begin{array}{r} 3x + 4y = 20 \\ -3 \times (x + 3y = -4) \\ \hline = ? \end{array} \quad \Rightarrow \quad \begin{array}{r} 3x + 4y = 20 \\ -3x - 9y = 12 \\ \hline = ? \end{array}$$

*Step 3:* Add the two equations together by adding the  $x$  terms, the  $y$  terms, and the numbers.<sup>16</sup>

$$\begin{array}{r} 3x + 4y = 20 \\ -3x - 9y = 12 \\ \hline 0x - 5y = 32 \end{array}$$

<sup>15</sup> It is impossible to find the values for two unknowns with less than two equations.

<sup>16</sup> The reason we multiply one of the equations by a constant (a number) and then add them together is to eliminate one of the unknowns. Whether you add or subtract the equations is simply a matter of choice.

Multiplying both sides of the resulting equation by negative one according to Rule 1, we get the following:

$$5y = -32$$

$$y = -32 / 5 = -6.4$$

*Step 4:* Having determined the value for one of the unknowns, we can insert it back into either of the original equations to solve for the second unknown. Using the first of the two original equations, we have:

$$3x + 4y = 20 \Rightarrow 3x + 4(-6.4) = 20 \Rightarrow 3x - 25.6 = 20$$

$$3x = 45.6$$

$$x = 15.2$$

*Step 5:* Plug both values back into the original equations to be sure your answers are correct. Does  $(3x = 4y = 20)$  and  $(x + 3y = -4)$ ?

$$3x + 4y \Rightarrow 3(15.2) + 4(-6.4) = 45.6 - 25.6 = 20$$

$$x + 3y = 15.2 + 3(-6.4) = 15.2 - 19.2 = -4$$

Since plugging the values into either equation gives us the same values we began with, our answers for  $x$  and  $y$  are indeed correct.

Solving for two unknown variables requires at least two equations. To solve two equations simultaneously, add or subtract them to eliminate one of the variables and find the value of the remaining variable. Then substitute the value of that variable back into one of the equations to solve for the other. In the afore mentioned example, we have demonstrated only one set of steps. There are probably countless other ways to solve this example, which would give the same answer.

## TIME VALUE OF MONEY

In this section, we will use **timelines** to calculate the present and future values of lump sums and annuities. Timelines will help you keep cash flows organized and allow you to see the timing of one cash flow in relation to the other cash flows and in relation to the present (i.e., today).

Before we set up a timeline, however, let's look at more of the terms we will utilize throughout the discussion.

## Quantitative Methods

A **lump sum** is a single cash flow. Lump sum cash flows are one-time events and therefore are not recurring.

An **annuity** is a finite number of equal cash flows occurring at fixed intervals of equal length over a defined period of time (e.g., monthly payments of \$100 for three years).

**Present value** is the value today of a cash flow to be received or paid in the future. On a timeline, present values occur before (to the left of) their relevant cash flows.

**Future value** is the value in the future of a cash flow received or paid today. On a timeline, future values occur after (to the right of) their relevant cash flows.

A **perpetuity** is a series of equal cash flows occurring at fixed intervals of equal length forever.

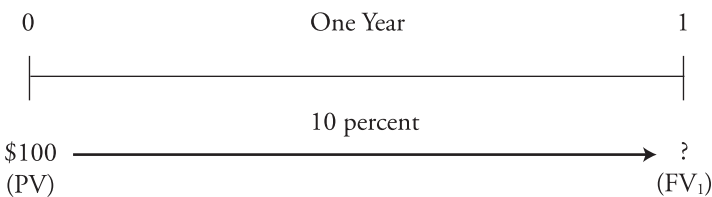
The **discount rate** and **compounding rate** are the rates of interest used to find the present and future values, respectively.

## LUMP SUMS

### Future Value of a Lump Sum

We'll start our discussion with the future value of a **lump sum**. Assume you put \$100 in an account paying 10% and leave it there for one year. How much will be in the account at the end of that year? The following timeline represents the one-year time period.

Figure 2: Determining Future Value at  $t = 1$



In one year, you'll have \$110. That \$110 will consist of the original \$100 plus \$10 in interest. To set that up in an equation, we say the future value in one year,  $FV_1$ , consists of the original \$100 plus the interest,  $i$ , it earns.

$$FV_1 = \$100 + (i \times \$100)$$

Since \$100 is the *present value*, the original deposit, we can substitute  $PV_0$  for the \$100 in the equation.

$$FV_1 = PV_0 + (i \times PV_0)$$

Factoring  $PV_0$  out of both terms on the right side of the equation we are left with:

$$FV_1 = PV_0(1 + i) \quad (1)$$

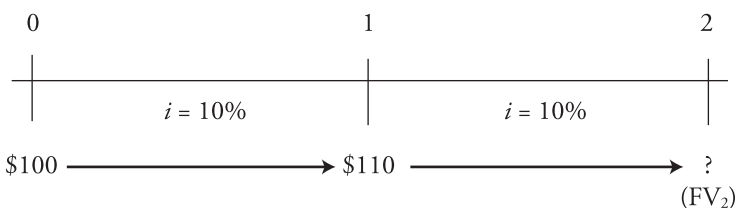
The result of these mathematical manipulations is the general equation for finding the future value of a lump sum invested for one year at a rate of interest  $i$ . Had it not been so easy to do the calculation in our heads, we would have substituted for the variables in the equation and gotten the following:

$$FV_1 = \$100(1.10) = \$110$$

What if you leave the money in the account for two years? After one year you'll have \$110, the original \$100 plus \$10 interest (or 10% times 100). At the end of the second year you will have the \$110 plus interest on the \$110 during the second year. The interest earned in the second year equals 10% times the \$110 balance with which you began the year. Therefore, the interest earned in the second year consists of interest on the original \$100 plus interest earned in the second year on the interest earned during the first year but left in the account. When interest is earned or paid on interest, the process is referred to as **compounding**. This explains why future values are sometimes referred to as **compound values**.

Now our timeline expands to include two years. Although the numbering is totally arbitrary and we could have used any number to indicate today, we are assuming we deposit \$100 at time 0 on the timeline. We already know the value after one year,  $FV_1$ , so let's start there.

**Figure 3: Determining Future Value at  $t = 2$**



## Quantitative Methods

We know from Equation 1 that the future value of a lump sum invested for one year at interest rate  $i$  is the lump sum multiplied by  $(1 + i)$ . We simply find the future value of \$110 invested for one year at 10% by using Equation 1 (adjusted for the different points in time), which gives us \$121.00.

$$FV_1 = PV_0(1 + i)$$

$$FV_2 = FV_1(1 + i)$$

$$FV_2 = \$110(1.10) = \$121$$

To take this example a step further, we make some additional adjustments. We know from Equation 1 that  $FV_1$  is equal to  $PV_0(1 + i)$ . Let's further develop relationships between future and present value.

We start with:  $FV_2 = FV_1(1 + i)$

Substituting, we get:  $FV_2 = PV_0(1 + i)(1 + i)$

And we end with:  $FV_2 = PV_0(1 + i)^2$  (2)

Equation 2 is the general equation for finding the future value of a lump sum invested for two years at interest rate  $i$ . In fact, we have actually discovered the general relationship between the present value of a lump sum and its future value at the end of any number of periods, as long as the interest rate remains the same. We can state the general relationship as the following:

$$FV_2 = PV_0(1 + i)^2$$
 (3)

Equation 3 says the future value of a lump sum invested for  $n$  years at interest rate  $i$  is the lump sum multiplied by  $(1 + i)^n$ . Let's look at some examples. We'll assume an initial investment today of \$100 and an interest rate of 5%.

Future value in 1 year:  $\$100(1.05) = \$105$

Future value in 5 years:  $\$100(1.05)^5 = \$100(1.2763) = \$127.63$

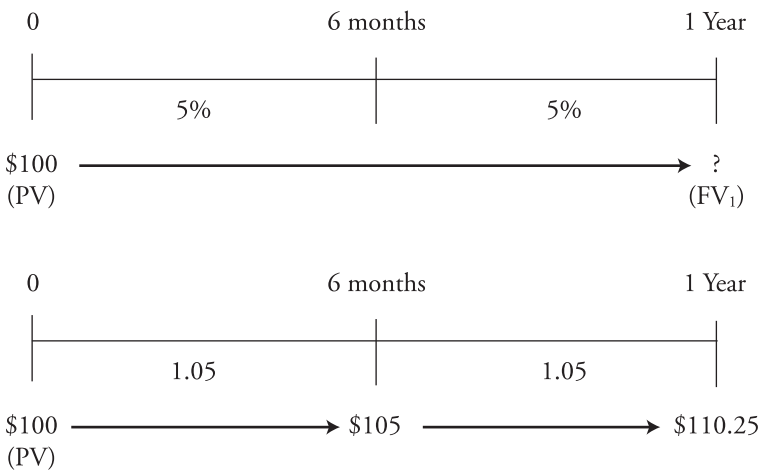
Future value in 15 years:  $\$100(1.05)^{15} = \$100(2.0789) = \$207.89$

Future value in 51 years:  $\$100(1.05)^{51} = \$100(12.0408) = \$1,204.08$

Regardless of the number of years, as long as the interest rate remains the same, the relationship in Equation 3 holds. Up to this point we have assumed interest was paid annually (i.e., annual compounding). However, most financial institutions pay and charge interest over much shorter periods. For instance, if an account pays interest every six months, we say interest is “compounded” semiannually. Every three months represents quarterly compounding, and every month is monthly compounding. Let’s look at an example with semiannual compounding.

Again, let’s assume that we deposit \$100 at time zero, and it remains in the account for one year. This time, however, we’ll assume the financial institution pays interest semiannually. We will also assume a **stated** or **nominal** rate of 10%, meaning it will pay 5% every six months.

**Figure 4: Future Values With Semi-Annual Compounding**



In order to find the future value in one year, we must first find the future value in six months. This value, which includes the original deposit plus interest, will earn interest over the second six-month period. The value after the first six months is the original deposit plus 5% interest, or \$105. The value after another six months (one year from deposit) is the \$105 plus interest of \$5.25 for a total of \$110.25.



## Quantitative Methods

The similarity to finding the FV in two years as we did in Equation 2 is not coincidental. Equation 2 is actually the format for finding the FV of a lump sum after any two periods at any interest rate, as long as there is no compounding within the periods. The periods could be days, weeks, months, quarters, or years. To find the value after one year when interest is paid every six months, we multiplied by 1.05 twice. Mathematically this is represented by:

$$FV = \$100(1.05)(1.05) = \$100(1.05)^2$$

$$FV = \$100(1.1025) = \$110.25$$

The process for semiannual compounding is mathematically identical to finding the future value in two years under *annual compounding*. In fact, we can modify Equation 2 to describe the relationship of present and future value for any number of years and compounding periods per year.

$$FV_n = PV_0 \left( 1 + \frac{i}{m} \right)^{m \times n} \quad (4)$$

where:

$FV_n$  = the future value after n years

$PV_0$  = the present value

$i$  = the stated annual rate of interest

$m$  = the number of compounding periods per year

$m \times n$  = the total number of compounding periods

(the number of years times the compounding periods per year)

For semiannual compounding  $m = 2$ ; for quarterly compounding  $m = 4$ ; and for monthly compounding  $m = 12$ . If you leave money in an account paying semiannual interest for four years, the total compounding periods would be  $4 \times 2 = 8$ . Interest would be calculated and paid eight times during the four years. Let's assume you left your \$100 on deposit for four years, and the bank pays 10% interest compounded semiannually. We'll use Equation 4 to find the amount in the account after four years.

$$FV = \$100 \left( 1 + \frac{0.10}{2} \right)^{4 \times 2}$$

$$FV = \$100(1.05)^8$$

$$FV = \$100(1.4775) = \$147.75$$

where:

$n = 4$ , because you will leave the money in the account for four years

$m = 2$ , because the bank pays interest semiannually

$i = 10\%$  (the annual stated or nominal rate of interest)

If the account only paid interest annually, the future value would be the following:

$$\begin{aligned} \text{FV} &= \$100 \left( 1 + \frac{0.10}{1} \right)^{4 \times 1} \\ \text{FV} &= \$100(1.10)^4 \\ \text{FV} &= \$100(1.4641) = \$146.41 \end{aligned}$$

The additional \$1.34 (i.e.,  $\$147.75 - \$146.41 = \$1.34$ ) is the extra interest earned from the compounding effect of interest on interest. Although the differences do not seem profound, the effects of compounding are magnified with larger values, greater number of compounding periods per year, or higher nominal interest rates. In our example, the extra \$1.34 was earned on an initial deposit of \$100. Had this been a \$1 billion deposit, the extra interest differential from compounding semiannually rather than annually would have amounted to \$13,400,000!

To demonstrate the effect of increasing the number of compounding periods per year, let's look at several alternative future value calculations when \$100 is deposited for one year at a 10% nominal rate of interest. In each case,  $m$  is the number of compounding periods per year.

$m = 1$ (annually)	$\text{FV} = \$100(1.10) = \$110$
$m = 2$ (every 6 months)	$\text{FV} = \$100(1.05)^2 = \$110.25$
$m = 4$ (quarterly)	$\text{FV} = \$100(1.025)^4 = \$110.38$
$m = 6$ (every 2 months)	$\text{FV} = \$100(1.0167)^6 = \$110.43$
$m = 12$ (monthly)	$\text{FV} = \$100(1.008333)^{12} = \$110.47$
$m = 52$ (weekly)	$\text{FV} = \$100(1.001923)^{52} = \$110.51$
$m = 365$ (daily)	$\text{FV} = \$100(1.000274)^{365} = \$110.52$

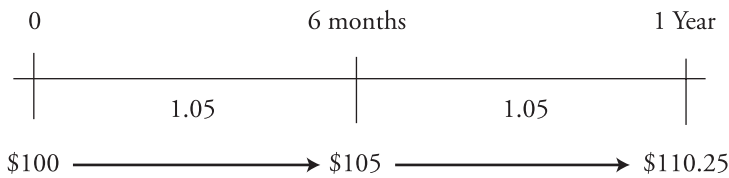
## Quantitative Methods

You will notice two very important characteristics of compounding:

1. For the same present value and interest rate, the future value increases as the number of compounding periods per year increases.
2. Each successive increase in future value is less than the preceding increase. (The future value increases at a decreasing rate.)

**Effective Interest Rates.** The concept of compounding is associated with the related concept of **effective interest rates**. In our semiannual compounding example, we assumed that \$100 was deposited for one year at 10% compounded semiannually. We represented it graphically using a timeline as follows:

**Figure 5: Effective Interest Rates**



The stated (nominal) rate of interest is 10%. However, determining the actual rate we earned involves comparing the ending value with the beginning value using Equation 5. You can determine the actual or “effective” rate of return by taking into consideration the impact of compounding. Equation 5 measures the change in value as a percentage of the beginning value.

$$\text{effective return} = \frac{V_1 - V_0}{V_0} \quad (5)$$

where:

$V_0$  = the total value of the investment at the beginning of the year

$V_1$  = the total value of the investment at the end of the year

You will notice Equation 5 stresses using the values at the beginning and the end of the year (actually, any twelve month period). By convention, we always state effective interest rates in terms of one year.

Returning to our previous example, let’s substitute our beginning-of-year and end-of-year values into Equation 5. Because the interest was compounded semiannually instead of annually, we actually earned 10.25% on the account rather than the 10% stated rate.

$$\text{effective return} = \frac{V_1 - V_0}{V_0} \quad (5)$$

$$\text{effective return} = \frac{\$110.25 - \$100}{\$100} = 0.1025 = 10.25\%$$

Let's employ algebraic principles and rewrite Equation 5 in the following form:

$$\text{effective return} = \frac{V_1 - V_0}{V_0} \Rightarrow \frac{V_1}{V_0} - \frac{V_0}{V_0}$$

$$\text{effective return} = \frac{V_1}{V_0} - 1 \quad (6)$$

Now let's restate Equation 6 in terms of FV and PV:

$$\frac{V_1}{V_0} - 1 \text{ is equivalent to } \frac{FV_1}{PV_0} - 1, \text{ and } FV_1 = PV_0 \left(1 + \frac{i}{m}\right)^{m \times 1}$$

$$\frac{FV_1}{PV_0} - 1 \text{ can be rewritten as } \frac{PV_0 \left(1 + \frac{i}{m}\right)^{m \times 1}}{PV_0} - 1 \quad (7)$$

Remember, we always state effective returns in annual terms. Thus we can set  $n = 1$  in the equation and the  $PV_0$  in the numerator and denominator cancel each other out. Substituting Equation 7 back into Equation 6 we get the following:

$$\text{effective return} = \left(1 + \frac{i}{m}\right)^m - 1 \quad (8)$$

We have arrived at the general equation to determine any effective interest rate in terms of its stated or nominal rate and the number of compounding periods per year. Let's investigate a few examples of calculating effective interest rates for the same stated interest at varying compounding assumptions. Notice that the effective rate increases as the number of compounding periods increase.

$$m = 1 \text{ (annual compounding)} \left(1 + \frac{0.12}{1}\right)^1 - 1 = (1.12)^1 - 1 = 0.12 = 12\%$$

$$m = 2 \text{ (semiannual)} \left(1 + \frac{0.12}{2}\right)^2 - 1 = (1.06)^2 - 1 = 0.1236 = 12.36\%$$

$$m = 4 \text{ (quarterly)} \left(1 + \frac{0.12}{4}\right)^4 - 1 = (1.03)^4 - 1 = 0.1255 = 12.55\%$$

$$m = 12 \text{ (monthly)} \left(1 + \frac{0.12}{12}\right)^{12} - 1 = (1.01)^{12} - 1 = 0.1268 = 12.68\%$$

## Quantitative Methods

$$m = 365 \text{ (daily)} \left( 1 + \frac{0.12}{365} \right)^{365} - 1 = (1.0003288)^{365} - 1 = 0.1275 = 12.75\%$$

**Geometric Mean Return.** The geometric mean return is a compound annual growth rate for an investment. For instance, assume you invested \$100 at time 0 and that the investment value grew to \$220 in 3 years. What annual return did you earn, on average? Using our future value formula from Equation 3:

$$100(1+i)^3 = 220 \tag{7}$$

The interest rate in Equation 7 is the geometric mean or compound average annual growth rate earned on the investment. Solving Equation 7, gives  $i = 30\%$ .<sup>17</sup>

More formally, the geometric mean is found using the following equation:

$$GM = \sqrt[n]{\frac{FV_n}{PV}} - 1 \text{ or } \left( \frac{FV_n}{PV} \right)^{1/n} - 1 \tag{8}$$

where:

GM = the geometric mean

$FV_n$  = future value of the lump sum investment

PV = present value, or the initial lump sum investment

n = the number of years over which the investment is held

Thus, if you invested \$500 in an mutual fund<sup>18</sup> five years ago and now the original investment is worth \$901.01, we would find the geometric mean return as follows:

$$\left( \frac{\$901.01}{\$500} \right)^{1/5} - 1 = (1.80202)^{1/5} - 1 = 1.125 - 1 = 12.5\%$$

The mutual fund provided an average annual return of 12.5%.

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<sup>17</sup> To solve this problem, divide both sides of the equation by 100, then take the third root and subtract 1;  $i = [220/100]^{0.33} - 1$ .

<sup>18</sup> A mutual fund is an investment company that gathers together the investments of many individuals and invests the total amount for them. This provides professional management and greater diversification of individual investments. Diversification will be discussed in a later chapter.

The geometric mean can also be stated using returns over several periods *of equal length* using the following formula:

$$GM = \sqrt[n]{(1 + x_1)(1 + x_2)(1 + x_3)\dots(1 + x_n)} - 1 \quad (9)$$

where:

GM = the geometric mean

$x_i$  = the  $i^{\text{th}}$  return measurement (the first, second, third, etc.)

$n$  = the number of data points (observations)

We add 1 to each observation's value, which is a percentage expressed as a decimal, multiply all the observations together, find the  $n^{\text{th}}$  root<sup>19</sup> of the product, and then subtract one.

Let's return to our mutual fund example. This time we will calculate the geometric mean return differently. Assume that over the last five years the fund has provided returns of 15%, 12%, 14%, 16%, and 6%. What was the geometric mean return for the fund?

$$GM = \sqrt[n]{(1 + x_1)(1 + x_2)(1 + x_3)\dots(1 + x_n)} - 1$$

$$GM = \sqrt[5]{(1 + 0.15) \times (1 + 0.12) \times (1 + 0.14) \times (1 + 0.16) \times (1 + 0.06)} - 1$$

$$= \sqrt[5]{(1.15)(1.12)(1.14)(1.16)(1.06)} - 1$$

$$= \sqrt[5]{1.80544} - 1 = 0.125$$

$$GM = 12.5\%^{20}$$

The geometric mean shows the average annual growth in your *cumulative* investment for the five years, assuming no funds are withdrawn. In other words, the geometric mean assumes compounding. In fact, when evaluating investment returns, the geometric mean is often referred to as the compound mean.

<sup>19</sup>  $n^{\text{th}}$  refers to the number of observations. If there were two observations, you would find the square root. With twenty observations you find the 20th root.

<sup>20</sup> You will notice that, although they are close, the geometric mean is smaller than the arithmetic mean. In fact, the geometric will always be less than or equal to the arithmetic mean.

## Quantitative Methods

### Present Value of a Lump Sum

Recall that Equation 3 showed us the relationship between the present and future values for a lump sum.

$$FV_n = PV(1 + i)^n \quad (3)$$

In Equation 3 the future value is determined by multiplying the present value by  $(1 + i)^n$ . To solve for the present value, we can divide both sides of the equation by  $(1 + i)^n$ .

$$PV = \frac{FV_n}{(1 + i)^n} \quad (3)$$

Finding a present value is actually deducting interest from the future value, which we refer to as **discounting**. The present value can be viewed as the amount that must be invested today in order to accumulate a desired amount in the future. The “desired amount in the future” is known as the “future value.” Returning to the future value examples used earlier, we can demonstrate how to calculate present values. We will assume the same discount rate of 5%.

(FV) The value in 1 year of \$100 deposited today:

$$\$100(1.05) = \$105$$

(PV) The value today of \$105 to be received in 1 year:

$$100 = \frac{\$105}{(1.05)}$$

(FV) The value in 5 years of \$100 deposited today:

$$\$100(1.05)^5 = \$127.63$$

(PV) The value today of \$127.63 to be received in 5 years:

$$100 = \frac{\$127.63}{(1.05)^5}$$

(FV) The value in 15 years of \$100 deposited today:

$$\$100(1.05)^{15} = \$207.89$$

(PV) The value today of \$207.89 to be received in 15 years:

$$100 = \frac{\$207.89}{(1.05)^{15}}$$

(FV) The value in 51 years of \$100 deposited today:

$$\$100(1.05)^{51} = \$1,204.08$$

(PV) The value today of \$1,204.08 to be received in 51 years:

$$100 = \frac{\$1,204.08}{(1.05)^{51}}$$

## ANNUITIES

Recall that an annuity is a series of equal payments that occur at fixed intervals of equal length through time. The series may consist of two or more cash flows. We begin by using Equation 3 for each cash flow in the series. For instance, consider the timeline and associated cash flows shown in Figure 6.

**Future value of an annuity due.** Assume the cash flows represent deposits to an account paying 10%, and you want to know how much you will have in the account at the end of the fifth year. Point zero on the timeline is when the first deposit is made. Each successive deposit is made at the beginning of each year, so the last deposit is made at the beginning of year five. When cash flows come at the beginning of the period, the annuity is known as an annuity due. An annuity due is typically associated with leases or other situations where payments are made in advance of services or products received or rendered. Our goal is to determine the amount we will have in the account at point five (i.e., the end of year five).

Consider an equipment lease. The user of the equipment (the lessee) pays the owner of the equipment (the lessor) a fee for the right to use the equipment over the next period. That is, leases are prepaid. An example of an ordinary (or deferred) annuity is a mortgage loan (or any debt) with end of period payments. When the money is borrowed, the borrower makes payments at the end of set periods, usually every month or semi-annually. Each payment will include interest, which



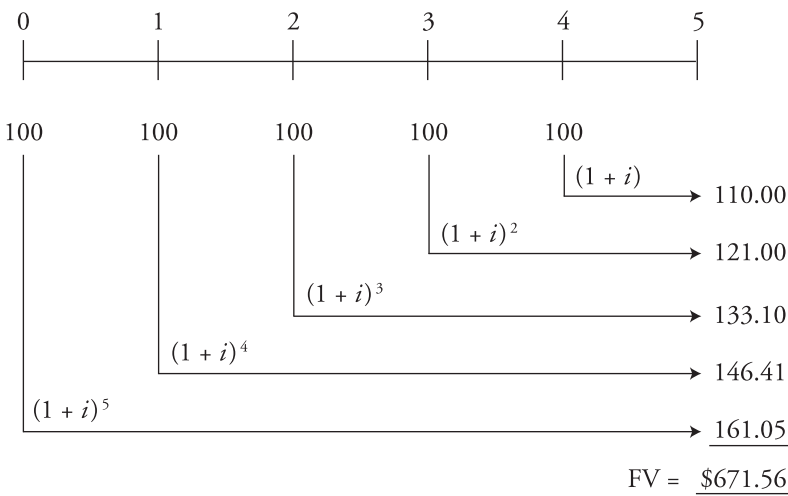
## Quantitative Methods

reflects the cost of the money over the previous period. When a mortgage is fully amortized, each of the fixed payments includes interest on the outstanding principal as well as a partial repayment of principal. Since the outstanding principal decreases with each payment, the proportion of each successive payment representing interest decreases, and the proportion representing principal increases.

The process of finding the future value of an annuity in this manner is equivalent to summing the future value of each individual cash flow. The cash flow stream is illustrated in Figure 6. Each cash flow is assumed to earn a 10% return for each of the indicated number of years. For example, the \$100 at time 0 will remain on deposit for a total of five years, so we multiply \$100 by  $(1 + i)^5$  and obtain \$161.05. When all the cash flows are compounded to find their future values, we add them to find the total future value of the five cash flows, which totals \$671.56. In other words, if you deposit \$100 per year in an account paying 10%, you will have \$671.56 in five years.

**Figure 6: Future Value of an Annuity Due**

Future Value of Five Deposits of \$100 Made at the Beginning of Each Year



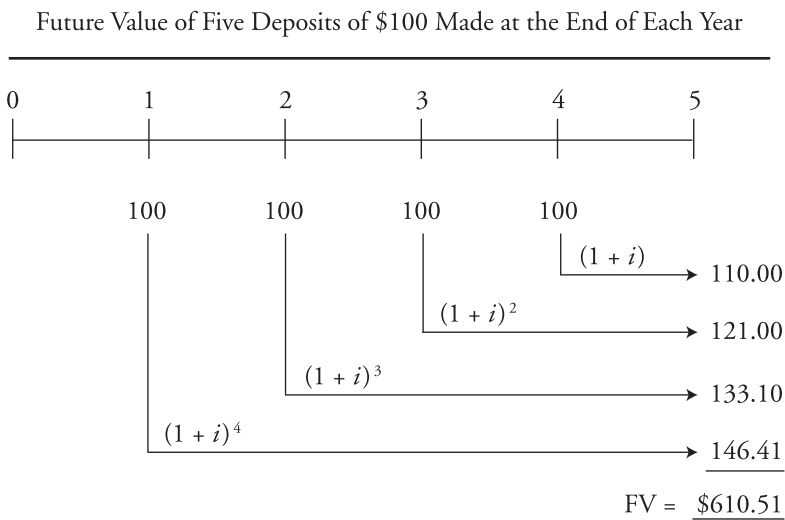
**Future value of an ordinary annuity.** Now we'll consider the same annuity of five \$100 deposits, but we'll assume that the deposits occur at the end of each year. When cash flows are at the end of the period, the annuity is known as an **ordinary annuity**. An ordinary annuity represents the typical cash flow pattern for loans, such as those for automobiles, homes, furniture, fixtures, and even businesses. The time line and cash flows are illustrated in Figure 7.

You will notice that the future value of the ordinary annuity is less than the future value of the annuity due, although the number and amount of the deposits is the same. The difference results from the fact that the deposits for the annuity due are all received exactly one period earlier than the corresponding deposits for the ordinary annuity. Each deposit for the annuity due accrues an additional period's (in this case, a year's) interest. The value of an **annuity due** is equal to the value of an **ordinary annuity plus one period's interest**. In our example the length of the period is a year, so  $\$610.51 + (\$610.51 \times 0.10) = \$671.56$ .

Looking at the relationship from another perspective, you will notice that the four deposits at points one through four are exactly the same for both annuities. The difference between the two types of annuities is the treatment of the remaining deposit. With the annuity due (Figure 6), the remaining deposit is made at time zero and earns interest for five years.

With the ordinary annuity (Figure 7), the remaining deposit is made at the end of year five immediately before the account is closed and the money is withdrawn. Since the last deposit in the case of the ordinary annuity earns no interest, the difference between the future values of the two annuities must be the interest earned on the deposit made at time 0 in the case of the annuity due. The future value of the ordinary annuity is \$610.51. The future value of the annuity due is \$671.56, exactly \$61.05 larger.

**Figure 7: Ordinary Annuity**



The solutions presented for both annuities were more for demonstration purposes than for actually calculating the future values. If you find yourself needing to

## Quantitative Methods

calculate the future value of an annuity, you will use one of two other approaches: a financial calculator or a formula.

Let's illustrate the use of a financial calculator to find the future value of an annuity. Our example is based upon the keystrokes required for a Texas Instruments (TI) Business Analyst II Plus<sup>®</sup> calculator.

**Ordinary annuity:** Your calculator should be set to end of period payments and one payment per year. To set to end of period payments, press  $2\text{nd} \Rightarrow \text{BGN}$  and press  $2\text{nd} \Rightarrow \text{SET}$  until END is displayed, then  $2\text{nd} \Rightarrow \text{QUIT}$ . (Since end is default, the display will not indicate end of period payments.) To set to one payment per year, press  $2\text{nd} \Rightarrow \text{P/Y} \Rightarrow 1 \Rightarrow \text{ENTER}$ ,  $2\text{nd} \Rightarrow \text{QUIT}$ . The keystrokes<sup>21</sup> to find the future value are the following:

```
-100 PMT [The calculator assumes one of the
      payments is an outflow (from the perspective of
      the investor) and one is an inflow. The negative
      sign indicates an outflow (the deposit)].
   5  N
  10  I/Y
CPT  FV = $610.51
```

**Annuity due.** For an annuity due, set the financial calculator for beginning-of-period payments and one payment per year. To set to one payment per year, press  $2\text{nd} \Rightarrow \text{P/Y} \Rightarrow 1 \Rightarrow \text{ENTER}$ ,  $2\text{nd} \Rightarrow \text{QUIT}$ . To set to end-of-period payments, press  $2\text{nd} \Rightarrow \text{BGN}$  and press  $2\text{nd} \Rightarrow \text{SET}$  until BGN is displayed, then  $2\text{nd} \Rightarrow \text{QUIT}$ . (BGN will show in the calculator display.) The keystrokes to find the future value are the following:

```
-100 PMT [The calculator assumes one of the
      payments is an outflow (from the perspective of
      the investor) and one is an inflow. The negative
      sign indicates an outflow (the deposit)].
   5  N
  10  I/Y
CPT  FV = $671.56
```

---

<sup>21</sup> Calculator Tip: Note that PV, FV, PMT, I/Y, and N are just memory registers. If there are values in these registers from previous computations, you will need to clear them. There are two ways to do this. The brute force method is to simply put a zero into the register that you're not using. The second and more elegant method is to press  $2\text{nd} \Rightarrow \text{CLR TVM}$  to completely clear the time value of money register before you enter the next set of data points.

While using a financial calculator is the recommended procedure, you can also use a formula for calculating the future value of an annuity. To find the future value of an **ordinary annuity**, multiply the cash flow (payment) by the formula and the result is the future value. Assume the same \$100 deposits made each year for five years earning a 10% return, compounded annually.

$$FV_n = PMT \frac{(1+i)^n - 1}{i}$$

$$FV_5 = \$100 \frac{(1.10)^5 - 1}{0.10}$$

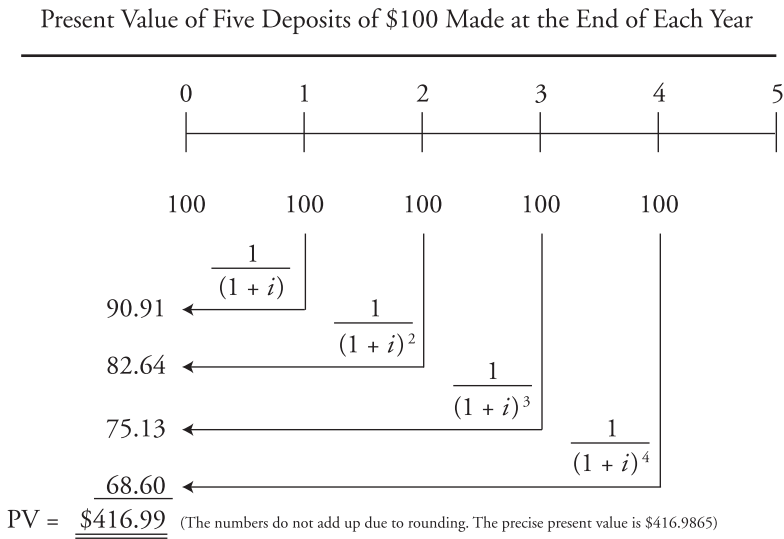
$$FV_5 = \$100 \left( \frac{1.6105 - 1}{0.10} \right) = \$100(6.1051) = \$610.51$$

The same process would be used to calculate the future value of an annuity due. We multiply the future value of the ordinary annuity by  $(1 + i)$  to adjust for the fact that each cash flow is received one period earlier, and thus earns an extra period's interest, with the annuity due.

$$\begin{aligned} FV_5 &= \$100 \left( \frac{1.6105 - 1}{0.10} \right) = \$100(6.1051) = \$610.51(1 + 0.10) \\ &= \$671.56 \end{aligned}$$

**Present value of an annuity due.** When we found the future value of an annuity, we compounded each cash flow individually and summed them at a future date. To find the present value of an annuity, we discount all the future values and sum them up. We'll start with the *annuity due* we used before. The cash flows are assumed to be paid/received at the beginning of each year, and we want to find the aggregate present value of the five cash flows at point zero on the timeline. Again we assume an interest rate of 10%. The process of finding the present value of an annuity due is equivalent to summing the present value of each individual cash flow. The cash flow stream is illustrated in Figure 8. Each cash flow is discounted at a 10% rate for each of the indicated number of years. For example, the present value of \$100 at time 4 equals 100 divided by  $(1 + 0.10)^4 = \$68.30$ ; the present value of \$100 at time 3 equals 100 divided by  $(1 + 0.10)^3 = \$75.13$ , and so on. When all the cash flows are discounted to find their present values, we add them to find the total present value of the five cash flows, which totals \$416.99. Figure 8 illustrates the calculation of the present value of an annuity due.

Figure 8: Present Value of an Annuity Due



To find the present value of this annuity due using a TI Business Analyst II Plus calculator, you would make the following key entries (be sure to turn on BGN):

-100 PMT  
 5 N  
 10 I/Y  
 CPT PV = \$416.99

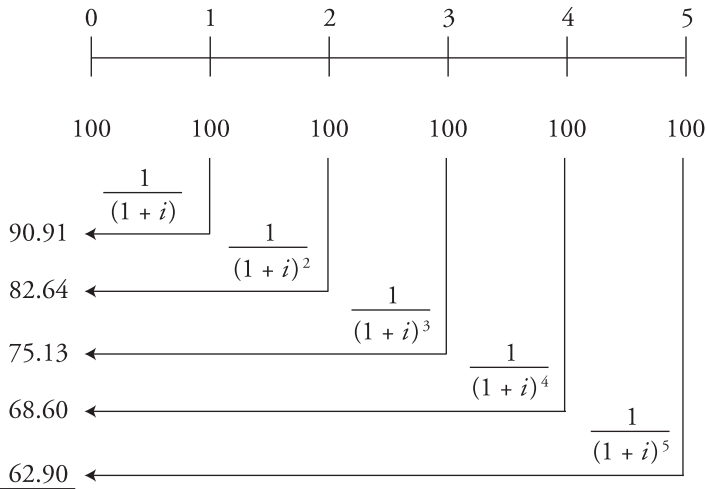
There are several ways of interpreting the \$416.99. The \$416.99 is the *present value* of the five \$100 payments/receipts, but what does “present value” really mean? A simple interpretation is that if you put \$416.99 in an account paying 10% interest, you will be able to withdraw \$100 per year for five years. Another somewhat more sophisticated interpretation is that \$416.99 is the maximum you would pay for an investment paying \$100 per year with a required return of 10%. A third interpretation is that if you borrow \$416.99 to be paid in five equal annual payments, you will pay \$100.00 per payment. Regardless, assuming a 10% interest rate, \$416.99 today is equivalent to five annual \$100 cash flows, the first cash flow occurring today.

**Present value of an ordinary annuity.** Likewise, the present value of an ordinary annuity may be calculated by summing the present value of each individual payment. Figure 9 illustrates the cash flow stream for the ordinary annuity. In this case the first cash flow occurs one year from today with the other four coming

yearly after that. The present value of each cash flow is found and then added to the others to get a total of \$379.08.

**Figure 9: Present Value of an Ordinary Annuity**

Present Value of Five Deposits of \$100 Made at the End of Each Year



PV = \$379.08 (The numbers do not add up due to rounding. The precise present value is \$379.07868)

The keystrokes to find the present value of an ordinary annuity are [be sure to turn BGN mode off (i.e., you are in END mode)]:

```
-100 PMT
  5 N
  10 I/Y
CPT PV = $379.08
```

In addition, we may calculate the present value of an ordinary annuity using the following formula:

$$PV_A = PMT \left[ \frac{1}{i} - \frac{1}{i(1+i)^n} \right]$$

$$PV_A = \$100 \left[ \frac{1}{0.10} - \frac{1}{0.10(1.10)^5} \right]$$

$$PV_A = \$100[10 - 6.2092]$$

$$PV_A = \$100[3.7908] = \$379.08$$

## Quantitative Methods

Thus we are able to confirm the present value of the ordinary annuity is \$379.08.<sup>22</sup>

When both principal and interest are included in a loan payment (rather than a series of interest payments followed by the repayment of principal at the maturity of the loan), we say the loan is fully amortized. Let's use the ordinary annuity of \$379.08 as an example. Let's assume you have borrowed \$379.08 for the purchase of a household item and have agreed to pay for it in five equal payments at 10% interest. We know from the previous example that the payments will be \$100 each. Figure 10 shows how each payment would be broken down into interest and principal if the loan were fully amortized.

**Figure 10: Amortized Loan (amortization of \$379.08 for five years at 10%)**

	<i>Payment</i>	<i>Interest</i>	<i>Principal</i>	<i>Balance</i>
				\$379.08
1	\$100	37.91	62.09	316.99
2	\$100	31.70	68.30	248.69
3	\$100	24.87	75.13	173.56
4	\$100	17.36	82.64	90.92
5	\$100	9.09	90.91	0*

\*slight rounding error

Each of the five payments repays a portion of the principal borrowed and pays interest on the balance remaining after the previous payment. The first payment includes 10% interest on the entire loan amount of \$379.08, or \$37.91. The remainder of the payment ( $\$100 - \$37.91 = \$62.09$ ) is applied to the principal, leaving a balance of \$316.99. The second payment includes 10% interest on the new balance of \$316.99, or \$31.70. Again, the remainder of the payment, \$68.30, is applied to the principal. This process continues until the loan is fully paid. You can see in Figure 5 that the interest in each payment decreases while the principal increases.

**Present value of a perpetuity.** As explained earlier, a perpetuity is a series of equal cash flows occurring at the same interval forever. The present value of a perpetuity equals the periodic cash amount divided by the discount (interest) rate. For example, assume you own a stock paying \$2 dividends forever. Further, assume the appropriate discount rate on the stock is 10%. The present value of the perpetuity equals \$2 divided by 10%, which equals \$20. In other words, you should be willing to pay \$20 for a stock expected to pay \$2 dividends per year forever, using

<sup>22</sup> Similar to calculations related to future value, the present value of an annuity due can be calculated from the present value of the ordinary annuity by multiplying the present value of the ordinary annuity by  $(1 + i)$ .

a discount rate of 10%. More formally, we can state the formula for finding the present value of a perpetuity as follows:

$$PV_{\text{perpetuity}} = \frac{CF}{i}$$

where:

CF = the periodic cash flows of the perpetuity

i = the discount rate

## AN INTRODUCTION TO STATISTICS

*Webster's Collegiate Dictionary*<sup>23</sup> defines statistics as “a branch of mathematics dealing with the collection, analysis, interpretation, and presentation of numerical data.” When we are trying to determine certain characteristics about a large population, we take a sample from that population. We then use that sample to derive certain statistics and make inferences about the population. In order to understand what this means, it is useful to become familiar with some of the more valuable statistics vocabulary.

### Population

A **population** is the collection of all possible individuals, objects, measurements, or other items; (e.g., the population of the U.S. is all people who call the U.S. their home country). The population of carp in the world is not just those that live in beautifully maintained and landscaped pools in Japan. The carp population would include every single carp, no matter where it lives.

An important characteristic of a population is its size. For instance, there are approximately 300 million people living in the U.S. That means there are about 300 million members of that particular population. If you wanted to estimate the average height of an American, you would not want to go around to every single person and measure his or her height. Even if you could afford the extreme cost in both time and money, it would be a logistical nightmare. How about weighing every carp in the world to find the average weight of an adult carp?

The field of statistics allows us to estimate these values without actually measuring each member of the population.

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23 Merriam Webster's Collegiate Dictionary, 10th Edition, 2000.



### Sample

A **sample** is a portion or subset of a population that is used to estimate characteristics of (i.e., make inferences about) the population. If we were interested in the average height of a U.S. citizen, we could select people from all over the U.S. (a sample of people), measure them, and find the average height. The average height of the individuals in the sample is then used to infer the average height of all people in the United States.

An important characteristic of samples is randomness. You don't want to force the sample to yield statistics that are biased because of the way the sample is taken. For example, if you are trying to estimate the percentage of people in the U.S. who are over age 65, you would not take your sample observations<sup>24</sup> from a retirement community. If you were to do that, you could estimate that nearly 100% of the U.S. population is over 65! An appropriate sample would be drawn from many different areas across the country in a random, unbiased way.

Another characteristic important to the sample is size. When an extremely large sample is drawn, the costs can be very high, and the inferences not significantly stronger than those of a somewhat smaller sample. However, if the sample is too small, the inferences drawn from the sample may not be trustworthy. Even though we will not pursue ideal sample size in this chapter, it is important to remember that sample size is very important to the value (the confidence) you can place on the inferences you make about a population.

### Variables

A **variable** is an unknown quantity (measurement) that can have different values. For example, if you were estimating average height and measured every person in your sample, the first value of the variable "height" would be the height of the first person measured. The second value would be the height of the second person; the third value would be the height of the third person, and so on. The variable "height" would have as many values (observations) as there are people in your sample. For example, we might use the letter  $x$  to denote *height*.  $x_1$  is the notation used to denote the height of the first person sampled.  $x_2$  is the notation used to denote the height of the second person sampled, etc. For instance, if the first person is 70 inches tall, then  $x_1 = 70$ . If the second person is 71 inches tall, then  $x_2 = 71$ .

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<sup>24</sup> An observation is one member of a sample. A sample of size 30 has 30 observations.

There are two main categories of variables: qualitative and quantitative. A qualitative variable measures attributes. These could include gender, religious preference, eye color, type of running shoe preferred, and place of birth. In other words, qualitative variables do not use numbers.

Conversely, quantitative variables are expressed numerically. These could include the average number of children in the typical household, the average height of American females, the percentage of people in the population with false teeth, or the average number of computers sold daily.

Quantitative variables can be categorized as discrete or continuous. Think of discrete as meaning that the variable can only have a countable number of easily identified values. If the variable can only take on a whole number value from 1 to 10, it would be considered discrete. Its only possible values are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. You'll notice that you can easily count each possible value.

Now let's say the variable can have any incremental value between 1 and 10. In this case the variable can assume an infinite number of possible values and is called a continuous variable. When the variable was discrete, two of its possible values were three and four, and it could not have a value between three and four. As a continuous variable, it can have the values three and four, but it can also take on any of the infinite values *between* three and four.

To illustrate, consider a continuous variable that is measured in inches. If the value of the first measurement is 3.0 inches, the next possible value could be so close to 3.0 that we have no instrument capable of measuring the distance. Consider the maximum number of zeros you can place between the decimal point and a one. An example would be 3.00001, but of course you can place many more than four zeros between the decimal and the one. In fact, the number of zeros you could place between the decimal point and the one is infinite.

An example of a discrete variable is the outcome of the roll of a die (1, 2, 3, 4, 5, or 6). An example of a continuous variable is the amount of rainfall during July in a city.

Let's turn our attention to the terminology that is used to describe data. If we want to estimate the average height of a large group of people, we can measure a sample

## Quantitative Methods

of them and write down each measurement.<sup>25</sup> Let's assume we collect the following measurements:

<i>Person</i>	<i>Height in Inches</i>
1	70
2	71
3	73
4	66
5	62
6	70

These measurements are referred to collectively as the sample data (plural), while each individual measurement or observation is a data point. One observation would be the value 70 inches. Another would be 66 inches. Once we have collected our data, we will look for ways of describing them as a whole (i.e., we will describe the distribution of the data).

## Frequency Distribution

Often, we do not want to see all the data points (especially if the sample size is large) but rather are interested in seeing merely a summary of the data. One popular way to summarize the data is to tabulate the frequency of observations falling in various categories. The tally of observations falling in equally spaced intervals is called the **frequency distribution** of the data. The frequency distribution shows how the data are scattered. For instance, consider the following frequency distribution.

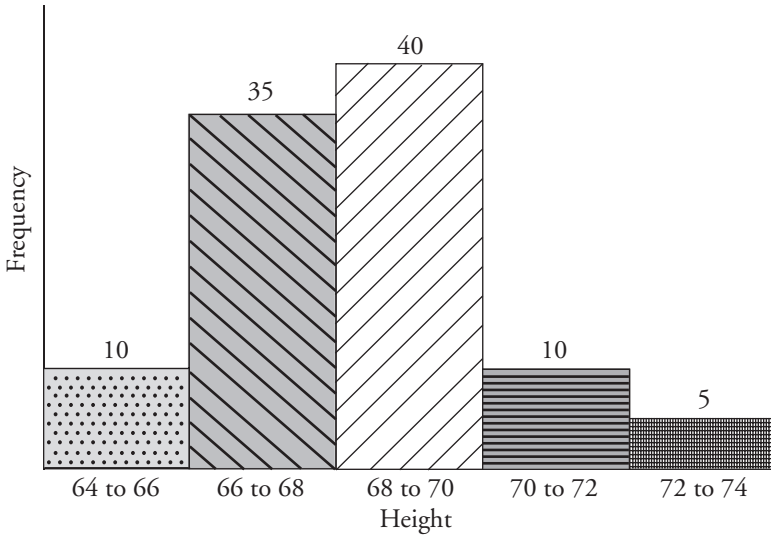
<i>Height Interval (inches)</i>	<i>Frequency Tally</i>
64 up to 66	10
66 up to 68	35
68 up to 70	40
70 up to 72	10
72 up to 74	5

The table indicates that from our sample of 100 individuals, 10 individuals are 64 up to 66 inches tall; 35 individuals are 66 up to 68 inches tall, et cetera. One way to summarize the data visually is by graphing the frequency distribution as follows:

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<sup>25</sup> We are assuming a very small sample size to make the example easier. In reality, you would measure anywhere from several dozen to several thousand people, depending on the size of the population.

Figure 11: Frequency Distribution Histogram



A graph of a frequency distribution is called a **histogram**. Notice how the histogram illustrates how the data are scattered. There is a center (at the 3rd category) with a pattern around the center. In this example, the distribution is skewed, meaning that there are not an equal number of observations on either side of the middle interval. Alternatively, a symmetric distribution is one in which there are an equal number of observations on either side of the middle interval (i.e., follows a bell-shape).

To describe the data further, we often want to find the center point of the data. The center point of the data is known as the central tendency of the distribution. There are a few statistical measures of central tendency, the most popular of which are the mean, median, and mode.

## MEASURES OF CENTRAL TENDENCY

### Mean

Mean is just another word for *average*, or the center of the data. For instance, when we previously mentioned estimating the average height, we could have used the expression “mean height.” The most common measure of central tendency is the arithmetic mean.

## Quantitative Methods

The arithmetic mean for a sample is denoted by:<sup>26</sup>

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (7)$$

Earlier, to estimate the average height of the U.S. population, we took a sample of people and measured their heights. By substituting the data (individual measurements) into the equation to find the sample mean, we get the following:

$$\bar{x} = \frac{70 + 71 + 73 + 66 + 62 + 70}{6} = 68.7 \text{ inches}$$

## Median

The median is the middle observation of the ranked data. If we ranked our height observations from the smallest to the largest, the same number of observations will fall above and below the median value. Our observations are 62, 66, 70, 70, 71, and 73. The median of the six observations falls between the third- and fourth-ranked observations. Thus, the simple average of the third and fourth observations, 70, represents the median of our sample. There are two observations greater than 70 (71 and 73), and there are two observations less than 70 (62 and 66).

A note is necessary at this point. You probably questioned why the mean and median, both measures of the center of the distribution, are different. In fact, you might even be asking yourself, “Why do we need so many measures of the center of the distribution in the first place?”

This question deserves some attention. The median finds the center of the distribution by number of observations. There are an equal number of observations above and below the median, regardless of their values. The mean, on the other hand, actually adds all the observations together and divides by the number of observations to find the mathematical center. You probably noticed that the mean of the sample observations, 68.7, isn’t even one of the observations. This is quite frequently the case. The reason we calculate both the mean and median of a data set is to get an idea of where the true center lies. Since the median and mean find the center in different ways, a more accurate estimation of where the center is and what is influencing its location can be gained by observing both the mean and median.

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<sup>26</sup> Sometimes the summation symbol,  $\Sigma$ , is used to define the arithmetic mean as  $\Sigma x/n$ . Also, the Greek symbol  $\mu$  is often used to represent the mean of the entire population;  $\bar{x}$  is used when deriving the mean of a sample. The method of calculating the population and sample means is the same, however.

## Mode

The mode is the observation that appears most often. In our case both the median and the mode are 70. The mean, median, and mode are all measures of *central tendency*. They all locate the center of the observations or population.

## MEASURES OF DISPERSION

Measures of central tendency show the location of the center of the distribution. We often also want a measure of how spread out or dispersed the data are. There are several popular measures of dispersion. Here, we will discuss the range, mean absolute deviation, and standard deviation.

### Range

The range is simply the “distance” between the lowest and the highest observed values. In our case we say the observations *ranged* from 62 inches to 73 inches, or 11 inches. The larger the range, the wider the dispersion of observations, and the smaller the range, the narrower the dispersion of the data.

We now have information about the central tendency and the dispersion of the data. We can look at these statistics together and learn much about the sample. For instance, we see its center is around 70 inches, and there is a range of 11 inches between the smallest and largest observations in the sample.

A weakness of the range is that it uses only two data points. In contrast, the mean absolute deviation, variance, and standard deviation use all the data points.

### Mean Absolute Deviation

The mean absolute deviation is a measure of the dispersion of the sample observations around the center of the distribution. It measures the average deviation from the mathematical mean. A deviation is measured as the distance from the mean to each observation.

## Quantitative Methods

With a mean of 68.7", the deviations for our sample are:

$$70.0'' - 68.7'' = 1.3''$$

$$71.0'' - 68.7'' = 2.3''$$

$$73.0'' - 68.7'' = 4.3''$$

$$66.0'' - 68.7'' = -2.7''$$

$$62.0'' - 68.7'' = -6.7''$$

$$70.0'' - 68.7'' = 1.3''$$

and the mean absolute deviation, MAD, can be calculated using the following:

$$\text{MAD} = \frac{\sum |x - \bar{x}|}{n} \quad (8)$$

where:

$x$  = the value of each observation

$\bar{x}$  = the arithmetic mean of the observations

$n$  = the number of observations

$|x - \bar{x}|$  = the absolute value of each deviation<sup>27</sup>

$$\text{MAD} = \frac{|70 - 68.7| + |71 - 68.7| + |73 - 68.7| + |66 - 68.7| + |62 - 68.7| + |70 - 68.7|}{6}$$

and

$$\text{MAD} = \frac{1.3 + 2.3 + 4.3 + 2.7 + 6.7 + 1.3}{6} = 3.1$$

The mean absolute deviation for our sample is 3.1 inches. Therefore, on average, the sample observations fall 3.1 inches from the sample mean. If the mean absolute deviation had been 1.0 inch, the observations would be much more closely grouped around the mean, or less dispersed. Had the mean absolute deviation been 6 inches, the observations would be more spread out or dispersed.

## Variance and Standard Deviation

Another way to measure the dispersion of our sample is with the **variance** and **standard deviation**. Both the variance and standard deviation are measures of dispersion of the data around the mean of a distribution. The calculation of the variance is similar to the mean absolute deviation calculation. Once again we begin with the deviations of each data point from the mean. Instead of averaging the

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<sup>27</sup> Since summing the negative and positive deviations would cancel them out, we have to ignore their signs and sum their absolute values.

deviations, however, we average the *squared* deviations. This also gives us all positive numbers because squaring a negative number results in a positive product.

Look at the following equation, which gives us the population variance:

$$\sigma^2 = \frac{\sum(X - \mu)^2}{N} \quad (5)$$

where:

$\sigma^2$  = the population variance (the arithmetic mean of the squared deviations from the population mean)

$\mu$  = the mean of the population

$X$  = an individual member of the population

$N$  = the number of observations in the population

After we calculate the variance for the population, we can find the standard deviation. The standard deviation,  $\sigma$ , is simply the square root of the variance,  $\sigma^2$ .

$$\sigma = \sqrt{\sigma^2}$$

or

$$\sigma = \sqrt{\frac{\sum(X - \mu)^2}{N}} \quad (6)$$

If we assume that our earlier data set of heights for six people included the entire population, the standard deviation would be calculated as follows:

$$\begin{aligned} & \sqrt{\frac{(70 - 68.7)^2 + (71 - 68.7)^2 + (73 - 68.7)^2 + (66 - 68.7)^2 + (62 - 68.7)^2 + (70 - 68.7)^2}{6}} \\ & \sigma = \sqrt{\frac{1.69 + 5.29 + 18.49 + 7.29 + 44.89 + 1.69}{6}} = \sqrt{13.22} = 3.64 \end{aligned}$$

Thus, the standard deviation of our population is 3.64 inches.

The equations for variance and standard deviation of a sample are slightly different than those for the population. Since a sample is smaller than the population, finding the arithmetic mean of the squared deviations tends to underestimate the true value. We have to adjust the formula slightly to account for this effect. The variance for a sample is as follows:

$$s^2 = \frac{\sum(x - \bar{x})^2}{n - 1} \quad (7)$$



## Quantitative Methods

where:

$s^2$  = the sample variance

$x$  = an individual observation in the sample

$\bar{x}$  = the sample mean

$(n - 1)$  = the number of sample observations minus 1

Notice we divided by  $(n - 1)$  rather than  $n$ . This is the modification for the size of the sample compared to the population. As with the population, the standard deviation of the sample is the square root of its variance.

$$s = \sqrt{s^2}$$

or

$$s^2 = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \quad (8)$$

For our previous example related to estimating the average height of a large population of people, we calculate the standard deviation as follows:

$$\sqrt{\frac{(70 - 68.7)^2 + (71 - 68.7)^2 + (73 - 68.7)^2 + (66 - 68.7)^2 + (62 - 68.7)^2 + (70 - 68.7)^2}{6 - 1}}$$
$$s = \sqrt{\frac{1.69 + 5.29 + 18.49 + 7.29 + 44.89 + 1.69}{5}} = 3.98$$

Thus, the standard deviation of our sample is 3.98 inches.

The larger the standard deviation, the greater the dispersion of the observations about the mean. This means the observations are relatively spread out. This is often referred to as a “loose” distribution. The smaller the standard deviation, the more the observations are grouped around the mean. The observations are less spread out. We would describe this as a “tight” distribution.

## THE NORMAL DISTRIBUTION AND THE EMPIRICAL RULE FOR PROBABILITIES

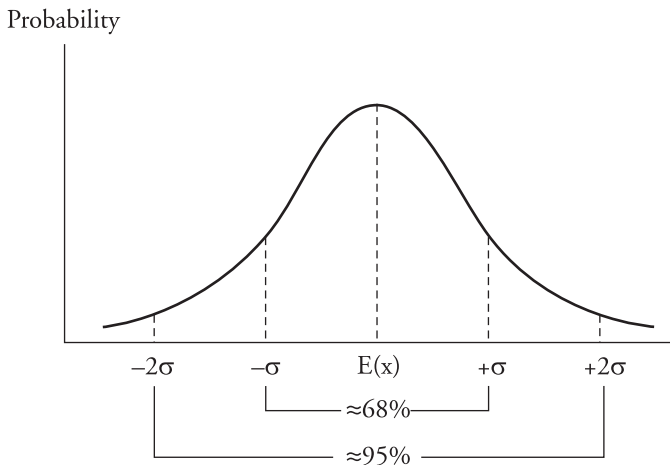
Used extensively in financial analysis, the normal distribution is the famous “bell-shaped curve” of which statisticians and others speak. In a normal distribution, the mean, median, and mode are equal. Although the normal distribution may not precisely describe the pattern of returns for common stocks, for example, it is commonly used because of its statistical properties, especially symmetry.

Perfect symmetry implies:

- The center point of the distribution is the mean, median, and mode.
- 50% of all possible returns are to the right of (above) the mean
- 50% of all possible returns are to the left of (below) the mean.

Figure 12 shows the normal distribution. Of particular interest are the symmetry of the distribution and the counting of standard deviations from the mean. To the right of (above) the mean, lines are drawn that would approximate the returns within one and two standard deviations of the mean. Symmetry implies we can include lines on the other side of (left of) the mean that are the same distance from the mean as their counterparts to the right.

**Figure 12: Normal Distribution**



In many business settings, we wish to estimate the likelihood of certain events (e.g., likelihood of positive earnings, of debt rating change, etc). A helpful rule for estimating the likelihood of events using the normal distribution is that approximately 68% of the data lie within 1 standard deviation of the mean, and 95% of the data lie within 2 standard deviations of the mean.

For instance, assume you have been hired to manage a portfolio for a client. Assume your portfolio has an expected mean annual return of 10% and a standard deviation of 5%. Your client asks you to estimate the probability that she will lose money next year on your portfolio.

You can use the empirical rule to answer the question. For instance, the empirical rule states that there is approximately a 95% chance that next year's portfolio return will lie within 2 standard deviations of the mean. Since the mean is 10% and the standard deviation is 5%, then 95% of the time the annual return will range from 0% to 20% (i.e., the mean return plus and minus 2 standard deviations). The

## Quantitative Methods

remaining 5% of the time, the portfolio return will lie *either* below 0% or above 20%. If we assume the portfolio returns are symmetric (equal probabilities for returns above and below the mean), there is a 2.5% chance the return will lie below 0% and another 2.5% chance the return will lie above 20%. This client should feel pretty safe about her investment: There is only a 2.5% chance she will lose money next year.

## SUMMARY

## ALGEBRA

- A. The sign of the sum of a positive number and a negative number is determined by the absolute values of the two numbers.
- B. When subtracting two numbers, change the sign of the second number and add the two.
- C. When multiplying negative numbers, the sign of the product depends upon the number of negative numbers.
  1. An even number of negative signs results in a positive product.
  2. An odd number of negative signs results in a negative product.
- D. An equation shows (defines) the relationships among the terms (variables) in the equation.
- E. An equal sign ( $=$ ) shows that all the items (terms) on the left side of the equation are equal to all the terms on the right side of the equation. The unknown terms can have only one value.
- F. An equal to or less than sign  $\leq$  shows that the value of all of the terms on the left side of the equation must be equal to or less than the value of all of the terms on the right side of the equation.
- G. A greater than or equal to sign  $\geq$  shows that the value of all the terms on the left side of the equation must be equal to or greater than the value of all of the terms on the right side of the equation.
- H. When a term moves from one side of the equation to the other, its sign changes, either from negative to positive or from positive to negative.
- I. Multiplying or dividing all terms in an equation by the same number or letter does not change the structure of the equation or the value of the unknown, but it does have the following two effects:
  1. All terms are increased or decreased by the same proportion.
  2. The value of the unknown term(s) is unaffected.
- J. Multiplying or dividing by one does not affect the value of a number or letter.
- K. Changing the order in which terms are multiplied does not affect the equation or the value of the unknown terms (i.e., the product of  $a \times b$  is the same as the product of  $b \times a$ ).
- L. When the same letter or number is added to or subtracted from all terms in an equation, neither the structure of the equation nor the value of the unknown term(s) changes.
- M. Changing the order of the equation affects neither the equation nor the value of the unknown term(s).
- N. A set of parentheses and all terms within them are referred to as a quantity.
- O. The parentheses and their contents can be treated as a single term.
- P. When multiplying a number or letter by a quantity inside a parentheses, you must multiply the number or letter by each term inside the parentheses.

## Quantitative Methods

- Q. A quantity within a set of parentheses can be the result of factoring. To factor, look for the largest number that goes evenly into two or more terms in the equation. Then divide the applicable terms by this number and place parentheses around the remaining numbers and letters.
- R. An exponent of a number or letter is referred to as its *power* (e.g.,  $x^3$  says  $x$  cubed, or  $x$  to the third power).
- S. The power refers to the times the letter or number is multiplied by itself.
- T. An exponent can be any value  $i$ . In order to solve for an unknown variable that has an exponent, the exponent of that variable must be removed (i.e., made equal to *one*).
- U. To remove an exponent, multiply it by its root. To remove any exponent  $i$ , take the  $i$ th root [i.e.,  $(x^i)^{1/i}$  or  $\sqrt[i]{x^i} = x$ ].
- V. Quantities within parentheses can have exponents.
- W. Exponents with parentheses are removed as any other exponent:  
$$\left[(x+1)^i\right]^{1/i} = (x+1) = x+1 \quad \text{or} \quad \sqrt[i]{(x+1)^i} = (x+1) = x+1$$
- X. There must be at least two equations in order to solve for the values of two unknown variables.
- Y. Set up the two equations as if you are adding them.
- Z. Multiply one of the equations by a number, which will enable you to eliminate one of the unknowns.
- AA. Add the two equations together by adding the  $x$  terms, the  $y$  terms, and the numbers.
- BB. Solve for the remaining unknown variable.
- CC. Plug the value for the first unknown variable into one of the original equations and solve for the second unknown variable.
- DD. Plug both values back into both of the original equations to check your answers.

## TIME VALUE OF MONEY

- A. A lump sum is a single cash flow.
1. The present value of a lump sum is the equivalent value today of a lump sum to be received or paid sometime in the future.
    - a. As interest rates increase (decrease), present values decrease (increase).
    - b. As the number of periods increase (decrease), present values decrease (increase).
  2. The future value of a lump sum is the equivalent value in the future of a lump sum to be received or paid today.
    - a. As interest rates decrease (increase), future values decrease (increase).
    - b. As the number of periods increase (decrease), future values increase (decrease).

- B. The effective interest rate (i.e., return) depends upon the nominal rate and the number of compounding periods per year. As the number of compounding periods increases, the effective rate increases.
- C. The geometric mean return is a compound annual growth rate for an investment. It represents the average annual growth rate earned on an investment over time. Geometric mean return takes into account the effects of compounding.
- D. An annuity is a countable number of equal cash flows occurring at fixed intervals of equal length over a defined period of time.
  1. The future value of an annuity is the equivalent lump sum value in the future of a series of cash flows to be received or paid. As interest rates decrease (increase), the future value of an annuity decreases (increases).
  2. The present value of an annuity is the equivalent lump sum value today of a series of cash flows to be received or paid. As interest rates increase (decrease), the present value of an annuity decreases (increases).
- E. When loan payments include both principal and interest, we say the loan is amortized. With each successive payment, the proportion of the payment applied to interest decreases, and the proportion applied to principal increases.
- F. The present value of a perpetuity (an annuity that never ends) is the cash flow divided by the discount rate.

**AN INTRODUCTION TO STATISTICS**

- A. A population is the collection of all possible individuals, objects, or other items (e.g., all the people in a country).
- B. A sample is an unbiased, randomly selected representative portion of the population that is used to estimate characteristics of the population.
- C. Qualitative variables measure attributes (e.g., eye color, gender).
- D. Quantitative variables are expressed numerically, (e.g., the number of children per household).
- E. Quantitative variables can be discrete or continuous.
  1. Discrete quantitative variables are countable.
  2. Continuous quantitative variables have an infinite number of possible values.
- F. The arithmetic mean is the simple average of the values (i.e., the sum of the values divided by the number of values).
 
$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \tag{7}$$
- G. The median is the middle observation of the ranked data.
- H. The mode is the observation that occurs the most.
- I. The range is the distance between the largest and smallest data points.
- J. The mean deviation is the average distance from the mean.

## Quantitative Methods

K. The variance and standard deviation are measures of dispersion of the data around the mean of the distribution.

1. Standard deviation equals the square root of the variance.
2. The population variance equals the average squared deviation from the mean:

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N} \quad (5)$$

3. The sample variance is similar to the population variance but has n-1 in the denominator instead of N:

$$s^2 = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \quad (7)$$

L. The empirical rule states that 68% of the data will lie within 1 standard deviation of the mean and that 95% of the data will lie within 2 standard deviations of the mean. This rule can be used to construct confidence intervals.

## PRACTICE QUESTIONS: QUANTITATIVE METHODS

*Note: All dollar values are rounded to the nearest whole dollar.*

1. A student has \$25,000 in her bank account, and the University charges a total of \$500 per credit hour. How many credit hours can she purchase before she must borrow money?
  - A. 5.
  - B. 12.
  - C. 50.
  - D. 150.
2. If  $\frac{15}{c} = 3$ , which of the following represents the value of  $c$ ?
  - A. 3.
  - B. 5.
  - C. 15.
  - D. 45.
3. If  $p \leq \frac{25}{5}$ , which of the following represents the value of  $p$ ?
  - A. Less than or equal to 5.
  - B. Greater than or equal to 5.
  - C. Equal to 5.
  - D. Equal to 25.
4. In the equation  $3(x + 5) = 45$ , which of the following represents the value of  $x$ ?
  - A. 10.
  - B. 15.
  - C. 20.
  - D. 25.
5. If  $4x + 4y = 24$  and  $2x + 3y = 24$ , which of the following statements is TRUE?
  - A.  $x = 6$ .
  - B.  $x = 12$ .
  - C.  $y = 6$ .
  - D.  $y = 12$ .
6. If  $x = 2 - y$  and  $y = x - 4$ , which of the following relationships is TRUE?
  - A.  $x = 3$ .
  - B.  $x = 6$ .
  - C.  $y = 1$ .
  - D.  $y = 14$ .



## Quantitative Methods

7. Jill invested \$100,000 in stocks and bonds. Equities earned a total return of 12%, and the fixed income component earned 8%. If she had invested twice as much in equities, she would have made \$1,800 more. How much was invested in equities?
- A. \$45,000.
  - B. \$10,000.
  - C. \$90,000.
  - D. \$55,000.
8. A client invested \$1.5 million both in stocks earning 13% total return and in bonds earning 5%. Total earnings for the clients was \$143,000. What percentage was invested in fixed income?
- A. 17.2%.
  - B. 25.4%.
  - C. 43.3%.
  - D. 85.9%.
9. For a given present value and interest rate, the future value:
- A. increases as the number of compounding periods per year increases.
  - B. decreases as the number of compounding periods per year increases.
  - C. remains the same as the number of compounding periods per year increases.
  - D. remains the same as the number of compounding periods per year decreases.
10. For a given future value and interest rate, the present value:
- A. increases as the number of compounding periods per year increases.
  - B. decreases as the number of compounding periods per year increases.
  - C. remains the same as the number of compounding periods per year increases.
  - D. remains the same as the number of compounding periods per year decreases.
11. Jim Wilson is planning to purchase a high performance sports car for \$100,000. He will finance the purchase with a 5-year fully amortized loan at an interest rate of 5.0% with payments due at the end of each year. What is the interest portion of the payment in year three and the remaining principal balance at the end of year three?
- |    | <u>Interest</u> | <u>Principal</u> |
|----|-----------------|------------------|
| A. | \$5,000         | \$42,948         |
| B. | \$3,145         | \$30,708         |
| C. | \$5,000         | \$30,708         |
| D. | \$3,145         | \$42,948         |

12. Samantha Tyson must decide which of four investments are the most attractive in terms of future value. The details of each investment opportunity are as follows:
- 1: \$1,000 annuity due with an interest rate of 7.1% and annual payments for three years.
  - 2: \$2,800 invested at an interest rate of 7.0% compounded monthly for three years.
  - 3: \$1,000 ordinary annuity with an interest rate of 7.1% and annual payments for three years.
  - 4: \$2,800 invested at an interest rate of 7.0% compounded semiannually for three years.
- Tyson has decided to rank each of the investments in order from highest future value to lowest. Which of the following answers correctly ranks the investments?
- A. 2, 4, 1, 3.
  - B. 1, 2, 4, 3.
  - C. 1, 2, 3, 4.
  - D. 2, 1, 4, 3.
13. What is the value of \$1,000 after 12 years at a *semiannually* compounded stated annual rate of 10%?
- A. \$2,200.
  - B. \$3,138.
  - C. \$3,225.
  - D. \$3,600.
14. What is the value of \$1,000 after 12 years at a *quarterly* compounded stated annual rate of 10%?
- A. \$3,271.
  - B. \$3,304.
  - C. \$2,200.
  - D. \$3,385.
15. What is the value today for a lump sum of \$1,000 to be received 5 years from now, using a 10% rate of interest?
- A. \$500.
  - B. \$621.
  - C. \$667.
  - D. \$909.

## Quantitative Methods

16. If \$5,000 is deposited into an account paying 6%, compounded monthly, what is the expected effective rate of return?
- A. 6.00%.
  - B. 6.17%.
  - C. 6.33%.
  - D. 6.50%.
17. For any nominal rate of interest, when the number of compounding periods per year increases, the effective rate of interest:
- A. increases.
  - B. decreases.
  - C. remains the same.
  - D. decreases at an increasing rate.
18. Cliff Bernstein is about to inherit his grandfather's estate. Over the next twenty years Cliff will receive \$17,250 at the end of each year as part of the trust set up in his name by his grandfather. In addition, Cliff will receive two one-time payments of \$250,000 six years from now and \$675,000 thirteen years from now. What is the present value of Cliff's inheritance using a 12% interest rate?
- A. \$425,660.
  - B. \$528,117.
  - C. \$224,740.
  - D. \$410,198.
19. An investor plans to make five year-end deposits of \$10,000 into an account paying 8%, compounded annually. At the end of five years (at the time of the last deposit) how much will be in the account?
- A. \$50,000.
  - B. \$54,000.
  - C. \$58,666.
  - D. \$63,359.
20. An investor plans to make deposits of \$10,000 into an account paying 8%, compounded annually. If she makes the deposits at the *beginning* of each year for the next five years, how much will she have in the account at the end of five years?
- A. \$50,000.
  - B. \$54,000.
  - C. \$58,666.
  - D. \$63,359.

21. Suppose a 30-year, \$200,000 mortgage loan is taken from a bank charging 6% interest, with annual compounding. What are the 30 year-end payments?
- A. \$6,667.
  - B. \$7,067.
  - C. \$13,707.
  - D. \$14,530.
22. An investment is expected to provide cash flows of \$100 one year from today, \$200 two years from today, and \$500 three years from today. If the required return is 8%, the value of this investment today is *closest* to:
- A. \$600.
  - B. \$661.
  - C. \$740.
  - D. \$800.
23. Mike will retire in 30 years and wants to have \$2.0 million for his retirement years. Mike expects to earn 10% (compounded annually) on annual deposits to an investment account starting one year from today. How much should Mike deposit annually for each of the next 30 years to reach his goal?
- A. \$11,010.
  - B. \$11,053.
  - C. \$12,159.
  - D. \$212,158.
24. Ben purchased a computer on credit. The store will charge an annual interest rate of 15% compounded annually. Payments are made at the end of each year. The term of the loan is two years. The annual cost to Ben is \$750. How much did Ben pay for the computer?
- A. \$1,154.
  - B. \$1,219.
  - C. \$1,304.
  - D. \$1,500.
25. Your money market account advertises a nominal rate of 2% with an effective annual rate of 2.0184%. Which of the following is the compounding frequency being used?
- A. Annual.
  - B. Semiannual.
  - C. Quarterly.
  - D. Monthly.

## Quantitative Methods

26. You are considering an investment with the following expected cash flows over the next 5 years (the first cash flow is 1 year from today).

Annual Cash Flow

\$10,000

\$12,000

\$14,000

X

\$18,000

The present value of the following cash flow stream is \$44,381.54 and assumes a discount rate of 8%. Calculate the missing cash flow amount (denoted by X in the table).

- A. \$2,000.  
B. \$4,000.  
C. \$8,000.  
D. \$16,000.
27. If \$5,000 is invested today and \$3,000 is invested one year from today, both at the annual interest rate of 6% compounded annually, the total amount in the account two years from today is *closest* to:
- A. \$8,000.  
B. \$8,671.  
C. \$8,798.  
D. \$8,800.
28. Suppose your client owns a perpetuity with a present value of \$1 million. What annual amount can your client withdraw every year, using an annual interest rate of 10%?
- A. \$100,000.  
B. \$100,100.  
C. \$500,000.  
D. \$1,000,000.
29. At a growth rate of 7.2%, approximately how long does it take a lump sum to double?
- A. 5 years.  
B. 1 year.  
C. 10 years.  
D. 8 years.

30. Assume you invest a lump sum of \$100 today. Further assume your investment value grows to \$210 in 5 years. What is your geometric mean annual return?
- A. 16%.
  - B. 20%.
  - C. 22%.
  - D. 25%.
31. If returns for the last five years are 2%, 15%, 17%, 19%, and 23%, which of the following represents the geometric mean?
- A. 12.39%.
  - B. 14.97%.
  - C. 15.21%.
  - D. 18.03%.

Use the following information regarding Rector, Inc. to answer Questions 32 through 34.

<i>Year</i>	<i>Return</i>
1999	35%
2000	10%
2001	5%
2002	-20%
2003	35%

32. What is the arithmetic mean for the annual returns of Rector's stock over the past five years?
- A. 10%.
  - B. 11%.
  - C. 13%.
  - D. 15%.
33. What is the geometric mean for the annual returns of Rector's stock over the past five years?
- A. 10%.
  - B. 11%.
  - C. 13%.
  - D. 15%.

## Quantitative Methods

34. What are the range, mode, and median for the annual returns of Rector's stock over the past five years?

	<u>Range</u>	<u>Mode</u>	<u>Median</u>
A.	35%	10%	55%
B.	55%	10%	35%
C.	35%	55%	10%
D.	55%	35%	10%

35. All of the following are measures of dispersion EXCEPT:
- A. mean average deviation.
  - B. standard deviation.
  - C. median.
  - D. variance.
36. All of the following are properties of the arithmetic mean EXCEPT:
- A. the sum of the deviations of each value from the mean will always be zero.
  - B. a set of data has only one mean.
  - C. the mean is not affected by extremely large or small values.
  - D. all values are included in computing the mean.
37. Joe has received a simple random sample of a partial year's sales for Oryx Company. The five months of sales are as follows: \$940,000, \$980,000, \$870,000, \$940,000, and \$920,000. The sample standard deviation is:
- A. \$40,000.
  - B. \$60,000.
  - C. \$80,000.
  - D. \$100,000.
38. All of the following are correct statements about a sample EXCEPT:
- A. a sample should be representative of the population.
  - B. the sample should be unbiased.
  - C. a sample is used to estimate characteristics of the population.
  - D. a sample includes all possible items.

Use the following information to answer Questions 39 through 44.

Below are the monthly salaries for six people. Round all answers to the nearest whole number.

<i>Person</i>	<i>Salary per Month (\$)</i>
1	5,000
2	3,000
3	3,500
4	4,000
5	4,250
6	4,300

39. Which of the following represents the arithmetic mean salary for the population?
- A. 4,000.
  - B. 4,008.
  - C. 4,200.
  - D. 5,280.
40. Which of the following represents the mean deviation of the salaries for the population?
- A. 508.
  - B. 793.
  - C. 1,492.
  - D. 4,008.
41. Which of the following represents the variance of the salaries for the population?
- A. 400,347.
  - B. 716,480.
  - C. 741,680.
  - D. 768,410.
42. Which of the following represents the standard deviation of the salaries for the population?
- A. 525.16.
  - B. 632.73.
  - C. 816.20.
  - D. 861.20.



## Quantitative Methods

43. Which of the following represents the variance of the salaries if the data represent a sample?
- A. 400,347.
  - B. 480,417.
  - C. 890,016.
  - D. 891,600.
44. Which of the following most closely represents the standard deviation of the salaries, if the data represent a sample?
- A. 632.
  - B. 693.
  - C. 934.
  - D. 940.

Use the following information to answer Questions 45 and 46.

Assume you examine the population of oil drilling projects. The projects follow a normal distribution (e.g., a symmetric distribution) with a mean profit of \$100 million and a standard deviation of \$50 million.

45. What is the probability that a randomly selected oil drilling project is profitable?
- A. 2.5%.
  - B. 16%.
  - C. 95%.
  - D. 97.5%.
46. What is the probability that a randomly selected oil drilling project will earn more than \$150 million?
- A. 2.5%.
  - B. 16%.
  - C. 95%.
  - D. 97.5%.

## PRACTICE QUESTION ANSWERS: QUANTITATIVE METHODS

1. C Let  $n$  represent the number of credit hours (the unknown). We know that the number of hours multiplied by the cost per hour, \$500, yields the total spent, which cannot be more than \$25,000. We represent this in equation form as the following:

$$\$500n = \$25,000$$

$$n = \frac{\$25,000}{\$500} = 50$$

2. B Multiplying both sides of the equation by  $c$ , we are left with the following:

$$15 = 3c$$

$$c = \frac{15}{3} = 5$$

3. A Dividing 25 by 5, we are left with  $p \leq 5$ . The  $\leq$  sign indicates “less than or equal to,” so the interpretation of the equation is  $p$  is less than or equal to 5.
4. A First we multiply through the parentheses by 3 and are left with  $3x + 15 = 45$ . We then subtract 15 from both sides and get  $3x = 30$ . Dividing both sides by 3 leaves us with  $x = 10$ .
5. D First set up the equations as simultaneous equations:

$$\text{Equation 1: } 4x + 4y = 24$$

$$\text{Equation 2: } 2x + 3y = 24$$

Next multiply both sides of Equation 2 by  $-2$  and add the two equations:

$$\begin{array}{r} 4x + 4y = 24 \\ -4x - 6y = -48 \\ \hline -2y = -24 \end{array}$$

Dividing both sides of the result by  $-2$  leaves us with  $y = 12$ . Substituting this value of  $y$  into our first equation gives us  $4x + 4(12) = 24$ . Subtracting 48 from both sides and dividing by 4 leaves us with  $x = -6$ .

6. A First set up the simultaneous equations:

$$x = 2 - y$$

$$y = x - 4$$

## Quantitative Methods

Now get all the variables on the left and all the numbers on the right. (Remember that a number or variable changes signs when it “crosses the bridge.”)

$$y + x = 2$$

$$y - x = -4$$

Add the two together:

$$2y = -2$$

$$y = -1$$

Substitute  $-1$  for  $y$  in the first equation:

$$x = 2 - (-1) = 3$$

7. **A** Define the variable, set up an equation based on the information, and solve for the variable.

$x$  = amount of money invested in equities

$100,000 - x$  = amount of money invested in fixed income securities

$$0.12x + 0.08(100,000 - x) + 1800 = 0.12(2x) + 0.08(100,000 - 2x)$$

$$0.12x + 8,000 - 0.08x + 1,800 = 0.24x + 8,000 - 0.16x$$

$$-0.04x = -1,800$$

$$x = 45,000$$

8. **C** Define the variable, set up an equation based on the information, and solve for the variable.

$x$  = bonds also known as fixed income

$1,500,000 - x$  = stocks also known as equities

$$0.05x + 0.13(1,500,000 - x) = 143,000$$

$$0.05x + 195,000 - 0.13x = 143,000$$

$$-0.08x = -52,000$$

$$x = 650,000$$

The question asks for the percent invested in fixed income as follows:

$$\frac{650,000}{1,500,000} = 0.433 \text{ or } 43.3\%$$

9. **A** As illustrated in the equation  $\left(1 + \frac{i}{m}\right)^m - 1$ , the effective interest rate increases as the number of compounding periods per year,  $m$ , increases. As the effective rate increases, the future value increases since you are compounding at a higher rate.
10. **B** As the effective rate increases, the present value must decrease since you are discounting at a higher rate.
11. **D** When a loan is fully amortized, the payments are typically equal for the life of the loan, and each payment includes interest on the amount of the loan still outstanding (remaining principal) with the rest of the payment applied to the principal balance. The payment is found using the ordinary annuity method as follows:

-100,000 PV  
 5 I/Y  
 5 N  
 CPT PMT = \$23,097

Interest in each year equal the interest rate (5.0%) times the principal balance at the end of the previous year. For the third year, the interest portion of the payment is  $0.05 \times 62,900 = \$3,145$ . The principal portion of the payment is  $23,097 - 3,145 = \$19,952$ . Thus the principal balance gets reduced to  $62,900 - 19,952 = \$42,948$  at the end of year three. The following amortization table demonstrates the interest, principal, and outstanding balance for each of the five years the loan is outstanding.

<i>Year</i>	<i>Payment</i>	<i>Interest</i>	<i>Principal</i>	<i>Balance</i>
0				100,000
1	23,097	5,000	18,097	81,903
2	23,097	4,095	19,002	62,900
3	23,097	3,145	19,952	42,948
4	23,097	2,147	20,950	21,998
5	23,097	1,100	21,998	0

## Quantitative Methods

12. D Begin by calculating the future value of each investment as follows:

Investment 1:

Calculator strokes:	2nd ⇒	BGN
	2nd ⇒	SET
	2nd ⇒	QUIT
	-1,000	PMT
	7.1	I/Y
	3	N
	CPT	FV = \$3,447

Investment 2:

Calculator strokes:	-2,800	PV
	7 / 12	I/Y
	3 * 12	N
	CPT	FV = \$3,452

Investment 3:

Calculator strokes:	-1,000	PMT
	7.1	I/Y
	3	N
	CPT	FV = \$3,218

Investment 4:

Calculator strokes:	-2,800	PV
	7 / 2	I/Y
	3 * 2	N
	CPT	FV = \$3,442

Therefore the investments in order of largest to smallest future value are 2, 1, 4, 3.

For Questions 13 and 14 use the following formula:

$$FV_n = PV \left( 1 + \frac{i}{m} \right)^{m \cdot n}$$

where:

m = the number of compounding periods per year

i = the stated or nominal rate of interest

n = the number of years

PV = the beginning amount or present value

FV = the ending amount or future value

Alternatively, you can plug the values into your TI BA II Plus® as shown. If you prefer to do that, set your P/Y value to 1.0 and enter the values. Remember, the calculator will assume one cash flow is an inflow and the other an outflow.

13. C  $FV = 1,000(1 + 0.10/2)^{12 \times 2} = 3,225.10$ , or

PV = 1,000, years = 12, compounding period per year = 2, interest rate = 10%

Calculator strokes:    -1,000    PV  
                                   10/2        I/Y  
                                   12 × 2     N  
                                   CPT        FV = \$3,225.10

14. A  $FV = 1000(1 + 0.10/4)^{12 \times 4} = \$3,271.49$ , or

PV = 1000, years = 12, compounding periods per year = 4, interest rate = 10%

Calculator strokes:    -1,000    PV  
                                   10/4        I/Y  
                                   12 × 4     N  
                                   CPT        FV = \$3,271.49

15. B  $PV = \frac{1000}{(1.10)^5} = 620.92$ , or

FV = 1000, years = 5, compounding periods per year = 1, interest rate = 10%

Calculator strokes:    1,000     FV  
                                   10/1        I/Y  
                                   5            N  
                                   CPT        PV = \$620.92

16. B The effective return for an account is the same regardless of the amount of money deposited. Using the effective interest rate formula:

$$\left(1 + \frac{i}{m}\right)^m - 1.0 = 12, i = 0.06, \text{ effective rate} = 1.0617 - 1 = 6.17\%$$

17. A See the formula in the answer to Question 8.

## Quantitative Methods

18. D The key to a problem such as this one is to find the present value of each investment separately and then add the present values together. There are three investments in this problem: an annuity of \$17,250 that will last 20 years, a lump sum payment of \$250,000 that will occur in six years, and a lump sum payment of \$675,000 that will occur in thirteen years. We find the present values as follows:

Annuity (make sure the calculator is in end mode):

Calculator strokes:	-17,250	PMT
	12	I/Y
	20	N
	CPT	PV = \$128,848

First Lump Sum:	-250,000	FV
	12	I/Y
	6	N
	CPT	PV = \$126,658

Second Lump Sum:	-675,000	FV
	12	I/Y
	13	N
	CPT	PV = \$154,693

Thus, the total value of the inheritance is  $\$128,848 + \$126,658 + \$154,693 = \$410,198$ .

19. C These are end-of-year payments. Your calculator should be set to END.

Note: Since it is default, END does not display on your calculator. If BGN is displayed, perform the following keystrokes first:

2nd  $\Rightarrow$  BGN  
2nd  $\Rightarrow$  SET  
2nd  $\Rightarrow$  QUIT

Calculator strokes:	-10,000	PMT
	8	I/Y
	5	N
	CPT	FV = \$58,666.01

20. **D** These are beginning-of-year payments. Your calculator should be set to BGN.

Note: BGN must be displayed on your calculator to use beginning-of-year cash flows. If BGN is not displayed, perform the following keystrokes:

2nd ⇒ BGN  
 2nd ⇒ SET  
 2nd ⇒ QUIT

Calculator strokes:    -10,000    PMT  
                                   5            N  
                                   8            I/Y  
                                   CPT        FV = \$63,359.29

21. **D** The present value of the payments must equal the amount borrowed. Be sure your calculator is set to END.

Keystrokes:            -200,000    PV  
                                   6            I/Y  
                                   30            N  
                                   CPT        PMT = \$14,529.78

22. **B** 
$$PV = \frac{100}{(1.08)^1} + \frac{200}{(1.08)^2} + \frac{500}{(1.08)^3} = 92.59 + 171.47 + 396.92 = \$660.98$$

23. **C** Be sure your calculator is set to END.

Keystrokes:            2,000,000    FV  
                                   10            I/Y  
                                   30            N  
                                   CPT        PMT = \$12,158.50

24. **B** Be sure your calculator is set to END.

Keystrokes:            750            PMT  
                                   15            I/Y  
                                   2            N  
                                   CPT        PV = \$1,219.28



## Quantitative Methods

25. D Effective return = 0.02 =  $\left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{0.02}{m}\right)^m - 1$

To use trial and error, plug different choices for m:

annual = 1, gives an effective rate of 2%, easy choice to eliminate

semiannual = 2, gives an effective rate of 2.01%, not the correct answer

quarterly = 4, gives an effective rate of 2.0151%, not the correct answer

monthly = 12; gives an effective rate of 2.0184%, the correct answer

26. A 
$$PV = \left(\frac{FV_1}{(1+i)^1}\right) + \left(\frac{FV_2}{(1+i)^2}\right) + \left(\frac{FV_3}{(1+i)^3}\right) + \left(\frac{FV_4}{(1+i)^4}\right) + \left(\frac{FV_5}{(1+i)^5}\right)$$
$$44,381.54 = \left(\frac{10,000}{(1+.08)^1}\right) + \left(\frac{12,000}{(1+.08)^2}\right) + \left(\frac{14,000}{(1+.08)^3}\right) + \left(\frac{X}{(1+.08)^4}\right) + \left(\frac{18,000}{(1+.08)^5}\right)$$
$$44,381.54 = 9,259.26 + 10,288.07 + 11,113.65 + \frac{X}{1.3605} + 12,250.50$$
$$44,381.54 = 42,911.48 + \frac{X}{1.3605}$$
$$1,470.06 = \frac{X}{1.3605}$$
$$X = 2,000$$

27. C  $FV = CF_1(1+i)^2 + CF_2(1+i)^1$

$$FV = 5,000(1.06)^2 + 3,000(1.06)^1$$

$$FV = 5,618 + 3,180 = \$8,798$$

28. A The present value of a perpetuity equals the periodic amount divided by the interest rate.

$$PV = PMT/i$$

$$PV \times i = PMT = \$1,000,000 \times 0.10 = \$100,000$$

29. C  $FV = PV(1 + i)^n$

Note if an investment doubles, the future value of each \$1 will be \$2. Therefore:

$$FV = 2 = 1(1.072)^n$$

Use trial and error, plugging for n:

n = 1, FV = 1.072, too low—not the correct answer

n = 5, FV = 1.42, too low—not the correct answer

n = 8, FV = 1.74, too low—not the correct answer

n = 10, FV = 2.004, close enough! The answer is 10 years.

30. A Geometric mean return =  $\sqrt[n]{\frac{FV}{PV}} - 1$  or  $\left(\frac{FV}{PV}\right)^{1/n} - 1$

$$\text{Geometric mean return} = \left(\frac{210}{100}\right)^{1/5} - 1 = 16\%$$

31. B The geometric mean, GM, is:

$$GM = \sqrt[n]{(1 + x_1)(1 + x_2)(1 + x_3)\dots(1 + x_n)} - 1$$

$$GM = \sqrt[5]{(1 + 0.02) \times (1 + 0.15) \times (1 + 0.17) \times (1 + 0.19) \times (1 + 0.23)} - 1$$

$$= \sqrt[5]{(1.02)(1.15)(1.17)(1.19)(1.23)} - 1$$

$$= \sqrt[5]{2.0088} - 1 = 1.1497 - 1 = 0.1497 = 14.97\%$$

32. C  $\frac{[35\% + 10\% + 5\% + (-20) + 35\%]}{5} = 13$

33. B  $(1.35 \times 1.10 \times 1.05 \times 0.80 \times 1.35)^{1/5} = 11\%$

34. D Range (high minus low) =  $35\% - (-20\%) = 55\%$

Mode (most observations) = 35%

Median (in the middle) = 10%

35. C Median is a measure of central tendency. Median is a descriptive statistic that measures the center of the distribution. Mean average deviation measures the average deviation from the mathematical mean. Average is calculated using the number of observations and deviation is measured as the distance from the mean of each observation. Standard deviation is the square root of the variance. Variance is the squared deviations from the mean.

Be aware that a sample variance would have a denominator of n – 1, rather than n.

## Quantitative Methods

36. C Statistically described as outliers, these values have a distorting impact not only on the mean, but on all other descriptive statistics. Advanced statistical methods can be used to lessen the problem.

$$37. A \quad \text{mean} = \frac{940,000 + 980,000 + 870,000 + 940,000 + 920,000}{5} = 930,000$$

$$\sqrt{\frac{[(940,000 - 930,000)^2 + (980,000 - 930,000)^2 + (870,000 - 930,000)^2 + (940,000 - 930,000)^2 + (920,000 - 930,000)^2]}{5 - 1}} = 40,000$$

38. D The inclusion of all possible items is the definition of a population. All other statements accurately describe a sample.

$$39. B \quad \frac{5,000 + 3,000 + 3,500 + 4,000 + 4,250 + 4,300}{6} = 4,008.3$$

40. A The mean deviation, MD, is:

$$MD = \frac{\sum |x - \bar{x}|}{n}$$

where:

$x$  = the value of each observation

$\bar{x}$  = the arithmetic mean of the observations

$n$  = the number of observations

$|x - \bar{x}|$  = the absolute value of each deviation

$$MD = \frac{(|5,000 - 4,008| + |3,000 - 4,008| + |3,500 - 4,008| + |4,000 - 4,008| + |4,250 - 4,008| + |4,300 - 4,008|)}{6}$$

$$MD = \frac{992 + 1,008 + 508 + 8 + 242 + 292}{6} = \frac{3050}{6} = 508.3$$

41. A The variance of a population is found using:

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

where:

$\mu$  = the mean of the population

$X$  = an individual member of the population

$N$  = the number of observations in the population

$$\begin{aligned} \sigma^2 &= \frac{\left[ (5,000 - 4,008)^2 + (3,000 - 4,008)^2 + (3,500 - 4,008)^2 \right. \\ &\quad \left. + (4,000 - 4,008)^2 + (4,250 - 4,008)^2 + (4,300 - 4,008)^2 \right]}{6} \\ &= \frac{(992)^2 + (-1,008)^2 + (-508)^2 + (-8)^2 + (242)^2 + (292)^2}{6} \\ &= \frac{984,064 + 1,016,064 + 258,064 + 64 + 58,564 + 85,264}{6} \\ &= \frac{2,402,084}{6} \\ &= 400,347 \end{aligned}$$

42. B The standard deviation is the square root of the variance, so:

$$\sigma = \sqrt{400,347} = 632.73$$

43. B The variance of a sample is found using:

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

where:

$s^2$  = the sample variance

$x$  = an individual observation in the sample

$\bar{x}$  = the sample mean

$(n - 1)$  = the number of sample observations minus 1

Since the numerator is exactly the same as the numerator in Question 41, we need only change the denominator to find the variance of the sample:

$$\begin{aligned} s^2 &= \frac{984,064 + 1,016,064 + 258,064 + 64 + 58,564 + 85,264}{5} \\ &= \frac{2,402,084}{5} \\ &= 480,417 \end{aligned}$$

## Quantitative Methods

44. **B** Again, the standard deviation is the square root of the variance, so:

$$s = \sqrt{480,417} = 693.12$$

45. **D** The empirical rule states that approximately 95% of the data lie within 2 standard deviations of the mean (e.g., from zero up to \$200 million). Using the rule, notice that a zero profit is 2 standard deviations below the mean. Thus, there is a 2.5% chance that a randomly selected oil venture will lose money (profit is less than zero) and a 97.5% chance that a randomly selected oil venture will make money. Similarly, there is a 97.5% chance that a randomly selected oil drilling project will make less than \$200 million.

46. **B** The empirical rule also states that approximately 68% of the data lie within 1 standard deviation of the mean. Using the rule, notice that \$150 million is 1 standard deviation above the mean. Thus, there is a 16% chance that a randomly selected oil drilling project will make more than \$150 million. Similarly, there is a 16% chance that a randomly selected oil drilling project will make less than \$50 million.

# ECONOMICS

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Economics is the study of how people make choices. Every time a person makes a choice, he or she does so in terms of utility. That is, people always choose the selection that maximizes satisfaction or pleasure. For the individual, the choice is typically one of weighing the costs and potential outcomes (i.e., benefits) of two or more alternatives. Business decisions are also driven by some form of cost/benefit analysis. Although the outcomes of business decisions are usually fairly straightforward (e.g., increased revenues, reduced costs, or some combination of the two), the manager must determine whether the reduced costs and/or increased revenues are worth the investment of time and money.

## SUPPLY AND DEMAND

Supply and demand are probably two of the most commonly used words in our economics vocabulary, so a discussion of each is appropriate. Although the textbook definition for supply might include such words as aggregate production, marginal cost, or marginal revenue, let's think of supply as the amount of an item available for purchase.

## SUPPLY

The supply of any good or service is the amount available for consumption in the marketplace. The amount of an item available for purchase is based on factors such as the number of producers, the profit from producing the item, the number of customers, the selling price, and the costs for shipping the item around the country or globe. Of course, the typical consumer could care less about these factors. What *is* important is whether the item we want is available when and where we want it at a price we are willing to pay. Many variables must be considered, so let's take a look at an example.

Assume you are a wholesaler and your company supplies two products: gerbil bedding and gold necklaces.<sup>1</sup> You purchase gold chain directly from the manufacturer in 1,000-inch rolls at \$2.00 per inch. It costs you another \$25.00 per necklace to cut the chain to length, add clasps, and prepare it for shipping. The bedding material is actually wood chips and mulch left over from a lumberyard. It costs you \$0.10 per pound and comes by truckload. (A pound of the bedding material is about one cubic foot, which is about right for the typical gerbil cage.) It costs you another \$0.15 to put one pound in a labeled plastic bag and prepare it for shipping.

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<sup>1</sup> Obviously, this odd mix of products is only for demonstration purposes.

## Economics

Shipping presents an interesting problem for these vastly different products. Since shipping costs are based on weight and size (volume), a small, heavy item costs no more to ship than a large (bulky), light item. This makes gold necklaces value-intensive products, meaning they are valuable, light, and small. The bedding, on the other hand, is cost intensive. It is very light and inexpensive but must be shipped in much larger containers. The result is that 100 necklaces can be shipped for about the same cost as 20 bags of bedding. Let's assume the shipping cost for either is \$10.00.

The gold for a 20-inch necklace costs \$40.00 (\$2 per inch), and the clasps and packaging cost another \$25.00, resulting in a total of \$65.00 to assemble and prepare one necklace for shipping. If jewelers place orders for 100 necklaces at a time, and 100 necklaces can be shipped for \$10.00, shipping adds only an additional \$0.10 to the cost of each necklace. This brings the total cost to assemble and ship one necklace to \$65.10. In turn, we assume that we can sell the necklace to a jeweler for \$100.

Next, let's look at the bedding. The costs for the material and packaging for one bag of bedding is \$0.25. Since it costs \$10.00 to ship 20 bags, the cost to ship one bag is \$0.50. This brings the total to \$0.75 per bag. Let's assume that you can sell them to pet stores for \$0.65. The result? It is cost effective for you to ship gold necklaces just about anywhere. The bedding, on the other hand, can only be sold locally, since to package and ship it costs more than the price for which you can sell it.

The bottom line is that you will not ship your gerbil bedding any distance. In fact, unless you can distribute the bedding locally at a total cost that is lower than your selling price, you won't supply gerbil bedding at all. And as long as costs are about the same for all suppliers of gerbil bedding, all suppliers are faced with the same situation.

Now, let's assume it costs about \$0.10 per bag to distribute bedding locally, as opposed to \$0.50 to ship it longer distances. That means it costs a total of \$0.35 per bag (\$0.25 material and packaging plus \$0.10 transportation) to supply bedding locally. If you can sell it for \$0.65, a gross profit of \$0.30 per bag will be realized. As long as you don't try to charge local retailers more than \$1.00 or so, out-of-town suppliers won't be enticed to compete with you. (Remember, out-of-town suppliers can ship it in to your local market at a total cost of \$0.75 per bag. As long as the local selling price doesn't make it profitable for outside suppliers to sell in your market, they won't.)

Local suppliers want to maximize profits, so they want to sell at the highest price possible, without reaching the estimated \$1.00 that would attract non-local competition. Note that price is the only thing that matters when gerbil-bedding suppliers compete among themselves in the local market. Gerbil bedding is gerbil

bedding! Customers don't care about the label on it or who provides it, so suppliers can't compete on quality or service.

The bottom line is that the supply of bedding is constrained by the amount the local suppliers can provide. Gold chains, on the other hand, are a totally different product. Since it is profitable to ship them anywhere, the gold chains that consumers see in stores can be from anywhere in the world. This makes the supply of gold chains limited only by the worldwide supply of gold.

The preceding arguments depend upon the price suppliers receive. If the total cost to make and ship an item is sufficiently below the selling price, that item will be supplied. When the selling price does not provide revenues sufficient to cover all costs and provide a profit, the item is not supplied. This leads us to the **law of supply**, which states that as the price of a product moves higher, more of that product will be supplied. The higher price will make it more attractive for producers to produce and deliver more of the product.

## DEMAND

Demand is how much society wants of a good or service. Demand will be a function of how many potential buyers there are for a product, as well as whether there are other products that could serve the same purpose. The selling price will usually affect demand, but not always. The demand for soft drinks would probably change if the price changed very much. If the price went up, consumers would shift to other brands of soft drinks, or possibly to juice or water. If the price declined, consumers would likely purchase more. However, demand for a good like milk would be less subject to changes in price. Increases in price may not affect demand much because there are no good substitutes for milk. Similarly, a decrease in price might not create more demand for milk. This difference in sensitivity of demand to price changes is called **elasticity**.

Let's return to our gold necklace and gerbil bedding example. Demand for gold necklaces will depend to some extent on fashion trends. If it became trendy to wear gold necklaces, demand would increase. The desire to be in fashion could cause many consumers to buy a gold necklace, even though they had not owned one before. This increase in demand would probably lead to higher prices, as producers realized that they could charge more. Why are they able to charge more? Because there are more potential buyers for their product. More buyers means higher demand.

Demand could also decrease if gold necklaces became unfashionable. If such necklaces were considered an ostentatious display of wealth, for example, consumers would hesitate to wear them. Fewer buyers would exist, and demand would fall.



## Economics

Demand for gold necklaces would also depend on prices. If gold prices increased dramatically, consumers would buy fewer gold necklaces. Consumers might even consider other types of necklaces, such as necklaces made of silver. If gold prices dropped, consumers would likely buy more. For one thing, more consumers could afford to buy gold necklaces, and demand would increase due to the higher number of potential buyers. In fact, the **law of demand** states that consumers will buy more of a good as the price of that good declines.

Demand for gerbil bedding will also be a function of the number of potential buyers, and of the price. For example, if a popular children's movie starred a gerbil in the leading role, more children would want gerbils. Presumably, more gerbils would mean a greater need for gerbil bedding, and demand would increase. Gerbil bedding probably has less price sensitivity than gold necklaces. A decrease in price would be unlikely to inspire consumers to rush out and buy a gerbil because bedding was so cheap. However, a price increase would affect demand. High prices for gerbil bedding would push consumers to find other products that would serve the same purpose. Since there are several cheap alternatives (e.g., shredded newspaper, leaves), demand for gerbil bedding would fall.

The alternative products previously mentioned (silver necklaces, shredded newspaper) are called **substitutes**. They are products that serve the same purpose. If the price of a product changes, this will affect demand for substitutes, and vice versa. With substitutes, an increase in demand for one product will lead to a decrease in demand for the other.

There are also situations where demand for one product will affect demand for a related product that is not a substitute. For example, we mentioned how increased popularity of gerbils would lead to an increase in demand for gerbil bedding. Gerbils and gerbil bedding would be considered **complements**. With complements, an increase in demand for one product leads to an increase in demand for another.

## EQUILIBRIUM

From our discussions of supply and demand, we can see that the interaction of these two economic conditions sets prices. Neither supply nor demand alone is sufficient. Regardless of whether a product is expensive or cheap to produce, if no market exists (no demand for the product), it won't sell at any price. If a strong market exists for the product, it will sell. However, profits depend on the price received. That price, in turn, depends on the number of buyers relative to the number of sellers (i.e., the demand relative to the supply).

In the gold chain and bedding discussions we took the selling price as a given. Whether gerbil bedding was shipped or not depended on the total cost to package

and ship the bags compared to the given selling price. We assumed a sufficiently low price such that it was not feasible to ship the bags of gerbil bedding cross-country. On the other hand, the gold necklaces sold at a high enough price that you could ship them anywhere and still make a profit. Just what determined the selling prices for those items?

Gold, as you probably are aware, is considered a precious metal. There is a limited (actually, carefully controlled) supply of gold on the world market at any time. Since gold is highly sought after worldwide, there is a more or less constant demand. As long as the supply of gold does not run ahead of demand, its price will remain relatively high.

Gerbil bedding, on the other hand, is not a precious commodity. Although gerbil owners might consider commercial bedding superior, they will resort to newspaper if the commercial product gets too expensive. This means there is a maximum retail price that consumers are willing to pay, so the costs to package and ship gerbil bedding become extremely important. Even though gerbil bedding is inexpensive to produce, its limited demand constrains possible sales opportunities and profits.

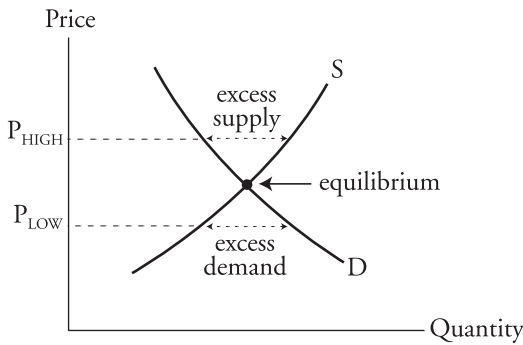
We've discussed supply, demand, and the relationships among them. Another term you will often hear is **equilibrium**, which can be thought of as two equal and opposing forces. Supply and demand must be in equilibrium for prices to be stable. For example, if everyone in the world suddenly wanted a gold necklace, the demand would far outstrip the supply of necklaces, and gold prices would increase dramatically. In a similar fashion, if increases in gold prices caused an abnormally large number of gold suppliers to increase their supply, the price would stabilize, or maybe even drop. As long as demand and supply are equal, the price for gold will remain about the same.

Without resorting to too much economic jargon to explain equilibrium, let's just leave it as meaning equal forces. In our example, the demand and supply for gold are equal. Suppliers are willing to provide the amount of gold that consumers are willing to purchase (i.e., the upward pressure on price caused by demand and the downward pressure on price caused by supply are perfectly balanced). Prices are stable because there is no pressure to push them higher or lower.

## Economics

Equilibrium is shown graphically in the following. Notice how higher prices will create excess supply, while lower prices generate excess demand.

Figure 1: Supply and Demand Equilibrium



Think of an equilibrium price as the price to which the good returns if there are no forces acting upon it. That is, unless some factor or combination of factors causes a disequilibrium, supply and demand tend to converge at the equilibrium price. As a very simple example, think of a ball sitting in the middle of a room. Unless some force acts directly on the ball, it remains still. A force can cause it to roll, but if the force isn't continued, the ball again comes to rest. In other words, you can think of just sitting still as the ball's equilibrium state. It is the state to which the ball will always return, unless some force causes it to move. In a like fashion, without some force, such as a temporary shortage or surplus, the price of a good will always tend to return to its equilibrium level.

For example, a drought in a coffee producing region can artificially reduce supply (create a temporary shortage), causing the price of coffee to rise temporarily above its long run equilibrium price. When weather conditions return to normal, the supply returns to normal and the price falls back to the equilibrium price. In a similar way, due to a surplus the market price of a good could temporarily fall to a level below the long-run equilibrium price. Once again, as conditions return to normal, the price rises back to the equilibrium price.

In the microeconomics section, we discuss some of the factors that affect equilibrium prices.

## DISTORTIONS TO EQUILIBRIUM

**Price controls** are maximum or minimum prices for goods, established by a government. A maximum price is a **price ceiling**, and a minimum price is a **price floor**. If price controls are set very far from equilibrium prices, they can have

significant negative economic consequences. A price ceiling below the equilibrium price for a product will create higher demand, while simultaneously reducing the available supply. Assume that a price ceiling is set at the low price ( $P_{\text{LOW}}$ ) shown in Figure 1. Demand will be higher than at the equilibrium price because the lower price will make more consumers willing to buy the good. At the same time, the low price discourages suppliers from producing the good. The result will be a shortage of the good.

Price floors also create problems. If a price floor ( $P_{\text{HIGH}}$ ) is set above the equilibrium price, demand will fall due to the high price, while supply will increase. There will be an excess supply of the good, and consumers will pay an artificially high price for whatever amount of the good they purchase.

Sometimes, artificial pricing for a good will give rise to a **black market** that operates outside legal channels. Items that are illegal or heavily taxed often develop black markets, as do items with controlled prices set artificially high or low. Illegal drugs are traded in a black market, and there are black markets for copyrighted materials such as music and software. Because black markets operate outside the law, the risks are higher for buyers and sellers alike. Buyers must be wary of poor quality goods, and sellers face criminal prosecution if they are caught. Also, because there are no legal remedies, any disputes between buyer and seller must be settled in other ways, often involving violence. A black market sacrifices economic efficiency and rewards the parties willing and able to use the most force to support their position.

## EFFECTS OF TAXES

Equilibrium prices are also distorted by taxes. Taxes that are imposed on a good can affect both the buyer and the seller. **Tax incidence** measures how the tax actually paid is allocated between buyers and sellers. Buyers will probably pay a higher price for a good that is taxed, and sellers will probably receive less from selling the good. The **statutory incidence** of a tax identifies the party responsible for paying the tax. This may or not be the party who bears the true economic burden of the tax.

For example, assume that the government imposes a 2% tax on house paint to help cover the negative environmental impact of disposing of old paint. The producer of paint would like to pass the entire tax along to consumers by raising the price of paint by 2%. It is quite possible, however, that raising all prices by 2% would reduce demand for paint. Consumers might seek substitutes, such as stains, or they might postpone painting jobs. Consumer resistance to higher prices may allow the producer to raise prices by only 1%. In this case, the tax burden would be borne equally by producers and consumers. Consumers would pay 1% more for paint, and producers would receive 1% less. A gallon of paint that cost \$10.00 before the imposition of the tax would now sell for \$10.10. If a producer made a profit of \$1.00 on each gallon, the profit would be reduced by \$0.10 to \$0.90 per gallon.

## Economics

The tax incidence will depend on how sensitive supply and demand are to changes in price. The sensitivity of supply and demand to changes in price is called elasticity, which is discussed in the microeconomics section. The more elastic the demand (or supply) is, the more sensitive quantity demanded (supplied) will be to changes in price. Graphically, more elastic demand (supply) will have a flatter demand (supply) curve, where small changes in price result in large changes in quantity demanded (supplied). If demand is inelastic relative to supply (i.e., the demand curve is steeper than the supply curve), the tax incidence will fall more heavily on buyers. This makes sense, as inelastic demand means buyers are not that sensitive to changes in price, so they would be willing to pay more of the tax burden. If supply is inelastic relative to demand (i.e., the supply curve is steeper than the demand curve), then the tax incidence will fall more heavily on sellers, as they will be less able to pass along any price increase.

## MACROECONOMICS

Macroeconomics refers to large or economy-wide forces. A macroeconomic variable (e.g., inflation, unemployment, or monetary policy) is a factor that affects the entire economy or a major portion of it.

Macroeconomic variables are beyond the control of individuals or individual companies. For instance, no matter how much money you save or spend, you will have no material effect on the rate of economic growth in your country. However, the aggregate spending or saving habits of the entire population of a country greatly affect that country's economic growth.

For an individual or company, macroeconomic variables are simply something with which to contend. For example, an increase in inflation increases input costs, which increases the total cost to manufacture goods. Whether the company can raise the selling price to absorb the increased cost is not certain, since that will depend on consumers' willingness to pay a higher price. Consumers tend not to blindly accept price increases, whether caused by inflation or not.

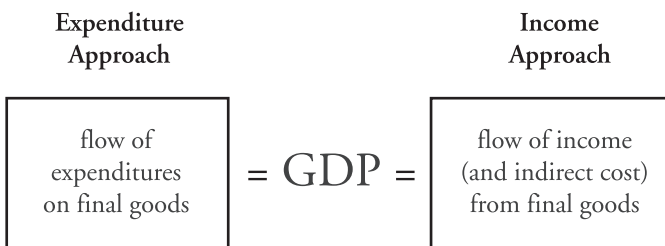
Let's take a look at some macroeconomic factors, starting with gross domestic product (GDP), inflation, and unemployment. We will also discuss aggregate demand and aggregate supply. We will conclude our macroeconomics section by exploring macroeconomic policy, specifically fiscal and monetary policy.

**Gross domestic product (GDP)** is the most commonly used measure of economic performance. It is the total market value of all domestically produced final goods and services during a given year. GDP is designed to measure the market value of production that flows through the economy.

- GDP includes only goods and services purchased by their final or ultimate users, so GDP measures final production. Intermediate stages of production are not included. For example, the price of flour sold to a baker is not included in GDP. Value added during the intermediate steps (i.e., harvesting, milling, and baking) are all included in the final selling price of the baked goods sold by the baker.
- GDP counts only the goods and services produced within the country's borders during the year, whether by citizens or foreigners. Therefore, goods produced at a factory in the United States and sold in the United States would be included in GDP, regardless of whether the factory was owned by a U.S. company or a Japanese company. Sales of used or secondhand goods are excluded, but sales commissions charged on the sale of used products are included.
- GDP excludes financial transactions and transfer payments since they do not represent current production. For example, stock and bond sales along with welfare and social security payments are excluded.

GDP may be measured using either the expenditure approach or the income approach. Both approaches yield the same results because aggregate expenditures must equal aggregate income, as illustrated in Figure 2.

Figure 2: GDP Measures



These two approaches are described as follows.

The **expenditure approach** considers total spending on all final goods and services produced during the year. The expenditure approach is a demand-based concept measured by summing the following expenditure items:

- *Personal consumption.* This represents household purchases for consumption and represents the largest component of GDP. Over two-thirds of GDP is accounted for by personal consumption. This category of expenditures includes durable goods, nondurable goods, and services.

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- *Gross private investment (expenditures of business)*. This is an important component of GDP because it provides an indicator of the future productive capacity of the economy. Fixed investment (investment in capital goods) is a key component of future economic growth. GDP includes replacement purchases plus net additions to the stock of capital assets plus investments in inventories. Inventory investments are the changes in the stock of unsold goods held by a business during the period.
- *Government consumption and gross investment*. Purchases of goods and services by federal, state, and local governments are included in GDP. Transfer payments are excluded. Hence, not all government spending is included in GDP.
- *Net exports of goods and services*. Since we only want to measure domestic production, net exports are calculated as total exports (domestically produced goods and services purchased by consumers outside the country) minus total imports (foreign-produced goods and services purchased by domestic consumers).

The **income approach** is a supply (i.e., production) oriented approach and measures GDP by summing the following components:

- Employee compensation.
- Proprietors' income.
- Rents.
- Corporate profits.
- Interest income.
- Indirect business taxes.
- Depreciation.
- Net income of foreigners (the income foreigners earn domestically minus the income that domestic citizens earn abroad).

The sum of the first five items (employee compensation, proprietors' income, rents, corporate profits, and interest income) equals **national income**. National income includes income earned by domestic citizens abroad as well as what they earn in their home country.

Gross domestic product measured at current prices is called **nominal GDP**. When we discuss economic growth, however, we want to measure the increase in actual production, or **real GDP**. Thus, we have to adjust nominal GDP for inflation

The **GDP deflator** is a *price index* that corresponds to the price change exhibited by all final goods and services produced. The GDP deflator is useful for measuring economy-wide inflation.

To calculate real GDP from nominal GDP, we need three pieces of information: (1) the GDP deflator in the base year, (2) the GDP deflator in the current period, and (3) the nominal GDP in the current year. These values are related as follows:

$$\text{real GDP}_{\text{current period}} = \text{nominal GDP}_{\text{current period}} \times \frac{\text{GDP deflator}_{\text{base year}}}{\text{GDP deflator}_{\text{current period}}}$$

GDP is the most comprehensive measure of the aggregate output of the economy, but there are several activities that are not included:

- Household goods and services—domestic chores such as cleaning, washing the car, or gardening are not captured in GDP unless someone is paid to do them. This may distort historical GDP comparisons if, for example, over time more households employ outsiders to perform these tasks.
- Black market activities—these goods and services sold in illegal markets are not captured in GDP.
- Quality of life—GDP does not reflect productivity or the quality of goods produced and sold. A computer produced in 1980 and sold for \$5,000 counts the same in GDP as a computer produced in 2005 and sold for \$5,000, despite the fact that the 2005 computer was considerably more powerful. GDP also does not reflect that the 1980 computer may have taken 50 hours to assemble, while the 2005 computer may have taken only 6 hours.
- Pollution—GDP does not adjust output for any damaging side effects, such as air or water pollution. Cleaning up pollution will add to GDP as it occurs, but there is no adjustment to current GDP for creating the problem in the first place.

## Inflation

**Inflation** is a very commonly used word, but does everyone who uses it really know what it means? *Inflation is the continuing rise in the general level of prices of goods and services. The purchasing power of the monetary unit, such as the (American) dollar, declines when inflation is present.*<sup>2</sup> It is easier to think of inflation as the artificial increase in prices due to excess demand, usually caused when there is too much money present in the economy.

Let's think about that for a moment. If you unexpectedly received a large sum of money, an inheritance or lottery prize for example, wouldn't you spend at least some of it? Now, are you contributing anything of value to the economy that warrants your receipt of this extra money? Is your increase in spending due to an increase in your productivity or an increase in your work output? If not, you could say the spending creates demand that is related not to need, but to excess spending ability.

<sup>2</sup> Gwartney, Stroup, and Sobel, *Economics: Private and Public Choice*, 9E, (The Dryden Press, 2000) 272.



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This excess spending usually increases demand for luxury goods such as cars, electronic equipment, and housing. The increased demand for those items increases their prices. Meanwhile, the rest of us are sitting around wondering why in the world prices are going up. There hasn't been an increase in the overall output of the economy. No more goods and services are being supplied that can absorb the excess spending. The result is an increased demand for a relatively constant supply.

Of course, you could argue that the inheritances of one or even several individuals shouldn't be sufficient to cause such dire results, and you would probably be correct. The point is that whenever we see more money in the system than the system justifies on the basis of its productive output, the result is inflation (i.e., a general rise in the level of prices in the economy caused by excess demand for goods and services). Whenever the level of spending in the economy is not due to economic reasons such as increased productivity (increased output from increased effort), the result is "artificial" demand. In other words, if wage increases are not due to economic reasons (e.g., increased efficiency, better or more work), the money added to the system causes "unearned" increased demand. Let's look at a very simple example.

Bob is a carpenter who works rather slowly and is not all that dependable, but when the rest of the carpenters on the job get a raise, Bob also gets one. Is Bob getting a raise because he deserves it? Has his work improved? Does he do more in a given day that would justify the increase in his wages? Whether or not Bob deserves the raise, his spending will increase, causing increased demand. This is the "artificial" demand referred to previously. Whether real or artificial, increased demand causes prices to rise.

There is a seemingly logical argument in support of Bob receiving a raise along with his productive and dependable coworkers. If Bob doesn't get a raise, his "real" income declines. Due to the inflation already in the economy, the dollars Bob earns are worth slightly less each week. After a while, without a raise, this constant erosion of his buying power will reduce Bob's ability to pay his living expenses.

Does this mean that inflation causes inflation? No. When raises are only sufficient to cover inflation, there is no new demand. Bob is only able to buy the amount of goods and services he could before the raise. Only if Bob's raise exceeds the current rate of inflation (the current rise in prices) does it cause increased demand.

Now, even if it is more than the current rate of inflation, Bob's raise doesn't necessarily have to cause inflation. If Bob has become more productive or if he does more or better work in a given day, his raise is due to increased productivity. He has increased his input to the system, thus justifying the extra money. He has not only increased his demand, he has also added more value (goods and/or services) to the economy. His work is "worth" more. We don't see more money "chasing" the

same amount of goods and services. We see more money “chasing” more goods and services. The result? Stable prices—no inflation.

Inflation is measured and tracked using the Consumer Price Index (CPI).<sup>3</sup> The CPI is a basket of consumer goods and services, the total price of which is recorded across time. Changes in the CPI (sometimes referred to as changes in the price level) indicate the amount of inflation (or deflation) in the economy. As the CPI increases over time, an increased price level of goods and services signals positive inflation in the economy.

Using the CPI, inflation is calculated as follows:

$$\text{inflation} = \frac{\text{CPI}_T}{\text{CPI}_{T-1}} - 1$$

Assuming:

$$\text{CPI}_{12/31/04} = 185.5$$

$$\text{CPI}_{3/31/05} = 187.3$$

Inflation for the first quarter of 2005:

$$\text{inflation} = \frac{187.3}{185.5} - 1 = 0.0097 = 0.97\%$$

As demonstrated in the preceding example, inflation is simply a measure of the overall change in the price level of common goods and services representative of the economy in general.

There is a very important difference between anticipated inflation and unanticipated inflation. When most economic decision makers expect a given level of inflation, they can plan accordingly. When inflation is unanticipated, however, the potential effects on economic activity are much worse.

## Unemployment

The U.S. civilian workforce is the total number of people over 16 years of age who are either employed or unemployed. Notice the distinction between the total population over 16 and those who are either employed or unemployed. To be considered unemployed an individual must be actively seeking employment or waiting to go back to work after a layoff. Although they may be at least 16 years of age, individuals who have never been employed or have no intention of seeking

<sup>3</sup> For a thorough discussion of the Consumer Price Index and other inflation indicators, see <http://stats.bls.gov/cpi>.

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employment are not considered among the country's workforce. These individuals would include children living at home, students, and retirees. Also missing from unemployment data are those classified as *discouraged workers*. An individual is classified as discouraged if he or she is unemployed but has given up trying to find work.

**Frictional unemployment** is the result of employers not being aware of qualified workers and workers not being aware of available jobs. In other words, frictional unemployment exists because the location of the jobs and the workers has not been conveyed to interested parties, so they don't connect (lack of communication). The implication is that with proper information and willingness to relocate, this form of unemployment is avoidable.

When the structural characteristics of the economy change, the result is **structural unemployment**. For a variety of reasons, economic conditions often shift such that jobs are lost in one sector, while new jobs are created in another sector. It becomes difficult for those seeking employment to find jobs because they are not qualified. A historical example was the shift in the early 20th century United States from an agrarian society to an industrial society. Millions of farm workers were unemployed because they did not have the skills to work in factories. Technological advances, such as the increased use of computers, often lead to situations where workers have to acquire new skills or training. Until such training is widely available, unemployment will increase. Under these circumstances, employers also find it difficult to find qualified workers because they need people with different skills.

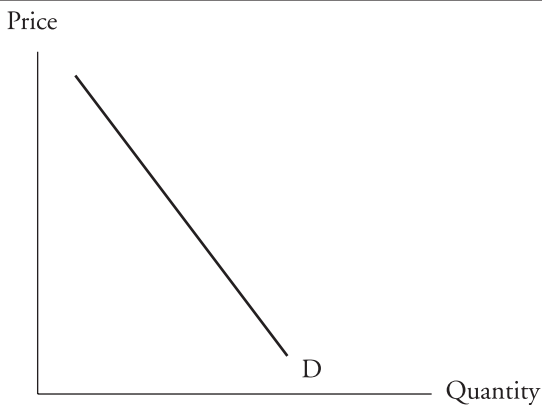
The third type of unemployment, **cyclical unemployment**, which derives its name from the business cycle, is due to decreases in the aggregate demand for goods and services. During such periods, firms produce less output and need fewer employees. Employees are typically laid off and return to their positions when the economy improves.

**Full employment** is the rate of employment resulting from the efficient utilization of the total labor force. This takes into consideration such factors as the levels of frictional and structural unemployment, which are to be expected in an expanding and evolving economy. In the United States full employment is thought to be around 95%. This means that at full employment, approximately 5% of the U.S. labor force will be unemployed. Related to full employment is the concept of a **natural rate of unemployment**, which is the long-run average unemployment rate caused by structural and frictional factors.

## Aggregate Demand and Supply

As shown in Figure 3, the aggregate demand (AD) curve depicts all equilibrium combinations of goods and services demanded at a given price level. The aggregate demand curve, which represents the effects of decisions made by all market participants, is downward sloping. The downward slope is indicative of the inverse (opposite direction) relationship between price and quantity demanded (in the aggregate) by consumers.

**Figure 3: Aggregate Demand**



One interesting characteristic of the aggregate demand curve is that it represents the quantity demanded of a good at all price levels, *given the current level of wealth (income) in the economy*. For example, the number of \$25.00 steak dinners you are willing to purchase will depend to some degree on how much money you have. In other words, a general increase in wealth will cause an increased demand for goods and services at every price level, and the AD curve will *shift* to the right. By “shift to the right,” we simply mean that the entire curve will move to the right, maintaining the same downward slope, indicating that at every price consumers will demand (consume) more.

The aggregate supply curve depicts the amount of goods and services that firms, in the aggregate, are willing to produce (supply) at any given price level. Once again, we are concerned with factors that can shift the *long-run* aggregate supply curve. As with demand, we must always make a clear distinction between long-run aggregate supply (LRAS) which represents the long-run productive capacity of the economy, and short-run aggregate supply (SRAS) which represents short-run deviations from the long-run potential.

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When an economy experiences a shock, there are naturally occurring self-correcting forces that tend to “push” the economy back toward equilibrium. Three mechanisms that are responsible for self-correcting the economy after a shock are:

1. **Consumption demand.** Demand is relatively stable over the business cycle. During periods of economic expansion, the incomes of many households increase significantly. However, households tend to save a larger proportion of their income instead of spending it all. This helps avoid a strong increase in demand that would aggravate the inflationary pressures. During recessions, households save less or even dip into their savings. Thus, demand will not decline as much as the decline in income suggests. Both situations tend to stabilize the economy.
2. **Real interest rates.** Changes in real interest rates help to stabilize aggregate demand and reduce economic fluctuations. During periods of economic expansion, real interest rates rise due to increased demand for borrowed funds. The higher real interest rates reduce consumer borrowing and capital spending by businesses. During recessions, real interest rates decline, encouraging consumers to borrow for purchases and businesses to raise capital for investment projects.
3. **Resource prices.** Changes in real resource prices will also dampen economic fluctuations. When the economy is operating at greater than full employment capacity, prices of resources increase and discourage continued expenditures. For example, when housing starts and new construction activity is high, the prices of lumber and other building materials tend to increase. Conversely, when the economy is operating at less than full capacity, the costs of resources fall, encouraging increased spending by businesses.

## Fiscal Policy and Monetary Policy

The government may try to stimulate or dampen the economy. There are two primary methods that can be used to control the economy: fiscal policy and monetary policy.

**Fiscal policy** refers to the government’s use of taxation and spending policies to achieve various macroeconomic goals. Taxation affects disposable income. To stimulate the economy, the government can reduce taxes, which increases disposable income, thus increasing expenditures (demand) in the private sector. Alternatively, increasing taxes reduces disposable income and private sector spending.

Government spending can be used to smooth fluctuations in aggregate demand. Fiscal policy is what sets government spending levels. **Expansionary fiscal policy** attempts to stimulate the economy by either reducing taxes or increasing

expenditures. **Restrictive fiscal policy** attempts to restrain (i.e., slow) the economy by increasing taxes or reducing government expenditures.

The central bank uses **monetary policy** to help achieve macroeconomic goals by changing the amount of money in circulation. Like any other product, the higher the demand for money relative to its supply, the higher its cost. The cost of money is the rate of interest. **Expansionary monetary policy** seeks to stimulate the economy by increasing the money supply. **Restrictive monetary policy** seeks to slow the economy by decreasing the money supply.

The U.S. Federal Reserve (The Fed) can change the supply of money in the economy by buying and selling U.S. Treasury securities in *open market operations*. To reduce the supply of money in circulation, the Fed can sell Treasury securities from its inventory.<sup>4</sup> The money used to purchase these securities is no longer in circulation. The resulting reduction in the money supply causes interest rates to rise. Consequently, companies and individuals who need to borrow money may have to postpone expansion plans or large purchases until interest rates decline.

If the economy is weak and unemployment is high, the Fed can buy Treasury securities, effectively increasing the money supply. This will cause interest rates to fall, and hopefully the companies that put expansion plans on hold and the individuals who decided to delay their large purchases can go ahead with their plans. This increases expenditures in the private sector and helps the economy move back to full employment.

Another way the Fed can control the money supply is by changing the *discount rate*, the rate the Fed charges banks in need of short-term funds. By increasing the discount rate, the Fed effectively makes money more expensive. The increase in the discount rate ultimately affects all other interest rates, so we see an economy-wide increase in rates. Since money is now more expensive, we again have a situation where some borrowers may have to postpone their plans. There is less money in the economy, money is now more expensive, and the economy slows. To stimulate the economy, the Fed can lower the discount rate, causing the general level of interest rates to decline.

## Marginal Propensity to Consume and the Expenditure Multiplier

As your income increases, you consume some of the increase and save some of the increase. The proportion of each additional dollar of income spent on personal consumption is called the marginal propensity to consume (MPC).

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<sup>4</sup> One reason for the Fed to reduce the money supply is to slow the economy in times of rising inflation.

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Mathematically:

$$\text{MPC} = \frac{\text{additional consumption}}{\text{additional income}}$$

Think for a moment about this increase in income. Assume you receive an additional, unexpected \$1,000 payment. You consume some of it ( $\$1,000 \times \text{MPC}$ ) and save the rest. What happens to the amount you spent? It represents someone else's additional income ( $\$1,000 \times \text{MPC}$ ). They will spend some  $[(\$1,000 \times \text{MPC}) \times \text{MPC}]$  and save the rest. This process continues indefinitely. You can see that the original \$1,000 of income expands or multiplies into more than \$1,000 of total income for the economy. The amount of the expansion is called the *expenditure multiplier* and is equal to:

$$M = \frac{1}{1 - \text{MPC}}$$

Therefore, if investment spending increases by \$1,000, aggregate spending will increase by  $M \times \$1,000$ .

## Classical Economics

Before the time of John Maynard Keynes, classical economists stressed the importance of aggregate supply and paid little attention to demand. The supply-side approach was spelled out in Say's Law which suggested it was impossible to overproduce relative to total demand because supply (production) creates its own demand. The idea is that production generates the income necessary to buy things. If producers make too much of one thing and not enough of another, the price of one will decline, and the price of the other will increase.

According to classical economists, markets always adjust quickly to direct the economy toward full employment. When unemployment is high, wages decline, which reduces costs and prices and pushes the economy back to full employment. Meanwhile, interest rates bring savings and investment into balance. The supply side emphasis of classical economics gained popularity with the economic policies of Ronald Reagan.

## Keynesian Economics

During the depression of the 1930s, John M. Keynes put forth an economic theory that tried to explain the inability of the economy to regain its long-run output level. His theory emphasized the importance of aggregate demand in determining the overall level of output in the economy. If spending decreases due to pessimism

on the part of consumers and investors, business will respond by cutting output. Unlike the classical view, Keynes felt that resource prices, especially wages, were highly inflexible in a downward direction. In other words, wages would be slow to decline, and employers would instead be forced to lay off more workers. Hence, in Keynes' view, the economy would languish for an extended period of time with high unemployment.

The Keynesian equilibrium occurs when spending is equal to output. Since extreme downward price rigidity is present, prices do not play a role in the Keynesian aggregate expenditure (AE) model. Hence, if demand is slack, there are no automatic forces capable of assuring full employment. Instability in a Keynesian world is driven from the demand side of the economy. The main sources of economic instability are consumer spending, private investment, and government expenditures. You know from our discussion of gross domestic product (GDP) in the macroeconomics section that these are the main components of GDP based on the expenditures approach of GDP measurement. Keynes believed that fluctuations in aggregate demand were the main cause of economic disruption. Keynes thought that an active fiscal policy, where the government varied its expenditures depending on economic conditions, could help smooth fluctuations in aggregate demand and thus promote economic stability.

Keynes believed that economies tended to move in cyclical patterns of expansion and contraction. Assume there is an increase in AD (perhaps from higher incomes abroad or an increase in consumer or business optimism). The multiplier magnifies the increased demand. The higher demand leads to income growth. The income growth leads to additional consumption and growing business sales. This leads to declining inventories, so businesses expand their output. Unemployment declines and the economy experiences a boom.

Eventually, the economy reaches full employment, which constrains additional growth. As growth slows, consumers and businesses become less optimistic and cut back on their expenditures. The multiplier magnifies the reduction in expenditures. Business inventories begin to build, so businesses cut back on production, and people are laid off. Some businesses experience bankruptcy.

Keynes thought the primary problem was wide fluctuations in private investment. He also thought that recessions would be long because lower interest rates and falling resource prices are insufficient to offset the decline in incomes and spending.

## **MICROECONOMICS**

Microeconomics deals with the decisions made by the individual firm. When we study microeconomics, we study how macroeconomic factors such as aggregate



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supply and demand, inflation, government actions, et cetera, affect the decisions and, hence, the operations of the individual firm. We know, for example, that as prices increase, consumers demand less of the good. This would imply that the individual supplier will try to produce at the lowest possible cost in order to sell products at a better price than its competitors.

For example, as a supplier of gerbil bedding and gold necklaces, you employ 20 people to process the products and prepare them for shipping or delivery. Ten of these people work on bedding and ten on necklaces. In times of high inflation, increases in your wholesale costs cause you to increase your price to retailers, which causes demand for your products to decline.

The bedding is far more sensitive to inflationary pressures, and the demand for it will almost disappear if its price rises. The result is that 10 people who used to work processing the bedding for delivery are now idle. As owner/manager of the firm, you don't like to see people unemployed, but you must think of your family and other obligations. You hold on as long as you can, but ultimately you must lay off most of the bedding workers. You reassign the most senior bedding processors to the gold necklace line for as long as possible. Finally, as the effects of inflation begin to show on the demand for gold necklaces, you must lay off the reassigned bedding processors as well as some of the gold processors.

Assume the gerbil bedding is processed in a separate building. You can close down that building to avoid paying for heating and cooling and some maintenance. This reduces much of your utility expenses and allows you to reduce your maintenance costs too. Hopefully, increased inflation and unexpected cost increases are temporary, and you can bring back the people you laid off and reopen your gerbil bedding building and production line.

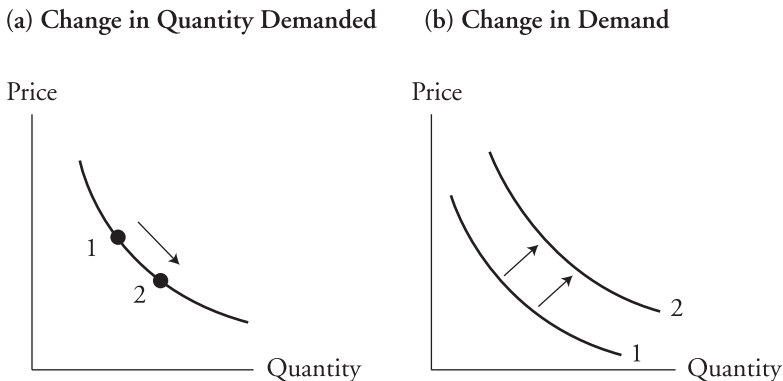
We could continue with this example, but the point should be clear: macroeconomic factors (variables), such as inflation, are out of your control. With adverse changes, you simply try to maintain operations, absorbing profit reductions and increasing prices as much as you can. Microeconomic factors, such as the number of employees and certain overhead costs, are within your control to some degree. In difficult times you may have some room to navigate; you can reduce some costs and temporarily eliminate others.

In the macroeconomics section, we discussed aggregate demand and aggregate supply, which are measures of overall economic activity. With microeconomics, we need to analyze demand and supply from the perspective of the individual firm.

## Shifts in Demand vs. Movements Along Demand Curves

The demand curve isolates the impact that price has on the amount of a product purchased. As indicated in panel (a) of Figure 4, a movement along a specific demand curve represents the change in quantity demanded resulting from a change in price.

Figure 4: Shifts in Demand



Some factors, however, cause the entire demand curve itself to shift. This is illustrated in panel (b) of Figure 4.

Demand curve shifts are called changes in demand and are caused by the following:

- *Changes in the levels of consumer income.* When consumers have more money, they can buy more of everything at a given price, so the aggregate demand curve shifts to the right.
- *Changes in the prices of substitutes and/or complements.* The price of butter goes up, so consumers buy less butter and more margarine. The demand curve for margarine shifts to the right.
- *Changes in expectations.* Consumers expect the price of cars to rise next month, so they buy a new car now before the price increases. The demand curve for new autos temporarily shifts to the right.
- *Changes in the size of the market.* As cities grow and shrink and as international markets open to domestic markets, the change in the number of customers shifts the demand curves of many products.
- *Changes in demographic trends.* In recent years, the number of people ages 15 to 24 has declined. This change will shift demand curves to the left for such things as jeans and pizza.
- *Changes in the popularity of goods.* As consumer tastes and preferences change, the demand curves for various products will shift.

## Elasticity of Demand

The demand curve that represents the entire market for a product is downward sloping, indicating that demand changes as price changes. As price increases, consumers as a whole demand less. As price decreases, consumers as a whole demand more. **Elasticity** refers to the amount of the decrease or increase in demand, when prices are changed. In this section we will discuss two measures of elasticity: income elasticity of demand and price elasticity of demand.

**Price elasticity** is the percentage change in demand given a percentage change in price and is described as follows:

$$E_p = \frac{\% \text{ change in quantity demanded}}{\% \text{ change in price}} = \frac{\% \Delta Q}{\% \Delta P}$$

$E_p$  = price elasticity of demand

When elasticity is greater than 1, demand is said to be **elastic**. When elasticity is less than 1, demand is **inelastic**.

### Example:

Let's assume you have been watching a product for a few months and have been counting the number of the product sold at different prices. You find that when the price increases by 8%, sales decrease by 22%. Calculate the product's price elasticity of demand.

**Answer:**

$$E_P = \frac{\% \Delta Q}{\% \Delta P}$$

$$E_P = \frac{-0.22}{0.080} = -2.75$$

There are two important implications of this calculation:

1. Notice that the elasticity is *negative*. This means that prices and quantity demanded move in opposite directions. As price increases, demand decreases, and as price decreases, demand increases.
2. The percentage change in quantity demanded is 2.75 times the percentage change in price. The demand for this product is fairly elastic, meaning the demand for the good is strongly affected by its price. There are other goods, for example gasoline for your car, that are much less sensitive to price changes. Another example is food. Since people must eat, the amount consumed might be virtually unaffected by price changes. Note, however, that consumers will begin to substitute lower cost foods for the ones with the greatest increases (e.g., chicken for beef).

Demand for a product is also affected by consumer income. Generally, as income increases, quantity demanded will also increase. **Income elasticity of demand** is defined as:

$$E_I = \frac{\% \text{ change in quantity demanded}}{\% \text{ change in price}} = \frac{\% \Delta Q}{\% \Delta I}$$

$E_I$  = income elasticity of demand

Although you will rarely see a product whose demand increases as its price increases, there are actually different types of reactions to changes in income:

1. *Inferior goods* have negative income elasticity of demand. Their demand actually goes down as income increases. For example, people may stop riding public buses and start driving their own cars as their income increases.

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2. *Normal goods* have income elasticities that are positive and can be divided into two categories.
  - a. A *necessity* has income elasticity less than 1.0. An example might be food.
  - b. A *luxury good* has income elasticity greater than 1.0. An example might be recreation (vacations).

## Capital

Any resource that has value because it assists in the production or supply of goods and services is considered *capital*. **Human capital** refers to the characteristics of people (e.g., physical strength, intelligence, education, manual dexterity, honesty, and reliability) that make them valuable to a company. Obviously, at your gerbil bedding and gold necklace company, you value reliability and honesty above strength. Shipping companies tend to place value on strength for most of their employees in addition to reliability and honesty. Engineering firms value intelligence and education, much like universities.

**Physical capital** refers to the nonhuman resources employed by the company (e.g., equipment, buildings, tools, and raw materials). Obviously, the amount of physical capital employed by a firm depends upon the industry. For instance, steel producers and auto manufacturers employ great amounts of physical capital. On the other hand, real estate development firms or brokers have little need for equipment, buildings, and tools. Firms often face a choice of producing a product by incurring high variable costs (e.g., labor), or incurring large fixed costs (e.g., robots).

The value of a firm depends on its ability to fill a need in society (i.e., to provide a necessary function or product). In turn, that ability depends on how well the firm uses capital (i.e., people, equipment, and other resources). When you value a firm, whether currently operating or not, you value that firm as if it is performing at its full potential. That is, you value it as though it is using all of its employed capital in the most efficient and productive manner.

To illustrate this concept of value, consider the following situation. You are at the beach one day, and you notice that a sandwich and ice cream shop (right on the beach) has gone out of business. You say to yourself, “How in the world could that place go out of business? An ice cream parlor in this location would have to be a goldmine!”

Your mental estimate of the value of the sandwich and ice cream shop depends on the collective potential value of all the capital it employed (i.e., the location, the building, the equipment, the people, the supplies, and the money). If the shop

went out of business, it is a good indication that management was not using its capital in the most efficient and productive manner.

## Costs (Factors) of Production

Let's categorize manufacturing costs (factors) as either fixed or variable. **Fixed costs** are incurred through the passage of time and are independent of production output. They occur each period regardless of whether your firm is actually producing anything. Rent is a good example of a fixed cost. Assume you are leasing (renting) your building and equipment. It doesn't matter if you ever turn the equipment on to produce your product, you must still pay the rent. When you begin production, the amount of rent you pay does not increase or decrease, so it is independent of production.

**Variable costs** are associated with production, not the passage of time. Variable costs include the direct labor and material costs incurred in assembling the final product. Consider the production line you use to assemble gold necklaces and prepare them for shipping. There is a station where the chain is measured and cut. The 20-inch pieces then go to another station, where clasps are attached. At the last station, the necklaces are packaged for shipping, and then moved to the shipping area.

Now consider the individual costs that are incurred in assembling one necklace: the cost of the material (gold chain), the cost of the clasps, the wages paid to the individuals at each station, the packaging material, and the shipping costs. All of these costs are related directly to the production of the gold necklaces, not the passage of time. They clearly are variable, not fixed, costs of production—if you produce no gold necklaces, none of these costs is incurred.

Classifying costs as fixed depends on the time frame. In the **short run**, there is nothing that can be done to alter the classification of fixed costs. Fixed costs, such as rent and mortgage payments, cannot be changed in the short run. Given a long enough time period, however, fixed costs can change. For instance, we can pay off or refinance our mortgage, or we can negotiate a different rent. Thus, we refer to the period of time necessary for us to change our fixed costs as the **long run**. In the long run, all costs are variable.

There are several different ways of looking at costs, beyond just fixed versus variable:

- *Average fixed costs* is total fixed costs divided by the number of units produced (output), so in the short run it declines as output increases.

## Economics

- *Average variable cost* equals the total variable cost divided by the number of units produced (output). We will see that variable cost per unit can increase or decrease as output changes.
- *Average total cost* equals the total costs (fixed and variable) divided by the number of units produced.
- *Marginal cost* is the cost of producing one additional unit of output. When you see the word “marginal” in economics, think of the word “next.” A firm’s marginal cost, therefore, is the total (additional) cost associated with producing the next unit of output.

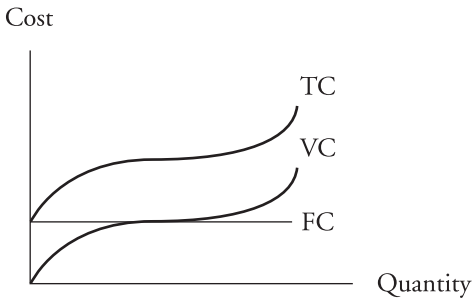
A firm should produce up to the point where marginal costs equal marginal revenues. That is, it should produce up to the quantity at which the revenue received from producing the next unit of production equals the costs to produce that unit.

The **law of diminishing returns** states that as more and more resources (e.g., labor) are devoted to the production process, they increase output, but at a decreasing rate. For example, assume you have an acre of corn that needs to be harvested, and you have started by yourself. The addition of a second and third worker is highly beneficial. That is, adding a second worker doubles output and adding a third worker increases output by an additional 40%. But if you already have 300 workers in the field, the increase in harvesting capacity of adding a 301st worker is not near the increase that was achieved when the second worker was added. The 301st worker will want to be paid just as much as everyone else, but his production will be very low. The result is that you have increased your costs by a greater percentage than you have increased your output.

The concept of diminishing marginal returns and the short-run average/marginal cost structure are illustrated in Figure 5. In Figure 5, total cost (TC) is equal to variable cost (VC) plus fixed costs (FC). You can see that as production increases, VC and TC increase at a decreasing rate (arch downward), flatten out, then increase at an increasing rate (arch upward).

The VC curve is at first upward-sloping and concave because at very low levels of output, variable costs per unit produced are high. Think of this as representing an inability to negotiate low input prices when purchasing small quantities or having too many workers in relation to the quantity produced. As production increases, you will reach a level of output where you can start to negotiate input prices (buy in bulk) and utilize workers more efficiently, so the VC *per unit* actually declines. As production continues to increase, you reach a point where there is a lot of waste and variable costs start to increase at an increasing rate (the curve turns upward and is convex).

Figure 5: Total, Variable, and Fixed Cost Curves



In Figure 5, fixed costs are represented by a horizontal line because they are assumed constant with respect to output. Remember, they are a function of time, not production. The total cost curve is simply the addition of fixed costs to the variable cost curve at every point, so the total cost curve looks identical to the variable cost curve, only shifted upward.

Given enough time, even fixed costs can change (are variable). With enough time, you can negotiate a different lease, change the amount of equipment you are using, or even change the number of managers you employ. Also, fixed costs are only fixed over a range of output. If your existing equipment can produce a limited number of units of output, to attain higher levels of output you will have to add more equipment, and you might even have to add another manager.

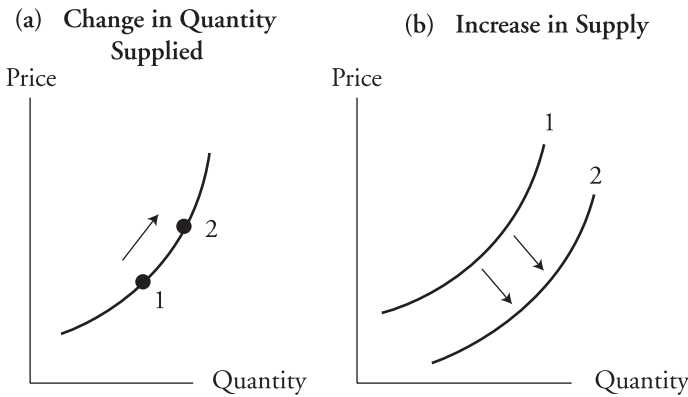
### Movement Along and Shifts In the Supply Curve

Other things held constant, the supply curve summarizes the willingness of producers to offer a product at a given price. The change in the quantity supplied as price changes represents movements along the supply curve. This is illustrated in panel (a) of Figure 6.

Some factors may cause producers to change the quantity they are willing to supply at all price levels simultaneously. These changes will shift the aggregate supply curve as shown in panel (b) of Figure 6.



Figure 6: Changes in the Supply Curve



The following are some of the reasons for a shift in the supply curve:

- *Changes in resource prices.* Higher costs in the resource markets will reduce supply (shift the supply curve left) and increase price in the product markets based on these resources.
- *Changes in technology.* The discovery of new, lower-cost production techniques will reduce the costs of production and increase supply in the product markets.
- *Natural disasters and political disruptions.* Natural disasters and changing political conditions can also alter (shift) supply.

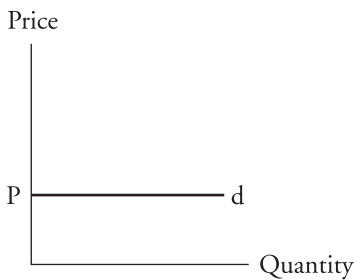
### The Purely Competitive Market

In a **purely competitive market**, there is intense competition, and prices are determined through an equilibrium process. The theory of pure competition assumes the following market and participant characteristics:

- All the firms in the market produce a homogeneous product. This simply means that the products produced by the firms in the market are more or less identical. As an example, the corn from one farmer is about the same as the corn from another farmer.
- There are a large number of independent firms. This means the firms do not collude in any way; they have to compete for your business (i.e., there is no oligopoly).
- Each seller is small relative to the total market. In other words, no seller controls a large enough portion of the market to manipulate the market in any way (i.e., there is no oligopoly or monopoly).
- There are no barriers to entry or exit. It is easy for a new competitor to enter the market (or leave) (i.e., the market is not too expensive to enter or regulated too heavily). You can probably think of industries that are somewhat difficult or impossible to enter because of high capital requirements or regulations (e.g., auto manufacturing or public utilities).

Competitors in a purely competitive market are **price takers**, meaning they have no control over the price they receive for their output. Their product is identical to that of their competitors, and they have no advantage in terms of size or pricing power. Since each produces a small amount of output relative to the total market output, each faces a horizontal (*perfectly elastic*) demand curve. This means they can sell all of their output at the prevailing market price. If they try to set their price higher than the market price, however, they sell nothing.

**Figure 7: Demand Curve for the Individual Competitor**



Notice that the demand curve for the individual competitor is different from the aggregate demand curve for the entire market. The demand curve that represents the entire market is downward-sloping, indicating that demand changes as price changes. As price increases, consumers as a whole demand less. As price decreases, consumers as a whole demand more. The amount demand changes with a given price change is determined by the elasticity of demand for the product, and the demand curve will generally have a negative slope (it will be downward-sloping). The individual competitor, on the other hand, faces a perfectly elastic (flat) demand curve, as in the preceding, because demand for that competitor's output goes to zero if he tries to raise his price.

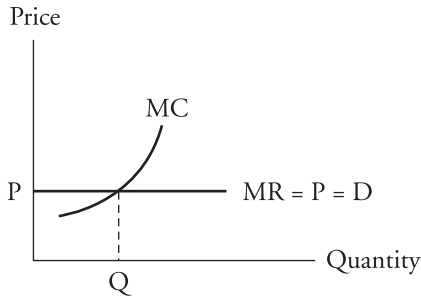
How does the individual firm in a purely competitive market determine the amount it will produce and sell? An individual firm will continue to expand production (output) until marginal revenue (MR) equals marginal cost (MC). (MR is the revenue obtained from selling one more unit of output, and MC is the additional cost of producing the last unit of output.) If the producer goes beyond that amount, the price received will not cover expenses. If he produces less, he is not maximizing profits because the marginal costs of producing units is less than the price that will be received. This production level should make intuitive sense. The firm will produce as many units as it can produce and sell for a profit. If producing another unit will cost more than the unit can be sold for, it makes no sense to produce that unit. The problem with applying this model in the real world is that it is often difficult to know the exact marginal cost of each single unit.

## Economics

In pure competition with a constant market price, the firm's marginal revenue is constant and equal to the selling price. In other words, no matter what amount the firm sells, it receives the market price ( $P$ ) per unit. (This explains why the firm's MR curve presented in Figure 8 is flat.) The graph indicates increasing marginal costs, as we have already discussed. When the firm's marginal cost exactly equals its marginal revenue, the firm is at the optimal output level ( $Q$ ).

**Figure 8: Profit Maximizing Output for the Purely Competitive Firm**

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## Price Searcher Markets

In a **price searcher market**, firms enjoy a certain amount of control over the supply of the good. They must search for the optimal price at which they maximize their profits. The reason some firms search for the optimal price rather than accepting the market price is *entry barriers*. That is, for some reason competitors cannot readily enter the market, so the existing firms experience limited competition.

Some examples of entry barriers include the following:

- *Economies of scale*. Sometimes bigger is better. The ability to produce more efficiently (cheaply) based upon volume is known as economies of scale. Large firms are able to spread their fixed costs across a higher volume of products, thus allowing them to sell those products at lower prices. In industries that have high fixed costs, competition will be limited by the fact that any new competitors cannot easily enter the market. New firms would need time to build sales to a level where the fixed costs could be effectively reduced on a per unit basis. The economies of scale are therefore a barrier to entry for new competition. Consider the automobile industry. There are large fixed costs associated with automobile production: research into the engineering and design, plus the production process itself. In order for auto producers to be profitable, they must be able to spread these high fixed costs across a high number of automobiles produced. If a plant only produced one vehicle, the price of that vehicle would have to cover all of the high fixed costs. If the plant produced one thousand vehicles, the fixed costs could be allocated across all of them, and the vehicles would be more affordable to consumers.

- *Government licensing and legal barriers* create barriers to entry in certain industries such as television broadcasting, where required government licenses restrict new competition.
- *Patents or exclusive rights of production* are granted to producers of new and innovative products. These encourage research and development because firms with new, innovative products know they will be protected from competition for a while, giving them the opportunity to recover the substantial costs associated with research and development.
- *Resource control* relates to the single firm that has sole control over a resource essential for entry into an industry, effectively eliminating potential competitors. For example, you don't see many new, successful diamond mines.

Problems associated with markets having high barriers to entry include the following:

- *Unsatisfied demand.* Monopolists (discussed in the following) can suppress supply and charge relatively high prices while not meeting the total demand of a population.
- *Limited consumer options.* For example, there is typically only one cable television operator in any region. If service is bad, customers can either keep subscribing or not. They don't have the option of switching operators.
- *Entry and exit are not properly motivated by profit.* Since barriers to entry are high, inefficient producers may be able to survive.
- *Legal protection* can allow a monopolist to seek abnormal profits. Spending time and money in search of favors from the government (called *rent seeking*) can be a waste of valuable resources.

## Monopolies and Oligopolies

When only one supplier of a product exists, that supplier is said to have a **monopoly**. Typically, there are very high barriers to entry in monopoly markets. For instance, assume you have control over land that holds a vast supply of gold. Next, assume that searching for gold in other parts of the world is extremely expensive because the world's supply of gold is nearly depleted and any new supplies are extremely deep underground. In a situation such as this, the possibility of new suppliers exploring for gold and being able to compete with you is very low. You are a monopoly because you effectively control the world's supply of gold.

Now let's make the simplifying assumption that your costs to supply gold are more or less constant, so your profit equals total revenues less some constant cost, where total revenue equals the amount of gold you sell multiplied by its selling price. Your goal, as a monopolist, is to find the combination of price and quantity supplied that maximizes *total revenue*. Since your costs are more or less constant, your only concern is maximizing the revenues you receive.

## Economics

This optimal price-quantity combination is at the point where the price increase (resulting from a reduction in the amount of gold you supply) exactly offsets the accompanying reduction in demand (caused by the increase in price). Thus, you have maximized your profit, and you didn't even consider competition.

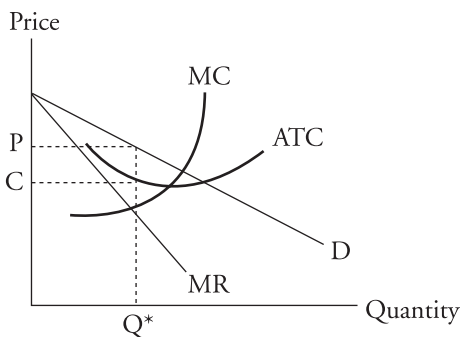
We showed previously that the individual firm in a competitive market faces a perfectly elastic (flat) demand curve, meaning the competitive firm must accept the market price. From macroeconomics we also know that the *aggregate* demand curve for the whole market is downward-sloping, indicating that consumers as a whole demand less (more) as prices rise (fall).

Since the monopolist is the only supplier of the good, the demand curve faced by the monopolist is effectively the demand curve for the whole market. The monopolist, therefore, faces a downward-sloping demand curve. The monopolist can raise or lower prices, but demand will be affected when it does so.

But even if the monopolist can set the price anywhere it wants, does it charge the highest possible price? The answer is no, because the monopolist wants to maximize profits, not price. Monopolists will not make profits if the ATC line is always above the demand curve. For example, if someone held a patent on a machine that could be used just one time to convert a \$5 bill into a \$10 bill, but the ATC of producing the machine was \$6, the monopolist would not make a profit.

Figure 9 shows the revenue-cost structure facing the monopolist. Note that, just as with the pure competitor, production will expand until  $MR = MC$  at optimal output  $Q^*$ . To find the price at which it will sell  $Q^*$  units, you must go to the demand curve. Note that the demand curve itself does not determine the optimal behavior of the monopolist. Just as with the pure competition model, optimal quantity is where  $MR = MC$ . For a profit to be ensured, the demand curve must lie above the ATC curve at the optimal quantity point (i.e.,  $P_1 - C_1 > 0$ ).

**Figure 9: Revenue-Cost Structure of the Monopolist**



**Oligopoly** is a market structure characterized by the following:

- A small number of sellers.
- Interdependence among competitors (decisions made by one firm affect the demand, price, and profit of others in the industry).
- Large economies of scale.
- Significant barriers to entry.
- Products may be similar or differentiated.

As with a monopoly, producers in an oligopoly market seek that combination of supply and price that maximizes profits. In contrast to a monopolist, oligopolists are highly dependent upon the actions of their rivals when making business decisions. Price determination in the auto industry is a good example. Automakers tend to play “follow the leader” and announce price increases in close synchronization. They are not working explicitly together, but the actions of one producer have a large impact on the others. The barriers to entry are large. It would take an enormous capital investment to start a new auto company because the large economies of scale that are achieved by the existing firms pose a significant barrier.

Oligopolists recognize that they cannot maximize profits when they are in fierce competition with one another. Hence, they may attempt to form associations or *cartels* to set prices and output so as to maximize profits.

Although not technically an oligopoly, OPEC is a familiar example of the concept of a cartel. OPEC faces an interesting problem, since they do not control the entire supply of crude oil. Member countries only control a very large portion of it, which they maintain carefully to manipulate prices. However, if they limit supplies too much, prices will rise to the point where potential competitors (drillers in Texas, the Gulf of Mexico, the North Sea, etc.) will be enticed to reopen their drilling and exploration sites.

Another interesting aspect of cartels is that each member is better off by secretly increasing its supply (i.e., cheating). Industry profits are maximized when all producers agree to restrict supply (thus mimicking monopoly conditions). This collusive agreement among producers will establish prices at a high level. Once the supply level is agreed to, individual firms will maximize their own profits by increasing their own production and thus increasing their share of the industry profit. For instance, if OPEC agrees on a certain supply from each country, all member countries are bound by that agreement. Once the agreement has been reached, the price of crude oil reacts based on the expected supply. Since each member's best interests are served by maximizing revenues, each member will want to do so. The way a member nation maximizes revenues is by increasing supply above that specified in the agreement. In other words, what is best for the group is not necessarily best for the individual member, and vice versa. Because each participant has an incentive to cheat, cartels tend not to be sustainable over time.

## Economics

Such collusive behavior is also limited for the following reasons:

- *Number of oligopolists.* When the number of oligopolists is large, effective collusion is less likely.
- *Monitoring partners.* Collusion is less attractive when it is difficult to detect and eliminate price cuts. In other words, if cheating is difficult to detect, fewer firms will be willing to enter into collusive agreements.
- *Low entry barriers.* New entrants will see the “premium” pricing that is available and enter the market (outside the oligopoly) with more attractive pricing in an attempt to steal market share.
- *Unstable demand conditions.* Unstable demand can lead to differing opinions between oligopolists as to how to best serve the industry’s clientele.
- *Vigorous antitrust action.* Antitrust actions may increase the cost of collusion.

## INTERNATIONAL ECONOMICS

Although there is sometimes substantial trade among governments, the majority of international trade is done by businesses.

When we enter the international arena, we open up a new set of considerations. First of all, there is the question of which country will produce what. For example, why does the United States purchase so much electronic equipment from Asia? Next, since trade is across borders, paying for international purchases becomes a concern. That is, how does a U.S. business pay for goods purchased from a company in Japan?

### Comparative and Absolute Advantages

**Comparative advantage** is the ability of one country to produce a good at a lower *opportunity cost* than its trading partners. Opportunity cost refers to the opportunities that must be foregone in order to produce the good. For example, a college student incurs opportunity cost because he or she is choosing to go to school rather than start a career immediately. The student’s resource (in this case, time) can only be used for one opportunity. The missed opportunities resulting from the choice to attend college are the opportunity costs of going to college.

Comparative advantage is the reason there is a differential impact from international trade among industries. Relative cost is the key to having comparative advantage. When trading partners specialize in producing products for which they have comparative advantage, costs are minimized, output is greater, and both trading partners benefit.

For example, consider the following chart, which indicates the production possibilities of food and drink per worker per day in Country A and Country B.

<i>Units of Output Per Day</i>		
	<i>Country A</i>	<i>Country B</i>
<i>Food</i>	2	6
<i>Drink</i>	4	9

Mutual gains could be realized from trade if A specializes in drink production and B specializes in food production. The reason centers on comparative advantage. The preceding table indicates that Country A can produce either 2 units of food or 4 units of drink per worker per day. Therefore, Country A must give up 2 units of drink to produce 1 unit of food. Country B can produce 6 units of food or 9 units of drink per worker per day. Country B must give up 1.5 units of drink to produce 1 unit of food. The opportunity cost for producing food is greater for A than B. If B produces 6 units of food and A produces 4 units of drink, and each country can agree on a trading price somewhere between 1.5 to 2.0 units of drink for 1 unit of food, they will both benefit compared to other possible combinations of productions.

For example, if the trading rate (called the terms of trade) is 1.75 units of drink for one unit of food, then B could produce 6 units of food and trade 2 units of food to A for  $2 \times 1.75 = 3.5$  units of drink. A ends up with 2 units of food and 0.5 units of drink. With no trading, A would have had only 1.75 units of food if enough resources had been diverted to produce 0.5 unit of drink, so its food consumption is increased 17% by specializing and trading.

B ends up with 4 units of food and 3.5 units of drink. With no trading, B would have had 4 units of food only by diverting resources from drink production that would have limited drink output to 3 units. Thus, both countries are better off. This makes intuitive sense—the most efficient producer of a good should produce that good and trade its output for goods that are more efficiently produced elsewhere.

**Absolute advantage** describes the situation in which a nation, as the result of its previous experience or natural endowments, can out-produce another nation using the same resources.

The law of comparative advantage holds that trading partners are better off if they specialize in the production of goods for which they are the low-opportunity cost producer and trade for (purchase) those goods for which they are the high-opportunity cost producer.



## Economics

A country gains (i.e., realizes expanded consumption possibilities) from international trade when it exports those goods for which it has a comparative advantage and imports those goods for which it does not. For example, suppose we can produce wheat at \$2 per bushel. If the production cost is \$3 in all other nations, it is to our advantage to produce more wheat and sell it on the world market. Hence, our exports increase and the other nations get cheaper wheat.

### Impact of Trade Restrictions

Although the gains from trade are apparent, countries often erect barriers to trade, including tariffs and quotas. A **tariff** is a tax imposed on imports, while a **quota** is an import quantity limitation. Both types of trade barriers artificially distort the costs of goods, potentially eliminating the benefits of comparative advantage.

Tariffs benefit domestic producers of products because the level of imports will be reduced due to an effective increase in the price of importing the good. For example, if the world price of semiconductors is \$40, and domestic producers can only profitably sell semiconductors at \$45, foreign producers have a comparative advantage. Hence, domestic producers will not be able to compete in their own domestic semiconductor market. However, if the government places a \$5 tariff on imported semiconductors, local producers will become competitive with foreign producers in the local market, and local semiconductor production will rise. Tariffs will also benefit the government because the government will collect the \$5 tax on all foreign semiconductors sold in the domestic market.

A quota is a limit (maximum) set on the import of a specific product and has the same effect as a tariff. The supply of imported goods is reduced, and a lower supply means a higher price domestically. With a quota, the government does not directly benefit because an explicit tax is not involved. Rather, the foreign producers who are granted import permits benefit because they are allowed to sell their goods in the domestic market at artificially high prices. Clearly, domestic producers also benefit from quotas because competition from foreign producers is limited. The bottom line is quotas can be more harmful than tariffs because (1) the government does not receive any funds from the imposition of quotas and (2) the foreign producers receive the revenue transfer (due to higher prices received for all goods sold under the import license).

Reasons that nations adopt trade restrictions include the following:

- *National defense.* Some industries are highly sensitive to national security, and their products should, therefore, remain in the country.
- *Infant industries.* These industries should be protected with tariffs or import quotas for a time, while they develop and reduce costs.

- *Anti-dumping.* Dumping occurs when a foreign firm sells products below cost in an attempt to gain market share by driving out domestic suppliers. The practice is sometimes supported by the foreign government.

**Do trade barriers protect jobs?** Trade restrictions are typically supported by local governments and citizens who believe they protect jobs and help maintain high wage levels. In the long run, however, trade restrictions cannot protect the net number of jobs in a country. The number of jobs protected by import restrictions will be offset by jobs lost in the import/export industry. Import/export firms will be unable to sell the overpriced domestic product abroad or import and sell the lower-priced, restricted foreign-made product.

**Do trade restrictions create jobs?** Trade restrictions may create jobs in the short run, but not in the long run. First of all, trade restrictions prevent your trading partners from developing the purchasing power needed to buy import goods from you, thus depressing your own export industry. Secondly, the higher price of the protected domestic good dampens domestic aggregate purchasing power, taking sales away from other domestic products. Finally, the jobs that would have been created in the import industry are typically never created.

**Does trade with low-wage countries depress wage rates in high-wage countries?** The belief that trading with low-wage countries depresses wages is based on a misunderstanding of the law of comparative advantage. A high hourly wage does not necessarily mean high per-unit labor costs. Labor *productivity* must be considered. The worker's skill level, the amount of invested capital, and production methods may produce labor costs per unit of output below that found in low-wage countries.

Consider the law of comparative advantage (not absolute advantage). When each country produces goods for which it has a comparative advantage, both countries will benefit. High-wage countries will have an advantage in high-tech manufacturing, and low-wage countries will have an advantage in labor-intensive goods. When both produce the goods in which they have an advantage, total output and the availability of goods increase.

## Foreign Exchange

The second major consideration for international traders is foreign exchange (i.e., exchanging the domestic currency for the foreign currency). To purchase products from a foreign country, the firm must have the currency of that country. Since foreign trade is so prevalent, there must be an equilibrium "price" for each currency in terms of all other currencies.

## Economics

An **exchange rate** is a ratio that describes how many units of one currency you can buy with one unit of another currency. Note that an exchange rate is quoted relative to another currency.

For example, if the Australian dollar (AUD) is trading at 0.60 U.S. dollars (\$0.60), each AUD will buy 60 U.S. cents. Remember the following:  $\text{AUD} = \$0.60 = 0.60$  dollars per AUD. Also, exchange rates between countries are the inverse of one another. Thus, the U.S. dollar quote in terms of AUD is:

$$\text{U.S. dollar} = 1/0.60 = \text{AUD } 1.66 = 1.66 \text{ AUD per U.S. dollar}$$

Therefore, if you are given dollars per AUD, you can easily get AUD per dollar by inverting the original quote.

### Example:

A U.S. importer has agreed to purchase 200 dozen long-stem roses from a Japanese flower supplier. The latest foreign exchange quote is  $\text{¥}111.02/\text{\$}$  or  $\text{\$}0.009007/\text{¥}$ . Specify the total dollar price for the roses if they are quoted at  $\text{¥}650$ , including all shipping costs.

### Answer:

First, if the roses cost  $\text{¥}650$  per dozen, their total cost in yen is:

$$\text{¥}650 \times 200 = \text{¥}130,000$$

Next, since the U.S. importer must pay in yen, he must exchange dollars for yen. That means we have to find the number of dollars that are equivalent to  $\text{¥}130,000$ :

$$\text{¥}130,000 \times \text{\$}0.009007/\text{¥} = \text{\$}1,170.91$$

The importer would go to the foreign exchange markets and exchange  $\text{\$}1,170.91$  for  $\text{¥}130,000$  to pay the Japanese supplier.

The trading of currencies takes place in foreign exchange markets, whose primary function is to facilitate international trade and investment. Knowledge of the operation and mechanics of these markets is important for a fundamental understanding of international financial management.

The foreign exchange market permits the transfer of purchasing power denominated in one currency for that of another currency. This market is not a physical place but rather an electronically linked network of banks, foreign exchange brokers, and dealers whose function it is to bring together buyers and sellers of foreign exchange. Transactions occur over the phone, telex, or SWIFT system (Society for Worldwide Interbank Financial Telecommunications).

Participants in the foreign exchange (interbank) market are large commercial banks, foreign exchange brokers, major multinational corporate customers, and central banks. Most of the trading in the United States goes through foreign exchange brokers, who match buyers and sellers for a small commission (1/32 of 1%). If small amounts of foreign currency are needed, individuals typically deal with their local banks.

The foreign currency **spot market** is where currencies trade for immediate delivery, although in practice, the settlement date (also called the value date) is set at two working days after the date the transaction is concluded. For example, if a spot deal is concluded on Thursday, the settlement date is Monday.

The foreign currency **forward market** is where contracts are used to buy or sell currencies for future delivery. For example, a forward contract between a bank and a U.S. customer might call for the delivery of a specified amount of a foreign currency on a specified future date in exchange for a specified amount of dollars. The exchange rate is fixed at the time the contract is signed. Forward contracts are generally for 30-, 90-, 180-, or 360-day periods.

Assume a U.S. firm must pay a Swiss firm CHF150,000 (CHF is the symbol for Swiss francs) in 90 days. The U.S. firm has dollars now but needs Swiss francs in the future. In order to offset the risk that the value of the Swiss franc might change (i.e., the exchange rate between Swiss francs and U.S. dollars), the U.S. firm will buy Swiss francs in the forward market. An opposite situation is a U.S. firm that agrees to receive payment of CHF200,000 30 days from now. To offset the risk of changing value, the firm will contract to sell Swiss francs in the forward market.

**Example: Foreign currency forward transaction**

A U.S. firm has entered into a contract in which it will receive CHF100,000 from a Swiss firm in 60 days. Forward contracts on Swiss francs maturing in 60 days specify a rate of CHF1.7530/\$. The current exchange rate is CHF1.7799/\$. One way or another, the U.S. firm has to exchange its Swiss francs for U.S. dollars in 60 days.

**Part 1:** Assume the U.S. firm chooses to take its chances with exchange rates and will simply take the rate of exchange on the date it receives the francs. Compared to the current exchange rate, how much will the U.S. firm gain or lose if, when it receives the francs, the exchange rate is 1.6556 Swiss francs to the dollar?

**Answer:**

- At the original exchange rate, assuming no change before the payment date, the U.S. firm would receive \$56,182.93 for the CHF100,000:

$$\text{CHF}100,000 \times \$0.561829/\text{CHF} = \$56,182.93$$

$$1/1.7799 = 0.561829$$

- However, at the payment date the exchange rate has changed, and the firm actually receives \$60,401.06:

$$\text{CHF}100,000 \times \$0.604011/\text{CHF} = \$60,401.06$$

$$1/1.6556 = 0.604011$$

By waiting, the U.S. firm receives more U.S. dollars for its Swiss francs. This is due to the *strengthening* of the Swiss franc. The Swiss franc *strengthened* with respect to the dollar, because on the agreement date it took 1.7799 Swiss francs to purchase one dollar, but it only took 1.6556 Swiss francs to purchase a dollar on the payment date.

Another way of looking at this is that the dollar *weakened* with respect to the Swiss franc over the period. It took \$0.561823 (1/1.7799) to purchase one Swiss franc originally, but at the payment date it took \$0.604011 (1/1.6556) to purchase one Swiss franc.

**Part 2:** Using the same exchange rates, assume the firm entered into the forward contract to sell Swiss francs at CHF1.7530/\$.

**Answer:**

- With the forward contract, the firm has locked in a rate of exchange. It has agreed to receive \$57,045.10 for their CHF100,000 (at CHF1.7530/\$):

$$\text{CHF}100,000 \times \$0.570451/\text{CHF} = \$57,045.10$$

$$1/1.7530 = 0.570451$$

Even though the Swiss franc has appreciated in value (strengthened) with respect to the U.S. dollar, the firm receives the rate specified in the forward contract. Note that by selling its francs in a forward contract, the firm receives fewer U.S. dollars than if it had simply waited.

**Part 3:** Now assume at the time of payment the exchange rate has risen to CHF1.8250/\$.

- Remember that at the original exchange rate (CHF1.7799/\$) and assuming no change before the payment date, the U.S. firm would receive \$56,182.93 for its CHF100,000:

$$\text{CHF}100,000 \times \$0.561829/\text{CHF} = \$56,182.93$$

$$1/1.7799 = 0.561829$$

**Answer:**

- At the payment date the exchange rate has changed and the firm actually receives \$54,794.52:

$$\text{CHF}100,000 \times \$0.547945/\text{CHF} = \$54,794.52$$

$$1/1.8250 = 0.547945$$

By waiting, the U.S. firm receives *less*, due to the *weakening* of the Swiss franc. The Swiss franc *weakened* with respect to the dollar because on the agreement date it took 1.7799 Swiss francs to purchase one dollar, and it took 1.8250 Swiss francs to purchase a dollar on the payment date.

Another way of looking at this is that the dollar *strengthened* with respect to the Swiss franc over the period. It took \$0.561829 (1/1.7799) to purchase one Swiss franc originally, but at the payment date it took only \$0.547945 (1/1.8250) to purchase one Swiss franc.

The reason we looked at two different exchange rates at the payment date is to point out a crucial characteristic of forward contracts. That is, the purchase price (rate of exchange between the two currencies) is locked in at the agreement date, regardless of whether or not exchange rates change in the future. In Part 1 the U.S. firm received \$60,401.06 because it waited to exchange its Swiss francs until it actually received them. In that case the Swiss franc strengthened, so the firm received more money by not entering into a forward contract.

In Part 2, however, the firm agreed to receive \$57,045.10 by selling its Swiss francs through a forward contract. If it had not entered into the forward contract but instead just waited, it would have received \$60,401.06. It would appear the firm lost money by entering the forward contract.

Why would the firm enter into a forward contract, when if it just waited and exchanged its francs when it received them it would get more U.S. dollars? The answer is that the firm *did not know* at the agreement date whether the Swiss franc would strengthen, weaken, or remain the same. Since it had no way of knowing the direction of exchange rate movements, if any, the firm was willing to lock in at the forward contract rate (a known price) to avoid the uncertainty associated with waiting (i.e., the exchange rate risk).

## Currency Appreciation or Depreciation

Exchange rates are largely a function of supply and demand for currencies. If demand for a currency rises relative to the supply, the currency should appreciate in value. If supply increases relative to demand, the currency should depreciate. Three major factors cause a country's currency to appreciate or depreciate.

1. Differential **income growth** among nations will cause nations with the highest income growth to demand more imported goods. Countries experiencing faster growth than its trading partners will have an increase in demand for foreign currencies to pay for the imported goods. There will be an increase in the supply of the fast growth country's currency together with more demand for foreign currencies. The result will be that the currency of the faster growth nation will depreciate relative to the currencies of the trading partners.
2. Differential **inflation rates** will reduce demand for the currencies with higher inflation rates. If a country's inflation rate is higher than its trading partner's, the demand for the country's currency will be low, and the currency will depreciate.
3. Differential **interest rates** will cause a flow of capital into those countries with the highest available real rates of interest. Therefore, there will be an increased demand for those currencies, and they will appreciate relative to countries whose available real rate of return is low.

## Balance of Payments Components

**Balance of payments** (BOP) accounting is used to keep track of transactions between a country and its international trading partners. The BOP accounts reflect all payments and liabilities to foreigners and all payments and obligations received from foreigners. The BOP equation is:

$$\text{current account} + \text{capital account} + \text{official reserve account} = 0$$

The **current account** measures the exchange of merchandise goods, the exchange of services, the exchange of investment income, and unilateral transfers (gifts to and from other nations). Basically, the current account includes all international transactions for goods and services. If an American buys a pair of shoes made in Italy, the transaction shows up in the current account. If a Japanese citizen hires an American lawyer, that transaction shows up in the current account. When the media reports on the "trade deficit," it is usually referring to the balance of merchandise trade, which is the goods component (and the largest piece) of the



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current account. The balance on current account summarizes the balance on goods and services, the exchange of investment income, and unilateral transfers.

The **capital account** measures the flow of funds for debt and equity investment into and out of the country. Instead of goods and services, the capital account includes transactions involving the assets of another country. If an American buys land in Italy, that transaction would be in the capital account. Similarly, if a Japanese investor bought U.S. Treasury securities (or stock in a U.S.-based company), that transaction would be in the capital account.

The **official reserve account** represents government-controlled assets in the form of gold, foreign currency-denominated assets, and SDRs (special drawing rights at the IMF). The easiest way to think of the official reserve account is as a cushion. Note that the balance of payments sums to zero. A negative balance in the current account must be offset by a positive balance in the capital account, and vice-versa. The official reserve account acts as a cushion to absorb a small imbalance between the current and capital accounts.

In order for the balance of payments to sum to zero, changes in the current account must be offset by changes in the capital account. This makes sense when you think about it. Assume that an American buys a bottle of French wine using dollars. The French wine seller now has dollars. Those dollars are no good in France, but they can be used either to buy American goods or to buy American assets. Either way, the dollars have to come back to the United States because they represent U.S. assets. In recent years, the United States has had large merchandise trade deficits, and large current account deficits, meaning that U.S. citizens have bought a lot of foreign goods. If foreigners bought lots of U.S. goods, the current account deficit could be reduced. Otherwise, those dollars will come back to the United States through the purchase of U.S. assets, which will create a surplus in the U.S. capital account. In fact, this is just what has happened, and the large proportion of foreign ownership of U.S. stocks and bonds reflects the capital account surplus of recent years.

## SUMMARY

### SUPPLY AND DEMAND

- A. Supply is defined as the amount of an item available for purchase. An item is supplied only if the price received from selling it is sufficiently greater than the total cost to provide it. An increase (decrease) in supply usually decreases (increases) price.
- B. Demand is defined as the total amount of an item society wants to purchase, which depends upon several factors (the number of potential consumers, price, the availability of substitutes, whether a necessity or a luxury, and the amount of money in the economy). An increase (decrease) in demand usually increases (decreases) price. The combination of supply and demand sets prices.

### MACROECONOMICS

- A. Gross domestic product (GDP) is the total market value of all final goods produced *within a country*, regardless of the nationality of the producer. GDP measures both output and income, which are equal. Two approaches to measuring GDP include the *expenditure approach* (a demand-based concept) and the *income approach*. GDP Deflator is a general price index that corresponds to the price change in all final goods and services produced.
- B. Inflation is an artificial increase in prices due to excess demand. Inflation is usually caused by too much money in the economy.
- C. Unemployment.
  - 1. Frictional unemployment is due to the inability to convey information that qualified workers for particular jobs are available.
  - 2. Structural unemployment is due to changes in the structure of the economy.
  - 3. Cyclical unemployment is caused by a decrease in aggregate demand.
- D. Fiscal policy. Governments use taxation and spending to achieve macroeconomic goals by either slowing or stimulating an economy.
- E. Monetary policy. The money supply in most countries is controlled by the central bank (the Fed in the United States). Increasing the money supply reduces interest rates, which stimulates borrowing, stimulates the economy, and increases expenditures.
- F. Marginal propensity to consume. The proportion of each additional dollar of income spent on personal consumption is the marginal propensity to consume (MPC).

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- G. Classical economics. Economists before John Maynard Keynes (1883–1946) believed in a laissez-faire approach and thought that it was impossible to overproduce relative to total demand because supply (production) creates its own demand. During a recession, prices and wages will drop quickly enough to bring the economy back to full employment.
- H. Keynesian economics. Instability in a Keynesian world is driven from the demand-side of the economy. The main sources of economic instability are consumer spending, private investment, and government expenditures.

## MICROECONOMICS

- A. Shifts in demand vs. movements along demand curves. A movement along a specific demand curve represents the change in quantity demanded resulting from a change in price. Demand curve shifts are called changes in demand.
- B. Elasticity refers to the amount of the decrease or increase in demand, when prices and/or income change.
- C. There are several different types of costs:
  - 1. Fixed costs remain unchanged in the short run.
  - 2. Variable costs are related to the level of production, not the passage of time.
  - 3. Average total cost equals the total costs (fixed and variable) divided by the number of units produced.
  - 4. Marginal cost is the cost of producing one additional unit of output.
- D. The law of diminishing returns states that as more and more resources (e.g., labor) are devoted to the production process, they increase output, but at an ever decreasing rate.
- E. Shifts in supply vs. movements along supply curves. The change in the quantity supplied as price changes represents movements along the supply curve. Shifts in the supply curve represent changes in the quantity suppliers are willing to supply at all price levels.
- F. In a purely competitive market:
  - 1. All the firms in the market produce a homogeneous product.
  - 2. There are a large number of independent firms.
  - 3. Each seller is small relative to the total market.
  - 4. There are no barriers to entry or exit.
  - 5. Competitors are price-takers.
- G. In a price-searcher market, firms search for the price at which they maximize profits.
- H. The monopoly market structure has the following characteristics:
  - 1. There is one seller of a specific, well-defined product that has no good substitutes.
  - 2. Barriers to entry are high.

- I. Oligopoly is a market structure characterized by:
  - 1. A small number of sellers.
  - 2. Interdependence among competitors.
  - 3. Significant barriers to entry.
- J. Regardless of the industry market structure, an individual firm will continue to expand production (output) until marginal revenue (MR) equals marginal cost (MC).

### INTERNATIONAL ECONOMICS

- A. Comparative advantage is the ability of one trading partner to produce a good at a lower opportunity cost than others can produce it. Trading partners are better off if they specialize in the production of goods for which they are the low-opportunity cost producer and trade for (purchase) those goods for which they are the high-opportunity cost producer.
- B. Tariffs benefit domestic producers of products because the level of imports will be reduced due to an effective increase in the price of importing the good. Quotas limit the quantity of an imported good. Quotas can be more harmful than tariffs.
- C. Trade restrictions create jobs only in the short run. Trade restrictions prevent your trading partners from developing the purchasing power needed to buy imported goods from you, thus depressing your own export industry.
- D. The trading (exchanging) of currencies takes place in foreign exchange markets. Participants in the foreign exchange (interbank) market are large commercial banks, foreign exchange brokers, major multinational corporate customers, and central banks.
- E. The foreign currency spot market is where currencies trade for immediate delivery. The foreign currency forward market is where contracts are used to buy or sell currencies for future delivery.
- F. Exchange rates are determined by the relative supply of and demand for currencies, which are driven by interest rates, inflation rates, and relative growth rates.
- G. The balance of payments captures all of a country's transactions in international markets, and includes the current account, capital account, and official reserve account.

## PRACTICE QUESTIONS: ECONOMICS

- Lekross Dairy can produce and package cheese for \$1.20 per pound. Lekross can ship this cheese locally for \$0.15 per pound, but the cost to ship over distances greater than 50 miles is \$0.55 per pound. A local grocer (Localmart) has offered to buy Lekross cheese for \$1.30 per pound, while a grocer 80 miles away (Far Stores) has offered \$1.80 per pound. Given this situation, you should:

  - accept Localmart's offer, but decline the offer of Far Stores.
  - accept Far Stores' offer, but decline the offer of Localmart.
  - accept both offers.
  - accept neither offer.
- Lionel Mandrake has been tracking sales of various products at a local grocery store. He has noticed that when prices of orange juice increase, sales of fruit punch increase. He has also noticed that when prices of cookies increase, sales of milk decrease. Based on this information, Mandrake should conclude that:

  - orange juice and fruit punch are substitutes, and milk and cookies are substitutes.
  - orange juice and fruit punch are substitutes, and milk and cookies are complements.
  - orange juice and fruit punch are complements, and milk and cookies are substitutes.
  - orange juice and fruit punch are complements, and milk and cookies are complements.
- Which of the following statements *most accurately* reflects the law of supply?

  - Businesses will produce only the quantity of goods they believe consumers will buy.
  - Consumers will buy more of a good as the price of that good declines.
  - As the price of a good increases, more of that good will be supplied.
  - As more inputs are devoted to production, output increases at an ever decreasing rate.
- Gasoline is a product for which demand is inelastic relative to supply in the short run. If the government imposes a new tax on gasoline producers:

  - the statutory incidence of the tax will fall more heavily on gasoline consumers than gasoline producers.
  - the tax incidence will fall more heavily on gasoline consumers than gasoline producers.
  - the tax incidence will fall more heavily on gasoline producers than gasoline consumers.
  - the tax incidence will most likely be split evenly between producers and consumers.

5. If full employment in the United States is 95% and the current rate of unemployment is 6%, which of the following statements is FALSE?
- A. The natural rate of unemployment is 5%.
  - B. Structural unemployment is less than 5%.
  - C. Frictional unemployment is less than 5%.
  - D. Cyclical unemployment is greater than 5%.
6. Assume that nominal GDP for 2004 was \$11,466 billion. If the GDP deflator for 2000 (base year) was 100, and the deflator for 2004 is 107.24, what is real GDP for 2004?
- A. \$12,296 billion.
  - B. \$11,668 billion.
  - C. \$11,267 billion.
  - D. \$10,692 billion.
7. The President of the United States has recently proposed a broad-based tax cut. This is an example of all of the following EXCEPT:
- A. restrictive monetary policy.
  - B. Keynesian fiscal policy.
  - C. an attempt to stimulate economic growth.
  - D. macroeconomic policy.
8. If the Consumer Price Index (CPI) was 184.3 at the end of 2003 and 190.3 at the end of 2004, then inflation in 2004 was *closest* to:
- A. 1.033%.
  - B. 3.15%.
  - C. 3.26%.
  - D. 6.00%.
9. Which of the following is an example of how the Federal Reserve (Fed) could implement an expansionary monetary policy?
- A. Sell U.S. Treasury securities in the open market.
  - B. Lower the marginal tax rate for wealthy taxpayers.
  - C. Lower the discount rate on short-term funds.
  - D. Raise the marginal tax rate for wealthy taxpayers.
10. The law of diminishing returns suggests that:
- A. firms producing near their maximum capacity will experience decreasing marginal revenue.
  - B. variable costs will increase as output increases.
  - C. average variable costs will increase at high levels of output.
  - D. average fixed costs will increase at high levels of output.

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11. Alchemy Inc. has just received government patent protection on an expensive process that converts copper into silver. This patent protection:
  - A. is a high entry barrier.
  - B. will allow Alchemy to recover its costs of research and development.
  - C. will grant Alchemy a monopoly position.
  - D. All of the above.
  
12. Joe's Cafe has found that the price elasticity of demand for desserts is  $-1.5$ . Based on this fact, Joe's cafe should expect that:
  - A. lowering the price of desserts would reduce demand by more than the percentage price reduction.
  - B. raising the price of desserts would reduce demand, but by less than the percentage change in price.
  - C. lowering the price of desserts would increase demand by more than the percentage price reduction.
  - D. raising the price of desserts would increase demand by more than the percentage change in price.
  
13. Protect is a liquid sealant used to protect metal surfaces from harsh weather environments. Protect is made from several chemicals, including oil and other petroleum derivatives. All of the following would be *likely* to cause a shift in the supply curve for Protect EXCEPT:
  - A. an increase in oil prices.
  - B. development of a less expensive production process for Protect.
  - C. an increase in the retail price of Protect.
  - D. a decrease in oil prices.
  
14. A purely competitive firm will maximize profits by setting output at the point where:
  - A. unit price equals marginal cost.
  - B. average cost equals marginal revenue.
  - C. average total revenue equals average fixed cost.
  - D. marginal revenue equals average variable cost.
  
15. According to the law of comparative advantage, trading partners should:
  - A. specialize in the production of goods where they have an absolute advantage.
  - B. impose tariffs on goods where they have low opportunity costs of production.
  - C. import goods for which they have high opportunity costs of production.
  - D. seek to import goods from nations with low wage rates.

16. Quotas are often considered to be more harmful forms of trade restrictions than tariffs because:
- A. quotas provide no cash flow benefits to the domestic government.
  - B. tariffs provide greater cash flow benefits to the foreign producers.
  - C. quotas benefit domestic producers at the expense of domestic consumers.
  - D. tariffs benefit domestic consumers at the expense of foreign producers.
17. Wakeup, Inc. is a U.S.-based firm that imports coffee from Brazil and packages it with muffins and sweet rolls in baskets sold as holiday gifts. Wakeup will need one million Brazilian Reals (BRL) in 30 days to pay for its latest shipment of coffee. The current exchange rate is 2.725 BRL/\$. The 30-day forward rate of exchange is 2.780 BRL/\$. If the spot rate of exchange 30 days from now is 2.745 BRL/\$, how much will Wakeup need to pay in dollars?
- A. \$359,712.
  - B. \$2,780,000.
  - C. \$364,299.
  - D. \$2,745,000.
18. All of the following transactions would affect the U.S. current account EXCEPT:
- A. U.S. oil refinery's purchase of oil from Saudi Arabia.
  - B. Japanese bank's purchase of a U.S. Treasury bond.
  - C. French consumer's purchase of U.S. manufactured DVD.
  - D. foreign aid payments made for disaster relief.



## PRACTICE QUESTION ANSWERS: ECONOMICS

1. **B** Lekross' cost for cheese delivered to Localmart is  $\$1.20 + 0.15 = \$1.35$  per pound. For Far Stores, the cost delivered is  $\$1.20 + 0.55 = \$1.75$  per pound. Far Stores' offer will cover Lekross' costs, while any cheese sold to Localmart would generate a loss.
2. **B** Complements are products where demand for one is directly related to demand for the other (i.e., an increase in demand for one leads to an increase in demand for the other), while substitutes are products where demand for one is inversely related to demand for the other. In this case, higher orange juice prices (which would lower demand) cause buyers to increase demand for fruit punch (substitute). Higher cookie prices (which would lower demand), cause lower milk demand (complement).
3. **C** The law of supply says higher prices for a good will lead to an increase in supply. Choice B is the law of demand, and choice D is the law of diminishing returns.
4. **B** If demand is inelastic relative to supply, quantity demanded will be less sensitive to price changes than quantity supplied. Consumers are willing to bear more of the price increase (or have less flexibility to alter their demand), so they will pay more of the tax. The statutory tax incidence refers only to who is responsible for actually paying the tax, and does not address who bears the economic burden of the tax.
5. **D** The natural rate of unemployment would include both frictional and structural unemployment and would be calculated as  $1 - 0.95 = 0.05$ , or 5%. Any excess would be cyclical in nature.
6. **D**

$$\text{Real GDP}_{2004} = \text{nominal GDP}_{2004} \frac{\text{GDP deflator}_{\text{base year}}}{\text{GDP deflator}_{2004}}$$

$$= 11,466 \times \frac{100.00}{107.24} = \$10,692 \text{ billion}$$
7. **A** Government taxing and spending is fiscal policy, which is macroeconomic policy. Tax cuts are believed to stimulate the economy, although research does not entirely support this belief.
8. **C** The formula for calculating inflation from the CPI is:
 
$$\text{inflation} = \frac{\text{CPI}_T}{\text{CPI}_{T-1}} - 1 = \frac{190.3}{184.3} - 1 = 0.326 = 3.26\%$$
9. **C** Lowering the discount rate would lower the cost of borrowing and hopefully stimulate the economy. This would be expansionary monetary policy. Selling Treasury securities in the open market would be a restrictive policy, as this would remove money from circulation. Adjusting tax rates would be fiscal policy authorized by Congress and carried out by the Treasury, not the Fed.

10. C The law of diminishing returns holds that as more and more resources are devoted to production, the output increases at a decreasing rate. Stated differently, more and more variable inputs will be required to produce another unit of output, which will raise average variable costs.
11. A The patent is a barrier to entry, but there are still other ways to produce silver (i.e., mining). Alchemy's ability to recover its expenses will depend on how expensive the process is; it may be cheaper to mine silver than to convert copper.
12. C The price elasticity of demand for desserts of  $-1.5$  indicates that changing price would lead to a more than commensurate change in demand (in the opposite direction). Therefore, lowering dessert prices would increase demand by more than the change in price.
13. C A change in the price of Protex would simply lead to a move along the supply curve. A change in the cost of the production process or a change in the price of raw materials would cause the entire curve to shift. Note that the question does not ask which way the curve is shifting.
14. A Remember that for the purely competitive firm, unit price equals marginal revenue—the firm faces a flat demand curve. Therefore profit will be maximized at the level where marginal revenue (or unit price) equals marginal cost.
15. C Comparative advantage holds that nations should produce and export goods where they have low opportunity costs of production and import goods where they have high opportunity costs of production.
16. A Tariffs provide some cash flow benefit to the domestic government, but quotas do not. Foreign producers get more income under quotas because there is no amount paid to the domestic government. Both types of restrictions allow domestic producers to charge higher prices at the expense of domestic consumers. Tariffs do not benefit domestic consumers.
17. C To buy 1 million BRL in 30 days, Wakeup will need  $1/2.745 \times 1,000,000 = \$364,299$ . The current spot and forward rates do not matter unless Wakeup is interested in setting up a hedge or evaluating its exchange risk.
18. B Purchasing the Treasury bond would affect the capital account, not the current account. The other transactions would all be included in the current account.

# FINANCIAL REPORTING AND ANALYSIS

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Financial statements are useful in helping investors and creditors evaluate firms. They give a snapshot of the firm's assets, liabilities, and equity at a point in time (the balance sheet) as well as a summary of the firm's operating performance over a specified time period (the income statement). They show the firm's operating, investing, and financing cash flows over a specified period (the statement of cash flows) and the amounts of and changes in ownership (the statement of owners' equity).

Financial reporting is done in periods of time. Companies tend to use the year as the primary length of the period but also report for periods less than a year (e.g., quarterly) on an interim basis.

The statements are prepared at the end of a uniform period to allow comparisons across time. Financial statements are prepared at the conclusion of each of these accounting periods, summarizing the activities that occurred during the period. The 12-month reporting period (i.e., the **fiscal year**) chosen by management does not have to coincide with the calendar year ending December 31.

Companies whose stocks are traded in public markets are required to report financial statements that meet widely accepted accounting rules. These rules are created by accounting standard-setting bodies.

One such body is the International Accounting Standards Board (IASB). The IASB's accounting standards are called **International Financial Reporting Standards** (IFRS) and are accepted in most of the world's major economies. In the United States, however, the accounting standard-setting body is the Financial Accounting Standards Board (FASB), which issues **Generally Accepted Accounting Principles** (U.S. GAAP).

The IASB and FASB have been working over time to converge their accounting principles into one uniform set, but as of this writing some key differences remain.

## DOUBLE ENTRY ACCOUNTING: DEBITS AND CREDITS

All accounting is based on a **double entry** system, where there are two sides to every transaction. When any transaction is entered into an accounting system, there must be at least two accounts affected (there can be more), one for each

side of the transaction. This is a result of the basic **accounting equation**, which is the underlying basis of the statement of financial position or balance sheet. The equation is as follows:

$$\text{assets} = \text{liabilities} + \text{owner's equity}$$

As with any equation, a change to one side must be offset by either an equal change in the same direction on the other side, or an opposite and offsetting change on the same side. Otherwise, the “equals” sign is violated and the equation fails. The accounting equation tells us what assets are owned or used by the firm, as well as where they came from. All of a firm’s assets must be financed either with some form of debt or equity. This should make sense. If you start a business, you must have capital in order to acquire assets. You might use your own money (equity), or you might borrow money (liabilities).

We use the terms **debit** and **credit** in double entry accounting, which are Latin for “left side” and “right side.” And because we are referring to the left and right sides of an equation, the left side must equal the right side, or the debits must equal the credits. How each of the different types of accounts is increased, whether on the left or right, is outlined in Figure 1.

**Figure 1: Debits and Credits**

<i>Accounts</i>	<i>Debit</i>	<i>Credit</i>
Assets	Increase	Decrease
Liabilities	Decrease	Increase
Owner’s Equity	Decrease	Increase
Revenues, Gains	Decrease	Increase
Expenses, Losses	Increase	Decrease

Let’s look at an example. Assume that a hardware store buys a case of hammers from a supplier for \$100. This transaction is a purchase of inventory (goods for resale). Inventory goes up while cash goes down by the same amount. The accounting entry for this sale would be to debit one account, Inventory, and credit another account, Cash.

	<i>Debit</i>	<i>Credit</i>
Inventory	100	
Cash		100

## Financial Reporting and Analysis

A debit to an asset account (inventory) produces an increase on the asset side of the accounting equation. A credit to an asset account (cash) produces a decrease on the asset side of the accounting equation. Here, the increase in one asset is offset by a decrease in another asset, so total assets remain unchanged and the accounting equation stays in balance.

What if the hardware store did not want to spend cash to buy the hammers and instead asked the hammer supplier to finance the purchase? In this situation, the hardware store would owe the supplier \$100. The account that results is called *accounts payable*, which represents a liability that must be satisfied at some point in the near future. In this case, instead of cash decreasing to offset the increase in inventory, a credit to accounts payable would increase liabilities (on the right side of the equation). Because liabilities increase by the same amount as assets, the accounting equation balances.

### ACCRUAL ACCOUNTING

The general definition of **accrual accounting** says that revenues are recorded when earned, regardless of when cash is collected. Likewise, expenses are recorded when incurred, regardless of when they are paid. The cash flow that results from a transaction may occur before, during, or after the transaction. For example, when sales are made on credit, a company delivers goods or services but allows the customer to pay later. Because the firm has provided goods or services, it can recognize revenue as earned, even though it has not received cash.

An important characteristic of accrual-based financial statements is the **matching principle**. The matching principle requires revenues to be matched or recorded with the expenses related to generating those revenues. If revenues and expenses are not properly matched within the same accounting period, it is difficult to identify which products or services are profitable and which ones are not.

Some expenses cannot be specifically identified with particular revenues. These should simply be recorded in the period they are incurred. One example might be the utility bill for office space.

An example of how accrual-based accounting works can be seen with credit sales. When a firm does not collect cash at the time of sale, the firm creates *accounts receivable*. Sales are reported on the income statement, and the amount owed by the customer is included in accounts receivable, a balance sheet asset.

To illustrate the concept, assume that a firm has an accounts receivable balance of \$1,000. This means that the firm has sold \$1,000 in products but has not yet collected the cash. Credit sales of \$1,000 would be recorded as follows:

	<i>Debit</i>	<i>Credit</i>
Accounts receivable	1,000	
Sales revenue		1,000

As the firm collects cash from its credit sales, the collections will be recorded as a debit (increase) to cash and a credit (decrease) to accounts receivable.

	<i>Debit</i>	<i>Credit</i>
Cash	1,000	
Accounts receivable		1,000

## THE BALANCE SHEET

The **balance sheet** shows the company's resources at a point in time, as well as claims to those resources. Claims can be categorized as **liabilities** (lenders' and creditors' claims) and **equity** (owners' claims). The balance sheet is also referred to as the **statement of financial position**. The following sample balance sheet illustrates the format along with a small variety of account types.

## Financial Reporting and Analysis

AAA Company, Inc.  
Balance Sheet  
December 31, 20X7

Current Assets		Current Liabilities	
Cash	\$100	Accounts Payable	\$175
Marketable Securities	300	Notes Payable	300
Accounts Receivable	800	Taxes Payable	75
Inventory	1,200	Current Portion Long-Term Debt	125
Supplies	200		
Prepaid Expenses	150		
<b>Total Current Assets</b>	<b>\$2,750</b>	<b>Total Current Liabilities</b>	<b>\$675</b>
Net Property, Plant & Equipment	5,000	Long-Term Debt	\$1,000
Other Assets	1,000	<b>Total Liabilities</b>	<b>\$1,675</b>
		Common Stock	\$2,000
		Additional Capital	1,000
		Retained Earnings	\$4,075
		<b>Total Equity</b>	<b>\$7,075</b>
<b>Total Assets</b>	<b>\$8,750</b>	<b>Total Liabilities &amp; Equity</b>	<b>\$8,750</b>

Just as in the accounting equation, assets are on the left side of the balance sheet, while liabilities and owners' equity (claims against the assets) are on the right. As must be the case, total assets equal total liabilities plus owners' equity.

The sample balance sheet for AAA Company, Inc. is an example of a **classified balance sheet**. "Classified" in this context means organized or sorted. Assets and liabilities are divided into current and long-term categories. **Current assets and liabilities** are those that will be converted to cash, used up, or satisfied (in the case of liabilities) within one year or within the operating cycle, whichever is longer. They are typically listed in decreasing order of liquidity. Those that are most liquid, at the top, will be converted to cash or used up sooner than those below them. Classifying the balance sheet in such a way allows users of financial information to quickly determine the firm's overall level of liquidity.

The difference between current assets and current liabilities is called **working capital** (or sometimes **net working capital**). The amount of working capital available is of particular interest to short-term creditors, because it is a measure of

the firm's ability to meet current obligations. Most businesses must have positive working capital to operate on a day-to-day basis. If working capital is consistently negative, the business must cover the shortfall with long-term debt or additional equity financing. However, positive working capital does not guarantee that short-term obligations will be met. Current assets consisting of obsolete inventory or uncollectible receivables would present a deceiving picture of the firm's short-term liquidity. The quality of the firm's current assets is important to short-term creditors.

Next, we will provide a basic overview of some common types of accounts.

**Assets** are resources that are owned or used by a business to produce a current or future benefit. The overall goal of most any business is to use assets to create more assets or revenues.

- *Cash*. This account not only includes cash available for immediate use, but *cash equivalents* such as certificates of deposit (CDs) and short-term government debt.
- *Marketable securities*. Securities owned by the firm are classified as available for trading, available for sale, or held to maturity. Their classification will determine whether they are shown on the balance sheet at current market value or historical cost.
- *Accounts receivable*. This account is created when credit sales are made to customers who agree to pay later.
- *Inventory*. Goods purchased or produced for resale. Inventory is carried on the balance sheet at the lower of cost or realizable value.
- *Supplies*. Supplies differ from inventory in that they are not for resale, but instead consumed by the business internally. Examples would include pens, pencils, paper, and toner for the copy machine.
- *Prepaid expenses*. Prepaid expenses are accruals that result when the firm pays in advance for items such as rent, insurance premiums, subscriptions, or fees. Prepaid expenses are an asset because they will bring a future benefit.
- *Deferred tax assets*. **Deferred tax** items result from timing differences between financial accounting and tax accounting. A deferred tax asset is created when these differences cause taxes due (on the company's tax form) to be greater than income tax expense (on the company's income statement).
- *Long-term fixed assets*. Long-term fixed assets are commonly referred to as *property, plant, & equipment*, and are carried on the balance sheet at a net book value.
  - *Land* is typically valued at its historical cost.
  - *Plant* represents the buildings that house the firm's production or selling activities. Plant is typically valued at historical cost less **accumulated depreciation**. Depreciation is an expense the firm recognizes in each period to reflect the fact that plant and equipment wear out over time.



## Financial Reporting and Analysis

- *Equipment* is the machinery or fixtures used to produce or sell inventory. Equipment is typically valued at historical cost less accumulated depreciation.

**Liabilities** represent the current and future obligations (debts) of the firm. These obligations can be satisfied by payment in cash, but in some cases also by providing goods and services. Following are some common types of liabilities.

- *Accounts payable*. Balances due to suppliers for goods and services purchased on trade credit.
- *Notes payable*. These debts differ from accounts payable in that they represent outright borrowing from lenders. Notes payable can be classified as either short or long term, depending upon how soon they must be repaid.
- *Unearned revenues*. Unearned revenues are an accrual account for advance payments made by customers. Once the firm provides the goods or services, it will reduce this liability and recognize sales revenue.
- *Long-term debt*. These are obligations that will not be repaid within the current year. Long-term debt is most often used to purchase long-term assets and requires repayment of principal plus interest.
- *Deferred tax liabilities*. A deferred tax liability is created when timing differences between financial accounting and tax accounting cause income tax expense (on the company's income statement) to be greater than taxes due (on the company's tax form).

**Shareholders' equity** is the owners' investment and the total earnings retained from the beginning of the business.

- *Contributed capital (paid-in-capital)* is the amount of the stockholders' investment in the firm's equity securities.
- *Common stock* is the portion of stockholders' investment valued at par or stated value.
- *Other paid-in-capital* is the excess of the shareholders' investment over the stock's par value.
- *Retained earnings* is the total net income less the amount distributed to the owners as dividends from the beginning of business. Retained earnings do not represent ready cash; rather, they represent earnings that the firm has reinvested in its business (for example, by buying plant and equipment).

## THE INCOME STATEMENT

The **income statement** allows the user of financial information to see the results of a company's day-to-day operations for the entire year. A condensed version in the most common format is as follows.

AAA Company, Inc.  
Income Statement  
Year ended December 31, 20X7

Sales
<u>Less: Cost of goods sold</u>
Gross margin
<u>Less: Selling, general &amp; administrative expenses</u>
Income from operations
<u>Less: Interest expense</u>
Income before taxes
<u>Less: Income taxes</u>
Net income

**Revenues** primarily include sales of goods and services, but may also include items such as interest and dividend income or rental income. **Gains** are created when companies sell assets (buildings, equipment, investments, etc.) for more than their book value.

**Expenses** represent use of resources. One of the greatest uses stems from the purchase or production of goods for resale. As the costs of producing inventory are incurred, they are recorded in the balance sheet as inventory for as long as the company owns the asset. Once the asset is sold, however, its cost is removed from the balance sheet and sent to the income statement as an expense, **cost of goods sold**. Subtracting cost of goods sold from revenue results in **gross profit**. This provides information about the company's primary source of profit—selling goods. Companies that sell services do not have an account called cost of goods sold; all costs incurred for selling services are expensed as incurred.

Almost all assets are written off to expense at some point, some more quickly than others. For example, when supplies are used up they are written off to supplies expense. Other assets may take years to use up. *Depreciation* is an expense firms recognize over an asset's useful life that represents its decline in value through use or age.

**Non-recurring items** are income (or losses) from outside the company's normal business operations and caused by events or transactions that do not typically occur. How these appear on the income statement depends on the nature of the items.

*Unusual or infrequent items* are events that are either unusual in nature or infrequent in occurrence, but not both. Some examples of unusual or infrequent items are gains or losses from the sale of assets; impairments; and restructuring costs. Unusual or infrequent items are included in income before taxes.

## Financial Reporting and Analysis

A *discontinued operation* is a part of the business that management has decided to dispose of but has not yet done so, or a part that was disposed of in the current year after it had generated income or losses. Any income or loss from a discontinued operation is reported separately in the income statement, net of tax.

*Extraordinary items* are events that are both unusual and infrequent. Examples of these include losses from an expropriation of assets or uninsured losses from natural disasters. Under U.S. GAAP, extraordinary items are reported separately in the income statement, net of tax, similar to discontinued operations. IFRS, however, does not allow items to be treated as extraordinary.

**Accounting changes** include changes in accounting principles, changes in accounting estimates, and prior-period adjustments.

A *change in accounting principle* refers to a change from one accounting method to another (e.g., a change from straight-line depreciation to accelerated depreciation). A change in accounting principle requires **retrospective application**. Any prior-period financial statements the firm presents must be restated to reflect the change.<sup>1</sup>

By contrast, a *change in accounting estimate* does not require the restatement of prior financial statements. An example of an accounting estimate is the useful life of an asset. Management may change the estimated useful life of an asset if new information indicates the asset will last longer than (or not as long as) originally expected.

A *prior-period adjustment* is a change from an incorrect accounting method to a correct one, or the correction of an error made in previous financial statements. Prior-period adjustments are made by restating results for all prior periods presented in the current financial statements. The company must disclose the nature of the adjustment and its effect on net income.

## THE STATEMENT OF CHANGES IN EQUITY

In the balance sheet we saw that owners' equity is composed of two parts: contributed capital and retained earnings. The **statement of changes in equity** shows how these two components changed throughout the year.

**Contributed capital** will show changes if the firm issued or repurchased stock. **Retained earnings** represents all net income to date that the firm has not paid to shareholders as dividends.

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<sup>1</sup> One exception involves inventory accounting. Under U.S. GAAP, a firm that changes to "last-in-first-out" from another inventory cost method does not apply the change retrospectively, but instead uses the carrying value of inventory as the first inventory layer.

The basic format of the statement of changes in equity is as follows.

AAA Company, Inc.  
Statement of Changes in Equity  
For the Year Ended December 31, 20X7

Equity Balance at January 1, 20X7	
+ Additional Capital Contributions	
+ Net Income for 20X7	
– Dividends Paid	
= Equity Balance at December 31, 20X7	

## THE STATEMENT OF CASH FLOWS

The primary purpose of the **statement of cash flows** is to provide information about a company's cash receipts and cash payments during an accounting period.

The statement of cash flows provides information on cash flows from operations, investing activities, and financing activities. Information on noncash activities must also be reported along with the statement.

**Cash flow from operations** represents changes in the working capital accounts (e.g., accounts receivable, inventory, and accounts payable) and all items that flow through the income statement (e.g., cash receipts from customers, payments for good sold, wages).

**Cash flow from investing** represents the purchase or sale of productive assets (physical assets and investments) for cash. Investing cash flow essentially deals with the items appearing on the lower left-hand portion of the balance sheet (fixed assets).

**Cash flow from financing** represents acquiring and dispensing ownership funds and borrowings. Financing cash flow deals with the lower right-hand portion of the balance sheet (long-term debt and equity).

**Noncash investing and financing activities** do not flow through the statement of cash flows because they do not require the use of cash. Examples are the following:

- Retiring debt securities by issuing equity securities to the lender.
- Converting preferred stock to common stock.
- Acquiring assets through a capital lease.
- Obtaining long-term assets by issuing notes payable to the seller.
- Exchanging one noncash asset for another noncash asset.
- The purchase of noncash assets by issuing equity or debt securities.

## Financial Reporting and Analysis

While these activities do not flow through the statement of cash flows, they should be disclosed in either the footnotes or on a separate schedule as investing or financing events that did not affect cash.

Presenting cash flow from operations using the **indirect method** starts with income after taxes (the bottom of the income statement) and adjusts backwards for noncash and other items. Changes in balance sheet items are used to adjust net income under the indirect method. Figure 2 identifies changes in balance sheet accounts as either sources of cash (added to net income) or uses of cash (subtracted from net income).

**Figure 2: Balance Sheet Items in the Cash Flow Statement**

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	<i>Increase</i>	<i>Decrease</i>
Current Assets	use	source
Current Liabilities	source	use

Net income

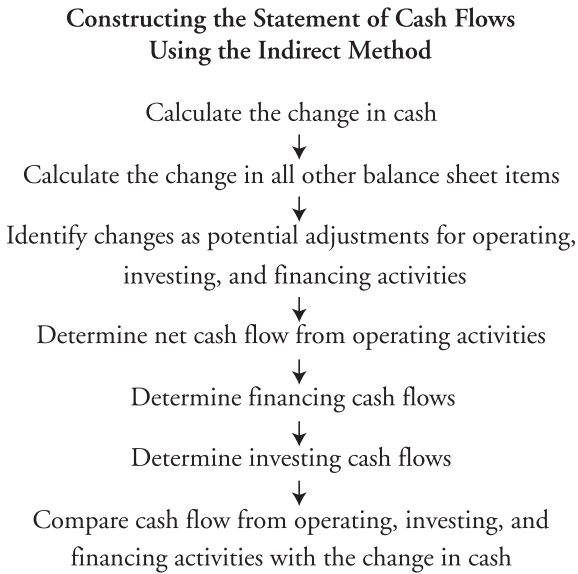
Adjust for:

- + Noncash expenses or losses
- Noncash revenues or gains

Adjust for changes in working capital:

- +/- Changes in operating asset accounts (e.g., accounts receivable)
- +/- Changes in operating liability accounts (e.g., accounts payable)
- = Cash flow from operations

Figure 3: Cash Flow Statement Construction

**Example: Statement of cash flows using the indirect method**

Given the following income statement and balance sheet information, calculate the statement of cash flows using the indirect method.

Income statement	
Sales	\$1,600
Cost of goods sold	(1,350)
Gross profit	250
Depreciation expense	100
Interest expense	47
Equity in earnings of investment	2
Gain on the sale of old machine	10
Income before taxes	\$115
Income taxes	
Current	35
Deferred	10
Net income after taxes	\$70

## Financial Reporting and Analysis

Balance Sheet	1/1/end	1/1/beg	Net Change
<b>Assets</b>			
Cash	\$292	\$100	\$192
Accounts receivable	280	200	80
Inventory	700	800	(100)
Property, plant, and equipment	1,020	1,000	20
Accumulated depreciation	(340)	(300)	(40)
Investments	12	10	2
<b>Total assets</b>	<b>\$1,964</b>	<b>\$1,810</b>	<b>\$154</b>
<b>Liabilities</b>			
Accounts payable	\$470	\$450	\$20
Mortgage	550	600	(50)
Bank note	100	0	100
Deferred taxes	90	80	10
Common stock	410	400	10
Retained earnings	344	280	64
<b>Total liabilities and equity</b>	<b>\$1,964</b>	<b>\$1,810</b>	<b>\$154</b>

Additional information:

- Dividends of \$6 were paid to shareholders.
- One new common share was sold at par value. Par is \$10 per share.
- Fixed assets (machinery) were sold for \$30. Original cost of these assets was \$80, and \$60 of accumulated depreciation has been charged to the original cost.
- New fixed assets were purchased for \$100. To pay for this acquisition, a 10-year, \$100 note was issued to a bank.
- The firm recognized a \$2 gain from a subsidiary using the equity method. No cash was received.

<b>Cash flow from operations (indirect method):</b>		
Net income		\$70
Add (subtract) adjustments		
Depreciation	\$100	
Deferred taxes	10	
Gain on the sale of machinery	(10)	
Equity in long-term investment	(2)	
Accounts receivable (use)	(80)	
Inventory (source)	100	
Accounts payable (source)	20	138
Net cash flow from operations		<u>208</u>

Let's scrutinize the statement of cash flows and talk a bit about its construction:

- Depreciation and deferred taxes are noncash expenses which reduce net income. Adding them back to net income eliminates any effect on cash flows.
- The gain on the sale of machinery is equal to the market value of the machine (\$30) minus the book value of the machine at the time of the sale (\$80 historical cost—\$60 accumulated depreciation). Observe that the full \$30 is listed as a cash flow from investing, so the \$10 gain is double counting and must be removed from net income when deriving CFO.
- The gain from the subsidiary is equity investment income that does not result in receipt of cash. Thus, it is eliminated from net income in deriving CFO.
- Accounts receivable and accounts payable are operating (working capital) accounts whose change is classified via sources/uses. That is, changes in accounts receivable, inventory, and accounts payable are adjustments to reflect funding from customers and suppliers.

You might be wondering what happened to the \$60 in accumulated depreciation written off when the old machine was sold. The answer is that the depreciation was entered into the cash flow computations in earlier periods. Only the current period's depreciation is considered on the statement of cash flows under the indirect method.

Determining cash flow from investing activities tends to be easier than calculating cash from operations. Each investing activity is classified as a cash inflow or a cash outflow. The individual items are then added together to compute cash from investing.



## Financial Reporting and Analysis

Continuing with our example:

Investing cash flows	
Purchase fixed assets (use)	(\$100)
Sale of old machine (source)	30
Net cash flow from investing	(70)

- The purchase of fixed assets is a cash expenditure and is reflected as a cash outflow.
- The entire amount received from the sale of the old machine (fixed assets) is reflected as a cash inflow.

Cash receipts and payments from each financing activity are analyzed individually and then totaled to compute cash flow from financing.

Financing cash flows	
10-year note (source)	\$100
Sale of common stock (source)	10
Dividends paid (use)	(6)
Repayment of mortgage note (use)	(50)
Net cash flow from financing	\$54

- The issuance of the 10-year, \$100 note and the sale of common stock represent cash inflows to the firm.
- Dividends paid flow through retained earnings and are classified as a financing cash outflow.
- The repayment of the mortgage note principal constitutes a use of cash.

**Completed Statement of Cash Flows**

Cash flow from operations (indirect method):	
Net income	\$70
Add (subtract) adjustments	
Depreciation	\$100
Deferred taxes	10
Gain on the sale of machinery	(\$10)
Equity in long-term investment	(2)
Accounts receivable (use)	(80)
Inventory (source)	100
Accounts payable (source)	20
	138
Net cash flow from operations	\$208
Investing cash flows	
Purchase fixed assets (use)	(\$100)
Sale of old machine (source)	30
Net cash flow from investing	(70)
Financing cash flows	
10-year note (source)	\$100
Sale of common stock (source)	10
Dividends paid (use)	(6)
Repayment of mortgage note (use)	(50)
Net cash flow from financing	\$54
Net cash flow	\$192
Net increase in cash	\$192

The net increase in cash is \$192. Note that this is equal to the net increase in cash shown on the balance sheet ( $\$292 - \$100 = \$192$ ).

**OTHER SOURCES OF FINANCIAL INFORMATION**

**Financial statement footnotes** include disclosures that help explain the information summarized in the financial statements. Footnotes are required by accounting standard-setting bodies and financial regulators. Footnotes allow users to assess

## Financial Reporting and Analysis

the amount, timing, and uncertainty of the estimates reported in the financial statements. Footnotes:

- Provide information about accounting methods and the assumptions and estimates used by management.
- Are audited, whereas other disclosures, such as supplementary schedules, are not audited.
- Provide additional information on such items as fixed assets, inventory, income taxes, pensions, debt, contingencies and commitments, marketable securities, significant customers, sales to related parties, and export sales.
- Often contain disclosures relating to contingent losses. Firms are required to accrue a loss when (1) it is probable that assets have been impaired or a liability has been incurred and (2) when the amount of the loss can be reasonably estimated. A range of possible losses from a minimum to a maximum range is estimated. If it is only reasonably possible that a loss has been incurred, then footnote disclosure of that loss contingency is required. Examples include litigation, expropriation, and repurchase agreements.

A firm may use *supplementary schedules* to report additional information outside the financial statements.

The **management's commentary** or **management's discussion and analysis** (MD&A) portion of a financial statement discusses the nature, past performance, and future outlook of the company. Issues addressed in this section may include the following:

- Results from operations, with a discussion of trends in sales and expenses.
- Capital resources and liquidity, with a discussion of trends in cash flows.
- A general business overview based on known trends.
- Discussion of significant effects of currently known trends, events, and uncertainties (may voluntarily disclose forward-looking data).
- Liquidity and capital resources and transactions or events with liquidity implications.
- Discontinued operations, extraordinary items, and other unusual or infrequent events.
- Extensive disclosures in interim financial statements.
- Disclosures of a segment's need for cash flows or contribution to revenues or profit.

An **audit** is an independent review of an entity's financial statements. Public accountants conduct the audit, examining the financial reports and supporting records. The auditor provides an opinion on the fairness and reliability of the financial reports. The independent certified public accountant employed by the board of directors is responsible for seeing that the financial statements conform to the accounting principles. The auditor examines the company's accounting and internal control systems, confirms assets and liabilities, and generally tries to be

confident that there are no material errors in the financial statements. Reading the auditor's report is important for an analyst.

The **standard auditor's opinion** contains three parts stating that:

- Whereas the financial statements are prepared by management and are its responsibility, the auditor has performed an independent review.
- Generally accepted auditing standards were followed, thus providing reasonable assurance that the financial statements contain no material errors.
- The auditor is satisfied that the statements were prepared in accordance with applicable accounting principles and that the estimates made are reasonable. The auditor's report must contain an additional explanation when accounting methods have not been used consistently between periods.

An *unqualified or clean opinion*<sup>2</sup> indicates that the auditor believes the statements are free from material omissions and errors. A *qualified opinion* may be issued if the firm has made any exceptions to the accounting principles, and the auditor will explain these exceptions. The auditor will issue an *adverse opinion* if the statements are not presented fairly. A *disclaimer of opinion* indicates that the auditor is unable to express an opinion.

The auditor's opinion will also contain an explanatory paragraph when a material loss is probable but the amount cannot be reasonably estimated. These "uncertainties" may relate to the going concern assumption, the valuation or realization of assets, or to litigation. This type of disclosure may be a signal of serious problems and calls for closer examination by the analyst.

In the United States, an auditor must express an opinion on a company's **internal controls**. Internal controls are the processes a company uses to present its financial statements accurately.

## A CLOSER LOOK AT THE BALANCE SHEET

In this section we will address several balance sheet accounts in more detail. The specific accounts covered are marketable securities, accounts receivable, inventory, long-term assets, current liabilities, long-term liabilities, and equity.

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2 These terms can be confusing because we usually think of "unqualified" as a negative thing and "qualified" as a positive thing. In this context, however, we should think of "unqualified" as meaning "accepted without qualification." Likewise, "qualified" is used here in the sense of making an exception to something, as in "Let me qualify that by saying..."

### Marketable Securities

Marketable securities are initially recorded at cost (cash price plus any acquisition costs, such as brokerage fees). For this discussion, assume a company owns a bond with historical cost of \$10,000 and current market value of \$12,000.

- If management intends to hold a bond to maturity, then they are reported at their original cost. These are called *investments in securities held to maturity*.<sup>3</sup> Here, the security's book value is \$10,000.
- If management classifies the bond as a *trading* investment (trading investments are always considered to be current assets), then the investment is reported on the balance sheet at its *fair market value*. Any unrealized holding gains or losses due to the appreciation or depreciation of the investment must be listed as a *gain or loss on the income statement*. Here, the unrealized holding gain is \$2,000 (\$12,000 – \$10,000).
- If management classifies the bond as an *available-for-sale* investment, then the investment is reported on the balance sheet at its *fair market value*. Any unrealized holding gains or losses due to the appreciation or depreciation of the investment must be listed as an *adjustment to stockholders' equity*, not as a gain or loss on the income statement.
- Interest or dividends received from these investments are recorded as interest or dividends earned (income).

### Accounts Receivable

Accounts receivable increase when a firm sells goods to customers on credit and decrease when the customers later pay the cash they owe.

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<sup>3</sup> Note that common and preferred stock cannot be classified as held-to-maturity because they do not have maturity dates.

**Example: Treatment of accounts receivable**

The SSS Corporation has \$10,000 of credit sales and \$2,000 of cash collections on its accounts receivable during the year:

The creation of accounts receivable is accounted for as:

	<i>Debit</i>	<i>Credit</i>
Accounts receivable	10,000	
Sales revenue		10,000

The payment by customers is recorded as:

	<i>Debit</i>	<i>Credit</i>
Cash	\$2,000	
Accounts receivable		\$2,000

The accounts receivable balance has increased by \$8,000.

**Uncollectible accounts** are accounts that customers cannot or will not pay. There are two approaches to accounting for uncollectible accounts: the direct write-off method and the allowance method.

- In the case of the *direct write-off method* of accounting for bad debts, bad debt expense is recorded as the accounts are written off as uncollectible.
- The *allowance method* for bad debts requires an estimate of the bad debts, generally made on the basis of the typical relation between either bad debt expense and credit sales or between accounts receivable and the allowance for doubtful accounts.
- The allowance for doubtful accounts method is more consistent with the matching principle and is preferred.

Estimating uncollectible accounts can be done using the percentage of net sales method (i.e., estimate percent of sales that will be uncollectible) or through the aging of accounts receivable (i.e., classifying accounts based on period outstanding and using a different percent from each aged category).

If a customer pays on an account that has been written off as uncollectible, the write-off is reversed, and the allowance account is credited for the amount (and cash is debited).

## Financial Reporting and Analysis

The direct write-off method is used for income tax calculations and by firms with immaterial bad debts. Here the firm does not use an allowance account (accounts receivable on the balance sheet is the amount owed by customers). It recognizes bad debt expense only when a particular account is written off as uncollectible.

## Inventory

The objective of **inventory accounting** is to determine the value of inventory that best achieves the matching of costs with revenues for the accounting period. The value of inventory determines the cost of goods sold, or COGS, on the income statement and the carryover of inventory to the next period on the balance sheet.

Management has choices regarding the systems and methods used for inventory accounting. Because inventory accounting directly affects the income statement, management's decisions about how to account for inventory will affect net income.

Management's choices in accounting for inventories include inventory processing systems and inventory costing methods.

Inventory processing systems:

- *Periodic*. Inventory is accounted for at the end of certain periods (e.g., quarterly). Only ending inventory is counted and priced, and COGS is determined by subtracting the cost of ending inventory from the cost of goods available for sale.
- *Perpetual*. A continuous record of the quantity and cost of merchandise is maintained as purchases and sales are made. COGS is accumulated as costs are transferred from inventory to COGS as sales are made.

Inventory costing methods:

- *Specific identification*. The units are priced according to the specific cost of each item in ending inventory. This method is mostly used for high-value items that can be identified easily by unit, such as automobiles.
- *Average cost*. Inventory is priced according to the average cost of the goods available for sale during the period (goods available for sale/number of units).
- *First-in, first-out (FIFO)*. The costs of the first items acquired by the firm are assigned to the first items sold. In other words, the oldest items are assumed to be sold first, and ending inventory reflects the cost of the items acquired most recently.
- *Last-in, first-out (LIFO)*. The costs of the last items acquired by the firm are assigned to the first items sold. In other words, the most recent items are assumed to be sold first, and ending inventory reflects the cost of the oldest items. The LIFO method is permitted under U.S. GAAP but is not permitted under IFRS.

Regardless of the cost method chosen, a company may not carry inventory on its balance sheet at a higher value than the inventory can be sold for. If the *net realizable value* (IFRS) or market value (U.S. GAAP) of inventory is less than its value on the balance sheet, the company must take a **writedown**. A writedown means the company must reduce the balance sheet value of inventory to its realizable or market value and recognize a loss on the income statement equal to the amount of the writedown. If the realizable value of the inventory later recovers, IFRS allows the firm to write its balance sheet value up again, but U.S. GAAP does not.

### Example: Inventory cost methods

Let the following be beginning inventory and purchases for the month of January for a firm that sells toasters:

01/01/20X8: Beginning inventory (1 unit, toaster #101)	\$12
01/06/20X8: 1 unit, purchase of toaster #102 @\$16	\$16
01/20/20X8: 2 units, purchase of toasters #103 and #104 @\$20	\$40
Ending inventory: 2 units (toasters #102 and #103)	

Therefore, two units were sold [number of units available (4) less number of units in ending inventory (2)]. The units sold were toasters #101 and #104.

**Specific Identification:** The units in ending inventory are valued at the specific invoice price of those units sold:

Ending inventory: 2 units (#102 @ \$16 and #103 @ \$20) = \$36  
 Cost of goods sold: 2 units (#101 @ \$12 and #104 @ \$20) = \$32

**Average Cost:** The units held in ending inventory and listed as sold are valued at the average cost of

$$\text{goods available for sale} = \frac{\text{beginning inventory} + \text{purchases}}{\text{number of units in inventory}}$$

Cost of goods available: 4 units at a total cost of \$68  
 Average cost = \$68 / 4 = \$17  
 Ending inventory: 2 units @ \$17 = \$34  
 Cost of goods sold: 2 units @\$17 = \$34



**First in, First out (FIFO):**

- Under FIFO, the unit cost of the one unit in beginning inventory and the January 6, 20X8, unit purchased (first in) are assigned as the two units sold to calculate the cost of goods sold.
- FIFO then assigns the remaining (or last in) costs to inventory. Hence, the two units in ending inventory will be valued at the unit price of the two units purchased on January 20, 20X8 (#103 and #104):

Ending inventory: 2 units @ \$20 = \$40

Cost of goods sold: 1 unit @ \$12 and 1 unit @ \$16 = \$28

**Last in, First out (LIFO)**

Ending inventory: 1 unit @ \$12 and 1 unit @ \$16 = \$28

Cost of goods sold: 2 units @ \$20 = \$40

- Under LIFO the unit cost of the two units of the January 20, 20X8, purchase (last in) is assigned to the two units sold.
- LIFO then assigns the initial (or first in) costs to inventory. Hence, the unit cost of the beginning unit and the unit purchased on January 6, 20X8, are assigned to the two units in ending inventory.

When using any valuation method, once ending inventory is valued under a particular cost flow assumption, COGS can be derived using the basic **inventory equation**:

$$\text{ending inventory} = \text{beginning inventory} + \text{purchases} - \text{cost of goods sold}$$

or

$$\text{cost of goods sold} = \text{beginning inventory} + \text{purchases} - \text{ending inventory}$$

	<i>AVG</i>	<i>FIFO</i>	<i>LIFO</i>
Beginning inventory	\$12	\$12	\$12
Plus purchases	\$56	\$56	\$56
Cost of goods available	\$68	\$68	\$68
Less ending inventory	<u>(\$34)</u>	<u>(\$40)</u>	<u>(\$28)</u>
Cost of goods sold	\$34	\$28	\$40

During periods of increasing inventory and rising prices:

- The FIFO method will result in the lowest COGS—first in, which goes to cost of goods sold, is the less expensive—and the highest net income. The higher the net income, the greater the inventories on the balance sheet [under FIFO, the last in (most expensive) goes to inventory]. Higher net income also means the firm will pay higher taxes, which in turn results in lower cash flows.
- LIFO will result in the highest cost of goods sold (last in is the most expensive) and lowest income. The lower the net income, the smaller the inventories on the balance sheet [under LIFO, the first in (least costly) goes to inventory]. Lower net income also means the firm will pay less in taxes, which in turn results in higher cash flows.
- The average cost method, being an average, is between the FIFO and LIFO valuations.
- Specific identification cannot be generalized because it depends on the specific units sold.

Figure 4 shows a comparison of the effects of LIFO and FIFO in periods of rising prices and stable or increasing inventory quantities.

**Figure 4: Comparisons of the Effects of LIFO and FIFO**

<i>LIFO results in . . .</i>	<i>FIFO results in . . .</i>
higher COGS	lower COGS
lower taxes	higher taxes
lower net income	higher net income
lower inventory balances	higher inventory balances
lower working capital	higher working capital
higher cash flows	lower cash flows

The opposite relationships hold for falling prices. If prices do not change, the income and inventory results are identical under all methods.

Because management’s various inventory accounting choices affect reported net income, they can have a significant impact on the company.

- *External evaluations* of the company by investors and creditors may be affected due to the levels of reported net income. Typically, higher net income is viewed more favorably than lower net income.
- *Internal evaluations* such as performance reviews that determine management compensation and bonuses may also be affected. In general, higher net income is viewed more favorably.

## Financial Reporting and Analysis

- A company's *cash flow* is influenced by inventory accounting choices through the amount of income taxes actually paid. As indicated in Figure 4, higher COGS will result in lower net income, which in turn will result in lower taxes payable. Yet, cash inflows and the actual cash cost of inventory acquired during the period are unaffected by the choice of inventory accounting methods. Consequently, because the firm is paying less cash in taxes, it will have higher operating cash flow than an otherwise identical firm choosing a different inventory accounting method and paying more in taxes.

## Long-Lived Assets

A **long-lived asset** is an asset that is typically employed in the production process of the firm and has a useful life of greater than one year. Long-lived assets are not made available for sale to the firm's customers (i.e., they do not represent inventory for sale).

Long-lived assets are classified in three main categories:

1. *Tangible assets* have a physical existence. Examples include land, buildings, and equipment. Allocating the cost of a tangible asset over its useful life is called *depreciation*.
2. *Natural resources* are purchased for the economic value that can be taken from the earth and used up over time. Examples include oil fields, timberland, and mines. Allocating the cost of a natural resource according to its use (e.g., cutting timber, pumping oil) is called *depletion*.
3. *Intangible assets* have no physical existence, but have a value that is based on rights or advantages that are conferred to the owner. Examples include copyrights, patents, trademarks, and franchises. The cost of most intangible assets is allocated to the periods over which it provides benefits through *amortization*.

Long-lived assets are generally reported at their **carrying value** or **book value** (i.e., historical cost less accumulated depreciation). However, if the asset has lost more of its revenue-generating ability than its accumulated depreciation reflects, it may have to be written down (referred to as **impairment**, in which case the amount of the writedown is recorded as a loss).

The decision to acquire long-lived assets is based on some type of present value analysis in which the present value of the asset's cash inflows is compared to the present value of the asset's cash outflows (e.g., initial outlay and operating costs).

Accounting issues pertaining to long-lived assets are how to spread the cost over the useful life, and how to represent the value of the asset each period on the balance sheet.

The cost of plant assets includes all expenditures (e.g., transportation and installation costs) that are necessary to acquire the assets and ready them for use. If the plant is constructed, *capitalized* costs (added to the purchase cost of the asset) include expenditures such as materials, labor, reasonable amounts of overhead, and interest cost during the construction period.

Included in the cost of land are expenditures such as search cost, real estate commissions, title transfer fees, back property taxes paid, surveying, and landscaping costs.

A sample entry to record the capitalized expenditures (assuming they were made in cash) is:

	<i>Debit</i>	<i>Credit</i>
Land	\$1,000,000	
Building	\$20,000,000	
Equipment	\$3,000,000	
Cash		\$24,000,000

**Depreciation** is used to allocate the cost of an asset over a period of time. *Depreciation expense* is the amount of this allocation for a given period.

Land is not depreciated. Land remains valued at its original cost.

Plant and equipment, however, have limited lives due to wear-and-tear or obsolescence. Because of this, plant and equipment costs must be allocated to expense over the plant and equipment's estimated economic life. There are several different methods of depreciation.

The **straight-line method** of depreciation allocates the depreciable cost of an asset evenly over the asset's estimated useful economic life. The following is an example of the straight-line method of calculating depreciation.

- A machine has a historical cost of \$12,000.
- The estimated useful life is ten years.
- After ten years, the machine will have an estimated salvage value of \$2,000.
- Cost less salvage value equals the depreciable value ( $\$12,000 - \$2,000 = \$10,000$ ).

## Financial Reporting and Analysis

The straight-line method of depreciation results in equal depreciation expenses each year over the equipment's 10-year life:

$$\begin{aligned}\text{straight-line depreciation} &= \frac{\text{cost} - \text{salvage value}}{\text{useful life}} = \frac{\$12,000 - 2,000}{10} \\ &= \$1,000 \text{ per year}\end{aligned}$$

Useful life of an asset is an accounting estimate by management. Let's assume that after three years of use (accumulated depreciation is \$3,000), management determines that the machine can only be used for two more years. To revise the depreciation schedule, the net book value of the machine (\$12,000 - \$3,000 = \$9,000) less the salvage value of \$2,000 will be depreciated over the remaining two years of useful life:

$$\begin{aligned}\text{straight-line depreciation} &= \frac{\text{net book value} - \text{salvage value}}{\text{useful life}} \\ &= \frac{\$9,000 - 2,000}{2} = \$3,500 \text{ per year}\end{aligned}$$

The **units-of-production method** allocates the depreciable cost of the asset as a function of the asset's use rather than time. The following is an example of the units-of-production method of calculating depreciation.

- A truck costs \$920,000.
- It has an estimated life of 300,000 miles.
- Salvage value is \$20,000.
- Cost of the truck per mile driven is:

$$\begin{aligned}\text{depreciation} &= \frac{\text{cost} - \text{salvage value}}{\text{estimated miles}} = \frac{\$920,000 - 20,000}{300,000} \\ &= \$3.00 \text{ per mile}\end{aligned}$$

If the truck is driven for 50,000 miles in year 1, the units-of-production depreciation expense is:

$$\begin{aligned}\text{depreciation} &= (\text{miles driven})(\text{depreciation per mile}) \\ &= (50,000 \text{ miles})(\$3.00 \text{ per mile}) = \$150,000\end{aligned}$$

Depletion of natural resources in a given period is determined using the units-of-production method.

**Example: Depletion**

Suppose a firm acquired mineral rights for \$1.5 million, and suppose it is estimated that the mineral deposits will produce 100,000 tons of ore. If 10,000 tons were extracted during the period, 1/10 of the cost is allocated to this period. The entry to record the depletion is:

	<i>Debit</i>	<i>Credit</i>
Depletion expense	\$150,000	
Accumulated depletion		\$150,000

**Accelerated depreciation** speeds up the recognition of depreciation expense in a systematic way so that more depreciation expense is recognized in the earlier years of the asset's life and less in the later years. Total depreciation expense over the life of the asset, however, will be the same as with straight-line depreciation. Accelerated depreciation is appropriate for assets that lose much of their economic value early in their lives, such as movie DVDs for rental.

**Declining balance (DB)** is a method that requires applying a constant rate to a declining book value. The most common declining balance method is the *double declining balance* method (DDB), which applies 200% of the straight-line rate to the declining balance. If an asset's life is 10 years, the straight-line rate is 1/10, or 10%. The DDB rate for this asset is 2/10, or 20%.

$$\text{DDB depreciation} = \frac{2}{\text{useful life}} (\text{cost} - \text{accumulated depreciation})$$

Declining balance depreciation does not explicitly use the salvage value in calculations, but depreciation is halted when the book value has been depreciated to the salvage value.

The following is an example of declining balance depreciation.

- A machine is purchased for \$12,000 on January 1 of year 1.
- The estimated useful life is five years.
- Estimated salvage value is \$2,000.

The depreciation expense using the double declining balance method is:

- Year 1:  $(2 / 5)(\$12,000 - 0) = \$4,800.00$ .
- Year 2:  $(2 / 5)(\$12,000 - \$4,800) = \$2,880.00$ .
- Year 3:  $(2 / 5)(\$12,000 - \$7,680) = \$1,728.00$ .

## Financial Reporting and Analysis

In years 1 through 3, the company has recognized cumulative depreciation expense of \$9,408. Since the total depreciation expense is \$10,000 (\$12,000 – \$2,000 salvage value), depreciation in year 4 is limited to \$592, rather than the  $(2 / 5)$   $(\$12,000 - \$9,408) = \$1,036.80$  using the DDB formula.

Year 5 depreciation expense is \$0 since the asset is fully depreciated to salvage value.

*Note:* The rate of depreciation is doubled (2/5) from straight-line, and the only thing that changes from year to year is the previous depreciation expense taken out. Declining balance depreciation can be based on double, 1.5, triple, or any other appropriate factor.

Assets that are worn out or no longer useful may be discarded or sold. When an asset is sold or discarded, its market value at the time of sale or disposal might be different from its book value. The book value of an asset is equal to its original historical cost minus accumulated depreciation.

- Discarded assets are simply disposed of, and the firm receives nothing in return. The market value for the asset is zero. If the asset has been depreciated to zero before being discarded, no gain or loss is recorded. However, if the asset has any remaining book value at the time of disposal, the book value amount is realized as a loss on the income statement. An asset that is disposed of is *derecognized* and removed from the balance sheet.
- When an asset is sold, the firm receives a cash payment in exchange for the asset. The asset's book value is compared to the sale price, and any difference is recognized as a gain or loss on the income statement.

**Intangible assets** have no physical existence, but legal rights confer benefits to the asset's owner. Intangible assets are distinguished from other assets that are classified as current assets (e.g., receivables) because intangibles are investments that are used in operations.

Examples of intangible assets include the following:

- Trademarks or brand names.
- Copyrights.
- Patents.
- Licenses or franchises.
- Leaseholds or leasehold improvements.
- Technology.
- Non-compete covenants.

Typically, intangible assets are only recorded on the balance sheet when they are purchased from another firm. Most costs for developing intangible assets internally are expensed as incurred.

When a company acquires an intangible asset (e.g., buys a patent), an asset is created by debiting the asset account for the acquisition cost. The cost of intangible assets is allocated over the estimated life of the asset. This allocation process is referred to as *amortization*. Amortization of intangibles uses the straight-line method.

**Goodwill** is an intangible asset created when a firm purchasing another business pays more than the fair market value of the business' assets if they were purchased individually. If the excess purchase price cannot be attributed to patents, brands, copyrights, or other intangible assets, it is recorded as goodwill. Goodwill reflects the factors that enable a company to earn an above-average rate of return, such as strong management, manufacturing efficiency, and customer approval.

Goodwill is assumed to have an indefinite life. Goodwill is not amortized, but is subject to an annual impairment review. Each year, a company must calculate the fair market value of its goodwill. If the fair market value is less than the carrying value on the balance sheet, the goodwill is said to be impaired. If impairment occurs, the carrying value of the goodwill account is reduced to its fair market value, and an impairment loss is recorded on the income statement.

## LIABILITIES

*Liabilities* are probable future payments of assets (usually cash) or services (in the case of prepaid revenue) that a firm is obligated to make as a result of previous operations. *Current liabilities* are obligations that the company expects to pay within one year or one operating cycle. *Long-term liabilities* are obligations that will be paid after the current year or operating cycle.

### Warranties

When a company sells products covered under **warranty**, future warranty expense associated with those sales must be estimated to achieve a matching of expenses with the revenue they generate. The amount of the future possible liability is estimated, most often, from past experience. This amount must consider the extent of the warranty—what it covers and the length of time covered. The *estimated warranty liability* (a.k.a. *estimated warranty payable*) is created upon the sale of the asset (debit warranty expense, credit liability).

Assume a toaster manufacturer sells 1,000 toasters, each with a \$10 warranty liability. The recognition and payment of warranties are noted in the following ways.



## Financial Reporting and Analysis

During period of sale:

	<i>Debit</i>	<i>Credit</i>
Warranty expense	\$10,000	
Estimated warranty payable		\$10,000

When repairs are made under warranty (assume 100 toasters at \$10 each):

	<i>Debit</i>	<i>Credit</i>
Estimated warranty payable	\$1,000	
Parts inventory		\$400
Cash		\$600

## Long-Term Debt

**Long-term debt** represents an obligation to repay a borrowed amount, plus interest, over a period greater than one year. All long-term liabilities, such as bonds, are recorded as the present value of future cash flows.

A bond's issue price does not always equal its par value. When bonds are issued at a price greater than par, the bonds are said to be issued at a **premium**. When bonds are issued at a price less than par, the bonds are said to be issued at a **discount**.

- A premium or discount arises because the *stated interest rate* (a.k.a. *coupon rate*) on the bonds is above or below, respectively, the *market rate of interest*.
- The accounting entry for bonds issued at a premium requires crediting the unamortized bond premium account. Suppose the \$1 million par value bond is issued at a premium of \$100,000. The entry is:

	<i>Debit</i>	<i>Credit</i>
Cash	\$1,100,000	
Unamortized bond premium		\$100,000
Bonds payable		\$1,000,000

- The accounting entry for bonds issued at a discount requires debiting the unamortized bond discount account. Suppose the \$1 million par value bonds are issued at a discount of \$100,000. The entry is:

	<i>Debit</i>	<i>Credit</i>
Cash	\$900,000	
Unamortized bond discount	\$100,000	
Bonds payable		\$1,000,000

*Total interest cost* consists of the total amount of coupon interest paid to bondholders over the life of the bond and any premium or discount that exists when the bonds are issued. Total interest cost can be calculated by subtracting the amount received by the firm when the bonds are issued from the total amount paid out to bondholders over the bond's life.

The total interest cost for a par bond is simply the amount of interest paid over the life of the bond. Suppose a company issues \$1,000,000 worth of bonds with a coupon rate of 6% and a maturity of five years. The total interest cost is calculated as:

$$\begin{aligned}
 \text{total amount paid to bondholders} &= \text{par value} + \text{interest payments} \\
 &= \$1,000,000 + (\$1,000,000 \times 0.06 \times 5) \\
 &= \$1,000,000 + 300,000 = \$1,300,000
 \end{aligned}$$

$$\text{total amount received at issuance} = \text{par value} = \$1,000,000$$

$$\begin{aligned}
 \text{total interest cost} &= \text{total amount paid} - \text{total amount received} \\
 &= \$1,300,000 - \$1,000,000 = \$300,000
 \end{aligned}$$

The total interest cost for a bond issued at a discount is equal to the amount of interest paid over the life of the bond *plus* the amount of the discount. This is because the discount effectively raises the (lower-than-market) interest rate the firm is paying on the bond. Suppose a company issues \$1,000,000 of bonds with a coupon rate of 6% and a maturity of five years, while the market rate of interest is 7%. Based on the 7% market rate, the bonds would sell for \$958,417.

$$\begin{aligned}
 \text{total amount paid to bondholders} &= \text{par value} + \text{interest payments} \\
 &= \$1,000,000 + (\$1,000,000 \times 0.06 \times 5) \\
 &= \$1,000,000 + 300,000 = \$1,300,000
 \end{aligned}$$

## Financial Reporting and Analysis

$$\begin{aligned}\text{total amount received at issuance} &= \text{par value} - \text{discount} \\ &= \$1,000,000 - 41,583 = \$958,417\end{aligned}$$

$$\begin{aligned}\text{total interest cost} &= \text{total amount paid} - \text{total amount received} \\ &= \$1,300,000 - \$958,417 = \$341,583\end{aligned}$$

The total interest cost for a bond issued at a premium is equal to the amount of interest paid over the life of the bond minus the amount of the premium. This is because the premium effectively lowers the (higher-than-market) interest rate the firm is paying on the bond. Suppose a company issues \$1,000,000 of bonds with a coupon rate of 6% and a maturity of five years, while the market rate of interest is 5%. Based on the 5% rate, the bonds would sell for \$1,043,760.

$$\begin{aligned}\text{total amount paid to bondholders} &= \text{par value} + \text{interest payments} \\ &= \$1,000,000 + (\$1,000,000 \times 0.06 \times 5) \\ &= \$1,000,000 + 300,000 = \$1,300,000\end{aligned}$$

$$\begin{aligned}\text{total amount received at issuance} &= \text{par value} + \text{premium} \\ &= \$1,000,000 + 43,760 = \$1,043,760\end{aligned}$$

$$\begin{aligned}\text{total interest cost} &= \text{total amount paid} - \text{total amount received} \\ &= \$1,300,000 - \$1,043,760 = \$256,240\end{aligned}$$

For bonds issued at a price above (i.e., at a premium) or below (i.e., at a discount) face value, the difference between the par value and the issue value must be *amortized* over the life of the bond as adjustment of interest expense in the income statement.

As a result of amortization, the carrying value of a bond issued at a premium or discount on a company's balance sheet will be equal to par on the bond's maturity date.

A premium or discount can be amortized either using the *straight-line method* (equal amount each period) or the *effective interest method* (the difference between the interest paid and the *effective interest*; effective interest is the *market interest rate that existed at the time the bonds were issued* multiplied by the carrying value of the bond).

**Example:**

Consider a bond issue of \$1,000,000 of 10-year bonds that have a coupon rate of 10% but were issued to yield 9%. Interest is paid semiannually. In this case, the premium is \$65,039. Today's bond price is \$1,065,039.

**Answer:**

Initially, the bond issue is recorded as:

	<i>Debit</i>	<i>Credit</i>
Cash	\$1,065,039	
Unamortized bond discount		\$65,039
Bonds payable		\$1,000,000

Using straight-line amortization, the entry for the first semiannual interest payment is:

	<i>Debit</i>	<i>Credit</i>
Bond interest expense	\$46,748	
Unamortized bond premium	\$3,252	
Cash (semiannual coupon payment)		\$50,000

Recognizing that there are 20 interest payments, the amortized amount of the bond premium =  $\$65,039 / 20 = \$3,252$ . The bond interest expense is computed as a plug figure (i.e.,  $50,000 - 3,252 = 46,748$ ).

Using the effective interest method, the interest expense at the end of the first period is:

$$\text{interest expense} = \$1,065,039(0.09)(0.5) = \$47,927$$

The amortized amount of the bond premium is the difference between the \$50,000 and the \$47,927, or \$2,073:

	<i>Debit</i>	<i>Credit</i>
Bond interest expense	\$47,927	
Unamortized bond premium	\$2,073	
Cash (semiannual coupon payment)		\$50,000

## Financial Reporting and Analysis

Non-interest bearing debt (i.e., *zero-coupon bonds*) should be initially recorded at discounted present value, using a discount rate equal to the company's normal borrowing rate. The issuer will recognize the interest expense for each period using the effective interest method, applying the discount rate to the book value of debt at the beginning of the period.

When a bond reaches its maturity date, the final interest payment is made and the principal is paid in full. A key point is that at maturity, the market value of the bond will equal its *book value*.

If a bond is retired prior to maturity, any difference between the market and book value of the bond is treated as a *non-recurring item*, and is shown on the income statement.

## Leases

A **lease** is an agreement that allows the *lessee* to use, for a limited time, a long-lived asset owned by the *lessor*. A lease must be classified as either an **operating lease** or a **capital (finance) lease**. A capital lease is a long-term liability. Operating leases are often used in an attempt to keep the liability off the balance sheet.

Under IFRS, a lease is classified as a finance lease if the rights and risks of ownership are substantially transferred to the lessee. Under U.S. GAAP, if a lease meets any of the following criteria, the lessee must classify it as a capital lease:

- Lease transfers ownership to property by end of lease.
- Lease contains bargain purchase option.
- Lease term is 75% or more of estimated economic life of the property.
- Present value of minimum lease payments at beginning of lease equals or exceeds 90% of the fair market value of property.

Operating leases are accounted for as rental agreements. No liability appears on the lessee's balance sheet.

- Rental payments are recognized as an expense by the lessee and as income by the lessor.
- Disclosures of cash payments due under operating leases for each of the next five years and in aggregate are required in the notes to financial statements.

In the case of a capital lease, the lessee treats the lease as if it were a purchase of property.

- An asset and a related long-term liability are recorded at the present value of future minimum lease payments.

- The value of the asset is allocated to expense in the income statement, on a straight-line basis, over the term of the lease.
- Each lease payment is treated as part interest expense and part payment of principal.

## Pensions

A **pension plan** is an agreement under which an employer agrees to pay monetary benefits to employees once their period of active service ends. The benefits normally depend on certain requirements, such as age and number of years of service.

A *pension fund* is an intermediary used by the employer to meet plan obligations. The employer makes payments to the fund. The fund makes investments and makes pension payments to employees. Most pension plans in the U.S. are funded because of Employee Retirement Income Security Act (ERISA) requirements and tax advantages.

A *defined contribution plan* is a pension plan that requires the employer to make only a specified contribution into the employee's retirement account. The contributions may be fixed or variable (profit-sharing). There is no promise of any specific level of future benefits, and *employees bear all the risk* of investment performance.

- Accounting for defined contribution plans is very straightforward. Pension cost equals the contributions made, and the employer will report an asset or liability reflecting the difference between actual payments made and the required payments.

With a *defined benefit plan*, the employer promises a specified monetary benefit upon retirement. The promised benefits may be fixed or pay-related. By promising a defined future benefit, the employer bears all the risk of investment performance.

- Determining the pension expense for defined benefit plans is more complicated than for defined contribution plans. Accounting for defined benefit plans is an important topic at Level II in the CFA program.

## OWNERS' EQUITY

**Contributed equity capital** consists of the par value (if applicable) of common stock, the par value of preferred stock, and paid-in capital in excess of par value.

- The *par value* of common stock is an amount that is arbitrarily set by management. The product of the par value per share and the number of shares issued represents the legal capital of the corporation. Par value is unrelated to the market value of a share of stock.

## Financial Reporting and Analysis

- *Paid-in-capital in excess of par* is the difference between what the corporation initially sold the shares for and the par value (if applicable).
- If the stock does not have a par value, the amount the stock is initially sold for is recorded simply as *common stock*.

**Preferred stock** (or **preference shares**) is equity ownership that has seniority (i.e., preference) over common stock with respect to claims on income and assets.

- Preferred stock dividends are usually a fixed amount, stated as either a fixed amount per share or a fixed percentage of the stock's par value.
- In the case of *cumulative preferred stock*, any dividends not paid when scheduled must be paid before any other preferred or common dividends can be paid. Dividends not paid when due are referred to as dividends in arrears.
- In the case of *noncumulative preferred stock*, the corporation is not obligated to pay any dividends not paid when scheduled.
- *Convertible preferred stock* may be exchanged into common stock at a stated rate. This is an option the investors have if they own the stock.
- *Callable preferred stock* may be bought back by the corporation at a specified price per share. This is an option that the corporation has if it issues callable preferred stock.

**Stock issuance.** If common stock has a par value, the issuance of the shares results in increasing both the common stock and paid-in capital accounts. For example, if a company issues 100,000 shares of \$1 par value stock for \$25 per share:

	<i>Debit</i>	<i>Credit</i>
Cash	\$2,500,000	
Common stock		\$100,000
Paid-in capital in excess of par value		\$2,400,000

If common stock has no par value, the issuance of the shares results in the common stock account. For example, if the company issues 100,000 shares of no par stock for \$25 per share:

	<i>Debit</i>	<i>Credit</i>
Cash	\$2,500,000	
Common stock		\$2,500,000

**Treasury stock.** Outstanding stock may be repurchased by the issuer and either held for future use (e.g., employee stock options) or retired.

Stock reacquired and held for future use is referred to as treasury stock. The purchase of treasury stock results in a debit to treasury stock and is typically recorded at cost. The treasury stock account is reported within contributed capital

as a reduction in capital. Suppose the company purchases 100,000 shares at \$50 each. The entry is:

	<i>Debit</i>	<i>Credit</i>
Treasury stock, common	\$5,000,000	
Cash		\$5,000,000

## FINANCIAL REPORTING CONCEPTS IN ACTION

Let's illustrate the basic function of financial statements with a simple example. Henry Hills has decided to open a book store in his home town and has decided to name the store "Hills of Books." The first step is for Henry to open a bank account. He takes \$50,000 of his own money and deposits the money in a new account in the name of Hills of Books. At this point, the balance sheet for Hills of Books would show the bank balance as an asset called cash. The offsetting entry would reflect the source of the cash asset. In this case, the source of funds was a contribution of capital by the owner and will be recorded as Owner's Equity. The entry is:

	<i>Debit</i>	<i>Credit</i>
Cash	\$50,000	
Capital contributions		\$50,000

Henry has arranged to open his store for business on January 1, 20X8, and has made several decisions regarding its day-to-day operations during November and December of 20X7.

- After locating vacant space in a local shopping mall, Henry signed a 2-year lease. At a rate of \$1,500 per month, the landlord required advance payment of the first three months' rent and that he also submit a deposit of one month's rent that will be refunded upon termination of the lease. Initially, all rent paid in advance will be treated as prepaid rent (an asset). As time passes and the rent expires the asset, prepaid rent, will be written off to rent expense. The deposit will also be recorded as an asset. The source for these payments is the cash account, which is reduced by  $(1,500 \times 3) + 1,500 = \$6,000$ . The entry to record this transaction is as follows:

	<i>Debit</i>	<i>Credit</i>
Deposits—rent	\$1,500	
Prepaid rent	\$4,500	
Cash		\$6,000



## Financial Reporting and Analysis

- Henry ordered books from a wholesale book distributor, at a cost of \$100,000. The distributor requires 40% of the total to be paid on delivery, another 30% paid within 30 days, and the remaining 30% within 60 days. The books will be Hills of Books' inventory. The amount paid on delivery ( $0.40 \times \$100,000 = \$40,000$ ) will reduce cash. The remaining balance ( $0.60 \times \$100,000 = \$60,000$ ) represents accounts payable.

	<i>Debit</i>	<i>Credit</i>
Inventory	\$100,000	
Cash		\$40,000
Accounts payable		\$60,000

- In order to allow customers to browse books before making a purchase, Henry bought shelves and some easy chairs. Combined with the cost of a cash register and various fixtures, the total Furniture and Fixtures outlay amounted to \$20,000. This cost will be recorded as Furniture and Fixtures, a long-term asset, and will reduce cash. Henry plans to depreciate these assets using the straight-line method of depreciation, assuming a four year useful life and no salvage value.

	<i>Debit</i>	<i>Credit</i>
Furniture & fixtures	\$20,000	
Cash		\$20,000

- Henry then hired a salesperson to help out so that he would not have to be at the store during all operating hours. The employee will work part-time, at a rate of \$1,200 per month. There is no accounting entry for the new employee until wages are paid.
- After arranging for a loan from the local bank, the business borrowed \$40,000. Henry signed a note payable that requires an annual payment of \$10,000 at the end of each of the next four years, plus annual interest of 8%, payable quarterly. The initial entry for this loan is (remember that the portion of debt due in one year is separated out and treated as a current liability):

	<i>Debit</i>	<i>Credit</i>
Cash	\$40,000	
Notes payable—current		\$10,000
Notes payable		\$30,000

Although the step-by-step process of accounting has been abbreviated in this example, the end result is the same. The balance sheet below shows the result of Hills' transactions prior to year-end. You can verify balances, such as the balance in cash, by totaling the debit (left) entries, totaling the credit (right) entries, and

taking the difference. Outside of cash, most accounts had only one change to them during the accounting period, so their ending balances are quite simple to determine.

Hills of Books  
Balance Sheet  
December 31, 20X7

Current assets		Current liabilities	
Cash	\$24,000	Accounts payable	\$60,000
Accounts receivable		Notes payable—current	10,000
Inventory	100,000		
Prepaid expenses	4,500		
Total current assets	\$128,500	Total current liabilities	\$70,000
Furniture & fixtures	20,000	Notes payable—long-term	30,000
Less: accumulated depreciation	0	<b>Total liabilities</b>	<b>\$100,000</b>
Net fixed assets	20,000		
		Capital contributions	50,000
Deposits—rent	1,500	Retained earnings	
		<b>Total equity</b>	<b>50,000</b>
<b>Total assets</b>	<b>\$150,000</b>	<b>Total liabilities &amp; equity</b>	<b>\$150,000</b>

Important points to remember about the balance sheet include the following:

- It is a snapshot of the business at one point in time.
- It shows all assets (what you have), liabilities, and equity (who has claims to what you have).
- Assets and liabilities are separated into short-term and long-term categories. Current assets are those that are expected to be consumed, sold, or converted to cash in less than one year. Current liabilities are expected to be paid within one year (e.g., accounts payable). Long-term assets and long-term liabilities have lives greater than one year.

We will look at the balance sheet for Hills of Books at the end of January, after one month of operation. Many of the changes over that time period will occur as a result of the revenues and expenses incurred in the day-to-day operation of the store. Those transactions are captured in the income statement.

Let's return to our example for Hills of Books. In January, Henry opened his store as planned. He bought supplies for his store (shopping bags, printed book marks to give to customers, gift wrap, plus some basic office supplies) at a cost of \$500.

## Financial Reporting and Analysis

Henry also paid \$3,100 for advertising in local newspapers and on a local radio station. The advertising paid off, as Hills of Books generated \$24,000 in sales revenue from selling books. The books sold cost Hills \$16,000. On January 25, Henry bought a sophisticated cash register that had inventory tracking capabilities for \$2,500. At the end of the month, Henry paid his employee the agreed upon \$1,200. Also at the end of the month, Henry became concerned about the dwindling cash balance, and contributed another \$5,000 of his own money to the business.

The income statement for January is shown in the following table:

Hills of Books  
Income Statement  
Month ended January 31, 20X8

Sales		\$ 24,000
Less: Cost of Goods Sold		(16,000)
Gross Profit		<u>\$ 8,000</u>
Selling, Gen. & Admin. Expenses		
Advertising	\$ 3,100	
Depreciation	417	
Rent	1,500	
Supplies	500	
Telephone	150	
Utilities	350	
Wages	<u>1,200</u>	
Total SG&A		(7,217)
Income From Operations		\$ 783
Less: Interest Expense		(267)
Net Income*		\$ 516

\*Since Hills of Books is a sole proprietorship, there will be no income tax for the business. Instead, any net profits are taxable as part of Henry's income on his personal income tax return.

The sales were mostly cash sales, but several customers established credit accounts. Their purchases will be billed to them at the end of the month. They are required to pay the balance due within 15 days of the date they are billed. The sales on credit for January totaled \$3,000.

The cost of goods sold literally represents the cost of the books Hills sold. The \$16,000 worth of inventory is removed from the inventory account at the time of sale and recorded as an expense. Therefore, on sales of \$24,000, Hills of Books earned a gross profit of \$8,000. Gross profit is what remains after covering the cost of the sale and is the amount available to pay operating expenses, interest expense, and hopefully allow some residual profit.

The SG&A expenses are the operating expenses required to keep the store up and running. Advertising falls into this category. It might be argued that advertising expenses should be allocated over multiple periods, since it may well be that customers will remember the advertisements for some time. However, the convention is to expense advertising costs immediately. For a large scale ad campaign that would take place over more than one accounting period, some capitalization across periods might be appropriate.

Depreciation is included as an SG&A expense because it applies to assets used for the day-to-day conduct of business (e.g., chairs, shelves). If this were a manufacturing company depreciating a machine that *produced* books, depreciation would probably belong in COGS. The calculation of depreciation in this case is the depreciable cost divided by four years, then divided by 12 to get a monthly amount ( $20,000 / 4 = 5,000$  per year;  $5,000 / 12 = \$417$  per month). Unless an asset is about to be disposed of, depreciation is generally only calculated once at year-end.

The rent expense was not really a cash outlay in January. Remember that the landlord required three months rent in advance. Rather than crediting the cash account on January 1, the credit would be to the prepaid rent (asset) account. Hence, as time went by, part of the asset expired and was written off.

The supplies, telephone, and utilities charges are just ordinary costs of doing business. Sometimes there will be a supplies account on the balance sheet, reflecting the fact that management believes those supplies will be used for several accounting periods, and possibly more than one year. In this case, given the nature of the supplies, it seems appropriate that they be expensed.

The wages expense is simplified for our example. In reality, there would be employment taxes to pay in addition to the wages, and payroll taxes to deduct from the employee's check. For simplicity, we have shown gross wages only.

The total SG&A expenses are deducted to derive *operating income*, or *EBIT*. As the name implies, this is net income remaining after deducting all operating expenses from the gross margin. Some firms would have a category below income from operations for other income and expense. This category would include non-operating income and expense items. For example, if Henry had opened an

## Financial Reporting and Analysis

interest-bearing account for his excess cash, the interest earned would be classified as other income.

Interest expense is also shown separately. The interest expense is not really an operating expense, since Henry could have elected to use 100% equity financing, and there would have been no interest expense. In this case, Henry will accrue one month's interest, even though it's not due until the end of the first quarter. We will make this adjustment for the sake of consistency in our example. The amount is calculated as  $\text{principal} \times \text{rate} \times \text{time}$ , or  $\$40,000 \times 0.08 \times 1/12 = \$267$ . Since the interest will not be paid until the end of the quarter, we will record an increase (credit) to interest payable rather than a decrease (debit) to cash.

The net income of \$516 is the amount taxable to Henry. Since Hills of Books is a sole proprietorship, this amount will be reported on Henry's personal tax return. However, it will only be the net profit at the end of the year that must be reported. There is no interim requirement for a monthly figure.

Notice that there were three transactions in January that do not appear in the income statement. The purchase of the new cash register is not an expense—this is an asset that should be capitalized. Henry will begin depreciating the cash register in February. Also, the additional capital contribution is not income. This contribution will merely increase the capital contributions equity account. Finally, the required payment to the inventory supplier does not appear in the income statement. The inventory is expensed (through COGS) only as it is sold. Don't forget the payment against accounts payable for 30% of the original \$100,000 purchase. We will now take another look at Hills of Books balance sheet to see how it has changed after one month of operation.

Hills of Books  
 Balance Sheet  
 January 31, 20X8

Current assets		Current liabilities	
Cash	\$ 12,200	Accounts payable	\$ 30,000
Accounts receivable	3,000	Current portion long-term debt	10,000
Inventory	84,000	Interest payable	267
Prepaid rent	3,000		
<b>Total current assets</b>	<b>\$ 102,200</b>	<b>Total current liabilities</b>	<b>\$ 40,267</b>
Furniture & fixtures	\$ 22,500	Long-term debt	30,000
Less: accumulated depreciation	(417)	Total liabilities	\$ 70,267
Net fixed assets	22,083		
		Capital contributions	55,000
Deposits—rent	1,500	Retained earnings	516
		<b>Total equity</b>	<b>55,516</b>
<b>Total assets</b>	<b>\$125,783</b>	<b>Total liabilities &amp; equity</b>	<b>\$ 125,783</b>

$$\text{Cash} = 24,000 + (24,000 - 3,000) - 3,100 - 500 - 150 - 350 - 1,200 - 2,500 - 30,000 + 5000 = 12,200$$

Let's take a look at some of the changes in the balance sheet. Cash has declined despite a good month of sales, largely due to the \$30,000 payment to the supplier. The \$3,000 balance in accounts receivable reflects the January sales on credit. Hopefully Hills of Books will be able to collect all of this balance. Most firms establish a contra-asset account called allowance for doubtful accounts to show that, unfortunately, not all customers will pay their bills. Over time a business can estimate this allowance fairly accurately as a percentage of credit sales. Of course, a firm can control this allowance to a certain extent by requiring customers to meet certain criteria before they are allowed to make purchases on credit.

The inventory decreased by \$16,000, the amount of the cost of goods sold. Prepaid rent also declined to reflect the expensing of January rent.

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Furniture and fixtures increased by the purchase of the cash register. January depreciation is also shown in the accumulated depreciation account. Depreciation is not a cash expense—the furniture and fixtures were already paid for. Depreciation is used to allocate the cost to expense over more than one accounting period, since the asset will be used and benefit more than one accounting period. The entry to record depreciation would be:

	<i>Debit</i>	<i>Credit</i>
Depreciation expense	417	
Accumulated depreciation		417

On the liability side, there are four changes: the reduction in accounts payable, the additional capital contribution, recorded interest payable, and the inclusion of current income in the retained earnings account. Current income must be included in retained earnings for the balance sheet to balance, since the profitability of the business increases assets as sales generate new cash and receivables, hopefully exceeding day-to-day costs. At the end of the year, current income must be added to the business' retained earnings in order to balance all the daily changes that have taken place in the rest of the balance sheet.

We mentioned above that some expenses do not require a current cash outlay, such as purchases on credit and depreciation expense. Furthermore, there were a number of cash payments/receipts that are not recorded as revenues and expenses. For example, borrowing from the bank and purchasing assets. Next we will look at Hills' statement of cash flows to see where Hills is generating and using cash. Remember, we would like to see a company that generates most of its cash internally, although that's not usually the case with a new company that's barely off the ground. Let's look.

Hills of Books  
Statement of Cash Flows  
Month ended January 31, 20X8

	Cash Flow
<b>Operating Activities:</b>	
Net income	\$ 516
Adjustments to NI:	
Depreciation add back	417
Reductions in prepaid items	1,500
Increase in interest payable	267
Decrease in accounts payable	(30,000)
Increase in accounts receivable	(3,000)
Decrease in inventory	16,000
Net cash from operations	\$ (14,300)
<b>Investing Activities:</b>	
Acquisition of assets	(2,500)
<b>Financing Activities:</b>	
Additional capital contribution	\$ 5,000
<b>Summary:</b>	
Net change in cash	\$ (11,800)
Cash beginning of January	24,000
Cash end of January	\$ 12,200

It seems confusing at first, when trying to decide how and why to adjust for all of the changes. Let's take a look back at the basic accounting equation for a fairly simple explanation. We saw earlier that the basic accounting equation is:

$$\text{assets} = \text{liabilities} + \text{owner's equity}$$

Since this is an equation, we also know that if there is a change on either side, there must be an equal and offsetting change on the other side. Therefore:

$$\Delta \text{assets} = \Delta \text{liabilities} + \Delta \text{owner's equity}$$



## Financial Reporting and Analysis

Furthermore, since we are interested in explaining the change in cash (where it came from and where it went) we must isolate cash on one side of the equation. We can do this by simply breaking cash down into two pieces: cash assets and non-cash assets. Therefore:

$$\Delta\text{cash} + \Delta\text{non-cash assets} = \Delta\text{liabilities} + \Delta\text{owner's equity}$$

and:

$$\Delta\text{cash} = \Delta\text{liabilities} + \Delta\text{owner's equity} - \Delta\text{non-cash assets}$$

Now let's use what we have to explain the statement of cash flows. The statement of cash flows presented above was prepared using the indirect method, starting with net income. Depreciation and any reductions in prepaid expenses are added back to net income because these expenses did not require any cash outlays during the current period. Next, changes in any current asset or current liability accounts are considered. Increases in current assets would indicate reductions in cash; for example, if accounts receivable increased, there were credit sales during the period, which are included in net income but which have not yet generated cash inflows. Similarly, decreases in current assets are sources of cash; a reduction of inventory means more goods were sold. A reduction in current liabilities is a use of cash, since cash was needed to pay them down. Note that all of these changes in current accounts arise out of daily operations: selling goods, collecting cash, paying bills.

What we've essentially done is convert an accrual-based income figure to one that is cash only. In Hills' case, cash flows from operating activities are not positive. Can you see why? In an effort to get the business up and running, Henry had to purchase the necessary inventory. More of the inventory has been paid for (use of cash) than has been sold (source of cash). Once Hills generates a steady source of business and chooses an optimal level of inventory, this trend should reverse.

To determine the true change in cash over the period, cash from operations must be adjusted for non-cash expenses, changes in working capital, and investing and financing cash flows. Non-cash expenses on the income statement include prepaid items and depreciation. They are expenses whose cash outflow takes place independently of the expense being incurred. Paying for the asset and using it are two different things.

Henry paid his landlord \$4,500 for three months' rent during December of the previous period. This resulted in a \$4,500 decrease in the cash account along with an entry for the prepaid item under current assets on the balance sheet. When Henry deducts the \$1,500 rent expense from January's operating income, the

deductions look like any other expense. Since this deduction reduces operating income but is not an actual cash flow, we must add the amount back to operating income to determine cash flow from operations for the month.

Generally speaking, if non-cash assets went up, they consumed cash in the process. On the contrary, if non-cash assets went down, they created cash in the process. Liabilities are just the opposite. Liabilities are only recorded when the company has done one of two things: they either borrowed cash (source) or incurred an expense that they have not yet paid. The increase from borrowing is obvious, but what you might have forgotten is what happens when a company incurs an expense that has not been paid for. The expense will be deducted as soon as it is incurred, but to show income from a cash flow perspective only, we must remove the change.

The statement of cash flows and the balance sheet must agree. The change in cash for the period as stated in the statement of cash flows plus the cash balance on the previous balance sheet must equal the cash balance on the current balance sheet.

Investing cash flows refer to cash transactions involving long-term assets. The purchase of the new cash register was an investing cash outflow. Since the purchase required a cash outlay, the amount on the statement of cash flows is a negative number.

Financing cash flows refer to transactions involving the capitalization of the business, regardless of whether the source of capital is debt or equity. In this case, Henry has contributed an additional \$5,000 of equity capital. This amount was a cash inflow and is therefore a positive number on the cash flow statement.

The summary line in the statement of cash flows shows a net change in cash for the month of  $-\$11,800$ , which brings our current cash balance to  $\$12,200$ . (This is confirmed on the balance sheet).

The statement of owners' equity shows the value of any assets the owner has invested in the business. On December 31, 20X7, Hills of Books had assets totaling  $\$150,000$ . Since Henry owed the bank  $\$40,000$  and his suppliers  $\$60,000$ , the business does not own all of the assets "free and clear." After deducting the  $\$100,000$  owed to others, Henry's equity totals  $\$50,000$  (his initial investment).

This is similar to the situation facing most homeowners. Consider a family who purchased a home last year for  $\$90,000$ . To make the purchase, the homeowners invested  $\$20,000$  of their own money and took out a mortgage for  $\$70,000$ . The house is currently worth  $\$100,000$ . The amount of the mortgage has not changed (ignoring whatever principal payments have been made). Thus, they now have an asset worth  $\$100,000$ , and a liability of  $\$70,000$ . This leaves them  $\$30,000$  of equity (assets minus liabilities) in the home. Notice that there is not necessarily any

## Financial Reporting and Analysis

relationship between the value of equity and the amount of cash on hand. Hills of Books' statement of owners' equity at the end of January is shown in the following table:

Hills of Books  
Statement of Owner's Equity  
January 31, 20X8

<b>Owner's Equity 1/1/20X8</b>	\$ 50,000
<b>Additions:</b>	
January net income	\$ 516
Additional capital contribution	\$ 5,000
<b>Withdrawals:</b>	
None	\$0
<b>Owner's Equity 1/31/20X8</b>	\$ 55,516

Net income has been transferred or "closed" to owners' equity. As a proprietorship, net income for Hills of Books belongs to Henry, but Henry chose not to withdraw any of it during January. Had he done so, the cash and owners' equity accounts would be reduced to reflect the withdrawal.

Let's consider our homeowners again. If they still have a \$70,000 outstanding mortgage and the value of their house drops below \$100,000, their equity in the home will also fall. In the same fashion, if Hills of Books shows an operating loss (negative net income), Henry's equity in the business declines and could even fall below his initial investment. To illustrate, assume a \$3,500 loss for the month of February, along with an additional contribution of \$2,000. The combined impact of this situation is shown in Hills of Books' February 28 statement of owners' equity:

Hills of Books  
Statement of Owner's Equity  
February 28, 20X8

<b>Owner's Equity 2/1/20X8</b>	\$ 55,516
<b>Additions:</b>	
February net income	\$ (3,500)
Additional capital contribution	\$ 2,000
<b>Withdrawals:</b>	
None	\$0
<b>Owner's Equity 2/28/20X8</b>	\$ 54,016

Note that even though Henry has invested a total of \$57,000 (\$50,000 initially plus \$5,000 in January and \$2,000 in February), his equity in the business is only \$54,016 due to the \$3,500 operating loss for the month of February. However, if the business shows a profit of \$1,200 in the month of March, the end-of-March statement of owners' equity (in the following table) would indicate the partial recovery of his lost equity. In this fashion, the value of owner's equity is subject to change from month to month.

Hills of Books  
Statement of Owner's Equity  
March 31, 20X8

<b>Owner's Equity 3/1/20X8</b>	\$ 54,016
<b>Additions:</b>	
March net income	\$ 1,200
Additional capital contribution	\$0
<b>Withdrawals:</b>	
None	\$0
<b>Owner's Equity 3/31/20X8</b>	<b>\$ 55,216</b>

A point worth remembering is the relationship between successive statements. Notice that the ending balance on the January 31 statement of owners' equity becomes the beginning balance for the next month's statement. Similarly, the ending balance on the February statement becomes the beginning balance on the March statement.

Although somewhat more complicated, the statement of owners' equity for a *publicly traded corporation* is similar to that for Hills of Books in many respects. Let's look at a statement of owners' equity for Hills of Books assuming it "goes public." You will probably notice first that *owners'* equity is now called *stockholders'* equity. This is because there are now many owners of the business, each of whom has purchased shares of stock on an organized exchange.

Let's move forward in time and assume that Hills of Books has been highly successful and has expanded to open several additional stores. Henry has incorporated the business and has invested \$300,000 of his own capital (this represents his cumulative investment to date). He currently holds 30,000 shares of common stock but has decided to issue 25,000 new shares at \$20 per share. The par value of Hills of Books stock is \$1.00 per share. Therefore, the sales price of \$20 will be allocated \$1.00 per share to common stock (par value) and \$19.00 per share to additional paid-in capital. Hills of books issued the new stock in January

## Financial Reporting and Analysis

20X1 and paid a \$0.50 per share dividend in December 20X1. The statement of stockholder's equity for 20X1 would be as follows.

Hills of Books  
Statement of Stockholder's Equity  
December 31, 20X1

<b>Stockholder's Equity 1/1/11</b>	\$ 378,622
<b>Additions:</b>	
Issued new stock at par	\$ 25,000
Additional paid-in capital	\$ 475,000
2008 net income	\$ 44,917
<b>Withdrawals:</b>	
Dividends paid	\$ 27,500
<b>Stockholder's Equity 12/31/11</b>	\$ 951,039

The beginning balance 1/1/20X1 reflects Henry's contributed capital of \$300,000, plus cumulative retained earnings (i.e., several years' net income) of \$78,622. The new stock issue (25,000 shares) is allocated to par value and additional paid-in capital. The 2011 net income is \$44,917, and the only withdrawal was the dividends paid to all shareholders—Henry with 30,000 shares plus the 25,000 new shares. The total dividend amount was therefore 55,000 shares @ 0.50 each, or \$27,500. Dividends reduce the equity account because they are a payment made to providers of equity capital. Dividends are not deducted in deriving net income, so they must be deducted separately in the equity section of the balance sheet. Dividends require a cash outlay and are included in the financing section of the statement of cash flows, as we saw earlier.

It is important to remember that these numbers reflect only the historical book value, which is the amount received at issuance of the stock. The market price of the stock will not be reflected in the Hills of Books financial statements. The actual value in the market place on December 31, 20X1 could have been \$5 per share or \$50 per share, and the Stockholder's equity numbers would still be the same. Also, just as with our discussion of owner's equity above, the equity balance provides no indication of how Hills of Books has used the money raised through equity sources—the money might be in fixed assets, inventory, accounts receivable, or even cash.

## SUMMARY

### PRINCIPLES AND PROCEDURES

- A. Accounting standards help assure consistency and comparability of accounting procedures.
- B. Financial statements in many countries are prepared in accordance with International Financial Reporting Standards (IFRS). Financial statements in the United States are created under Generally Accepted Accounting Principles (U.S. GAAP).
- C. Financial statements are prepared at regular, predetermined times.
- D. In the double entry accounting system, every debit has a corresponding credit.
  - 1. Debits increase asset, expense, loss, and dividend accounts. Debits decrease liability, equity, revenue, and gain accounts.
  - 2. Credits increase liability, equity, revenue, and gain accounts. Credits decrease asset, expense, loss, and dividend accounts.
  - 3. The basic accounting equation is as follows:  $\text{assets} = \text{liabilities} + \text{owners' equity}$ . The equation, with its two equal sides, is the underlying reason for the use of double entry accounting, wherein two equal changes (at least) must take place in an entry in order for the equation to maintain balance. The equation is also the underlying format of the balance sheet.
- E. Under accrual basis accounting, revenues are recorded when earned and expensed when incurred, regardless of the timing of their related cash flows. The matching principle requires that revenues and the expenses related to or incurred for the production of those revenues be recorded in the same accounting period. These two basic principles facilitate the most meaningful measure of profitability.

### FOUR PRIMARY FINANCIAL STATEMENTS

- A. The balance sheet is also known as the statement of financial position. It includes asset, liability, and equity accounts, which represent both the resources owned by the firm as well as claims to those resources as of a specific point in time—usually December 31 or whichever date represents the company's year end.
  - 1. Assets are those resources owned and/or used by the firm that bring a current or future benefit. Assets may be contributed to the business by the owner(s), borrowed, or generated by the business. They can be current (short-term) or fixed (long-term).
    - a. Current assets are those that will be used up, sold, or converted into cash within one year or the operating cycle, whichever is longer. They include cash, marketable securities, accounts receivable, inventory, supplies, and prepaid items.

## Financial Reporting and Analysis

- b. Long-lived assets are those with lives greater than one year and include property, plant, and equipment, long-term investments, natural resources, and intangibles.
    2. Liabilities are external claims on the firm's assets by lenders and creditors. They can be current (short-term) or long-term.
      - a. Current or short-term liabilities are payable within one year. These include wages payable, accounts payable, short-term notes payable, and the current portion of long-term debt.
      - b. Long-term liabilities are those that extend beyond one year. These include long-term notes payable, mortgages payable, leases, and bonds payable.
    3. Owners' equity represents the total value of the ownership claim(s) of the proprietor, partners, or stockholders. It represents what remains of assets (residual) after all debts are satisfied. Equity can be created in one of two ways.
      - a. Contributed capital represents claims to assets that were literally contributed by the owners of the company, whether by private capital contributions or public stock issues in exchange for assets.
      - b. Retained earnings represents the sum of all current and prior period earnings that have not been returned to the owners by way of dividend distributions. Retained earnings differs from current-year earnings (net income) in that it is a multi-period cumulative figure.
- B. The income statement measures the results of operations for a period of time, usually one year. Profitability is determined by taking the difference between revenues and gains and expenses and losses.
  1. Revenues represent an "inflow," or creation of new assets, that is typically created through the day-to-day operations of the business.
  2. Gains result from the disposal of long-term assets for an amount greater than their book or carrying value on the balance sheet (i.e., you received more than the adjusted cost basis).
  3. Expenses result from the consumption of assets. When the amount of assets created during the period (revenues) is greater than the amount of assets consumed (expenses) the difference is a positive net income or operating profit figure.
    - a. Cost of goods sold is an expense associated with the sale of inventory. When the inventory is sold, its cost is removed from the balance sheet and written off on the income statement against the revenue created in the sale (the matching principle).
    - b. Depreciation, depletion, and amortization are the systematic allocation of the cost of long-term assets to expense over the period of time benefited by the assets. Rather than removing long-term assets from the books as they are expensed, the total of depreciation taken is recorded as a credit to an account called accumulated depreciation. Accumulated depreciation is a contra-asset account

that is netted against long-term assets to create an indirect reduction in their book values.

- c. The cost vs. expense issue arises when costs are incurred: Do you record an asset or expense? If the cost results in an asset, the cost should be capitalized, or recorded as an asset. If the cost does not result in the creation of an asset, it should be expensed immediately.
  4. “Below the line” items are reported separately from net income from operations, and include gains and losses not related to normal ongoing operations of the firm. Gains or losses on discontinued operations are reported here, as are extraordinary items and effects of any changes in accounting methods.
- C. The statement of changes in equity provides a detailed look at the increases and decreases in equity during the year. Equity can be increased with additional capital contributions and positive earnings during the year, or it can be decreased by losses incurred during the year as well as dividends paid to the owners of the company.
- D. The statement of cash flows identifies the sources and uses of cash from three different business functions:
1. Cash flows from operating activities are created through the day-to-day conduct of business. This section of the statement of cash flows can be created in one of two different ways.
    - a. The direct method of determining cash flows from operating activities is a top-down approach whereby items on the income statement are adjusted for changes in related working capital accounts (current assets and current liabilities) to turn accrual-based income and expenses into cash basis figures.
    - b. The indirect method is a bottom-up approach that begins with net income and adjusts it back to a cash basis figure.
  2. Cash flows from investing activities are those sources and uses of cash created by the purchase and sale of long-term assets. All related dividends and interest received as a result and gains and losses created on disposal are included as adjustments in the operating activities section, since they are included in the determination of net income.
  3. Cash flows from financing activities are those cash flows created by borrowing from and repayment to the lenders, creditors, and owners of the business. Dividends paid to owners are also included as a use of cash in this section.
- E. Analysts should look further than the prepared financial statements when evaluating a publicly traded company. More extensive information is available in footnotes to the financial statements, management’s discussion and analysis, and other public sources such as filings with the SEC.



## A CLOSER LOOK AT THE BALANCE SHEET

- A. Marketable securities are classified as either securities held to maturity (reported at original cost), trading (reported at fair market value), or available for sale (reported at fair market value).
- B. Accounts receivable must be adjusted for bad debts. There are two methods to make this adjustment: the direct write-off method (where bad debt expense is recorded as the accounts are written off) and the allowance for doubtful accounts method (where bad debts are estimated). The allowance method is preferred.
- C. Inventory costing methods include specific identification, average cost, FIFO, and LIFO. LIFO is permitted under U.S. GAAP but not under IFRS. In normal periods of rising prices, FIFO will generate higher net income (lower COGS) and higher ending inventory. LIFO will generate lower net income (higher COGS) and lower ending inventory. The effects of inventory accounting choices can be evaluated using the basic inventory equation:
- $$\text{ending inventory} = \text{beginning inventory} + \text{purchases} - \text{cost of goods sold}$$
- D. Long-lived assets are capitalized, with their cost then expensed over their useful lives. This expense is called depreciation for tangible assets, depletion for natural resources, and amortization for intangible assets.
1. Straight-line depreciation allocates the cost evenly over the asset's life.
  2. Accelerated depreciation methods (e.g., double declining balance) allocate more of the cost to the earlier years of the asset's life.
- E. Contingent liabilities must be disclosed in the balance sheet if the future payment is probable and reasonably estimable.
- F. Long-term liabilities (e.g., bonds) are recorded at the present value of future cash flows. Any premium or discount from face value at issuance must be amortized annually over the life of the debt.
- G. Capital leases must be shown on the balance sheet. Operating leases are not recorded on the balance sheet, and firms sometimes try to use operating leases to keep lease obligations off the balance sheet.
- H. Preferred stock is stock with a claim on income and assets that is senior to common stock.
- I. Treasury stock is stock that has been repurchased by the issuing corporation.

**PRACTICE QUESTIONS: FINANCIAL REPORTING AND ANALYSIS**

1. Assume that Jill's Dress Shop paid cash to buy inventory of 50 dresses in January. The *most likely* effect on Jill's Dress Shop financial statements would be:
  - A. an increase in assets.
  - B. an increase in sales.
  - C. a decrease in owners' equity.
  - D. no change in assets.
  
2. Which of the following is INCORRECT with regard to the dual entry accounting system?
  - A. Additions to inventory are recorded as debits to inventory.
  - B. Increases in accounts payable are recorded as credits to accounts payable.
  - C. Increases in depreciation expense are recorded as credits to depreciation expense.
  - D. Cash payments for rent are recorded as credits to cash.
  
3. You sign a note to borrow \$10,000 from your banker. The \$10,000 will be repaid in one payment in one year. What are the original entries in the general journal that account for this transaction?
  - A. \$10,000 credit to cash account, \$10,000 credit to notes payable account.
  - B. \$10,000 debit to cash account, \$10,000 debit to notes payable account.
  - C. \$10,000 debit to cash account, \$10,000 credit to notes payable account.
  - D. \$10,000 credit to cash account, \$10,000 debit to notes payable account.
  
4. The balance sheet is constructed on the basic accounting equation which states that:
  - A. current liabilities plus long-term debt equal assets.
  - B. total liabilities plus equity equal assets.
  - C. current assets plus equity equal total liabilities.
  - D. current assets plus fixed assets equal total liabilities.
  
5. In a classified balance sheet, which of the following accounts would be listed first?
  - A. Land.
  - B. Intangible assets.
  - C. Inventory.
  - D. Prepaid expense.

## Financial Reporting and Analysis

6. All of the following would be reported on the income statement “below the line” EXCEPT:
  - A. discontinued operations.
  - B. income tax expense.
  - C. extraordinary items.
  - D. cumulative effect of changes in accounting methods.
  
7. Which of the following would NOT affect cash flow from operations?
  - A. An increase in accounts payable.
  - B. Payment of interest expense.
  - C. Payment of income taxes.
  - D. Payment of dividends.
  
8. Prepaid expense items, such as prepaid rent, are usually considered:
  - A. capital assets.
  - B. current assets.
  - C. current liabilities.
  - D. long-term liabilities.
  
9. MicroChip Technologies, Inc. owns 50,000 shares of stock in Zenathon Corp. Microchip paid \$6.00 per share for the stock, which had a value of \$9.00 per share on December 31, 20X7. If Microchip classifies the Zenathon investment as a trading investment, then the value reported on the 12/31/07 balance sheet would be:
  - A. \$300,000, and the unrealized gain would not be included in income.
  - B. \$450,000, and the unrealized gain would be shown on the income statement.
  - C. \$450,000, and the unrealized gain would be listed as an adjustment to stockholder’s equity.
  - D. \$450,000, and the unrealized gain would not be included in income or equity.

10. Jake's Spoons made the following inventory purchases in the first quarter:

<u>Month</u>	<u>Units</u>	<u>Cost per unit</u>
January	50	\$7
February	40	\$8
March	25	\$9

Assume that Jake's had no inventory at the beginning of January and that 95 units were sold during the first quarter. Ending inventory for the first quarter under FIFO and LIFO methods of inventory accounting would be:

	<u>FIFO</u>	<u>LIFO</u>
A.	\$140	\$140
B.	\$180	\$140
C.	\$140	\$180
D.	\$180	\$180

11. Expenses that are related to specific revenues but not with a specific product should be:
- deferred and amortized over the lifetime of the specific product.
  - expensed immediately.
  - expensed in the period in which revenues are recognized.
  - expensed as incurred.
12. An analyst is evaluating two firms in the same industry. Firm A uses the LIFO method of inventory accounting, while Firm B uses FIFO. In a period of rising prices, the analyst should expect that Firm A would report:
- higher net income and higher inventory than Firm B.
  - higher net income and lower inventory than Firm B.
  - lower net income and higher inventory than Firm B.
  - lower net income and lower inventory than Firm B.
13. A piece of industrial equipment costs \$100,000, and it will cost \$20,000 to ship and install. The equipment is expected to have a 6-year useful life and no salvage value. Calculate the annual depreciation expense and book value at the end of the fourth year using straight line depreciation:
- |    | <u>Depreciation</u> | <u>Book value</u> |
|----|---------------------|-------------------|
| A. | \$16,667            | \$40,000          |
| B. | \$20,000            | \$40,000          |
| C. | \$16,667            | \$33,333          |
| D. | \$20,000            | \$33,333          |
14. Depreciation:
- is a cash expense.
  - reduces income tax liability.
  - increases gross operating profits.
  - is considered a long-term liability.

## Financial Reporting and Analysis

15. Charles Corp has just purchased a new machine for \$400,000. Charles Corp plans to depreciate the machine over the next four years using the double declining balance method of depreciation. If the machine has a salvage value of \$80,000, what will Charles Corp report as depreciation expense in year 3?
- A. \$50,000.
  - B. \$40,000.
  - C. \$20,000.
  - D. \$0.
16. All of the following are examples of intangible assets EXCEPT:
- A. timberland.
  - B. patents.
  - C. goodwill arising from an acquisition.
  - D. licenses to use technology developed by another firm.
17. Gears Inc. issued \$10 million par value of bonds in 20X7 with an annual coupon rate of 8.0%. The bonds were issued at a price of \$1,050 per \$1000 par value, and they mature in ten years. The total interest cost of these bonds over their life is:
- A. \$8,500,000.
  - B. \$8,400,000.
  - C. \$8,000,000.
  - D. \$7,500,000.
18. Regal Corp has leased a machine under a long-term agreement that includes a bargain purchase option. Under U.S. GAAP, this transaction should be treated as:
- A. an operating lease, regardless of other features of the agreement.
  - B. a capital lease, so long as the term of the agreement is at least 75% of the estimated economic life of the machine.
  - C. an operating lease, so long as the term of the agreement is at least 75% of the estimated economic life of the machine.
  - D. a capital lease, regardless of other features of the agreement.
19. Treasury stock is best defined as:
- A. outstanding stock repurchased by the issuer.
  - B. stock with no par value.
  - C. additional paid-in capital.
  - D. stock payable to shareholders as part of a stock dividend.

20. Stockholder's equity would be increased by all of the following EXCEPT:
- A. issuing new stock at par value.
  - B. increase in market price of common stock.
  - C. positive net income.
  - D. issuing new stock above par value.

Use the following information to answer Questions 21 through 27.

Burle Industries is a manufacturer of heavy mining equipment. The company has been in operation for ten years, and during the last five years the company has been a publicly held firm. Financial data (in random order) for Burle Industries for the year ending 12/31/07 are presented as follows:

Gross Profit	\$20,000
Accum. Depreciation	\$60,000
Accounts Payable	\$6,000
Revenue	\$35,000
Depreciation Expense	\$6,000
Cash	\$9,000
Notes Payable	\$500
Paid-in Capital	\$10,000
Fixed Assets	\$90,000
Retained Earnings	\$21,500
Accounts Receivable	\$15,000
SG&A Expenses	\$2,000
Inventory	\$7,000
Long-term Debt	\$3,000
Interest Expense	\$270
Common Stock	\$20,000
Income Taxes	\$4,692

Over the course of the year, accounts receivable increased by \$1,200, inventory decreased by \$750, accounts payable increased by \$400, new fixed assets were purchased for \$10,000, and long-term debt was reduced by \$1,000.

21. Calculate the dollar value of Burle Industries' current assets and total assets as of December 31, 20X7.

	<u>Current assets</u>	<u>Total assets</u>
A.	\$24,000	\$61,000
B.	\$31,000	\$114,000
C.	\$31,000	\$61,000
D.	\$24,000	\$114,000

## Financial Reporting and Analysis

22. Calculate the net working capital for Burle Industries as of December 31, 20X7.
- A. \$17,000.
  - B. \$17,500.
  - C. \$24,000.
  - D. \$24,500.
23. Calculate the total value of equity for Burle Industries as of December 31, 20X7.
- A. \$41,500.
  - B. \$30,000.
  - C. \$61,000.
  - D. \$51,500.
24. Calculate the cost of goods sold for Burle Industries for the period ending December 31, 20X7.
- A. \$15,000.
  - B. \$20,000.
  - C. \$10,000.
  - D. \$5,000.
25. Assuming that Burle Industries did not pay any dividends to common stockholders, what was the addition to retained earnings in 20X7?
- A. \$4,692.
  - B. \$7,038.
  - C. \$11,730.
  - D. \$21,500.
26. Calculate the cash flow from operations for Burle Industries for the year ending December 31, 20X7.
- A. \$6,988.
  - B. \$7,088.
  - C. \$12,988.
  - D. \$13,088.
27. Calculate the total cash flow for Burle Industries for the year ending December 31, 20X7.
- A. \$1,988.
  - B. \$2,988.
  - C. \$3,988.
  - D. \$11,988.

**PRACTICE QUESTION ANSWERS: FINANCIAL REPORTING AND ANALYSIS**

1. **D** The increase in inventory would be offset by the payment from cash.
2. **C** Increases expenses are recorded as debits. Depreciation is handled like any other expense—an increase in depreciation would be a debit.
3. **C** Signing the note represents both an increase to cash (an asset) and an increase to notes payable (a liability).

	<i>Debit</i>	<i>Credit</i>
Cash	\$10,000	
Notes payable		\$10,000

4. **B** The accounting equation equates the two sides of the balance sheet—assets equal total liabilities plus equity.
5. **C** Land and intangible assets are long-term assets, while inventory and prepaid expense are current assets. Inventory is listed before prepaid expense in a classified balance sheet because inventory is expected to be converted to cash more quickly.
6. **B** “Below the line” items are reported after net income. Income taxes are deducted in determining net income.
7. **D** Accounts payable is a working capital account, and changes to working capital accounts are included in cash from operations. Interest paid and taxes paid also are included in cash from operations. Dividends paid are considered in cash from financing.
8. **B** Prepaid items are typically expenses that have been paid in advance. A good example is insurance, which may be paid for several months or a year a time, then expensed monthly. The prepaid amount, which has not been expensed, is carried on the balance sheet as a current asset.
9. **B** Trading investments are reported at fair market value, and any unrealized gain or loss is shown on the income statement.
10. **B** Ending inventory would be  $(50 + 40 + 25) - 95 = 20$  units. Under FIFO, the units remaining would be the most recent purchased, so inventory would be  $20 \times 9 = \$180$ . Under LIFO, the units remaining would be the first ones purchased, so inventory would be  $20 \times 7 = \$140$ .
11. **C** Matching principle requires that revenues and expenses are matched with each other and matched within the appropriate accounting period.



## Financial Reporting and Analysis

12. D In a rising price environment, LIFO would mean more expensive inventory was being included in cost of goods sold, so net income would be lower. The remaining inventory would be the less expensive inventory purchased earlier, so ending inventory would be lower also.
13. B The total capitalized cost of the equipment is \$120,000 (invoice \$100,000 plus shipping and installation). Since we are using straight-line depreciation, the depreciation expense in any given year is equal to  $\$120,000 / 6 = \$20,000$ . The following table will help demonstrate the depreciation of the asset over time:

<i>Year</i>	<i>Capitalized Cost</i>	<i>Depreciation Expense</i>	<i>Accumulated Depreciation</i>	<i>Book Value</i>
	120,000			
1	120,000	20,000	20,000	100,000
2	120,000	20,000	40,000	80,000
3	120,000	20,000	60,000	60,000
4	120,000	20,000	80,000	40,000
5	120,000	20,000	100,000	20,000
6	120,000	20,000	120,000	0

As the table demonstrates, the book value (i.e., original cost minus accumulated depreciation) at the end of year 4 is \$40,000.

14. B Although depreciation is a non-cash expense, it reduces taxable income and taxes. It also reduces gross operating profit.
15. C Depreciation expenses:
- $$\text{Year 1: } \left( \frac{2}{4\text{years}} \right) (\$400,000 - 0) = \$200,000$$
- $$\text{Year 2: } \left( \frac{2}{4\text{years}} \right) (\$400,000 - 200,000) = \$100,000$$
- $$\text{Year 3: } \left( \frac{2}{4\text{years}} \right) (\$400,000 - 300,000) = \$50,000$$

Under the declining balance method, salvage value is not used in the annual expense calculation, but the asset cannot be depreciated below salvage value. Therefore, after year 2 only \$20,000 of additional depreciation may be taken.

16. A Timberland is a tangible asset.

17. **D** Total interest cost is total coupon interest paid to bondholders less the premium received at issuance. In this case, the total interest cost would be:

$$(\$10\text{M} \times 8\% \times 10 \text{ years}) - \$500,000 = 7,500,000$$

18. **D** The agreement would be a capital lease so long as it included either a bargain purchase option or a term of 75% of the machine's life. The agreement would also be considered a capital lease if the agreement transferred ownership or if the present value of the lease payments were greater than or equal to 90% of the fair market value of the property.
19. **A** Treasury stock is stock that has been repurchased by the issuing firm.
20. **B** Issuing new stock would increase equity, regardless of whether the stock was issued at or above par value. Positive net income adds to retained earnings, which would also increase equity. Changes in the market price of the stock are not reflected in the firm's balance sheet.
21. **C** Burle's current assets consist of cash, accounts receivable, and inventory:  $\$9,000 + \$15,000 + \$7,000 = \$31,000$ . Total assets for the company consist of current assets plus fixed assets less accumulated depreciation:  $\$31,000 + \$90,000 - \$60,000 = \$61,000$ .
22. **D** Net working capital is defined as current assets minus current liabilities. Total current liabilities for Burle Industries is equal to the sum of its current liabilities, which include accounts payable and notes payable:  $\$6,000 + \$500 = \$6,500$ . For Burle Industries, the net working capital equals  $\$31,000 - \$6,500 = \$24,500$ .
23. **D** The total value of equity for Burle Industries is simply the sum of the equity accounts as given in the problem. Burle is a public company and therefore has the following equity accounts: common stock, paid in capital, and retained earnings. When we add all the accounts together we get a total value of equity equal to  $\$20,000 + \$10,000 + \$21,500 = \$51,500$ .
24. **A** The cost of goods sold can be determined from income statement data. We already know the revenue and gross profit for Burle Industries. Therefore we only need to substitute what we know into the following equation and determine the cost of goods sold figure: Revenue – COGS = Gross Profit;  $\$35,000 - \text{COGS} = \$20,000$ ;  $\text{COGS} = \$35,000 - \$20,000 = \$15,000$ .
25. **B** If a firm does not pay any dividends, the amount added to retained earnings is equal to the firm's net income for the period. Calculate net income as follows:

$$\text{revenue} - \text{COGS} - \text{SG\&A} - \text{depreciation} - \text{interest} - \text{taxes} = \text{net income}$$

$$\$35,000 - \$15,000 - \$2,000 - \$6,000 - \$270 - \$4,692 = \$7,038$$

## Financial Reporting and Analysis

26. C Cash flow from operations is calculated as follows: net income + depreciation – increases in current assets + decreases in current assets – decreases in current liabilities + increases in current liabilities. More specifically, for Burle the operating cash flows are calculated as follows: net income + depreciation – increase in accounts receivable + decrease in inventory + increase in accounts payable = operating cash flow:  $\$7,038 + \$6,000 - \$1,200 + \$750 + \$400 = \$12,988$ .
27. A Total cash flow is the sum of cash flows from operations, investing, and financing. Since Burle bought fixed assets in the amount of \$10,000, this represents a cash outflow for investment. Also, the company paid down debt (a cash outflow) by \$1,000 over the course of the year. Therefore investing and financing cash flows are negative for Burle. We sum all the cash flows as follows: operating cash flow + investing cash flow + financing cash flow:  $\$12,988 - \$10,000 - \$1,000 = \$1,988$ .

# CORPORATE FINANCE

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**Corporate finance** is the study of how corporations raise and use capital, how corporate financial managers evaluate possible capital investments, and how corporations are governed by their shareholders.

Unlike a proprietorship or partnership, a **corporation** is a separate, legal, tax-paying entity. Typically the owners, known as stockholders or shareholders, are numerous and may be scattered throughout in the world. They rely on managers, who may or may not be owners, to make the operating decisions. Shareholders elect a **board of directors** that is responsible for hiring and monitoring the managers.

Conflicts may arise from this separation of corporate management and ownership. The owners of the firm may or may not be interested in daily operations, but they are very interested in the value of their investment in the company (the stock they hold). On the other hand, management is very interested in daily operations, particularly as they relate to salary and job security. Management may have opportunities to use company resources in ways that are in their own best interest, rather than in the owners' best interest. How corporations are organized to ensure that management acts in the owners' interests is the subject of **corporate governance**.

## FORMS OF BUSINESS

Let's look at an example of the development of a business. Jim Smith owns and operates a portable hot dog kiosk in a large city. Since his business is flourishing, he purchases another inexpensive hot dog stand for cash and hires Jean Jones to manage the new stand.

Some time later, because of the growth of business, Jean approaches Jim with the suggestion that they rent a small building she has found. The location is excellent, and rent is inexpensive. Since sales are increasing and customer loyalty is high, Jim feels renting the building is something he should pursue. However, Jim will need \$100,000 to purchase new equipment and make repairs and alterations to the building. Jim has \$20,000 that he can invest in the business, and the bank will loan him \$60,000. Jean offers to invest the other \$20,000, but only in exchange for half ownership. Under this arrangement they both own the business, and they share equally in the profits.

## Corporate Finance

Eventually, Jim and Jean own and operate several hot dog shops and have hired a team of managers for each of the shops. With the number of locations and increasing obligations and liabilities, Jim and Jean's attorney, John, has recommended that they *incorporate* as J&J Dogs. Incorporation will allow J&J Dogs to continue operations as usual while freeing Jim and Jean from some of the liabilities of business ownership and the problems associated with the limited life of a partnership.

John has indicated to Jim and Jean that by forming a corporation, they effectively create a separate entity. The corporation has a life of its own, separate from those of Jim and Jean, while the life of their original partnership is tied directly to the lives of the partners. If either one of them dies, the partnership ceases to exist. If they choose to incorporate, J&J Dogs could continue indefinitely regardless of whether or not Jim and Jean survive.

In addition, by forming a corporation, Jim and Jean insulate themselves from some of the potential legal liability of operating a business. For example, Jim and Jean cannot possibly ensure that all sanitary and safety precautions are being followed all the time at all their shops. For example, a careless employee could leave hot dogs out of refrigeration too long before cooking them. From this oversight, a customer could contract an illness.

Under their current arrangement as a partnership, Jim and Jean could be held personally liable for the victim's illness. An ensuing lawsuit could take all the assets of J&J Dogs, as well as the personal assets of Jim and Jean. However, if J&J Dogs is incorporated, Jim and Jean's losses are limited to what they have invested in J&J Dogs. The lawsuit could take the business assets, but none of their personal assets. After considering the potential for an unexpected and possibly disastrous liability, Jim and Jean agree to form a corporation.

After incorporation, their business flourishes and they have people from all over the country clamoring to learn the secrets of their success. While Jim and Jean will not give away their secret recipes, they will let others lease the rights to use them by offering franchises locally.

J&J Dogs rapidly becomes a household name. Jim and Jean consider establishing a national franchise of their business. Of course, this requires a substantial investment. The only way they can come up with the millions of dollars needed to "go national" with their franchises is to go public. After the **initial public offering** (IPO) of shares, there are thousands of owners of J&J Dogs.

This example makes a very important point. Business as we know it could not have developed without finance. When Jim first expanded his operation from one to two kiosks, it was his own savings that made this possible. He financed the expansion. However, expansion into a fixed location was beyond his means. He needed the infusion of capital only a partner could provide.

Later, forming a corporation provided Jim and Jean with limited liability and protected the company from the death of either partner. Moreover, this provided the platform to gain access to the capital necessary for national expansion. Jim and Jean were thus able to sell a portion of their company to the public through an IPO. This provided a major infusion of capital as well as future access to capital markets.

Let's sum up what we know about the various business forms.

## Proprietorship

The least complicated form of business to establish is the **proprietorship**. When Jim had only one hot dog kiosk, his business was a proprietorship. Legally, the business and Jim were indistinguishable. When his kiosk earned a profit, it belonged exclusively to Jim, and he paid income taxes on it. If the business showed a loss for the period, Jim could use that loss to offset profits from other sources of personal income. He alone had total claim to all profits, but he alone also had to accept all losses.

Any liability incurred by the business was also directly tied to Jim. If the business did not pay a liability (e.g., a note to a bank or debt to a supplier) Jim could be forced to pay the debt from his personal assets. If Jim couldn't pay the liability, creditors could seize the business and all of Jim's personal assets, if necessary, to cover the amount owed. In a proprietorship, liability to the owner is unlimited. If someone sues the business, all personal and business assets are considered the same. The lawsuit could seize any or all of Jim's assets, including those in the business. In addition, the life of a proprietorship is limited to the proprietor's life. If the proprietor dies, the business effectively dies also. Its assets then become assets in the proprietor's estate.

One of the most frequently given reasons for starting and maintaining a proprietorship is freedom to make decisions. As the sole proprietor, Jim made all the business decisions. He decided where to locate the kiosk, the types of hot dogs he would sell, the price he would charge, and the hours of operation for the business.

### Partnership

A **partnership** is a more complicated form of business. It is a legal contract between two or more individuals that share the ownership interests in a business. For J&J Dogs, the partnership agreement specified that Jim and Jean each own 50% of the business. While J&J Dogs was being operated as a partnership, the partnership agreement defined how ownership in J&J Dogs was divided between Jim and Jean.

Legally, a partnership is very similar to a proprietorship. As with a proprietorship, we say profits and losses “flow through” to the owners. That is, the partnership itself does not pay income taxes. Jim and Jean divided any profits and losses equally and reported them on their personal income tax returns.

Also, like a proprietorship, partnership debts “flow through” to the owners according to their proportional ownership. Since Jim and Jean each owned 50% of the business, they each were liable for 50% of any debts or other claims against the business. If J&J Dogs went out of business with large debts, Jim and Jean would each be liable to repay 50% of the debt. Creditors would seize business and personal money and/or assets from both of the partners to satisfy the claims.

From Jim’s perspective, the primary benefit from forming the partnership was getting the capital necessary to expand the business. Expansion required additional equity<sup>1</sup> capital, which he did not have but which Jean could provide. There were other benefits (e.g., sharing management responsibilities and division of the liability exposure), but financing was paramount to the expansion and ultimate success of the business.

While he gained assistance in running the business, Jim was no longer free to make decisions on his own. Jean might not agree with him on the types of food to purchase, the hours of operation, prices to pay for inputs and prices to charge, along with many other daily business decisions. Disagreements among partners are one of the primary reasons for the dissolution of partnership agreements and consequent business failures.

### Corporation

In our previous example, J&J Dogs waited to incorporate its business until it was large enough that the benefits from forming a corporation outweighed the drawbacks from giving up the partnership. A **corporation** is a business entity that is

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<sup>1</sup> Equity means ownership. Equity capital is the owners’ contribution to the business in the form of money or other assets.

legally separate from the owners. This separateness affords the corporate form three specific advantages over the proprietorship and partnership forms of business.

1. **Unlimited Life.** Since the corporation is a legal entity separate from its owners, its life is not tied to that of the owners. If both Jim and Jean died, their ownership shares in the corporation, not the total assets of the corporation, would become part of their respective estates.
2. **Limited Liability.** In a proprietorship or partnership, all profits, losses, debts, and other liabilities “flow through” to the owner(s). In the case of a corporation, the owners are only liable for the amount they have invested in the corporation. For instance, a large lawsuit could easily claim more than the available assets of J&J Dogs, Inc. Since Jim and Jean are considered legally separate from the corporation, the suit cannot claim their personal assets. They can only lose their time, effort, and monetary investments in the business.
3. **Ease of Ownership Transfer.** Ownership interest in publicly traded corporations is obtained by purchasing their **common stock**. Common stock is sold in shares, with each share indicating a percentage ownership in the firm. If you hold 10% of the common stock of a firm, you own 10% of that firm. Ownership interest permits holders of common stock to vote at stockholder meetings, giving the owners of the firm a say in how the firm is managed. Stockholders usually have one vote per share of stock they own. Votes are cast on major questions faced by the corporation, such as the election of the Board of Directors.

Large corporations typically have millions of shares of stock outstanding and have thousands of owners. Since some owners would have to travel hundreds or even thousands of miles to attend the stockholder meeting to cast their votes, corporations send them proxies. A *proxy* is an absentee ballot. On the proxy, the stockholder indicates his choice for members of the Board of Directors and yes or no on other questions included in the proxy. If a stockholder fails to return the proxy by the indicated date, management typically has the right to cast that stockholder’s votes.

The charge of the Board of Directors is to elect, advise, and oversee the president of the corporation. Directors are paid and are expected to meet regularly, usually quarterly. The only prerequisite to being a member of the Board is interest and valuable expertise.

The profits of any firm belong to the owners of the firm. Rather than pay out all net income to the stockholders, the Board will ordinarily retain a portion for future investment. This means that net income is divided into two parts. The first is the portion paid to stockholders in the form of dividends, and the second is the portion



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reinvested by management for the stockholders. This portion is called **retained earnings** on the firm's balance sheet.

Since they are a part of net income, retained earnings represent profits generated by the firm that have not been paid to the stockholders. By retaining part of the firm's earnings, management is implicitly promising to use them to maintain or replace equipment or to invest it in profitable projects or expansion. As a general rule, firms with many investment opportunities tend to pay smaller dividends, while firms with fewer investment opportunities, such as public utilities, tend to pay larger dividends.

In the introduction to this chapter, we noted that certain conflicts can arise as a result of the separation of ownership and management of a corporate entity. In general, management is supposed to strive to maximize stockholder wealth by maximizing the firm's stock price. However, this assumes that management and the stockholders always have the same personal goals. This is not necessarily the case.

*Agency* arises from the separation of corporate ownership and management. Since the owners are numerous and scattered all over the country (and possibly the world), managers must act as their agents and run the firm for them. Every operating and investment decision the manager makes is actually made for the owners. It's similar to the relationship between homeowners and real estate agents. Agents represent homeowners to potential buyers, while homeowners must depend upon the agents to act in their best interest.

In large corporations, conflicts may arise between the interests of (1) the firm's stockholders and its managers or (2) the firm's stockholders and the firm's bondholders. Before we discuss these, however, let's be sure you understand the relationship between stockholders and bondholders and the sources of risk in a large corporation.

The characteristics of the firm issuing stocks and bonds determine a large portion of the risk of its securities. We divide these characteristics into two categories: **financial risk** and **business risk**. Financial risk comes from the way management finances the firm's assets and growth. Using common equity stabilizes the firm's cash flows, while using debt creates **leverage** which can lead to volatility in the firm's cash flows (i.e., using debt makes the firm riskier). There is also a certain amount of volatility in the firm's earnings due to business risks. Much of this risk is associated with the type of industry in which the firm operates, but management can make choices that will increase or decrease the firm's business risk.

Stockholders are primarily concerned with the price of their stock. They want management to make decisions that will cause the price to increase. Bondholders, on the other hand, are far more interested in the firm's ability to make the required

periodic interest payments and repay the principal when due. The ability to make interest payments as part of the normal operations of the firm depends on management's ability to consistently generate sufficient operating income or earnings before interest and taxes (EBIT). The appropriate level of firm risk is one source of disagreement between stockholders and bondholders.

Stockholders want management to undertake risky projects to maximize their expected returns, even though doing so also increases the firm's overall risk. A large portion of the increased risk is a result of unsystematic (company specific) factors which can be diversified away by stockholders who hold their stock as part of a large portfolio. Thus the stockholders can achieve higher expected returns without bearing all of the associated increase in risk.

Bondholders, on the other hand, would prefer that management be cautious in running the firm. They prefer less risky projects that are expected to maintain a stable level of operating profit that is sufficient to cover the requisite interest payments on the bonds. In addition, a firm's creditors want to limit further issuance of debt to protect the stability of their investment. Issuance of more and more debt will likely result in increasing the interest rates the firm must pay to borrow. These higher interest rates increase interest payments and the overall risk of the firm.

Stockholders will tolerate additional risk with its accompanying increased expected returns. Bondholders want stability with its lower, more consistent returns. If stockholders get their wish, they in effect transfer (i.e., take) value from the bondholders. The increased risk reduces the value of the debt claims on the firm while simultaneously increasing the value of the equity claims. If the bondholders get their way, their value is maintained, but the value of the equity claims is not maximized.

## RISK AND RETURN

When a corporate manager makes a decision to purchase equipment, expand operations, or start a new product line, the manager must consider both the costs and the benefits of that decision. We study costs and benefits in the framework of **risk** and **return**. Decisions made by the corporate financial manager will usually involve money, since money is both the cost and the benefit of most financial decisions. Risk is important because virtually no financial decision is immune from uncertainty.

We use the word **uncertainty** as a synonym for risk. In finance, whenever there is uncertainty about an outcome, that outcome is considered risky. Let's assume today is the day your best friend Chris promised to pay back the \$200 he borrowed from you last week. Are you certain Chris will pay you the \$200 today as promised? If

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Chris gives you the money today, you will have received the payment exactly as you expected. If you receive the money tomorrow or the next day, you will have received the amount expected but not at the original time expected.

Let's define risk as *the possibility of an unfavorable event*. Is receiving the \$200 late an unfavorable event? What if you needed the money today to pay your cable television bill? If you receive the money late, the result is interrupted cable service.

Since most financial decisions result in receiving (or paying) a cash flow in the future, and since the future is always uncertain, there is obviously risk associated with any future cash flow. In fact, as the cash flow occurs further into the future, it will generally tend to become riskier.

Given enough time, just about anything can happen. That means that the longer the time before you expect a cash flow, the higher the possibility that something will happen to affect the way it is received or even if it is received at all. Since no one can predict the future with certainty, we must accept that some amount of risk is inevitable.

### What Causes Risk?

Risk, as it applies to common stock, is defined the same way as it is for any other investment. It is the possibility of an unfavorable event. From the investor's perspective, a particularly unfavorable event is a decline in the price of the stock. We can define the unfavorable event for a common stock as a decline in price, but what causes stock prices to decrease?

The forces that can affect stock prices can be separated into two general categories: macroeconomic and microeconomic. **Macroeconomic variables** (e.g., inflation, the national unemployment rate, government policies) are economy-wide in nature and affect all stock prices to varying degrees. For instance, inflation causes overall price increases, resulting in decreased demand and reduced profits. As the unemployment rate increases, economy-wide consumption declines, also resulting in reduced profits. Macroeconomic factors such as these can be the most troublesome for investors.<sup>2</sup>

### Microeconomic (Firm-Specific) Variables

Microeconomic variables are those that are specific to each firm. They are the characteristics of the firm, such as its management, employees, products, and

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<sup>2</sup> We will concentrate our discussion here on microeconomic (firm-specific) risk factors. Macroeconomic (systematic) risk factors are discussed in the Portfolio Management chapter.

financing choices. Let's take a little time to discuss some of the most important microeconomic forces that affect stock prices.

**Business Risk.** Recall from the chapter on Financial Reporting and Analysis that several factors can affect income from operations, also known as operating income or earnings before interest and taxes (EBIT). Let's look at the top portion of Figure 1, the Income Statement for J&J Dogs, our rapidly expanding company from the first section of this chapter.

After deducting selling and operating costs from revenue, we observe that EBIT for J&J Dogs is \$1,614. A change in the value of any one of those factors, including revenue, will change EBIT. From the chapter on Quantitative Methods, we know that a factor with many different possible values has a distribution of possible values. A change in any of the factors leading to EBIT (revenue or expense) will yield a different value from the distribution of possible values for EBIT. In other words, EBIT is a variable.

**Figure 1: J&J Dogs—Income Statement**

Revenue	\$8,500
Less:	
<b>Cost of Goods Sold:</b>	
Hot dogs	975
Other	975
<b>Operating Expenses:</b>	
Wages	2,500
Utilities	800
Rent	1,000
Insurance	600
Depreciation	36
<b>Income From Operations</b>	<b>\$1,614</b>

We will define **business risk** as the uncertainty in EBIT or the forces that cause this uncertainty.<sup>3</sup> Whenever a factor has a distribution of possible values, we say the true value of that factor is uncertain. Remember, we are using uncertainty as a synonym for risk, so an uncertain factor is risky in a corporate finance context.

Of course risk doesn't apply to Figure 1 because this level of EBIT has already been realized. We encounter risk when we are trying to predict a future value. Whenever the true value of a future cash flow is not known with certainty, we call it an

<sup>3</sup> As with many other terms in finance, there are other definitions for business risk, but this definition will fulfill our needs in this chapter.

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*unknown.* We might have a pretty good idea about the range of values it could have, but we can't predict it with certainty.

We now turn to some of the better-known sources of business risk: revenue variability, cost and price structure, competition, and operating leverage.

**Variability in revenue.** If the firm's future revenues are highly variable (subject to extreme changes), future EBIT will be highly uncertain. Anything that causes sales variability is considered a source of business risk. This would include demand for the product or service (quantity sold) and the price at which it sells.

If demand for the firm's output is subject to seasonal swings or if the product has many substitutes, sales will be hard to predict. Also, if the product market is subject to severe price variability, as with the market for computer chips, there will be variability in sales revenues.

**Cost and input price structure.** Some firms face uncertainty in the costs of their inputs. This variability in input costs will obviously cause variability in the firms' operating and net income. Also at issue is the firms' ability to pass along increased input prices to the consumer through an increased sales price. Firms with little price flexibility and uncertain input costs will be exposed to higher levels of risk.

**Competition and new product development.** Some firms' revenues are very difficult to predict because competitors—domestic or international—may enter the market at any time with a competing or improved product. The degree to which competing products affect revenues will depend on how quickly and inexpensively the firm can develop another product.

**Operating leverage.** Operating leverage depends upon the proportion of the firm's costs that are fixed. In the short run, the higher the proportion of fixed costs, the less flexibility management has in lowering costs to accommodate weak sales. In the long run, all costs are variable and management can change the entire cost structure.

Operating leverage is measured as the percentage change in EBIT that results from a given percentage change in revenues. With no fixed costs, the change in EBIT is the same as the change in revenues. For example, if revenues increase (decrease) by 10%, EBIT will also increase (decrease) by 10%. Fixed costs act as a lever and cause the percentage change in EBIT to be greater than the percentage change in revenues. A high proportion of fixed costs could cause the percentage change in EBIT to be three or four times the percentage change in revenue. In this case, a 10% increase (decrease) in revenues would lead to a 30% or 40% increase (decrease) in EBIT. Later in this chapter we provide an example of breakeven calculations relating to operating leverage, as well as graphs further illustrating

the leverage effect. The main point is that if EBIT has been leveraged through the existence of fixed costs, its distribution of possible values has been widened and the firm's business risk has been increased.

Business risk depends on the industry as well as management's decisions. For example, the auto and steel industries can do little about their fixed cost structure since they have many large fixed assets, and labor costs are fixed for extended periods by unions. Alternatively, real estate development companies tend to own very few assets and use subcontracted labor, and therefore have low fixed costs and low operating leverage.

**Financial risk.** Whereas business risk is mostly a result of the firm's assets and its revenue and cost structure, **financial risk** is the risk of the firm due to management's choices regarding the use of debt financing. The effects of debt financing are shown on the income statement below EBIT in the form of interest expense. Interest expense has the effect of levering (i.e., magnifying) the percentage change in net income up or down. This levering of net income effectively increases or widens the distribution of its possible values (increases risk).

**Financial leverage** is the percentage change in net income resulting from a given percentage change in EBIT. With no interest charges, the change in net income is the same as the change in EBIT.<sup>4</sup> For example, if EBIT increases (decreases) by 10%, net income also increases (decreases) by 10%. Interest is a fixed cost and acts as a lever to make the percentage change in net income greater than the percentage change in EBIT. Much like the use of high levels of fixed operating costs, heavy dependence on debt financing could cause the percentage change in net income to be many times the percentage change in EBIT. Later, we provide an example of breakeven calculations relating to financial leverage, as well as graphs further illustrating the leverage effect. The key issue is that the resulting amplification in net income could be a boon for companies with high operating profit margins but could be a disaster for a struggling firm.

## CAPITAL STRUCTURE

Capital structure refers to the relative proportions of debt and equity the owners of a firm have used to finance its operations. We can observe a firm's capital structure by examining its balance sheet. In Figure 2, we have reproduced the balance sheet for J&J Dogs. The left side of the balance sheet shows the assets of the firm, including current and fixed assets. The right side is divided into liability and owners' equity accounts. These accounts represent the sources of capital utilized to pay for the assets.

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<sup>4</sup> This assumes taxes are a fixed percentage of taxable income.

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Capital structure gives information about the firm's long-term or permanent sources of capital. In the following balance sheet, we see the relationship between the long-term liabilities and the equity accounts.

**Figure 2: J&J Dogs—Balance Sheet, July 31, 200X (\$000)**

<b>Current assets</b>		<b>Current liabilities</b>	
Cash	\$2,434	Accounts payable	\$850
Inventory	1,150		
Supplies	750	<b>Long-term liabilities</b>	
Prepaid rent	1,000	Note payable	4,000
Prepaid insurance	3,000		
<b>Total current assets</b>	<b>8,334</b>		
<b>Fixed assets:</b>		<b>Total liabilities</b>	<b>\$4,850</b>
Equipment	3,100		
Less acc. depreciation	36	Owners' equity	6,548
Net fixed assets	3,064		
<b>Total assets</b>	<b>\$11,398</b>	<b>Total liabilities plus owners' equity</b>	<b>\$11,398</b>

The typical large corporation has several different issues of long-term debt (bonds) outstanding as well as a great deal of common stock and retained earnings. It is the relative proportions of these sources of financing that represent the capital structure of the firm.<sup>5</sup>

If a firm with \$1,000,000 in total assets is “financed” with \$500,000 long-term debt (usually simply referred to as debt) and \$500,000 common equity, we say the firm's debt ratio is 50% (i.e.,  $\$500,000/\$1,000,000 = 0.50$ ). Notice its equity ratio, the ratio of equity to total assets, is also 50%, and its debt-equity ratio is  $\$500,000/\$500,000 = 1.0$ .<sup>6</sup>

Capital structure will affect the rate at which the firm can borrow as well as the rate of return required by its equity holders. Moreover, management's choices for funding affect not only the firm's cost of capital but the value of the firm as well.

<sup>5</sup> Some argue the firm's capital structure includes all forms of permanent capital. This would include retained earnings, common stock, preferred stock, bonds, and any short-term sources that remain at a more or less constant level on the balance sheet. One short-term source could be trade payables that tend to stay at the same level or increase as the firm grows. In this case, the trade payables function as a permanent source of debt financing.

<sup>6</sup> These and other ratios are discussed and calculated in the Asset Valuation chapter.

A firm's **target capital structure** is the debt ratio that the firm tries to maintain over time and is typically similar to the average for the industry in which the firm operates. Should the firm's debt ratio fall below the target level, new capital needs will be satisfied by issuing debt. On the other hand, if the debt ratio is greater than the target level, the firm will raise new capital by retaining earnings or issuing new equity.

When setting its target capital structure, management must weigh the tradeoff between risk and return associated with the use of debt, since the use of debt increases the risk borne by both shareholders and bondholders. However, using debt also leads to higher expected rates of return for stockholders. The higher risk associated with debt may depress stock prices, while the higher expected return may increase stock prices. Thus, the firm's **optimal capital structure** is the one that balances the influence of risk and return and maximizes the firm's stock price. The optimal debt ratio will be the firm's target capital structure.

Several factors influence management's ability and desire to issue debt and, hence, the firm's optimal capital structure:

- Business risk is the risk inherent in the firm's basic operations due to the type of industry in which the firm operates. For example, a public utility could be viewed as fairly low risk. Compared to most industrial firms, a public utility firm's demand is fairly predictable. Therefore it is uncommon for utilities to face financial difficulties. On the other hand, a deep sea exploration firm might be considered very risky. Not only does the ship and crew continually face the perils of the open sea, the probability of finding anything of value is remote. Firms that face a high degree of business risk will typically maintain a larger proportion of equity in their capital structure.
- One of the reasons for using debt is the **tax shield**. Interest payments on debt issued by businesses are tax deductible in most countries, whereas dividends are typically not. Thus, the tax shield provided by debt financing lowers the effective cost of using debt relative to the cost of using equity. Other things equal, this would imply that firms should use as much debt as possible. However, when a firm has a low effective tax rate, additional debt is less advantageous.
- **Financial flexibility** refers to a firm's ability to go to the capital markets and raise funds at reasonable terms. If the firm already employs a good deal of debt, management might find it difficult to sell new debt.

## COST OF CAPITAL

On the right (liability) side of a firm's balance sheet, we have debt, preferred stock, and common equity, which are normally referred to as the **capital components** of the firm. Any increase in a firm's total assets will have to be financed through an increase in at least one of these components. The costs of the components are called the **component costs of capital**.



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We will focus on the three major capital components and their associated component costs:

- $k_d$  The interest rate (the required return) at which the firm can issue new debt; also, the yield to maturity on existing debt. This is the before-tax component cost of debt.
- $k_d(1 - t)$  After-tax cost of debt. Here,  $t$  is the firm's marginal tax rate. Remember, the interest payments made by a firm are tax deductible, which makes the after-tax cost of debt less than the before-tax cost.
- $k_{ps}$  Cost of preferred stock. Since all dividends, common and preferred, are issued out of net income (after-tax), they are not tax-deductible.
- $k_{ce}$  Cost of common equity. We can interpret this as the required rate of return on common stock. The cost of common equity is not directly observable and must be estimated.

**Cost of debt.** Since the interest expense on debt is tax-deductible, we consider the *after-tax cost of debt* [ $k_d(1 - t)$ ]. It is the interest rate at which the firm can issue new debt ( $k_d$ ) net of the tax savings resulting from the tax-deductibility of interest payments ( $k_d t$ ).

### Example: Cost of debt

J&J Dogs, Inc. is planning to issue new debt at a yield of 8%. J&J Dogs is in the 40% marginal tax rate. What is J&J Dogs' cost of debt capital?

**Answer:**

$$k_d(1 - t) = 8\%(1 - 0.4) = 4.8\%$$

**Cost of preferred stock.** Preferred stock is a perpetuity that pays a fixed dividend ( $D_{ps}$ ) forever. The cost of preferred stock [i.e., its required return ( $k_{ps}$ )]:

$$\text{price of preferred stock } P_{ps} = \frac{D_{ps}}{k_{ps}}$$

$$\text{required return } k_{ps} = \frac{D_{ps}}{P_{ps}}$$

where:

$D_{ps}$  = preferred dividend

$P_{ps}$  = price of preferred stock

### Example: Cost of preferred stock

Suppose J&J Dogs has preferred stock that pays an \$8 dividend per share and sells for \$95 per share. What is J&J Dogs' cost of preferred stock?

**Answer:**

$$k_{ps} = \frac{D_{ps}}{P_{ps}}$$

$$k_{ps} = \frac{\$8}{\$95} = 0.084 = 8.4\%$$

**Cost of common equity.** The cost of common equity ( $k_{ce}$ ) is the return that a firm's stockholders require on the equity that the firm retains from its earnings. If a stock is in equilibrium, the rate of return investors require is equal to the rate of return they expect to get.

The cost of common equity can be *estimated* using one of the following three approaches: the CAPM approach<sup>7</sup>, the bond yield plus risk premium approach, and the discounted cash flow approach.

#### 1. The capital asset pricing model (CAPM) approach.

*Step 1:* Estimate the risk-free rate,  $R_f$ . A short-term Treasury bill (T-bill) rate or a long-term Treasury bond rate can be used as an estimate of the risk-free rate.

*Step 2:* Estimate the stock's beta,  $\beta$ . This is the stock's systematic risk.

*Step 3:* Estimate the expected rate of return on the market ( $R_M$ ).

<sup>7</sup> The capital asset pricing model and the concept of systematic (beta) risk are explained in the chapter on Portfolio Management.

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*Step 4:* Use the CAPM equation to estimate the required rate of return.

$$k_{ce} = R_F + \beta(R_M - R_F)$$

### Example: Using CAPM to estimate $k_{ce}$

Suppose  $R_F = 6\%$ ,  $R_M = 11\%$ , and  $\beta = 1.1$ . What is the required rate of return for J&J Dogs' stock?

**Answer:**

$$k_{ce} = 6\% + (11\% - 6\%)(1.1) = 11.5\%$$

Results obtained with the CAPM approach should be interpreted with caution because (1) you must choose an appropriate estimate of the risk-free rate, (2) beta is an estimate of the security's risk, and (3) the market risk premium ( $R_M - R_F$ ) is difficult to estimate correctly.

## 2. The bond yield plus risk premium approach.

The method adds a risk premium (usually 3 to 5 percentage points) to the interest rate (required return) on the firm's long-term debt.

$$k_{ce} = \text{bond yield} + \text{risk premium}$$

### Example: Estimating $k_{ce}$ with bond yield plus a risk premium

J&J Dogs' required return on long-term debt is 8%. Suppose the risk premium is estimated to be 5%. What is J&J Dogs' cost of equity estimate?

**Answer:**

$$k_{ce} = 8\% + 5\% = 13\%$$

Since it is difficult to estimate the firm's appropriate risk premium,  $k_{ce}$  will be just a ballpark estimate.

### 3. The discounted cash flow approach.

If dividends are expected to grow at a constant rate,  $g$ , then the current price of the stock is given by the constant growth dividend valuation model:<sup>8</sup>

$$P_0 = \frac{D_1}{k_{ce} - g}$$

where:

$D_1$  = next year's dividend

$k_{ce}$  = the investor's required rate of return

$g$  = the firm's expected constant growth rate

Rearranging the terms, you can solve for  $k_{ce}$ ,

$$k_{ce} = \frac{D_1}{P_0} + g$$

But in order to use  $k_{ce} = \frac{D_1}{P_0} + g$ , you have to first estimate the expected growth rate  $g$ . This can be done by the following:

- Using a growth rate projected by security analysts.
- Using the following equation:

$$g = (\text{retention rate})(\text{ROE}) = (1 - \text{dividend payout rate})(\text{ROE})$$

#### Example: Estimating $k_{ce}$ using the dividend discount model

Suppose J&J Dogs' stock sells for \$21.00, next year's dividend is expected to be \$1.00, J&J Dogs' expected ROE is 12%, and J&J Dogs is expected to pay out 40% of its earnings. What is J&J Dogs' cost of equity?

**Answer:**

$$g = (\text{ROE})(\text{retention rate})$$

$$g = (0.12)(1 - 0.4) = 0.072 = 7.2\%$$

$$k_{ce} = \left( \frac{\$1}{\$21} \right) + 0.072 = 0.12 \text{ or } 12\%$$

<sup>8</sup> The constant growth model is explained in the chapter on Asset Valuation.

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Note that the three models give you three different estimates of  $k_{ce}$ :

CAPM → 11.5%  
Bond yield plus risk premium → 13.0%  
Discounted cash flow → 12.0%

Analysts must use their judgment in deciding which method is the most appropriate, depending upon the specific circumstances.

### Weighted Average Cost of Capital (WACC)

Generally, it is necessary to raise each type of capital (i.e., debt, common stock, or preferred stock) in large sums. The large issues may temporarily overemphasize the most recently issued capital in the firm's capital structure, but in the long run management will try to move toward target weights for each capital type.

Even though management typically issues only one type of capital at a time to fund operations and expansion, it is necessary to consider all the capital components when discussing the firm's cost of capital. Following is a simplified example of why this is necessary.

Assume a manager has \$100 in her pocket—\$50 from selling common stock, \$40 from selling bonds, and \$10 from selling preferred stock—and she starts a firm by purchasing several assets with the \$100. Can you determine which of those assets were purchased using cash received from selling common equity, which from debt, and which from preferred stock?

Obviously, it is impossible to allocate the capital sources to individual assets. The manager put all the cash together in her pocket and took out enough to pay for the assets, one at a time. Which specific funds were used each time is impossible to determine. Therefore, when we consider the cost of funds used to purchase the firm's assets, we must consider the average cost of the three capital components. We refer to the average cost of the funding sources as the firm's **weighted average cost of capital (WACC)**.

A firm's weighted average cost of capital is the weighted average of the individual component costs. The weight of each component is defined by the firm's capital structure:

$$\text{WACC} = (w_d)[k_d(1 - t)] + (w_{ps})(k_{ps}) + (w_{ce})(k_{ce})$$

where:

$w_d$  = the weight of debt in the capital structure

$w_{ps}$  = the weight of preferred stock in the capital structure

$w_{ce}$  = the weight of common equity in the capital structure

### Example: Computing WACC

Suppose J&J Dogs' target capital structure and its component costs of capital are as follows:

$$w_d = 0.45, w_{ps} = 0.05, \text{ and } w_{ce} = 0.50$$

$$k_d = 8.0\%, k_{ps} = 8.4\%, k_{ce} = 11.5\%$$

If J&J Dogs' marginal tax rate is 40%, what is its WACC?

**Answer:**

J&J Dogs' WACC is:

$$\begin{aligned} \text{WACC} &= (0.45)(0.08)(1 - 0.4) + (0.05)(0.084) + (0.50)(0.115) \\ &= 0.0833 \cong 8.3\% \end{aligned}$$

## CAPITAL BUDGETING

To maintain or expand its operating capabilities, every firm must weigh the costs and benefits of investing in their business. Evaluating and selecting potential investments (projects) is known as **capital budgeting**. These decisions may relate to building a new facility, buying a new machine, or acquiring another firm. The basic question to address is whether the potential return on the project justifies the initial investment (i.e., its cost), and will increase the long-term value of the firm. To determine which projects are worthwhile, we must first assess the cash flows of the project.

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In general, an investment project will require an initial investment to undertake the project, and then provide positive cash flow benefits during its working life. The tradeoff between the cash costs and benefits of the project will determine its relative attractiveness.

Initiating a project requires that a firm invest in certain assets necessary to complete the project. The firm may need to purchase a new piece of machinery, update a warehouse, hold a higher balance of accounts receivable and accounts payable, or utilize undeveloped real estate. The sum of these initial costs is called the **net investment**. The net investment may involve *direct costs* such as purchasing and installing an asset or *opportunity costs* such as using a previously idle asset instead of selling it in the market.

No rational firm would undertake an investment project unless the project provided a positive cash flow benefit. This cash benefit can come in the form of incremental (i.e., additional) cash inflow to the firm or incremental cash savings that would go unrealized without the project. The incremental cash benefit must be considered, however, net of any expenses and investments required to operate the project. These expenses may include operating costs, taxes, and investments in net working capital or fixed assets to support the project. The cash flow benefit after subtracting out expenses and continued investment is referred to as the project's **net cash flow**.

A project's cash flows must be evaluated on an incremental, after-tax basis. We are only interested in the cash flows that the project adds to the firm's total cash flow, not in the total cash flow itself. Also, taxes have a real cash implication and must be accounted for in the estimation of cash flows for the project.

Let's return to J&J Dogs to provide a capital budgeting cash flow example. After becoming a publicly traded corporation, J&J Dogs has been presented with two alternative investment projects.

Project A is an **expansion project** that entails the purchase of equipment designed to increase the company's current production capabilities. The net investment for Project A is \$500,000 and includes the cost of the machinery, shipping and installation costs, and an increase in net working capital. Project A is expected to increase the company's cash flows by \$100,000 in year 1, \$200,000 in year 2, and \$275,000 in years 3 through 5 of the project.

Project B is a **replacement project** that entails the replacement of an old machine with a new machine that will significantly reduce the company's operating costs. Project B will require a net investment of \$600,000 for the new machine, shipping, and installation. The project will provide a cash savings of \$245,000 per year for five years.

A comparison of the cash flows associated with Project A and Project B is provided in Figure 3.

**Figure 3: J&J Dogs—Project A & Project B Cash Flows**

<i>Undiscounted Values</i>		
<i>Year</i>	<i>Project A</i>	<i>Project B</i>
0	−\$500,000	−\$600,000
1	100,000	245,000
2	200,000	245,000
3	275,000	245,000
4	275,000	245,000
5	275,000	245,000
<b>Total</b>	<b>\$625,000</b>	<b>\$625,000</b>

According to Figure 3, we should be equally satisfied between investing in Project A and investing in Project B. The total net cash flows for the two projects are exactly the same. However, we have failed to take into account the time value of money and the timing of the cash flows. Therefore we are making an apples-to-oranges comparison. The capital budgeting process cannot be completed without an objective method to evaluate projects that take into account all of the project's cash flows, the project's level of risk, and the time value of money. We will examine two such methods in the next sections as well as a third method that, while commonly used, does not meet all of the criteria just listed. The three methods are the net present value (NPV) method, the internal rate of return (IRR) method, and the payback period method.

## Net Present Value

A project's **net present value** (NPV) is equal to the sum of the present values of the net cash flows minus the net investment required to initiate the project. To determine the present value of the project's future net cash flows, a discount rate must be used to discount the future net cash flows back to the present. If the risk of the project is equal to the overall risk of the company, then the marginal WACC (i.e., the cost to raise more debt and equity capital in their current proportions) can be used as the discount rate. If the project is more or less risky than the company as a whole, then an adjustment to the marginal WACC will be necessary.

In Figure 3 we presented the cash flows for two projects under consideration by J&J Dogs. Direct comparison of the two projects was not possible, however, because the cash flows were not stated on a comparable basis. We can restate the cash flows from the two projects on a present value basis by assuming that the appropriate



## Corporate Finance

discount rate for both projects is equal to J&J Dog's current marginal WACC of 10%. We will not go into detail on the present value calculations since this topic was covered in the chapter on Quantitative Methods. Just remember that each cash flow can be discounted as a lump sum value. The present value of each cash flow is summarized in Figure 4.

**Figure 4: J&J Dogs—Present Value of Cash Flows from Projects A & B**

Year	Present Values	
	Project A	Project B
0	-\$500,000	-\$600,000
1	90,909	222,727
2	165,289	202,479
3	206,612	184,072
4	187,829	167,338
5	170,753	152,126
NPV	\$321,392	\$328,743

Figure 4 shows the present value of each of the cash flows associated with Project A and Project B. For example, the present value of the year 3 cash flow for Project A is:

$$PV_{CF3,A} = \frac{275,000}{(1.10)^3} = 206,612$$

The net present value of each project is simply the sum of the present value of each project's cash flows, including the net investment. In the preceding table, the net investment is represented as a negative value at time zero. Remember the net investment is a cash outflow that reduces the NPV of the project.

According to Figure 4, J&J Dogs should undertake Project B, since it has a higher NPV and will add more value to the firm than Project A. If enough capital were available, however, the company could accept both projects. The reason is simple: any project with a positive NPV adds value to the firm and is acceptable.

This reasoning applies to all firms undergoing the capital budgeting process. A firm should rank, from highest to lowest, all projects with a positive NPV and, starting with the highest NPV project, select as many projects as the firm's capital budget will allow. Projects with a negative NPV should be rejected since these projects would reduce the overall value of the firm.<sup>9</sup> Projects with an NPV equal to zero can either be accepted or rejected but will not change the value of the firm.

<sup>9</sup> In some special cases, such as projects required for the firm to comply with a regulation, a firm may have to accept a project with a negative NPV. The firm can still use NPV analysis to identify the least costly alternative.

## Internal Rate of Return

The **internal rate of return** (IRR) of any project is its expected return, given its cost and expected cash flows. That is, IRR is the discount rate that makes the present value of the expected cash inflows equal to the cost of the asset and makes the NPV zero. Mathematically, we can represent the IRR as the discount rate that makes the following true:

$$\text{net investment} = \text{sum of the present values of cash inflows}$$

IRR can be determined using a financial calculator (recommended) or by trial and error. The trial and error method is performed by discounting the project's cash inflows at various discount rates until you find the one that satisfies the formula. If you use trial and error in our J&J Dogs example, you would start with a rate greater than 10% because 10% generated a positive NPV for both Project A and Project B.<sup>10</sup> However, it is much simpler and faster to use a calculator to find the IRR of a project.

Using a financial calculator, the IRR for Project A is 29.2% and the IRR for Project B is 29.7%. In Figure 5, you will observe that when we discount the cash inflows from Projects A and B at 29.2% and 29.7%, respectively, the net present values of both projects fall to zero.

**Figure 5: J&J Dogs—NPV of Projects A & B Using the IRR as the Discount Rate**

	29.2%	29.7%
Year	Project A	Project B
0	−\$500,000	−\$600,000
1	\$77,410	\$188,877
2	\$119,845	\$145,611
3	\$127,562	\$112,255
4	\$98,745	\$86,541
5	\$76,438	\$66,716
NPV	\$0	\$0

How should we interpret the IRR? As stated earlier, the IRR is the expected return on a project, given its cost and expected cash flows. This means that if J&J Dogs spent \$500,000 on Project A and \$600,000 on Project B and the cash flows from each project came as expected, Project A would generate a return of 29.2% and Project B would generate a return of 29.7%. Since both projects are expected to

<sup>10</sup> Remember the relationship of discount rates and present values. Since discounting at 10% yielded a present value for each project greater than the project's net investment, the discount rate would have to be increased to reduce the net present value to zero.

## Corporate Finance

generate a return greater than J&J Dogs' cost of capital of 10%, they are both acceptable investments and can be expected to increase the value of the firm.

Similar to the NPV rule, the IRR decision process applies to all firms undergoing the capital budgeting process. A firm should accept projects with an IRR greater than the cost of capital and reject projects with an IRR less than the cost of capital. Projects with an IRR equal to the cost of capital can either be accepted or rejected but will not change the value of the firm.

We cannot conclude, however, that the project with the highest IRR will add the most value to the firm. Differences in the sizes of projects and the timing of their cash flows can cause one project to have a higher IRR but a lower NPV compared to another. For that reason, when choosing among *mutually exclusive* projects (that is, when a firm can accept only one project from two or more alternatives), a firm should rely on the NPV method.

The IRR method is most useful for projects that have a *conventional cash flow pattern*, with an initial cash outflow followed by a series of cash inflows. Because the sign changes only once, the project has a single IRR. However, if the sign on a project's cash flows changes more than once (for example, if it has a disposal cost at the end of its life), the project can have *multiple IRRs*. Mathematically, each change in sign can produce another discount rate at which the NPV is zero. We don't have any good way to decide which of these IRRs is "correct."

## Payback Period

The **payback period** (PBP) is the number of years it takes to recover the initial cost of an investment. In other words, PBP is the time required for a project's net cash flows to repay the net investment. The formula used to calculate the payback period is as follows:

$$\text{PBP} = \text{years until full recovery} + \frac{\text{unrecovered cost at the beginning of full recovery year}}{\text{cash flow during full recovery year}}$$

An example using the net cash flows from J&J Dogs will help demonstrate the concept. Figure 6 lists the net cash flows expected from Projects A and B as well as the cumulative cash flows.

Figure 6: J&amp;J Dogs—Cumulative NCF of Projects A &amp; B

Year	Project A		Project B	
	NCF	Cum. NCF	NCF	Cum. NCF
0	−\$500,000	−\$500,000	−\$600,000	−\$600,000
1	100,000	−400,000	245,000	−355,000
2	200,000	−200,000	245,000	−110,000
3	275,000	75,000	245,000	135,000
4	275,000	350,000	245,000	380,000
5	275,000	625,000	245,000	625,000

You can see from the table that each project will take somewhere between two and three years to recover the initial investment cost. We observe this fact by noting that the cumulative net cash flows for both Project A and B turn positive in year 3. To be more exact, we calculate the payback period for both projects as follows:

$$\text{Project A: } \text{PBP} = 2 + \frac{200,000}{275,000} = 2 + 0.73 = 2.73 \text{ years}$$

$$\text{Project B: } \text{PBP} = 2 + \frac{110,000}{245,000} = 2 + 0.45 = 2.45 \text{ years}$$

In order to evaluate a potential project using the PBP, a firm would arbitrarily select a maximum payback period. Any project with a PBP greater than the maximum would be considered unacceptable. If J&J Dogs were only willing to accept projects with a PBP less than two and a half years, project B would be selected. In this example, Project B has the highest NPV and would add the most value to the firm (J&J Dogs got lucky). However, it is possible that a project with the lowest PBP could also have the lowest NPV.

As noted earlier, the payback period does not consider the risk of the project or the time value of money associated with the cash flows. As such, it is inferior to the NPV and IRR methods as a method of project evaluation.

## BREAKEVEN ANALYSIS

Breakeven analysis is a tool used by firms to quantify the effects of **operating leverage** on their investment projects and on the firm as a whole. Recall that operating leverage is the trade-off between variable costs and fixed costs. Operating leverage amplifies the earnings of the firm. High operating leverage will make years of good profitability look even better and years of poor profitability look even worse. These effects are created by the presence of fixed costs in the operating structure.

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Fixed costs cannot be changed in the short run. The costs must be paid no matter how many units of a particular product the firm chooses to produce. Variable costs, on the other hand, depend on the number of units the firm produces. If no production is undertaken, no variable costs are incurred. To make a profit, the firm must sell enough of its product to cover both variable and fixed costs. The level of sales at which a firm covers all of its fixed and variable costs is called the **breakeven point**. The breakeven sales quantity,  $Q_{BE}$ , can be defined as follows:

$$\text{sales revenue} = \text{operating costs}$$

or:

$$(\text{price per unit})(\text{quantity}) = (\text{variable cost per unit} \times \text{quantity}) + \text{fixed costs}$$

or:

$$PQ = VQ + F$$

At breakeven, operating profit is equal to zero:  $PQ - VQ - F = 0$ . Therefore  $Q_{BE} = F / (P - V)$ . We have simply solved for  $Q$  which leaves us with a ratio of fixed costs to what is known as the **contribution margin** ( $P - V$ ). The breakeven quantity of sales is the point at which operating profit (also known as EBIT) is equal to zero. Sales of a quantity greater than  $Q_{BE}$  will result in positive operating profit for the firm. Sales of a quantity less than  $Q_{BE}$  will result in operating losses for the firm.

To make the concept more concrete, let's revisit the two projects under consideration by J&J Dogs. J&J Dogs sells its product for a price of \$4.00 per hot dog. If selected, Project A would incur variable costs of \$3.00 per unit and total fixed costs of \$40,000. Project B, on the other hand, would only incur variable costs of \$2.00 per unit but total fixed costs would be \$120,000. Figure 7 summarizes all of this information.

**Figure 7: J&J Dogs—Project Costs**

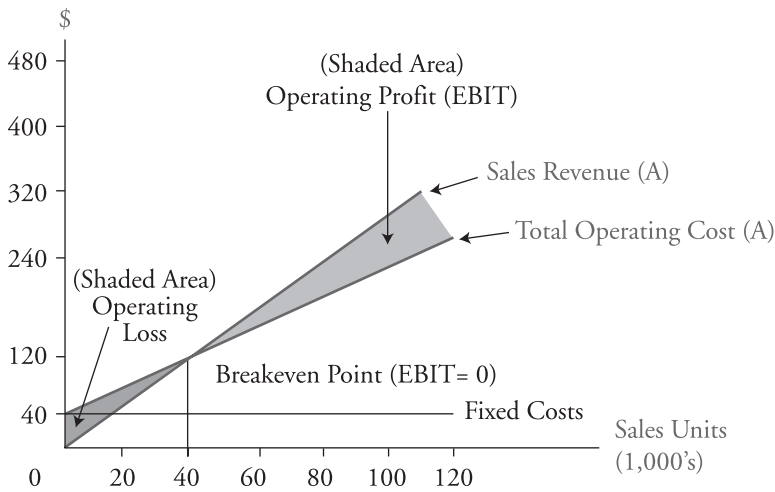
	<i>Project A</i>	<i>Project B</i>
Price	\$4.00	\$4.00
Variable costs	\$3.00	\$2.00
Fixed costs	\$40,000	\$120,000
Assets	\$500,000	\$600,000

For Project A, the breakeven quantity of sales is:

$$Q_{BE} = \$40,000 / (\$4.00 - \$3.00) = 40,000 \text{ units}$$

The breakeven quantity and the relationship between sales revenue, total operating cost, operating profits, and operating losses for Project A are illustrated in Figure 8.

**Figure 8: Breakeven Analysis for Project A**

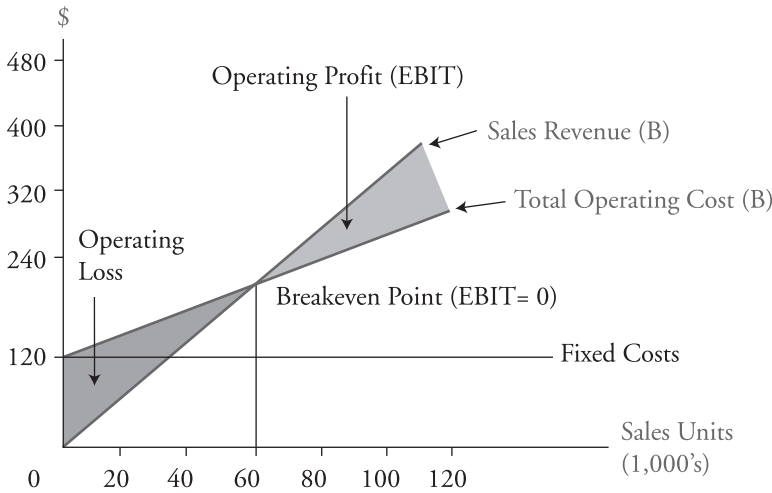


For Project B, the breakeven quantity of sales is:

$$Q_{BE} = \$120,000 / (\$4.00 - \$2.00) = 60,000 \text{ units}$$

The breakeven quantity and the relationship between sales revenue, total operating cost, operating profits, and operating losses for Project B are illustrated in Figure 9.

Figure 9: Breakeven Analysis for Project B



The calculations and figures given above demonstrate that Project B has more operating leverage than Project A. We observe this from the higher breakeven quantity of sales associated with Project B.

J&J Dogs, and any firm for that matter, should be concerned with the operating leverage associated with potential investment projects. If the firm already has a high level of fixed costs and thus operating leverage, the firm must carefully consider whether to take on projects that also have a high degree of operating leverage.

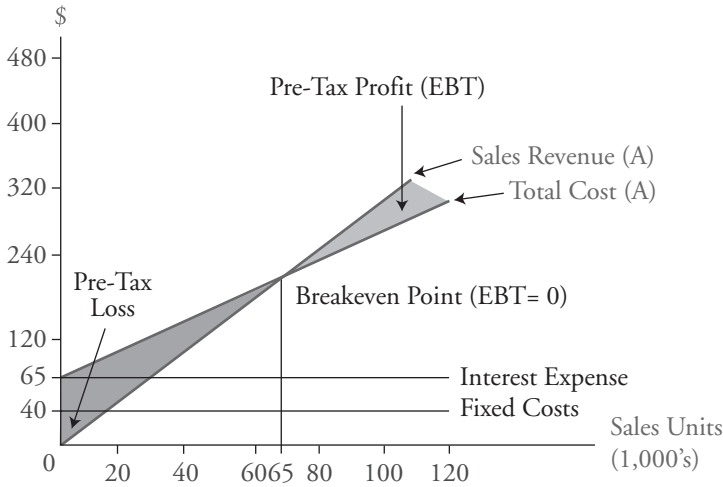
We can also assess the effects of using **financial leverage** in the capital structure. In the previous examples, J&J Dogs did not use debt to finance the proposed projects. Now let's assume that they finance one-half of each project with debt having a required return of 10%. The net result is that the firm would now have an annual fixed interest expense of \$25,000 (= \$500,000 × 0.5 × 0.10) if they accepted Project A, and \$30,000 if they accepted Project B.

The effect of the interest expense is to increase the fixed costs that must be covered to reach the breakeven level of EBT (earnings before taxes = EBIT – interest expenses).

For Project A, the new breakeven quantity of sales is:

$$Q_{BEA} = (\$40,000 + 25,000) / (\$4.00 - \$3.00) = 65,000 \text{ units}$$

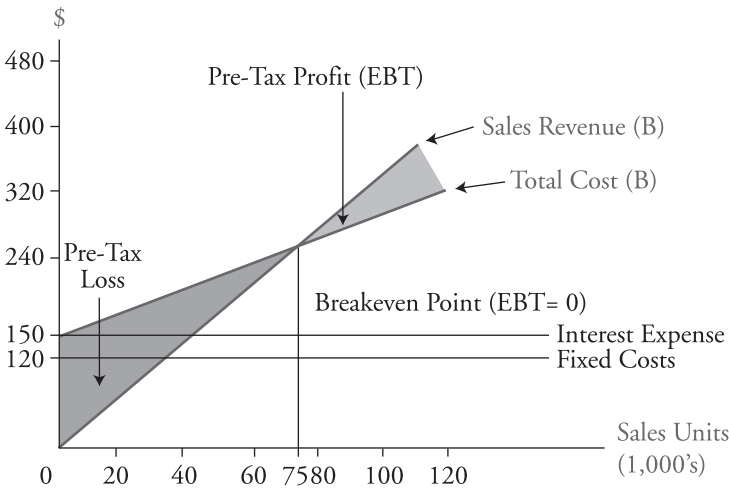
Figure 10: Breakeven Quantity for Project A with Interest Expense



For Project B, the new breakeven quantity of sales is:

$$Q_{\text{BEB}} = (\$120,000 + 30,000) / (\$4.00 - \$2.00) = 75,000 \text{ units}$$

Figure 11: Breakeven Quantity for Project B with Interest Expense



The effect of using financial leverage is that all of the business risk is now concentrated on a smaller pool of equity capital (because one-half of each project is financed with debt). We can illustrate this by assuming unit sales of 100,000 for Projects A and B, and a tax rate of 40%.



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EBT, net income, and ROE without financial leverage:

$$\text{EBT Project A: } 100,000 (\$4.00 - \$3.00) - \$40,000 - \$0 = \$60,000$$

$$\text{Net income Project A: } \$60,000 \times (1 - 0.4) = \$36,000$$

$$\text{ROE}_A: \$36,000 / \$500,000 = 7.2\%$$

$$\text{EBT Project B: } 100,000 (\$4.00 - \$2.00) - \$120,000 - \$0 = \$80,000$$

$$\text{Net income Project B: } \$80,000 \times (1 - 0.4) = \$48,000$$

$$\text{ROE}_B: \$48,000 / \$600,000 = 8.0\%$$

EBT, net income, and ROE with financial leverage:

$$\text{EBT Project A: } 100,000 (\$4.00 - \$3.00) - \$40,000 - \$25,000 = \$35,000$$

$$\text{Net income Project A: } \$35,000 \times (1 - 0.4) = \$21,000$$

$$\text{ROE}_A: \$21,000 / \$250,000 = 8.4\%$$

$$\text{EBT Project B: } 100,000 (\$4.00 - \$2.00) - \$120,000 - \$30,000 = \$50,000$$

$$\text{Net income Project B: } \$50,000 \times (1 - 0.4) = \$30,000$$

$$\text{ROE}_B: \$30,000 / \$300,000 = 10.0\%$$

Because the projects are both generating a level of sales in excess of their respective breakeven points, using financial leverage has resulted in an increase in the return on equity in both cases. If sales were less than the breakeven level, the losses would have been magnified relative to what they would have been without the use of financial leverage.

The bottom line is that the use of financial leverage does not change the degree of business risk present, but it does affect the firm's overall level of risk. Therefore, the appropriate degree of financial leverage will depend on the firm's inherent level of business risk and management's objectives for total firm risk.

## SUMMARY

### FORMS OF BUSINESS

- A. Proprietorship.
  - 1. A proprietorship is the easiest and least expensive way to start a business.
  - 2. The business and the owner are indistinguishable, with all profits and losses “flowing through” to the owner.
  - 3. It can be difficult to get necessary operating capital or capital for expansion, as collateral for loans is limited to assets in the business and assets held by the owner.
  - 4. The owner has unlimited liability, being personally responsible for any debts incurred by the business. If business assets are insufficient to satisfy business or legal claims, the owner’s assets can be seized.
  - 5. The life of the proprietorship is linked to the life of the proprietor.
- B. Partnership.
  - 1. A partnership is easy to form and is only slightly more expensive to form than a proprietorship.
  - 2. There can be two or more partners, but all will be bound by legal documentation that clearly specifies the proportional ownerships.
  - 3. Legally, the business and the owners are indistinguishable. All profits and losses “flow through” to the owners according to their proportional ownership.
  - 4. Operating and expansion capital are somewhat easier to obtain for a partnership than for a proprietorship.
  - 5. The owners have unlimited liability and are personally responsible for any debts incurred by the business.
  - 6. The life of the partnership is linked to each of the lives of the partners. If any partner dies, the partnership dies also.
- C. Corporation.
  - 1. Corporations can have one or more owners (usually a large number of owners).
  - 2. Regardless of the number of owners, all owners have limited liability.
  - 3. All forms of corporations have unlimited life (i.e., if an owner dies, his or her stock becomes part of the estate).
  - 4. The ease of raising capital depends upon the size of the corporation. It is usually much easier for large corporations to raise capital.
  - 5. Common stock of a corporation may pay dividends. Dividends are usually paid quarterly out of net income (after taxes) and require stockholders to pay taxes on dividends received, resulting in double taxation.

## AGENCY COSTS

- A. Agency costs are the actual or implied costs associated with the separation of ownership and management.
  - 1. Managers are the owners' agents (i.e., they operate the firm in the absence of the owners).
  - 2. The goals of the managers might not agree with those of the owners.
    - a. Owners want management to take risky projects to increase the expected return on the firm's stock.
    - b. Owners can diversify away the unsystematic risk by holding the firm's stock in a portfolio.
  - 3. Managers are concerned with the owners' wishes, but also with maintaining their employment.
    - a. If the firm does well, management and owners do well.
    - b. If the firm does poorly, the owners are partially protected by diversification, but management loses its jobs.
- B. There is also an agency relationship between stockholders and bondholders.
  - 1. Bondholders receive fixed interest payments and the face value of the bonds at maturity.
  - 2. Bondholders want the firm to maintain a low level of risk.
    - a. High firm risk increases stock prices but reduces bond prices.
    - b. Bondholders are concerned management will increase the firm's risk once they have purchased the bonds. The value lost by the bondholders is captured by the stockholders.

## RISK AND RETURN

- A. Risk is the possibility of an unfavorable event (usually a lower-than-expected return on the investment). Unfavorable events can include the following:
  - 1. Receiving a cash flow later than expected.
  - 2. Receiving a cash flow smaller than expected.
  - 3. Not receiving an expected cash flow.
- B. Risk in publicly traded securities is caused by microeconomic and macroeconomic factors.
  - 1. Microeconomic factors are characteristics of the issuing firm and the security itself. These cause the unsystematic or diversifiable risk of the security.
  - 2. Macroeconomic factors are economy-wide forces such as inflation. These cause the systematic or non-diversifiable risk of the security.

- C. Firm-specific factors consist of business risk and financial risk.
1. Business risk is due to characteristics of the industry (i.e., the type of business in which the firm operates).
    - a. Firms in the industry might be subject to variable revenues.
    - b. The firm's cost and pricing structures might not be flexible.
    - c. New product development might be slow.
    - d. Use of fixed assets causes operating leverage; the greater the amount of fixed assets, the higher the operating leverage.
    - e. Operating leverage is measured as the percentage change in EBIT given a percentage change in revenues.
  2. Financial risk is due to the utilization of fixed-obligation sources of financing, primarily bonds. Financial leverage results from the use of debt:
    - a. The more debt used and the greater the fixed obligation, the higher the financial leverage.
    - b. Financial leverage is measured as the percentage change in net income given a percentage change in EBIT.
- D. Capital structure refers to the way management has paid for the firm's assets.
1. Any long-term form of capital is considered part of the capital structure. Usually this means common stock, preferred stock, and bonds.
  2. Any permanent form of capital should be considered. Short-term sources of capital should be considered if they are significant in amount and maintained at a more or less constant level.
- E. The firm's capital structure affects the firm's cost of capital. The firm's cost of capital is a weighted average of the costs (returns required by investors) of the firm's sources of capital: common stock, long-term debt, and preferred stock.
- F. A firm's target capital structure is the debt-to-equity ratio that the firm tries to maintain over time. Several factors influence the firm's ability and desire to issue debt and, hence, its target capital structure.
1. Business risk is the risk inherent in the firm's basic operations due to the type of industry in which the firm operates. The greater the firm's business risk, the lower its optimal debt ratio, so risky activities like deep sea exploration would typically be financed with equity rather than debt.
  2. Using debt creates a tax shield. That is, interest payments on debt issued by firms are tax deductible, which lowers the effective cost of using debt.
  3. Firms maintain financial flexibility by not over-utilizing debt.
- G. The firm's cost of capital is the rate it must pay on the various capital components used to fund its assets:
1.  $k_d$  is the component cost of debt, equal to the required rate of return on new debt or the yield to maturity on existing debt.
  2.  $k_d(1 - t)$  is the after-tax cost of debt.

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3.  $k_{ps}$  is the component cost of preferred stock.

4.  $k_{ce}$  is the component cost of common equity.

H. The component cost of preferred stock is found by dividing its price by its annual dividend.

$$k_{ps} = \frac{D_{ps}}{P_{ps}}$$

I. The firm's cost of common equity can be estimated using different methods:

1. The capital asset pricing model (CAPM) approach.

$$k_{ce} = R_F + \beta(R_M - R_F)$$

2. The bond yield plus risk premium approach.

3. The discounted cash flow approach.

$$k_{ce} = \frac{D_1}{P_0} + g$$

a. Use the growth rate as projected by security analysts.

b. Estimate  $g$  using:

$$\begin{aligned} g &= (\text{retention rate})(\text{return on equity}) \\ &= (1 - \text{payout rate})(\text{ROE}) \end{aligned}$$

J. The firm's weighted average cost of capital (WACC) is the weighted average of the individual component costs. The weight of each component is its weight in the firm's target capital structure:

$$\text{WACC} = (w_d)[k_d(1 - t)] + (w_{ps})(k_{ps}) + (w_{ce})(k_{ce})$$

## CAPITAL BUDGETING

A. Capital budgeting is the planning process for allocating funds to long-term projects, such as expanding operations.

B. An essential step in the capital budgeting process is the estimation of the cash flows expected from potential capital investment projects. Only incremental (i.e., resulting from acceptance of the project) cash flows should be considered.

1. The cost to initiate a project is called the net investment and generally includes the cost to acquire new assets as well as investments in net working capital.

2. Cash benefits come in the form of positive cash flow from increased revenue and profits or in the form of substantial cost savings.

- C. Net present value (NPV) compares the cost of an asset to its expected cash flows.
1. The present value of expected cash flows is found by discounting them at the required return.
  2. The NPV is the difference between the total present value of expected inflows and the cost of the asset.
  3. There are three basic rules to remember with NPV:
    - a. A positive NPV indicates the project will add value to the firm.
    - b. A negative NPV indicates the project will subtract value from the firm.
    - c. A zero NPV indicates the project will meet its cost of capital but neither adds value to nor subtracts value from the firm.
- D. The internal rate of return (IRR) is the asset's expected return and is the discount rate that makes the present value of the inflows equal to its cost (i.e.,  $NPV = 0$ ).
1. An IRR greater than the cost of capital indicates the project will add value to the firm.
  2. An IRR less than the cost of capital indicates the project will subtract value from the firm.
  3. An IRR equal to the cost of capital indicates the project will neither add value to nor subtract value from the firm.
- E. The payback period (PBP) is the length of time required to recover a project's net investment. PBP is arbitrary and not recommended as a stand-alone method of evaluating capital budgeting projects.

### BREAKEVEN ANALYSIS

- A. Operating breakeven analysis is used to determine the quantity of sales that will just cover a project's fixed and variable operating costs and as such indicates the degree of operating leverage of the project.
1. The breakeven quantity of sales results in an operating profit of zero.
  2. Sales in excess of the breakeven quantity result in operating profits.
  3. Sales below the breakeven quantity result in operating losses.
- B. Firms that already employ a high degree of operating leverage should use caution when considering additional projects with high breakeven levels.
- C. Financial breakeven analysis includes interest cost as a fixed expense, and the firm is at the breakeven point when  $EBT (= EBIT - I)$  and net income are equal to zero.
- D. Financial leverage increases the unit sales quantity required to breakeven, and magnifies the effects of operating leverage.
- E. Managers should use financial leverage to magnify the firm's existing operating leverage only if they are comfortable with the resulting total firm risk.

**PRACTICE QUESTIONS: CORPORATE FINANCE**

1. All of the following are characteristics related to the ownership of a corporation EXCEPT:
  - A. the amount of control exerted by any individual over the firm is equal to the percentage of shares owned by the individual.
  - B. ownership is easily transferable in a public market.
  - C. owners of the firm generally are entitled to vote on major issues regarding the firm's management.
  - D. limited liability applies only to the minority shareholders.
  
2. Following are four statements given at a symposium on business structure. Which of the four statements *correctly* describes various attributes of the different forms of business?
  - A. A proprietorship offers the greatest level of owner control over business decisions but is also the most complicated business structure.
  - B. In a partnership the owners always share equal control over the operating decisions, making it the most complicated type of business.
  - C. A corporation is the most difficult business structure to form, and each owner has little influence on operating business decisions.
  - D. An owner's control over operating decisions is the same in every business structure, but corporations are the most difficult to form.
  
3. VolTech Inc. is considering a change in its capital structure, which includes only debt and equity. The company has decided to increase the proportion of debt financing in order to move the capital structure toward what the company perceives as the optimal structure. The company has also recently experienced a decline in its marginal tax rate. Assuming the costs debt and equity capital remain unchanged, what effect will the change in capital structure and tax rate have on VolTech's weighted average cost of capital?

<u>Capital structure</u>	<u>Tax rate</u>
A. Increase	Increase
B. Decrease	Increase
C. Decrease	Decrease
D. Increase	Decrease
  
4. Which of the following statements is FALSE?
  - A. A corporation is not legally required to pay dividends.
  - B. Dividends are subject to double taxation.
  - C. Liquidating dividends get paid out of retained earnings.
  - D. The Board of Directors declares dividends.

5. Williams Corporation has recently enacted a plan to hire a review board for the firm's senior management team. The review board will be responsible for ensuring that the senior managers stop using company resources, such as the corporate jet, for personal use. Which of the following is FALSE about Williams Corporation?
- A. The cost of hiring the review board for the management team is an explicit agency cost.
  - B. The review board is likely to increase management's operating efficiency.
  - C. Management's excessive use of company assets is an agency cost to the firm.
  - D. The firm's stockholders bear the agency costs associated with the firm's current situation.
6. Shamus Fitzgerald is a partner in a company that provides cleaning services to office buildings. When the company was formed as a partnership three years ago, Shamus provided \$5,000 in equity capital but has not provided any equity investment since then. Today, the company has a total of \$27,000 in common equity capital. The company also has \$11,000 in long-term debt. Using this information, determine which of the following statements is TRUE.
- A. If the company fails, Shamus' personal assets cannot be used to repay the company's obligations.
  - B. Incorporating the company would increase the Shamus' liability exposure.
  - C. Shamus is personally responsible for \$2,037 of the company's long-term debt.
  - D. In the event of company failure, Shamus is only responsible for repaying \$5,000 of the company's outstanding liabilities.
7. Sylvia Hall is estimating changes in the level of risk related to Tyler Corporation. Hall believes that several microeconomic factors have a direct influence on the riskiness of Tyler Corp. and has developed a model predicting the risk level using microeconomic factors. Which of the following should NOT be included as a factor in Hall's model?
- A. Unanticipated changes in the rate of inflation.
  - B. A change in the firm's management.
  - C. An unexpected decline in the economic viability of some of Tyler Corp.'s assets.
  - D. An increase in the firm's cost to acquire raw materials.



## Corporate Finance

8. Forrest, Inc. wants to amplify its ability to convert revenue increases into increases in net income. Which of the following *accurately* describes a method Forrest could use to accomplish this goal?
- A. Increase the financial leverage of the firm.
  - B. Utilize a smaller proportion of fixed assets in the asset structure.
  - C. Decrease the level of debt expense on the income statement.
  - D. Shift the firm's cost structure to include a greater proportion of variable costs.
9. Samantha Hurley assesses risk for Kowel & Associates, a risk management consulting firm. Hurley has observed that one of Kowel & Associates' most important clients is exposed to a large degree of risk related to two factors: (1) changes in the overall level of interest rates and (2) changes in the number of competing products. Hurley's risk estimates indicate that the client's risk rating has exceeded acceptable levels. What advice should Hurley give the client?
- A. Reduce the systematic risk related to competing products, and reduce the total risk related to interest rates.
  - B. Reduce the unsystematic risk related to competing products, and reduce the macroeconomic risk related to interest rates.
  - C. Reduce the unsystematic risk related to competing products, and reduce the stand-alone risk related to interest rates.
  - D. Reduce the stand-alone risk related to competing products, and reduce the macroeconomic risk related to interest rates.
10. Charleston, Inc. just experienced a decrease in revenue of 10%. In response, operating income and net income both decreased by 25%. Which of the following is TRUE about Charleston, Inc.?
- A. The firm has high operating leverage but no financial leverage.
  - B. The firm has no operating leverage but high financial leverage.
  - C. The firm has high operating leverage and high financial leverage.
  - D. The firm has no operating leverage and no financial leverage.
11. Which of the following statements about a corporation declaring a cash dividend is TRUE?
- A. The cash dividend must be approved for payment by the shareholders.
  - B. The cash dividend is a current liability to the corporation when declared.
  - C. The cash dividend does not affect the corporation's working capital.
  - D. The cash dividend is taxed as capital gains.

12. Bill Garrison has been asked to explain the basics of leverage and a firm's capital structure to a group of novice investors. Which of Garrison's statements *accurately* describes leverage and capital structure?
- A. If a firm does not rely heavily on operating leverage, then the firm's fixed costs are a relatively small proportion of total costs.
  - B. An increase in the corporate tax rate will not change the way a corporation raises capital.
  - C. A firm exposed to significant business risk is more likely to use financial leverage than a firm with little business risk.
  - D. If a firm increases its financial leverage, the operating leverage will increase in a similar proportion.
13. Pieter Reinhardt recently made the following statements to a group of investors regarding agency costs. Which of the statements does NOT accurately describe an aspect of agency costs?
- A. Agency costs arise as a result of management's desire to use company assets for its personal gain and the owners' desire to maximize the value of the firm.
  - B. Agency costs are minimized when the owners of a firm and the management of a firm are the same.
  - C. Agency costs arise as a result of stockholders' desire to increase the price of the common stock and debtholders' desire to secure interest payments.
  - D. Agency costs do not have any impact on the overall value of the firm.
14. Green & Company is a manufacturer of flooring materials including carpets and hardwoods. Recently, management at Green & Company announced its intention to finance a new investment project without altering the current capital structure of the firm. Which of the following *correctly* explains Green & Company's announcement?
- A. Green & Company intends to maintain the ratio of current liabilities to total liabilities as it initiates the new investment project.
  - B. Any equity required to finance the new investment project will only come from retained earnings.
  - C. Green & Company intends to maintain the ratio of current assets to total assets as it initiates the new investment project.
  - D. The assets required to initiate the investment project will be acquired without altering the proportions of debt and equity capital currently on the balance sheet.

## Corporate Finance

15. Madison Foods Company currently has \$2,500,000 of debt on its balance sheet and \$7,500,000 of equity. Madison has calculated that its long-term target capital structure should be 30% debt and 70% equity. Given the company's current capital structure, which of the following is TRUE?
- A. Any projects undertaken in the future will need to be financed with 70% equity.
  - B. Any projects undertaken in the future will need to be financed with more than 30% debt.
  - C. Any projects undertaken in the future will need to be financed with less than 30% debt.
  - D. Any projects undertaken in the future will need to be financed with more than 70% equity.
16. Which of the following is FALSE regarding dividends?
- A. The market price of a preferred stock is influenced by its dividend payout.
  - B. Stockholders pay double income taxes on the dividends they receive.
  - C. A utility company normally pays larger dividends.
  - D. Dividends are the portion retained by a company to grow operations.
17. Jerry Matthews is reviewing a report written by several managers reporting directly to him. The report details opportunities to be considered in the capital budgeting process this year. Which of the following opportunities listed by the managers is NOT a capital budgeting matter?
- A. The firm needs to replace certain pieces of its assembly line machinery but does not need to replace the entire line.
  - B. The firm can save significant amounts of money by increasing the relative proportion of equity on the balance sheet.
  - C. The firm has a large tract of land that could be used to build a warehousing facility.
  - D. The firm can reduce its costs significantly by investing in a just-in-time inventory system.
18. If a firm is considering a capital investment project that is significantly more risky than the firm itself, which of the following will be TRUE?
- A. The IRR necessary to make the project worthwhile will be lower than a project with risk equal to the firm as a whole.
  - B. The discount rate to find the project's NPV will be higher than the firm's weighted average cost of capital.
  - C. The discount rate to find the project's NPV will be lower than the firm's weighted average cost of capital.
  - D. The IRR will not be a meaningful measure in this instance.

19. In the capital budgeting process, which of the following is NOT relevant?
- The cost to acquire a new piece of equipment or machinery.
  - Cost savings that result from the investment project under consideration.
  - The risk of the cash flows associated with the project.
  - Total cash flow of the firm.
20. The following demonstrates the cash flow streams for Projects A, B, C, D, and E. Which of the projects has the shortest and longest payback periods, respectively?

<i>Cash Flows</i>				
<i>Year</i>	<i>Project A</i>	<i>Project B</i>	<i>Project C</i>	<i>Project D</i>
0	-15,000,000	-1,500,000	-8,750,000	-905,000
1	2,500,000	250,000	6,250,000	465,000
2	3,000,000	300,000	2,300,000	465,000
3	2,950,000	575,000	1,275,000	465,000
4	2,450,000	205,000	580,000	465,000
5	2,000,000	115,000	245,000	465,000
6	2,500,000	95,000	165,000	465,000

- | <u>Shortest</u> | <u>Longest</u> |
|-----------------|----------------|
| A. Project C    | Project A      |
| B. Project D    | Project B      |
| C. Project C    | Project B      |
| D. Project D    | Project A      |
21. Dodson Corp. expects a 10-year replacement project designed to increase the company's productivity to produce an NPV equal to \$1.5 million. The project will provide positive cash flows in years 1 through 10 of the project. If the project cash flows end unexpectedly after seven years, what will happen to the project's NPV and IRR?
- | <u>NPV</u>  | <u>IRR</u> |
|-------------|------------|
| A. Increase | Decrease   |
| B. Decrease | Decrease   |
| C. Decrease | Increase   |
| D. Increase | Increase   |

## Corporate Finance

22. Which of the following statements is FALSE?
- A. Operating leverage is a key determinant of break-even levels.
  - B. The degree of operating leverage tells us the sensitivity of operating cash flow to changes in sales volume.
  - C. In the short run, management has the flexibility to change a company's high fixed cost structure.
  - D. With no fixed costs, the change in EBIT is the same as the change in revenues.
23. Mark Kelley is assessing an investment project. He has determined that, given his cash flow and risk assumptions, the project will add value to his firm. Three days after deciding the project was acceptable, Kelley decided that the project was actually more risky than he originally thought. After reassessing the project's risk, what should happen to the cost of capital and the NPV of the project Kelley is considering?
- A. The cost of capital should decrease, while the NPV should increase.
  - B. The cost of capital should increase, and the NPV should increase.
  - C. The cost of capital should decrease, and the NPV should decrease.
  - D. The cost of capital should increase, while the NPV should decrease.
24. In a meeting with her colleagues, Martha Samuels detailed the ideal characteristics of measures to evaluate capital budgeting projects. Which of the following characteristics is NOT necessary?
- A. The measure should account for the timing of the cash flows.
  - B. The measure should account for the method of financing.
  - C. The measure should account for the risk of the project.
  - D. The measure should account for all of the project's cash flows.

Use the following information to answer Questions 25 and 26.

Barret Clothing Manufacturers, Inc. is considering adding a new piece of equipment that will significantly increase the production capabilities of their largest U.S. manufacturing facility. The project is expected to last for five years and will require an initial investment of \$1,500,000. The net cash flows expected for this expansion project are \$400,000 per year for each of the project's five years.

25. What is the NPV of the project at 10%?
- A. \$16,315.
  - B. \$31,422.
  - C. \$1,531,422.
  - D. \$1,522,415.

26. What is the project's IRR?
- A. 10.00%.
  - B. 11.02%.
  - C. 9.85%.
  - D. 10.42%.

Use the following information to answer Questions 27 through 31.

Jamestown Sporting Supplies is a large manufacturer of sporting goods equipment to the U.S. and Canadian markets. Jamestown has grown rapidly over the past few years, and as a result, one of their key pieces of machinery is wearing out. Jamestown is evaluating two potential projects, each of which would involve the purchase of a new machine to replace the old one. The incremental cash flows from each project including the net investment required today are summarized in the following table.

Year	<i>Undiscounted Values</i>	
	<i>Project A</i>	<i>Project B</i>
0	-\$175,000	-\$120,000
1	100,000	10,000
2	90,000	10,000
3	10,000	40,000
4	5,000	105,000
<b>Total</b>	<b>\$30,000</b>	<b>\$45,000</b>

Jamestown's capital structure consists of 40% debt and 60% equity. The after-tax cost of debt for the company is 5.5%, and the cost of equity is 8.0%.

27. Jamestown's weighted average cost of capital is equal to which of the following?
- A. 6.0%.
  - B. 6.5%.
  - C. 7.0%.
  - D. 7.5%.
28. If Jamestown is unwilling to accept any project with a payback period greater than 3 years, which of the following is TRUE?
- A. Jamestown should accept Project B, since it has a PBP of 1.83 years.
  - B. Jamestown should accept Project A, since it has a PBP of 1.83 years.
  - C. Jamestown should not accept Project B, since it has a PBP of 4.57 years.
  - D. Jamestown should not accept Project A, since it has a PBP of 4.57 years.

Corporate Finance

29. Calculate the net present value of each of the projects that Jamestown is considering.
- |    | <u>Project A</u> | <u>Project B</u> |
|----|------------------|------------------|
| A. | \$12,939         | \$10,411         |
| B. | \$10,386         | \$9,045          |
| C. | \$9,045          | \$10,836         |
| D. | \$10,411         | \$12,939         |
30. Using the net present values calculated previously, which of the following statements is TRUE?
- A. Because it adds more value to the firm, Project A should be preferred to Project B.
  - B. Jamestown should not accept either of the projects, since they will decrease the value of the firm.
  - C. Jamestown should be indifferent between Project A and Project B, since they both add value to the firm.
  - D. Because it adds more value to the firm, Project B should be preferred to Project A.
31. Calculate the IRR for each project.
- |    | <u>Project A</u> | <u>Project B</u> |
|----|------------------|------------------|
| A. | 10.5%            | 10.5%            |
| B. | 10.5%            | 9.8%             |
| C. | 9.8%             | 10.5%            |
| D. | 9.8%             | 9.8%             |

Use the following information to answer Questions 32 through 35.

Toppers, Inc. manufactures and sells baseball caps in the United States. The company sells its complete line of caps for \$25 apiece. The company is currently evaluating three different projects that would change the cost structure of its operations. Toppers is concerned about increasing the amount of operating leverage as a result of undertaking one of the three projects. Each of the projects' expected costs are summarized in the following table.

	<i>Project A</i>	<i>Project B</i>	<i>Project C</i>
Price	\$25.00	\$25.00	\$25.00
Variable costs	\$15.00	\$20.00	\$10.00
Fixed costs	\$80,000	\$60,000	\$120,000

32. What is the breakeven quantity of sales for Project B?
- A. 12,000 caps.
  - B. 8,000 caps.
  - C. 1,000 caps.
  - D. 333 caps.
33. What is the breakeven quantity of sales for Project C?
- A. 12,000 caps.
  - B. 8,000 caps.
  - C. 1,000 caps.
  - D. 229 caps.
34. Given Toppers' aversion to additional operating risk, which of the following statements is CORRECT?
- A. Project B has the lowest contribution margin and therefore the lowest operating leverage.
  - B. Project A has more operating leverage than Project C.
  - C. Project B will add the most operating leverage and should be avoided.
  - D. Project C has more operating leverage than Project A.
35. If Toppers sells 10,000 caps, what will be the operating profit or loss for Projects A, B, and C?
- A. Project A will have an operating loss of \$20,000, and Project C will have an operating profit of \$30,000.
  - B. Project C will have an operating profit of \$30,000, and Project B will have an operating loss of \$10,000.
  - C. Project B will have an operating profit of \$10,000, and Project C will have an operating profit of \$30,000.
  - D. Project A will have an operating profit of \$20,000, and Project B will have an operating profit of \$10,000.



**PRACTICE QUESTION ANSWERS: CORPORATE FINANCE**

1. **D** Limited liability extends to all of the owners of a corporation. This is one of the reasons firms undergo the time consuming and, in some cases, expensive process of incorporating the firm.
2. **C** Corporations must be registered within their state of incorporation, and they require more external reporting. In a proprietorship there is a sole owner. This individual has complete control over all decisions. In all other forms of business there are multiple owners which diffuse the control of any one owner.
3. **B** Since debt generally has a lower component cost of capital than does equity, increasing the proportion of debt in the capital structure will decrease the overall WACC. Consider the following example:

$$k_{\text{equity}} = 15\%, k_{\text{debt}} = 8\%, t = 40\%$$

$$\begin{aligned} \text{WACC} &= 0.70(15\%) + 0.30[8\%(1 - 0.40)] = 10.5\% + 1.4\% \\ &= 11.9\% \end{aligned}$$

$$\begin{aligned} \text{WACC} &= 0.60(15\%) + 0.40[8\%(1 - 0.40)] = 9.00\% + 1.92\% \\ &= 10.9\% \end{aligned}$$

Thus, we can see that as the proportion of debt increases, the WACC decreases. A decrease in the tax rate of a company will increase the WACC. Consider another example:

$$k_{\text{equity}} = 15\%, k_{\text{debt}} = 8\%, w_{\text{equity}} = 70\%, w_{\text{debt}} = 30\%$$

$$\begin{aligned} \text{WACC} &= 0.70(15\%) + 0.30[8\%(1 - 0.40)] = 10.5\% + 1.4\% \\ &= 11.9\% \end{aligned}$$

$$\begin{aligned} \text{WACC} &= 0.70(15\%) + 0.30[8\%(1 - 0.30)] = 10.5\% + 1.7\% \\ &= 12.2\% \end{aligned}$$

As the tax rate decreases, the WACC increases.

4. **C** Ordinary dividends are paid out of retained earnings. If the firm decides to pay dividends in excess of retained earnings, this is known as a liquidating dividend.

5. **B** Having the review board in place may eliminate the wasteful use of the company's assets, but the cost will be the payment made to the board for their services as well as decreased operating flexibility. With someone watching their every move, management will become overly cautious and may forego profitable opportunities for fear of scrutiny by the review board.
6. **C** In a partnership, each partner is personally responsible for company liabilities in proportion to the partner's ownership in the company. In this question, Shamus owns  $5,000 / 27,000 = 18.51\%$  of the company and is thus responsible for  $0.1851 \times 11,000 = \$2,037$  of the company's debt.
7. **A** Microeconomic factors include firm-specific factors. Inflation is a macroeconomic (system-wide) factor.
8. **A** Increasing financial leverage will amplify how revenue increases affect the firm's net income. This will also increase the firm's financial risk.
9. **B** Unsystematic risk or firm-specific risk is the risk caused by microeconomic forces (such as the risk associated with competing products). Macroeconomic risk relates to economy-wide forces that affect all companies (such as the level of interest rates).
10. **A** If a firm does not employ financial leverage, the percentage change in net income should be equal to the percentage change in operating income.
11. **B** The Board of Directors of the corporation has the authority to declare a dividend, and when a dividend is declared it becomes a current liability of the corporation. The current liability impacts the company's working capital (current assets minus current liabilities).
12. **A** Operating leverage comes from the use of fixed costs in the operating structure. As fixed costs decrease as a proportion of total costs, the operating leverage of the firm decreases.
13. **D** The origin of agency costs is the separation of ownership and management. Also, the goals of the stockholders, bondholders, and management are sometimes at odds. Agency costs are detrimental to the firm and prevent the firm from realizing its maximum value.
14. **D** Capital structure refers to the proportions of debt and equity financing used to pay for the firm's assets. If Green & Company wants to finance a new investment without changing the capital structure, it will need to raise the required investment according to the existing proportions. For instance, if the current capital structure consists of 40% debt and 60% equity, Green & Company must raise 40 cents of debt capital and 60 cents of equity capital for every dollar of investment required for the new project.

## Corporate Finance

15. **B** The firm's current capital structure includes a debt ratio of  $\frac{2,500,000}{(2,500,000 + 7,500,000)} = 25\%$ . Since the target capital structure includes a debt ratio of 30%, new projects will have to make up the extra 5% needed to get to the target structure. Therefore the proportion of debt financing for new projects will have to be greater than the target 30% to make up the deficiency. If the firm was already at the target structure, then new projects would be financed with 30% debt and 70% equity.
16. **D** Dividends are not retained by the corporation but are paid to shareholders. Earnings retained by the corporation are used to finance future projects, which, if successful, will allow the company to grow.
17. **B** Increasing the proportion of equity on the balance sheet is a financing decision, not a capital budgeting decision. Capital budgeting decisions relate to the opportunities for investment in the business to increase its value. Changing the mix of debt and equity doesn't change the value, just how it is divided among the stakeholders.
18. **B** A project that is riskier than the firm as a whole will need to be evaluated on a present value basis using a discount rate that is higher than the weighted average cost of capital.
19. **D** When considering capital budgeting projects, it is the incremental cash flow, not the total cash flow, that is relevant. The project needs to be evaluated in terms of how much value it will add to the firm. In order to do this, we must isolate only those cash flows that will result from the project itself.
20. **D** First calculate the cumulative cash flows for each project as demonstrated in the following table:

<i>Cumulative Cash Flows</i>				
<i>Year</i>	<i>Project A</i>	<i>Project B</i>	<i>Project C</i>	<i>Project D</i>
0	-15,000,000	-1,500,000	-8,750,000	-905,000
1	-12,500,000	-1,250,000	-2,500,000	-440,000
2	-9,500,000	-950,000	-200,000	25,000
3	-6,550,000	-375,000	1,075,000	490,000
4	-4,100,000	-170,000	1,655,000	955,000
5	-2,100,000	-55,000	1,900,000	1,420,000
6	400,000	40,000	2,065,000	1,885,000

Once the cumulative cash flows for a project turn positive (indicating full recovery of the initial project cost), we can calculate the payback period for the project using the following formula:

$$\text{PBP} = \text{years until full recovery} + \frac{\text{unrecovered cost at the beginning of the last year}}{\text{cash flow during the last year}}$$

Thus the payback period (PBP) for each project is as follows:

$$\text{PBP}_A = 5 + \frac{2,100,000}{2,500,000} = 5 + 0.84 = 5.84$$

$$\text{PBP}_B = 5 + \frac{55,000}{95,000} = 5 + 0.58 = 5.58$$

$$\text{PBP}_C = 2 + \frac{200,000}{1,275,000} = 2 + 0.16 = 2.16$$

$$\text{PBP}_D = 1 + \frac{440,000}{465,000} = 1 + 0.95 = 1.95$$

21. **B** If the positive cash flows from a project end before they are expected or are less than originally expected, then the NPV of a project will decrease since there is less overall cash benefit from undertaking the project. The IRR will also decrease for the same reason. Fewer cash flows or smaller cash flows than expected translates into smaller returns.
22. **C** Management can only effectively deal with high fixed costs over a longer-term time horizon.
23. **D** As the risk of a project increases, the cost of capital (the interest rate used to discount the project's cash flows to their present value) should increase to reflect the additional return required for the increased risk. An increased cost of capital would necessitate a decreased NPV. Discounting the project's cash flows at a higher rate decreases the present value of each cash flow and thus the NPV as well.
24. **B** How a firm chooses to finance a project is theoretically independent from the decision to accept or reject a project. The decision to accept a project should be based on an objective measure that takes into account all of a project's cash flows, the project's risk, and the time value of money.
25. **A**

<i>Year</i>	<i>Net Cash Flow</i>	<i>PV @ 10%</i>
0	-\$1,500,000	-\$1,500,000
1	400,000	363,636
2	400,000	330,579
3	400,000	300,526
4	400,000	273,205
5	400,000	<u>248,369</u>
		<b>\$1,516,315</b>

$$\text{NPV} = \Sigma \text{PV inflows} - \text{cost} = \$1,516,315 - \$1,500,000 = \$16,315$$

**Corporate Finance**

Note: The cash inflows could be treated as an annuity of \$400,000 for five years:

-400,000      PMT  
 5                N  
 10               I/Y  
 CPT PV        \$1,516,315

26. D Since the cash inflows are an annuity, solving for the IRR is fairly simple:

-400,000      PMT  
 5                N  
 1,500,000     PV  
 CPT I/Y        10.42%

27. C The WACC is the weighted average of the capital costs.

$$\text{WACC} = (0.40 \times 5.5\%) + (0.60 \times 8.0\%) = 2.2\% + 4.8\% = 7.0\%$$

28. B In order to calculate the payback period, we must first calculate the cumulative cash flows for each project.

Year	Project A		Project B	
	NCF	Cum. NCF	NCF	Cum. NCF
0	-\$175,000	-\$175,000	-\$120,000	-\$120,000
1	\$100,000	-75,000	\$10,000	-110,000
2	\$90,000	15,000	\$10,000	-100,000
3	\$10,000	25,000	\$40,000	-60,000
4	\$5,000	30,000	\$105,000	45,000

The table shows that for Project A, the PBP will be between one and two years since the cumulative cash flow turns positive in year 2. For Project B, the PBP will be between 3 and 4 years. More specifically:

$$\text{PBP}_A = 1 + \frac{75,000}{90,000} = 1 + 0.83 = 1.83 \text{ years}$$

$$\text{PBP}_B = 3 + \frac{60,000}{105,000} = 3 + 0.57 = 3.57 \text{ years}$$

Since the cutoff point for investment projects is three years, Jamestown should accept Project A.

29. **C** To calculate the NPV of each project, discount each project's cash flows using the 7.0% WACC as the discount rate. The cash flows (undiscounted and discounted) are presented in the following tables.

Year	Undiscounted Values		Year	Present Values	
	Project A	Project B		Project A	Project B
0	-\$175,000	-\$120,000	0	-\$175,000	-\$120,000
1	100,000	10,000	1	93,458	9,346
2	90,000	10,000	2	78,609	8,734
3	10,000	40,000	3	8,163	32,652
4	5,000	105,000	4	3,814	80,104
<b>Total</b>	<b>\$30,000</b>	<b>\$45,000</b>	<b>NPV</b>	<b>\$9,045</b>	<b>\$10,836</b>

Each cash flow gets discounted as a lump sum. As an example, the year 1 cash flow for Project A would get discounted as follows:

$$\text{Year 1 cash flow: } PV_A = \frac{\$100,000}{(1 + 0.07)^1} = \$93,458$$

The NPV of each project is simply the sum of the present values minus the net investment. The net investment is represented in these tables as the negative cash flow in year 0.

30. **D** Although any project with a positive NPV will increase the value of the firm, Jamestown only needs to replace one machine and thus can only choose one project. Since Project B has a greater NPV than Project A, the firm should choose Project B. Project B will increase the value of the firm by the greatest amount.
31. **B** The IRR is the discount rate that makes the NPV of the project equal zero. For example, for Project A, the IRR is the discount rate that, when substituted into the NPV formula below, makes the equation true:

$$0 = \frac{100,000}{(1 + \text{IRR})^1} + \frac{90,000}{(1 + \text{IRR})^2} + \frac{10,000}{(1 + \text{IRR})^3} + \frac{5,000}{(1 + \text{IRR})^4} - 175,000$$

## Corporate Finance

The IRR can be found by trial and error or by using a calculator such as the TI BA II+. In order to find the IRR using the TI BA II+, use the following keystrokes:

[CF] [2ND] [CLR WORK]	Clears the cash flow worksheet
175,000 [+/-] [ENTER]	Enters the net investment
[↓] 100,000 [ENTER]	Enters 1st year's cash flow
[↓] [↓] 90,000 [ENTER]	Enters 2nd year's cash flow
[↓] [↓] 10,000 [ENTER]	Enters 3rd year's cash flow
[↓] [↓] 5,000 [ENTER]	Enters 4th year's cash flow
[IRR] [CPT]	Computes the IRR

Using the method detailed above, the IRR for Project A is 10.5%, and the IRR for Project B is 9.8%.

32. A  $Q_{BE} = \frac{60,000}{(25 - 20)} = 12,000$

33. B  $Q_{BE} = \frac{120,000}{(25 - 10)} = 8,000$

34. C Project B has the highest breakeven level of sales (12,000) and therefore adds the most operating leverage. Project A and Project C both have a breakeven quantity of 8,000 and therefore have the same operating risk. Project B does have the lowest contribution margin ( $25 - 20 = 5$ ), but this does not indicate that it has the lowest operating leverage.

35. B Calculate the operating profit or loss as follows:

operating profit = sales – variable cost – fixed cost

$$\text{Profit}_A = (10,000 \times 25) - (10,000 \times 15) - 80,000 = \$20,000$$

$$\text{Profit}_B = (10,000 \times 25) - (10,000 \times 20) - 60,000 = -\$10,000$$

$$\text{Profit}_C = (10,000 \times 25) - (10,000 \times 10) - 120,000 = \$30,000$$

# SECURITIES MARKETS

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Markets provide the means for buyers and sellers to exchange goods and services. Exchange can be for money or for other goods and services (as in a barter system). Markets can have physical locations, where buyers and sellers actually meet, or they can be channels through which goods and services flow. In the latter case it is possible that neither the buyer nor the seller knows who is on the other end of the transaction.

There are three basic types of markets. First is a **direct market**, in which the buyer and seller must contact one another directly. When you visit a farmer's market, you are participating in a direct market. You see the items you want, and you purchase them directly from the seller. The obvious benefit with direct markets is the ability to inspect the item before you purchase it. Of course, this also presents a potentially large problem for both you and the vendor. You had to travel to the market, and the vendor can only sell to those who make the trip.

The second type of market is a **broker market**. In this case, a broker facilitates the transaction by bringing the buyer and seller together. Consider a real estate transaction. You may choose to sell your home by yourself (i.e., "by owner"). To be successful, you must be able to actively market your property, meet prospects at your home, and negotiate the sale terms. You can advertise your house in newspapers or other publications, but that can be expensive.

If you have a real estate broker handling the sale for you, she will advertise the sale through specialized real estate networks. The broker will help you establish a fair offering price, give you ideas on how to improve the appearance of the house, show the house to prospects, and act as the go-between during negotiations. In other words, the broker, in exchange for a **commission** on the sale, acts as your agent in presenting the home to a wide range of prospective buyers. Notice that a real estate broker does not actually take ownership of the home being sold. Rather, the function of the broker is to bring the buyer and seller together.

In the third type of market, a **dealer market**, the dealer takes ownership of the product before reselling it to the final user. The dealer can sell his products at a central location, such as a grocery store, or through a network or even over the internet. Dealers do not charge a commission. A dealer's profit comes from buying the product at one price and selling it for a higher price.



## Securities Markets

All three forms of markets exist today. You see cars by the side of the road or in parking lots with “for sale” signs in the windows. You see apartments for rent or houses for sale by owner. These are obvious examples of the direct market at work. Possibly next door to the “For Sale By Owner” sign in one front yard you will see a real estate company sign, an example of the broker market. And you have visited a grocery store or any of thousands of other retail stores, which are examples of dealer markets.

**Money markets and capital markets** are primarily broker and dealer markets. Buy or sell orders are placed with a broker, who takes the orders into the marketplace and tries to find a willing party to take the opposite side of the transaction. For the most part, this matching is done through computer systems, and trading happens almost instantly. For some stocks or bonds that are traded less frequently, it might take longer to find a willing buyer or seller, and there may be some amount of price negotiation.

A typical stock or bond has a **bid price** and an **ask price** any time the market is open for business. Assume that the stock of Smith Company has a bid price of \$15.000 and an ask price of \$15.125. This indicates that there are sellers willing to sell Smith stock for \$15.125 per share, and buyers willing to buy Smith stock for \$15.000 per share. If an investor enters an order to buy Smith at the market price, the broker will enter the order and it will be executed at the ask price of \$15.125. The investor in this case was willing to pay the market price of the seller, which is \$15.125. Similarly, an investor who enters an order to sell Smith at market would receive \$15.000 per share for his stock—the market price of buyers. The difference between the bid and ask price is called the **bid-ask spread**. Large stocks or bonds with high trading volumes typically have small spreads. Stocks and bonds that trade less frequently have wider spreads.

## SPOT, FORWARD, AND FUTURES MARKETS

A **spot market** is a market where securities, commodities, and other goods are traded for immediate delivery. The price you receive in a spot market is the current market value determined by the supply and demand for the product. The most common spot market is the retail store (e.g., shoe store or a grocery store). In this case, the customer selects the desired product and takes it home. In financial spot markets, investors buy currencies, securities, and commodities for “immediate delivery” of two to three days. This is equivalent to calling your broker and buying common stock or other securities. When a farmer sells corn in the spot market, he delivers it to his local grain elevator.

**Forward markets** were developed because of the uncertainty associated with future spot market prices. For instance, suppose a farmer has planted corn, which he

will harvest and deliver to the elevator in three months. He sees the price of corn today but isn't sure what it will be when he harvests. The price of corn when it is harvested will depend on the demand for corn compared to the quantity harvested worldwide. If farmers produce surplus crops, the price of corn may fall due to its abundance.

Regardless, the farmer would like an idea of the price he will receive before he determines how much corn to plant. To lock in a selling price for his corn, the farmer can enter into a forward contract to deliver a set amount of corn at a predetermined price. In this way, the farmer knows in advance the price he will receive. If he waits until delivery to find out the price he will receive, the price per bushel may have dropped significantly.

## PRIMARY AND SECONDARY MARKETS

Firms obtain long-term capital by selling their securities in the **primary markets**. They typically utilize the services of an investment-banking firm to either privately place their securities or sell them publicly. In a **public offering**, securities are offered to the general investing public through **investment bankers**. If the stock offering is the first time the company has made its shares available to the public, the offering is referred to as an initial public offering (IPO). If the company has issued stock previously but is now raising additional capital through selling new shares, the sale is referred to as a secondary stock offering. In either case, the transaction is said to be in the primary market because the securities are being issued for the first time and the net proceeds go to the issuer.

All securities sold publicly in the United States must be registered with the Securities and Exchange Commission (SEC). To register their securities, firms must publish a prospectus, which discloses information as specified by the SEC. The process can be costly both in terms of time and money, and a good deal of information about the firm must be disclosed to the general public. A syndicate of investment banking firms, with a few lead banks, will help the firm through the process. The lead banks actually meet with management to decide what type of security to issue, when to issue the security, how much to issue, and at what price.

The investment banks will advertise the sale in a **tombstone ad** in financial publications. The tombstone contains the names of the investment banking firms involved in the sale, the main characteristics of the security, and the date of the sale. The tombstone will direct interested investors to the prospectus, which is available through any of the listed investment banking firms.

The lead banks and others listed on the tombstone are part of the **underwriting syndicate**. When an issue of securities is **underwritten**, the investment banks

## Securities Markets

guarantee the proceeds of the issue to the firm before the securities are offered publicly. A fully underwritten issue transfers risk to the underwriter since the price and quantity of securities being issued has been fully guaranteed by the underwriter. The prospectus lists all involved investment banks along with the extent of their underwriting obligation, the number of shares or bonds each investment bank agrees to guarantee. Often, investment bankers will visit large institutional investors, such as pension funds, mutual funds, and insurance companies to determine their interest.

For their efforts, investment banks charge a fee known as the **underwriter spread**. This spread is the difference between the price investment bankers pay for the securities and the price at which they sell them to investors. The total costs of selling securities publicly (the spread, legal fees, accounting and financial costs, and printing costs) are known as **flotation costs**. The magnitude of the investment banks' fees will depend upon the work the banks have to do to sell the securities as well as the risk associated with selling them. For example, if a very large, well-known firm sells securities, flotation costs are minimal as a percentage of the proceeds. For a smaller, less well-known firm, the spread alone can be as high as 20% to 30% of the proceeds. This is mostly due to the extra effort the banks must expend to sell these securities as well as the risk that they might not sell their total allocation.

To speed up the process of issuing securities and maintaining a certain degree of privacy, many firms place their securities privately. A **private placement** is restricted to a limited number of very large investors, and the issue does not have to be registered with the SEC. This avoids the necessity of disclosing great amounts of information about the firm and its owners.

An investment banking firm, acting as a broker, brings together representatives of the firm and the investors to discuss the details of the issue. In this fashion, only those investors actually involved in the private placement get the inside information that would be disclosed in a prospectus for a public offering. Most privately placed securities are bonds. Private placement investors take large positions (i.e., buy a large amount of the debt) in the offering and typically hold the bonds to maturity. The interest rate (coupon) on the bonds is usually slightly higher than if the bonds were sold publicly because the investors cannot sell their bonds to the public. However, due to the flotation cost savings and the lack of publicly disclosed information, private placements are often used to sell corporate bonds.

In **secondary markets**, investors buy and sell securities that have already been issued by firms. Although firms will occasionally buy their own stock in secondary market transactions, the vast majority of secondary market transactions involve investors trading among themselves.

## EQUITY MARKETS

Many investors immediately think of the equity market when discussing investment opportunities. This comes as no surprise given the widely available information and relative ease of trading that characterize shares of common stock.

*Common stockholders* are the owners of the issuing firm. When you own the common shares of a company, you share in the company's successes and failures. Relative to fixed income securities, common stock investing is considered to be more risky since there is no guarantee of receiving any sort of payment in the future. Even though they are not guaranteed, some companies pay cash dividends to holders of their common stock. These dividends are typically paid quarterly.

The National Association of Securities Dealers Automated Quotation System (NASDAQ). NASDAQ, once referred to as the over-the-counter market, is actually a network of computers linked worldwide. Where listing on one of the national exchanges (e.g., New York Stock Exchange) was once considered a mark of distinction, many companies now choose to list on NASDAQ's National Market System (NMS) instead.

Rather than purchasing memberships to trade on the NASDAQ, membership is achieved by subscription. *Level 1* subscription is information only. The subscriber member receives only representative prices of listed stocks, not bid and ask prices, which are typically delayed by 15 minutes. *Level 2* members act as brokers. They receive information and can trade at live bid-ask quotes, but they are not allowed to enter their own quotes. *Level 3* subscribers are **market makers**.

*Making a market* means the subscriber/member is required to post bid and ask prices on the network. A bid is the price the market maker (dealer) will pay for the security. The market maker sells at the ask price. The bid-ask spread represents the market maker's profit.

By standing ready to take the other side of any buy or sell order, the market maker ensures a smooth-running market for their stocks. For example, if the market maker receives many buy orders and there are few or no corresponding sell orders, he will deliver stocks from his inventory to fill the orders. The market maker is required to post the transaction on the network so the investing public and other members know of it. Trading on the NASDAQ system can take place almost instantaneously. Often the broker will know the price at which the trade took place while the investor is still on the telephone.

The New York Stock Exchange utilizes a **specialist** system, in which each stock is allocated to one specialist. Members acting as specialists can trade only in their

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assigned stocks at a specific location on the trading floor called a **trading post**, where **floor brokers** come to present orders for their customers.

The most common type of trade is known as a **market order**, an order from a customer to trade at the best possible price. Brokers present bids to buy and offer to sell by open outcry to any interested party at the trading post. In this fashion, floor brokers and others<sup>1</sup> among the trading crowd have the opportunity to participate, thus facilitating the competitive pricing of stocks. When the highest bid (buy) meets the lowest ask (sell), a trade is executed and the customer's order is filled.

Specialists manage this auction process by electronically quoting and recording current bid and ask prices for the stocks assigned to them. This enables current price information to be transmitted worldwide, keeping all market participants informed of the total supply and demand for any particular NYSE-listed stock.

According to the NYSE, the role of the specialist can be divided into five vital functions:<sup>2</sup> (1) specialists can act as agents, (2) catalysts, and (3) auctioneers; and (4) they also stabilize prices and (5) provide capital.

1. **Act as Agents.** One of the specialist's jobs is to execute orders for floor brokers. A floor broker may get an order from a customer who only wants to buy a stock at a price lower than the current market price or sell it at a price higher than the current market price. In such cases, the broker may ask the specialist to hold the order, known as a **limit order**, and execute it if and when the price of the stock reaches the level specified by the customer. Limit orders are considered aggressive because they will be filled only at the specified price, which is always better than the market price when the order is placed. By holding limit orders, the specialist acts as an agent for the floor broker.

Likewise, the specialist acts as an agent by holding more defensive orders called **stop-loss orders**. These are orders to sell below the current market price. Let's assume you hold a stock, which has appreciated greatly while you've held it. To protect your profit, you can submit an order to sell the stock if it drops below a certain price. Once the stock falls to the stop price, the order is executed. The stop loss order is considered a defensive trade, since it is used for protection rather than for aggressive trading.

2. **Act as Catalysts.** A catalyst precipitates an event, without being directly affected by the outcome. Specialists serve as the contact point between brokers with buy and sell orders in the NYSE's two-way auction market. In this respect,

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1 In addition to the floor brokers who handle public orders, there are floor traders. These members trade their own accounts and help the floor brokers when asked.

2 Source: <http://www.nyse.com>.

the specialists act as catalysts, bringing buyers and sellers together so that they can trade with each other.

3. **Act as Auctioneers.** At the start of each trading day, specialists establish a fair market price for each of their stocks. They base that price on the supply and demand for the stock as indicated by the number of buy and sell limit orders and the limit prices. A large number of limit orders grouped around a certain price indicate what the market anticipates for that stock. For example, many sell orders at a price below the prior closing price would indicate the market (i.e., investors in aggregate) feels that the stock is going to decrease in price at the open. In addition, specialists quote the current bids and offers on their stocks to other brokers throughout the day.
4. **Stabilize Prices.** Specialists are also called upon to maintain “orderly markets” in their assigned stocks. That is, they ensure that trading in their stocks moves smoothly throughout the day, with minimal price fluctuations between trades. Large gaps in trading prices create uncertainty around the stock’s true value, so the specialist will trade intermittently at prices in between those of the limit orders.
5. **Provide Capital.** Specialists act as a market maker. If buy orders temporarily outnumber sell orders, or if sell orders outnumber buy orders, the specialist is required to use his firm’s own capital to minimize the imbalance. This is accomplished by buying or selling against the trend of the market until a price is reached at which public supply and demand are once again in balance. Specialists act as *dealers* in this capacity. However, specialists actually participate in only about 10% of all shares traded. The rest of the time, orders clear without the participation of specialists.

## International Equity Investment

U.S. investors often desire to gain exposure to international equity markets in hopes of offsetting domestic risk factors or capitalizing on international equity opportunities. Whatever the motivation, *international equity* investing may be accomplished in a number of ways.

For investors willing to accept the risks of investing directly in foreign markets, they can attempt a *direct purchase of foreign shares*. Here you buy the shares in the country where the firm is listed. This requires making payment in the foreign currency and transferring the certificates to your own country. Obtaining exposure to foreign markets in this manner is slightly more complicated than domestic investing.

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Investors who do not wish to incur the additional complexities of direct purchases of foreign securities may gain international exposure in three ways. First, an investor may purchase *American Depository Receipts* (ADRs). ADRs are shares issued by U.S. banks representing an ownership interest in the actual shares of foreign entities, which are held in deposit at a bank in the issuing firm's country. U.S. investors can also purchase mutual funds that invest in international equities.

## FIXED INCOME MARKETS

In addition to the markets for equity capital, there is an extensive secondary market for various types of debt securities (also referred to as fixed income securities), which are issued by corporations, states, municipalities, and federal governments. **Fixed income securities** have a contractual repayment schedule. By purchasing a fixed income security, you are lending money (called the principal) to the issuer. In return, the borrower promises to make periodic interest payments and, at maturity, pay back the principal.

**U.S. Treasury securities** include bills (T-bills), notes, and bonds. U.S. Treasury obligations are considered to be risk free due to their low probability of default, and they are extremely liquid. Long-term borrowing by the U.S. Treasury is conducted through Treasury notes and bonds with original maturities from one to ten years. They are issued in denominations starting at \$1,000, pay interest semiannually, and return the principal at the stated maturity.

Notes and bonds are quoted at a percentage of par or face value. For instance, you might see a 20-year T-Bond quoted at 97.27. The numbers to the left of the decimal (a dash may be used in place of the decimal) represent percent, in this case 97%. The numbers to the right of the decimal (or dash) are 32nds of a percent. This bond is therefore quoted at 97 and 27/32% of par (97.84375%). Thus a Treasury bond with a \$1,000 face value would sell for \$978.44 ( $0.9784375 \times \$1,000 = \$978.44$ ).

The U.S. Treasury does not issue zero-coupon bonds, but bond dealers can create zero-coupon securities known as **Treasury strips**. Zero coupon bonds (zeros) are pure discount instruments. You buy the zero at a discount from the face value and receive the face value at maturity, with no interest payments between the purchase and maturity dates. If you visualize the cash flows associated with T-notes and T-bonds, you will see a stream of coupon payments and a relatively large payment (the principal) at the end. For example, a 6%, 20-year T-bond would pay \$60.00 per year (\$30 every six months) and return the \$1,000 principal (face value) in 20 years. With strips, all the payments are "stripped" apart and sold separately. Investors can buy individual coupons, a specified set of coupons, or the principal

payment. Each individual cash flow, whether coupon or principal, becomes a zero coupon bond.

**Municipal bonds** (munis) are issued by local governments. Munis may be either *general obligation bonds* (GOs) or *revenue bonds*. Payments to general obligation holders come from the full taxing authority of the municipal issuer. Revenue bonds are serviced with the revenues from the project financed with the bond proceeds (e.g., toll highway bonds). Munis are generally exempt from federal income taxes, and in some cases they are exempt from state and local income taxes. For example, a municipal bond issued by a city in Wisconsin is typically exempt from Wisconsin state income taxes as well as federal income taxes. Because of differences in tax treatment between municipal bonds and other bonds, investors should compare them on an after-tax basis.

**Corporate bonds** represent borrowing by businesses. Most corporate bond trading in the secondary market takes place in the over-the-counter market through dealers who buy and sell the securities for their own accounts. Publicly traded corporate debt securities represent a much smaller proportion of the overall bond market than the government or government-sponsored sectors represent. Corporate debt securities range in maturity from short-term to long-term and are rated according to their credit quality. Higher credit quality issues carry a lower required return, and low credit quality issues carry a higher required return. The corporate debt sector is much less liquid than the government debt sector.

A corporate bond's indenture lists the terms of the loan, which include the payment schedule and any call or refunding provisions that allow the bond to be redeemed prior to maturity. The indenture also specifies sinking fund provisions. These require the issuer to redeem a given percentage of the outstanding issue prior to maturity to protect the bondholders.

Corporate bonds can be categorized as follows:

- *Senior secured bonds* include mortgage bonds, collateral trust bonds, and equipment trust certificates. Mortgage bonds are backed by a lien against a specific asset. If necessary, bankruptcy proceeds from the sale of the collateral assets are used to repay the bondholders. Collateral trust bonds have financial assets as collateral. Equipment trust certificates are bonds that are backed by the equipment that they financed; these are typical in the transportation industry.
- *Debentures* are simply corporate promises to pay interest and principal. No specific assets are pledged, so the bondholder is dependent on the issuer's ability to generate the cash flows necessary to make the promised payments.
- *Subordinated debenture bonds* have the same general features as debentures but have a lower priority claim against the assets of the company in case of default.



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- *Income bonds* pay interest only if the issuer earns enough income to make the payment. If the company has no income, it does not have to make the interest payment, and it cannot be declared bankrupt. Unpaid interest is classified as interest in arrears, which means it must be paid out of future earnings.
- *Convertible bonds* allow the holder to convert them into a set number of shares of common stock. Convertibles are attractive to some investors because they combine the features of a debt security with the capital gain potential of common equity. Consequently, convertible bonds generally offer lower interest rates than nonconvertible bonds of equivalent risk.
- *Debentures with warrants* provide investors with the right to purchase the firm's common stock at a specified price over a stated period of time. The warrant is said to be a "sweetener" because it makes the debenture more attractive. This sweetener results in a lower required yield. The warrant also provides issuing firms with potential future equity capital should the warrants be exercised.

**Preferred stock** is an equity security that pays a fixed dividend as a percentage of its face value, similar to the coupon payments on a bond. Unlike fixed income securities, however, preferred stock dividends are not legally binding. In reality, however, preferred dividends are indirectly binding because of the adverse credit implications of missing a dividend. From the company's point of view, preferred dividends come out of after-tax earnings, while interest payments come out of pre-tax earnings.

**International bonds** are fixed income instruments issued outside of the investor's home country, or by foreign entities within the investor's home country. From the perspective of a U.S.-domiciled investor, these types of securities represent over half of all fixed income securities available. There are three basic types of international bonds: Eurobonds, Yankee bonds, and international domestic bonds.

1. A *Eurobond* is a bond issued in a country outside of the country issuing the currency in which it is denominated. For example, a U.S. dollar-denominated bond issued in London is a Eurodollar bond.
2. *Yankee bonds* are U.S. dollar-denominated bonds issued in the United States by non-U.S. companies and governments. (Similar bonds issued in Japan are called *Samurai* bonds, while those issued in the United Kingdom are called *Bulldogs*, and so on.) Yankee bonds enable U.S. investors to buy the bonds of foreign companies but receive U.S. dollar-denominated payments, thus eliminating currency risk.
3. *International domestic bonds* is a catch-all category for the various domestic bond markets in foreign countries. As is the case in the United States, these include bonds issued domestically and denominated in the local currency. For example, a Canadian corporate bond issued in Canada in Canadian dollars is

an international domestic bond to a Japanese (or any non-Canadian) investor. Non-Canadian investors buying such a bond are exposed to exchange rate risk.

Short-term, low-risk debt securities are traded in the **money market**. As the name implies, securities traded in the money market are nearly “money” or cash. They are extremely liquid (i.e., they can be bought or sold very quickly at their fair market value). Because of their short-term nature, they are not as susceptible to interest rate risk as longer-lived debt securities. Money market are institutional investors because money market securities are typically traded in very large denominations. Small investors can invest in the money market by buying money market mutual funds.

U.S. **Treasury bills** are the most actively traded money market instrument. Individual investors can buy T-bills in \$1,000 denominations from government securities dealers, Federal Reserve Banks, brokers, financial institutions, or directly from the Treasury. Institutional investors can purchase T-bills in much larger denominations through government securities dealers or government auctions.

The following is a representative T-bill quote as it might appear in the *Wall Street Journal*. (Data in the quote would be closing figures for the previous trading day.)

<i>Maturity</i>	<i>Days to Maturity</i>	<i>Bid</i>	<i>Asked</i>	<i>Chg.</i>	<i>Ask Yld.</i>
June 15, '05	120	3.73	3.71	+0.01	3.81

The **maturity** of the bill is the day the bill matures or expires (i.e., the date the Treasury sends holders the face value of the bill).

Due to the **skip day settlement process**, it is assumed transactions will clear two business days later. The T-bill quoted above matures in 120 days. If you compared the actual date of the quote with the maturity date, it would appear two days longer than the stated maturity in days.

As with other dealer-handled securities, T-bill prices are quoted in bid and ask prices. These quotes are stated as a percentage discount, the **bank discount**, from face value. Also, the number of days in one year is assumed to be 360. Of course, the bid represents the price dealers will pay for a bill, and the ask is the price at which they will sell to you. Note that the bid yield exceeds the ask yield due to the inverse relationship between price and yield. In the preceding quotes, the bid price is lower than the ask price.

**Chg.** is the change from the previous day's bid discount. The closing bid discount for this bill on the previous day was 3.72%.

**Ask Yld.** (ask yield) is the **bond-equivalent yield** on the T-bill if purchased at the quoted ask discount, based on a 365-day year. This allows investors to easily

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compare T-bill yields with T-note and T-bond yields, which are quoted on a bond-equivalent basis.

Dollar discount is the discount off the face value based upon the stated bank discount rate. This is represented by:

$$\text{dollar discount} = r_b (\text{FV}) \frac{n}{360}$$

where:

$r_b$  = the annualized bank discount rate (bid or ask)

FV = the face value of the bill (we will assume \$1,000)

n = the number of days until maturity

If you want to purchase the bill, the price you pay is based upon the portion received of the **ask discount**, which in our example is equal to 3.71% off the face value of the bill.

Since the bill has 120 days remaining, you receive one-third of the full (annual) bank discount, calculated as follows:

$$\begin{aligned} \text{price} &= \text{face value} - \text{dollar discount (ask)} \\ &= \$1,000 - 0.0371(1,000) \frac{120}{360} \\ &= \$1,000 - 37.10(0.3333) \\ &= \$1,000 - \$12.37 = \$987.63 \end{aligned}$$

You will receive a dollar discount of \$12.37 and pay \$987.63 for the bill. When the bill matures in 120 days, you will receive the \$1,000 face value, and your return will be:

$$\frac{\$12.37}{\$987.63} \times \frac{365}{120} = 0.038096 = 3.81\%$$

You have probably noticed this is the **Ask Yld.** (i.e., bond-equivalent yield) from the quote. It is different from the *ask discount* for two primary reasons. First, the ask discount is a percentage of face value, while the ask yield is a percentage of price paid. Next, the actual yield is based upon a calendar of 365 days.<sup>3</sup> Let's take a closer look at how we calculated the ask yield.

Other money market instruments include certificates of deposit (CDs), commercial paper, banker's acceptances, Eurodollar deposits, repos, and federal funds. Most of these are listed daily in the *Wall Street Journal* in a column called "Money Rates."

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<sup>3</sup> Leap years assume a 366-day year.

CDs are short-term time deposits with commercial banks. They are available in almost any denomination and usually have maturities from 90 days to one year. There is typically a penalty for early withdrawal of the funds in a CD.

**Commercial paper** is a short-term security issued by a strong, creditworthy corporation with a short-term funding requirement. Since they are short term (270 days or less), very liquid, and very low risk, there is an extensive market for commercial paper. Often a firm will find itself with a temporary excess amount of cash. The firm can buy commercial paper issued by another firm, hold it for the desired time period (a few days to several months), and resell it in the commercial paper market. Commercial paper is generally not collateralized (i.e., no assets support the issue) and is not registered with the SEC.

**Bankers' acceptances** are utilized in international trade. They are short-term agreements with an importer's bank that guarantee payment of the exporter's invoice. For example, if a U.S. company wants to do business with a company in a less-developed nation, the U.S. firm (the exporter) would have the other firm (the importer) procure a guarantee in the form of a banker's acceptance. This eliminates the credit risk associated with the transaction. Of course, the exporter and importer also have to agree on exchange terms (such as whether the importer must pay in U.S. dollars or local currency).

**Repo** is short for *repurchase agreement*, which is a short-term loan collateralized by marketable securities, such as common or preferred stock, money market instruments, or Treasury securities. Institutional investors, states, and local governments use the repo market as a means to invest short-term excess cash at very low risk. Rather than liquidate inventories of short-term marketable securities and repurchase them later, firms with a short-term need for cash use the securities as collateral to borrow in the repo market. As with commercial paper, there is an extensive secondary repo market.

**Federal funds** are cash reserves a U.S. financial institution must keep on deposit at its regional Federal Reserve Bank. Larger banks also maintain federal funds deposits at correspondent banks throughout the world to facilitate the immediate transfer of funds among banks for very large transactions. Banks with short-term excess federal funds can lend (i.e., transfer by journal entry) federal funds to banks with a short-term deficit. Billions of dollars may change hands in a given day in the federal funds market.

## DERIVATIVES MARKETS

A **derivative** is any instrument that derives its value from the value of some other asset. Derivatives trading occurs on a number of U.S. and international

## Securities Markets

exchanges including the Chicago Mercantile Exchange, Chicago Board of Trade, and American Stock Exchange. Among the more common securities traded in the derivatives markets are options, forwards, and futures.

### Options and Warrants

**Warrants** are issued by corporations and convey to the holder the right, but not the obligation, to purchase a firm's common stock at a stated price over a stated time period. Warrants do not constitute ownership of the firm's stock unless they are exercised, meaning that the holder exercises his right to buy the stock in the future. If the warrants are not exercised within the stated time period, they expire worthless.

**Call options** are similar to warrants, but are created by investors willing to "write" or sell an option on an underlying asset, such as a stock. The buyer of a call option on a stock has the right to buy stock at a stated price on or before a stated date from the call seller. In general, a call buyer expects to profit if the price of the stock increases. Options ordinarily have shorter lives than warrants. Call options usually have maturities of less than a year, whereas the maturities of warrants may extend for many years.

The buyer of a **put option** on a stock has the right to sell the stock on or before a given date at a stated price to the put seller. Puts are purchased by investors who expect a stock price to decline or by investors who already own the stock but want to protect against a drop in the stock's price.

The common thread between all types of options contracts is that the holder (i.e., the buyer) has the *right but not the obligation* to either buy or sell the underlying asset (in our examples the underlying asset has been common stock). The seller (the "writer") is obligated to take the opposite side of the transaction if the buyer elects to exercise their option to buy or sell the underlying.

### Futures

**Commodity futures contracts** are contracts for the delivery of an underlying commodity at some future date. The current price of the futures contract is determined by the participants' beliefs about the future price of the commodity. If an investor expects the price of a commodity to go up, she will buy (go long) a futures contract today. A long position is obligated to accept delivery of the commodity in the future, paying the price agreed upon today. If she sells a similar contract before the delivery date, she can cancel her obligation (at a profit or a loss) without accepting delivery of the commodity. If the investor expects the price

to decline, she may sell (go short) a futures contract, expecting to buy a similar contract to cover her obligation after the price has dropped.

**Financial futures** are futures contracts for which the underlying asset is some financial instrument, such as T-bills, T-bonds, or a stock index.

The common thread between all futures contracts is that both counterparties to the futures contract are *obligated* to enter into the stated transaction on the expiration date of the contract if they have not previously closed their positions.

These contracts provide tools to protect fixed income portfolios from adverse price changes. This is called hedging. Similarly, currency futures may be used to speculate on price changes.

There are several differences between buying a futures contract and buying the underlying asset itself. One is leverage. When you buy a futures contract, you only have to put up a deposit equal to a small proportion of the value of the contract (called the margin). Consequently, when commodity prices change, the value of commodity futures contracts change by a lot relative to the required margin deposit. As with options, the term of the contract is short-term, typically expiring within a year.

## ALTERNATIVE INVESTMENTS

Instead of direct investment, many investors choose to acquire their investments indirectly by purchasing shares in **investment companies** or **mutual funds**.

Investment companies pool together many investors' funds to purchase a large, diversified portfolio that would otherwise be unavailable to the individuals. Some of the many types of investment funds available include the following:

- *Money market funds* offer investors alternatives to bank savings accounts and provide access to money market instruments that are normally only available to large institutional investors. These funds often offer check-writing privileges.
- *Bond funds* invest in various categories of fixed income securities: governments, corporates, and municipal bonds.
- *Common stock funds* invest in equities based on the fund's stated investment goals. Stock funds provide investors with limited financial resources access to the benefits of diversification and professional management. There are international funds that provide investors convenient access to foreign markets.
- *Balanced funds* represent investment combinations of money market, bond, and common stock funds, with the allocations based on the fund's stated investment objectives.

## Securities Markets

**Real estate** is an important investment option that can greatly improve the diversity of a portfolio, since real estate returns have a low correlation with stock and bond returns.

**Real estate investment trusts (REITs)** are real estate investment pools specializing in one of a variety of real estate assets. They are similar to mutual funds. An investor can purchase an interest in construction and development REITs, mortgage pool REITs, or REITs that own and manage properties.

REITs are required by law to pay out 90% of their income as dividends, which leaves little to invest in new acquisitions. Furthermore, they must keep at least 75% of their assets in real estate investments and hold each investment for at least four years. Like any investment fund, each REIT has certain stated investment objectives, which the individual or institutional investor should carefully consider before acquiring shares.

The major advantage of investing in an REIT is that the investor is not involved with the actual control and management of properties. In addition, REITs are traded like other securities, offering the investor more liquidity than a direct investment in real estate.

## MARKET INDEXES

A market index is used to estimate and monitor the performance of the stock market or a segment of the market. Three primary methodologies are used to construct market indexes: price-weighted, market value-weighted, and unweighted.

### PRICE-WEIGHTED INDEX

The value of a *price-weighted index* is the arithmetic average of the prices of the securities included in the index.

Computationally, a price-weighted index adds together the market price of each stock in the index and then divides this total by the number of stocks in the index.

$$\text{price-weighted index} = \frac{(\sum \text{of stock prices})}{(\text{number of stocks in index adjusted for splits})}$$

The divisor of a price-weighted index must be adjusted for stock splits and changes in the composition of the index (i.e., adding or deleting stocks) so that the value of the index is unaffected by the change. For example, if a stock split occurs, the divisor must be adjusted before calculating the next value of the index.

The best known price-weighted index is the Dow Jones Industrial Average (DJIA), developed by Charles H. Dow in 1896. Since determining the overall movement in the market is difficult due to frequent and varied changes in the prices of stocks, he collected trading data on 12 stocks, which served as a sample of the overall stock market. He used movements in the average price of his sample as an indicator of movements of the stock market as a whole. If the average price in his sample increased, for example, the market was up. In the same fashion, if the average price decreased, the market was down. Figure 1 shows the original 12 stocks in the DJIA.

**Figure 1: Original Twelve Stocks in the Dow Jones Industrial Average**

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American Cotton	Oil Laclede Gas
American Sugar	National Lead
American Tobacco	North American
Chicago Gas	Tennessee Coal and Iron
Distilling and Cattle Feeding	U.S. Leather Preferred
General Electric	U.S. Rubber

Figure 2 shows the composition of the DJIA as of December 2011. The list has changed significantly since its inception, primarily because most of the original companies no longer exist. General Electric is the only one of the original firms that is still in the DJIA. Today, the number of firms has increased to 30, 27 of which are NYSE-listed stocks with the remaining three coming from the NASDAQ. The Dow Jones Company publishes several other indexes, including its Transportation and Utilities Averages.



Figure 2: The 30 Stocks in the DJIA, December 2011

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3M Co.	Intel Corp.
Alcoa Inc.	International Business Machines Corp.
American Express Co.	Johnson & Johnson
AT&T Inc.	JPMorgan Chase & Co.
Bank of America Corp.	Kraft Foods Inc.
Boeing Co.	McDonald's Corp.
Caterpillar Inc.	Merck & Co. Inc.
Chevron Corp.	Microsoft Corp.
Cisco Systems Inc.	Pfizer Inc.
Coca-Cola Co.	Procter & Gamble Co.
E.I. DuPont de Nemours & Co.	Travelers Cos. Inc.
Exxon Mobil Corp.	United Technologies Corp.
General Electric Co.	Verizon Communications Inc.
Hewlett-Packard Co.	Wal-Mart Stores Inc.
Home Depot Inc.	Walt Disney Co.

Let's look at a very simplified example of this approach. We will assume there are only four stocks in the DJIA, and their price histories are as follows:

	<i>Stocks</i>				<i>DJIA</i>
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	
Day 1	20.00	15.00	20.00	30.00	21.25 <sup>b</sup>
Day 2	21.00	16.00	18.00	29.00	21.00 <sup>c</sup>
Day 3	12.00 <sup>a</sup>	17.00	19.00	31.00	22.57 <sup>d</sup>

<sup>a</sup> Stock A had a 2:1 stock split after the close of the market on day 2.

$$^b DJIA_1 = \frac{20 + 15 + 20 + 30}{4} = 21.25$$

$$^c DJIA_2 = \frac{21 + 16 + 18 + 29}{4} = 21.00$$

$$^d DJIA_3 = \frac{12 + 17 + 19 + 31}{3.5} = 22.57$$

At the end of day one, the closing prices of the four stocks are added together and divided by four, resulting in a DJIA of 21.25. This figure represents the average price of the stocks at the end of day one. At the end of day two we see mixed results. Stock A is up \$1.00, B is up \$1.00, C is down \$2.00, and D is down \$1.00. By simply looking at the individual price movements it is very difficult to determine the general movement of the market, if any. Since a non-economic event has not occurred, the DJIA is calculated using the same denominator as for

day one. The DJIA at the end of day two is 21.00, indicating a minor downward movement in the market.

On day three, we observe that stock B has increased \$1.00, C has increased \$1.00, and D has increased \$2.00. Stock A appears to have dropped \$9.00, but A has experienced a 2:1 stock split.<sup>4</sup> Theoretically, the stock split should affect only the market price and par value of the stock, not the value of the firm.<sup>5</sup> However, if we calculate the DJIA *without* adjusting for the split we get:

$$\text{DJIA} = \frac{12 + 17 + 19 + 31}{4} = 19.75$$

When observing a value of 19.75 for the DJIA, a casual observer would think the market is down. However, upon closer scrutiny, he would be somewhat perplexed. All the stocks in the average have increased in price except stock A, and the split caused the drop in A, not a drop in the market value of the firm. In fact, with a 2:1 split, the theoretical post-split price for A should be half of \$21.00, or \$10.50. Since A closed at \$12.00, it actually closed up along with the other stocks!

To account for the split in stock A, we have to adjust the divisor before calculating the DJIA. First, we use the day two DJIA value of 21.00 and adjust the price of A to its theoretically correct post-split price. Leaving the other stocks unchanged, we solve for the denominator that would leave the DJIA unchanged.

This is done in the following way:

$$\text{DJIA}_2 = 21.00 = \frac{10.50 + 16.00 + 18.00 + 29.00}{d_{\text{new}}}$$

$d_{\text{new}}$  is the new denominator (i.e., the denominator that would leave the DJIA unchanged if A had split, and all other prices remained unchanged). Rearranging and solving for the new denominator:

$$d_{\text{new}} = \frac{10.50 + 16.00 + 18.00 + 29.00}{21.00} = 3.50$$

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4 A 2:1 (2 for 1) stock split means you receive two new shares to replace every old share you hold. For example, if you hold 100 shares with a market value of \$50.00 each, your shares were worth a total of \$5,000. After the split you will hold 200 shares, but their price will be halved to account for the split. They will each be worth \$25.00, so your wealth has not changed. Before the split you held 100 shares of stock worth \$50 per share; now you hold 200 shares of stock worth \$25 per share. In each case you hold \$5,000 in stock.

5 In the absence of a split or stock dividend, changes in the price of a stock indicate changes in the market value of the company.

## Securities Markets

Using the closing prices for day three *and the adjusted divisor*, the true value of the DJIA at the end of day three is:<sup>6</sup>

$$\text{DJIA}_3 = \frac{12 + 17 + 19 + 31}{3.5} = 22.57$$

After adjusting the denominator to account for the split in Stock A, we confirm that the DJIA has actually increased, as we had expected.

As of July 2010, the divisor was 0.132129493. Obviously, there have been many adjustments made to the divisor over the years to account for stock splits and dividends as well as replacement of companies when they are acquired or removed from the index.<sup>7</sup> The divisor is listed every day in the third section of the *Wall Street Journal* in the headings for the Dow Jones Industrial, Transportation, and Utilities Averages.

Criticisms of the DJIA include the following:

- The limited number of stocks in the index (30). Considering that there are thousands of stocks traded in the U.S. markets, 30 does not appear to be a fully representative sample.
- The fact that the 30 stocks represent some of the largest firms listed on the NYSE and NASDAQ. Thus, small firms and their effect on the market are not represented in the DJIA.
- The downward bias in the computation of the index. This is a result of the decline in relative weighting within the index that occurs when a high-priced stock splits. Since successful firms tend to increase and split more frequently than less successful firms, their relative influence in the index is reduced through time.

Another price-weighted index is the Nikkei Dow Jones Stock Average. The Nikkei Dow is an arithmetic average of 225 stocks listed on the Tokyo Stock Exchange.

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6 Remember, Stock A split during the day, so its price halved and then increased somewhat by the end (close) of the day.

7 Adjusting the divisor for replacing a stock is done exactly as it is for splits. In this case the divisor is calculated for the previous day with the new stock's price in place of the stock being removed. The divisor typically gets smaller with each adjustment, but it will get larger if a stock is replaced with a higher-priced stock.

## MARKET VALUE-WEIGHTED INDEX

A market value-weighted index (or simply “value-weighted”) is calculated by summing the total market value (current stock price times the number of shares outstanding) of all the stocks in the index. This sum is then divided by a similar sum calculated during the selected base period. This ratio is then multiplied by the index’s beginning base value.

$$\text{MV index} = \frac{\text{current total market value}}{\text{base total market value}} \times \text{beginning index value}$$

where:

current total market value = the total market value of the stocks in the index today

base total market value = the total market value of the stocks in the index on the day the index was formed

beginning index value = the (arbitrary) starting value of the index

Most well-known market indexes are in fact market value weighted. The following are major *market value-weighted indexes*:

- Standard & Poor’s 500 Composite Index (500 firms).
- New York Stock Exchange Index considers all NYSE stocks in one of five value weighted indexes: (1) industrial, (2) utility, (3) transportation, (4) financial, and (5) the composite index.
- Other U.S. indexes are the NASDAQ index, the AMEX Market Value Index, the Dow Jones Equity Market Index, the Wilshire 5000 Equity Index, and the Russell 3,000 Index.
- Non-U.S. indexes include the *Financial Times* Actuaries Share Indexes, which represents stocks on the London Stock Exchange, and the Tokyo Stock Exchange Price Index, which represents stocks listed on the first section of the Tokyo Stock Exchange.

The major problem with a value-weighted index is that firms with greater market capitalizations have a greater impact on the index than do firms with lower market capitalizations.

## UNWEIGHTED (EQUAL-WEIGHTED) PRICE INDEX

Unweighted index computations are based on returns rather than stock prices. In an unweighted index all stocks are equally weighted (i.e., the index is computed as if an investor maintains an *equal dollar investment* in each stock in the index), and changes in the index are calculated as the average (either geometric or arithmetic) percentage price change in the listed stocks.

## Securities Markets

The formula to compute the *arithmetic average* is:

$$R_A = \frac{R_1 + R_2 + \dots + R_N}{N}$$

where:

$R_A$  = arithmetic average return

$R_i$  = return for stock  $i$ ,  $i = 1$  to  $N$

$N$  = number of stocks included

The formula to compute the *geometric average*:

$$R_G = [(1 + R_1)(1 + R_2)\dots(1 + R_N)]^{1/N} - 1$$

where:

$R_G$  = time-weighted return

$R_i$  = return for stock  $i$ ,  $i = 1$  to  $N$

$N$  = number of stocks included

Two equal weighted indexes:

1. The Value Line Composite Average is a geometric average of 1,695 stock returns.
2. The Financial Times Ordinary Share Index is a geometric average of 30 major stocks on the London Stock Exchange.

### **Example: Price-weighted, unweighted, and market value-weighted indexes**

Given the following information in Figures 3 and 4, calculate a *price-weighted*, *unweighted*, and *value-weighted* index.

Answer: Price-weighted index

Figure 3: Price/Market Value Data to Calculate a Price-Weighted Index

	<i>As of December 31, 20X7</i>			<i>As of January 31, 20X8</i>		
	<i>Share Price</i>	<i>Number of Shares</i>	<i>Market Value</i>	<i>Share Price</i>	<i>Number of Shares</i>	<i>Market Value</i>
Stock W	\$30	1,000	\$30,000	\$18	2,000	\$36,000
Stock X	\$10	3,000	\$30,000	\$12	3,000	\$36,000
Stock Y	\$20	1,000	\$20,000	\$25	1,000	\$25,000
Stock Z	\$60	500	\$30,000	\$58	500	\$29,000
		<b>Total</b>	<b>\$110,000</b>		<b>Total</b>	<b>\$126,000</b>

Note: Firm W has a split in this scenario (Figure 3) only.

December 31, 20X7:

$$PW_0 = \frac{30 + 10 + 20 + 60}{4} = 30$$

Since Stock W has experienced a 2:1 split (note doubling of shares) between the two points in time, the divisor (denominator) must be adjusted before calculating the value of the index on January 31, 20X8.

To adjust the divisor, go back to the previous day and halve stock W's price to adjust for the 2:1 split and calculate the divisor that would have left the average unchanged:

$$\frac{15 + 10 + 20 + 60}{\text{new divisor}} = 30$$

$$\text{new divisor} = \frac{15 + 10 + 20 + 60}{30} = 3.5$$

January 31, 20X8:

Using the new divisor and the prices for January 31, 20X8:

$$PW_1 = \frac{18 + 12 + 25 + 58}{3.5} = 32.29$$

The change in the price-weighted index from 12/31/2007 to 1/31/2008 represents a 1-month return of  $[(32.29 - 30) / 30] = 7.63\%$ .

**Answer: Unweighted index**

**Figure 4: Price/Market Value Data to Calculate an Unweighted Index**

Stock	As of December 31, 20X7			As of January 31, 20X8			Return Relative
	Share Price	Number of Shares	Market Value	Share Price	Number of Shares	Market Value	
W	\$30	1,000	\$30,000	\$32	1,000	\$32,000	1.0667
X	\$10	3,000	\$30,000	\$12	3,000	\$36,000	1.2000
Y	\$20	1,000	\$20,000	\$25	1,000	\$25,000	1.2500
Z	\$60	500	\$30,000	\$58	500	\$29,000	0.9667
		<b>Total</b>	<b>\$110,000</b>		<b>Total</b>	<b>\$126,000</b>	

Arithmetic return:

$$R_A = \frac{R_1 + R_2 + \dots + R_N}{N}$$

$$R_A = \frac{0.0667 + 0.2000 + 0.2500 + 0.0333}{4} = 0.1208 = 12.08\%$$

Geometric return:

$$R_G = [(1 + R_1)(1 + R_2) \dots (1 + R_N)]^{1/N} - 1$$

$$R_G = [(1.0667)(1.2000)(1.2500)(0.9667)]^{1/4} - 1$$

$$R_G = (1.5468)^{1/4} - 1 = 1.1152 - 1 = 0.1152 = 11.52\%$$

Assuming a value of 100 for the index on December 3, the value of the index on January 31 is:

$$100(1.1208) = 112.09 \text{ (arithmetic)} \text{ or } 100(1.1152) = 111.52 \text{ (geometric)}$$

**Answer: Value-Weighted Index**

$$MV \text{ index} = \frac{\text{current total market value}}{\text{base total market value}} \times \text{beginning index value}$$

To calculate the level of a market value-weighted index, you must have the total market value of the index at its inception and its assumed value at that point. We will assume the base total market value of the stocks in the index was \$50,000, at which time the value of the index was set at 100.

$$\text{MV index} = \frac{\text{current total market value}}{\text{base total market value}} \times \text{beginning index value}$$

$$\text{MV index}_{12/31/20X7} = \frac{\$110,000}{\$50,000} \times 100 = 220$$

$$\text{MV index}_{1/31/20X8} = \frac{\$122,000}{\$50,000} \times 100 = 244$$

$$\text{index return}_{\text{January}} = \frac{244}{220} - 1 = 10.91\%$$



## SUMMARY

### MARKETS

- A. In a direct market, the buyer and seller contact one another directly.
- B. In a broker market, the broker facilitates the meeting of the buyer and seller. The broker does not take an ownership interest in the product being sold.
- C. In a dealer market, a dealer facilitates the transaction by purchasing ownership in the product and reselling.
- D. In spot markets, securities, commodities, and other goods are traded for immediate delivery.
- E. In forward markets, traders agree to transact at a future date at a specified price.

### PRIMARY CAPITAL MARKETS

- A. Corporations, federal governments, and municipalities raise capital in primary markets by issuing new securities. That is, the issuing firm or organization receives the proceeds from the sale, net of the issuance (flotation) costs.
- B. In a public offering, organizations sell securities to the general public.
- C. In a private placement, the investment banking firm acts as a broker, and the issue (usually bonds) is sold to a small number of large investors.

### SECONDARY CAPITAL MARKETS

- A. In secondary markets investors trade securities among themselves.
- B. Proceeds from the sale of securities in the secondary market go to investors, not the original issuer of the security.
- C. Common stock represents ownership in the issuing corporation.
- D. Common stock is categorized by the industry and country in which the issuer operates.
- E. The best-known secondary markets deal in equity securities. They are traded over the counter and on physical exchanges worldwide.
- F. The New York Stock Exchange (NYSE) utilizes a specialist system. Specialists have the following responsibilities:
  - 1. Act as agents by executing orders for floor brokers and taking limit and stop-loss orders.
  - 2. Act as catalysts by bringing together buyers and sellers.
  - 3. Act as auctioneers by establishing a fair price when the market opens each day.
  - 4. Stabilize prices by trading intermittently to minimize price gaps.
  - 5. Provide capital by using company assets to eliminate order imbalances.
- G. NASDAQ, the over-the-counter market, is the most famous “cyber trading floor.” Market makers, similar to specialists on the NYSE, enter their own bid and ask price quotes to ensure smooth functioning and liquid markets.

- H. International equity investments allow investors to gain exposure to foreign markets. Investors can gain international exposure through purchase of foreign stocks, ADRs, American shares, or international mutual funds.
- I. There is an extensive secondary market for debt securities issued by corporations, states, municipalities, and federal governments.
- J. Savings accounts are a simple, low-risk method of fixed income investing and can be carried out through passbook savings accounts and certificates of deposit.
- K. The U.S. Treasury issues T-bills, T-notes, and T-bonds with maturities of less than one year, one to ten years, and ten to thirty years, respectively. T-bills are discount securities, while T-notes and T-bonds are interest bearing.
- L. Strips are T-note and T-bond interest and principal payments, which have been stripped apart and sold separately. Each payment becomes a zero-coupon bond.
- M. Munis are issued by local governments and can be backed by revenues generated from a public works project or by the government's taxing authority. Munis are exempt from federal income tax.
- N. Corporations also issue debt securities. Most corporate debt is sold privately.
- O. Corporate debt is categorized according to the security provided to investors and includes senior secured bonds, debentures, subordinated debentures, income bonds, convertible bonds, and debentures with attached warrants.
- P. Preferred stock is technically an equity security but behaves more like a fixed income security since it pays a fixed dividend.
- Q. International bonds are fixed income securities issued outside of the investor's home country, or issued by foreign investors within the home country. For the U.S. investor, these include Eurobonds, Yankee bonds, and international domestic bonds.
- R. Derivatives derive their value from some underlying asset. The derivatives market allows investors to manage the uncertainty of future events.
- S. Options convey the right but not the obligation to purchase or sell the underlying asset in the future. Types of options are warrants, call options, and put options.
- T. Futures are contracts that obligate both parties to transact in the future. Futures exist on both commodities and financial instruments. The relatively low margin requirements on futures allow the investor to leverage his investment.

## **MONEY MARKETS**

- A. Money market securities are short-lived, low risk, very liquid debt instruments.
- B. U.S. T-bills are the most heavily traded money market instruments.

## Securities Markets

- C. T-bills are pure discount instruments. They are purchased at a discount from face value and pay face value at maturity. T-bills are priced using the bank discount method.
- D. Commercial paper is uncollateralized, short-term (up to 270 days) borrowing by very large, creditworthy firms.
- E. Bankers' acceptances are used in international trade. They are short-term agreements with an importer's bank that guarantee payment of an exporter's invoice.
- F. Repurchase agreements (repos) are short-term loans collateralized by marketable securities.
- G. Federal funds represent the amount of money banks must keep on deposit as reserves at the Federal Reserve Bank in their region.

## ALTERNATIVE INVESTMENTS

- A. Investment funds pool the funds of many investors together to purchase a diversified portfolio of securities. Investment funds can be structured many ways, but some of the most common are money market funds, bond funds, common stock funds, and balanced funds.
- B. REITs invest in various types of real estate properties and/or real estate mortgages. REITs sell shares of stock to the investing public and have the following characteristics:
  1. REITs must use the proceeds, along with borrowed funds, to invest in a portfolio of real estate investments.
  2. REITs are required by law to pay out 90% of their income.
  3. REITs must keep at least 75% of their assets in real estate and must hold each investment for at least four years.

## MARKET INDICATORS

- A. A price-weighted index is simply an arithmetic average of the prices of the securities included in the index. Computationally, a price-weighted index adds together the market price of each stock in the index and then divides this total by the number of stocks in the index.
- B. The value-weighted index assumes you make a proportional market value investment in each company in the index. The major problem with a value-weighted index is that firms with greater market capitalization have a greater impact on the index than do firms with lower market capitalization.
- C. Unweighted index computations are based on returns rather than stock prices. In an unweighted index all stocks are equally weighted (i.e., the index is computed as if an investor maintains an equal dollar investment in each stock in the index), and changes in the index are calculated as the (either geometric or arithmetic) average percentage price change in the listed stocks.

**PRACTICE QUESTIONS: SECURITIES MARKETS**

1. All of the following are advantages of private placements for the issuing entity EXCEPT that private placements:
  - A. do not require a prospectus.
  - B. are issued in the third market.
  - C. do not require SEC approval.
  - D. have lower flotation costs.
  
2. Consider the following 4 stocks, all of which are included in the S&P 500 index:

<i>Stock</i>	<i>Price per share</i>	<i>Number of shares (in millions)</i>
A	\$8.00	100
B	\$45.00	10
C	\$3.00	200
D	\$8.00	10

- The return on the S&P 500 index would be *most* affected by a 10% change in the price of which stock?
- A. Stock A.
  - B. Stock B.
  - C. Stock C.
  - D. Stock D.
- 
3. Which of the following is the *primary difference* between a broker market and a dealer market?
    - A. In a broker market, the buyer and seller must meet face to face.
    - B. In the dealer market, dealers are legally prevented from taking possession of the product.
    - C. In a broker market, the broker actually takes ownership of the product.
    - D. In the dealer market, the dealer actually takes ownership of the product.
  
  4. Frances Russell expects to harvest her wheat crop in six months. In total she expects to harvest 10,000 bushels. If Russell wants to sell her bushels of wheat now, in which of the following markets would she *most likely* sell her wheat?
    - A. Money market.
    - B. Spot market.
    - C. Forward market.
    - D. Farmer's market.

## Securities Markets

5. Which of the following is TRUE concerning primary and secondary markets?
- A. Stocks of larger, more established firms are traded only in primary markets.
  - B. Stocks of newer, smaller firms are traded only in secondary markets.
  - C. Bonds are traded only in secondary markets.
  - D. Firms issue new securities in primary markets.
6. Miller & Company, a large clothing retailer, is preparing to make a secondary public offering of its common stock. The company's investment banking firm has sent a detailed list of the flotation costs the company can expect to incur. Which of the following items included in the investment banker's list is NOT an example of flotation costs?
- A. The costs of printing stock certificates.
  - B. The underwriter spread.
  - C. Legal and accounting fees.
  - D. Product advertising costs.
7. Which of the following have initial maturities of less than one year?
- A. U.S. Treasury notes.
  - B. U.S. Treasury bonds.
  - C. U.S. Treasury bills.
  - D. All can be issued with an initial life less than one year.
8. A large manufacturing firm often has short-term excesses of cash. The firm's CFO has mentioned several times that the firm should invest the excess cash in the money market to pick up a little extra return. In which of the following securities would the CFO NOT want to invest?
- A. U.S. Treasury bill.
  - B. Corporate bond.
  - C. Commercial paper.
  - D. Banker's acceptance.
9. Terrel Sporting Goods (TSG) is planning to make an initial public offering next month. The investment bank for TSG's IPO is planning to fully underwrite the issuance. Which of the following is TRUE of the underwriter in this situation?
- A. The underwriter faces the most risk.
  - B. The underwriter only promises best efforts in selling the issue.
  - C. The underwriter spread is the difference between the price the investors pay for the stock and the flotation costs.
  - D. The underwriter guarantees all the shares will be sold but not the price the issuing firm will receive.

Use the following information to answer Questions 10 through 12.

<i>Maturity</i>	<i>Days to Maturity</i>	<i>Bid</i>	<i>Asked</i>	<i>Chg.</i>	<i>Ask Yield</i>
June 15 '05	140	3.73	3.71	+0.01	?

10. Assuming an investor could trade at the discounts in the quote, what would he have to pay for the T-bill?
- \$962.70.
  - \$962.90.
  - \$985.49.
  - \$985.57.
11. Assuming an investor could trade at the discounts in the quote, what would he receive from selling the T-bill?
- \$985.49.
  - \$985.57.
  - \$962.70.
  - \$962.90.
12. What is the ask yield (bond-equivalent yield) for the T-bill in the quote?
- 3.71%.
  - 3.73%.
  - 3.82%.
  - 3.91%.
13. Jared Keyes strongly believes that the price of SCF Inc. will increase rapidly in the next several months. He has decided to invest a significant amount of his personal assets into the stock of SCF which he currently does not own. If Keyes submits a market order, which of the following *best* describes how the order should be executed?
- Highest ask available.
  - Lowest ask available.
  - Highest bid available.
  - Lowest bid available.
14. HMB, Inc., wishes to issue long-term fixed income securities to finance an expansion project but does not have specific assets to pledge to the issue. In addition, the company is concerned that future cash flow may become constrained, preventing the company from making any required interest payments. Given HMB's situation, which of the following would be the *least* attractive financing alternative?
- Preferred stock.
  - Income bonds
  - Subordinated debentures.
  - Zero coupon debentures.

## Securities Markets

15. Henrietta Higgins is a U.S. investor with a portfolio worth more than \$1 million. Recently, Higgins decided to decrease her exposure to the domestic equity market by reallocating her current portfolio. Which of the following would NOT help Higgins accomplish her goal?
- A. American depository receipts.
  - B. Long S&P 500 index futures.
  - C. U.S. real estate investment trusts.
  - D. Yankee bonds.
16. As a newly hired specialist for the New York Stock Exchange, Terry Quince will be expected to perform all of the following tasks EXCEPT:
- A. monitor and execute unfilled limit orders.
  - B. monitor compliance of margin requirements.
  - C. provide liquidity to the market.
  - D. act as a dealer for her company's account.
17. A U.S. government bond is selling in the market at 91-28. The dollar value of this bond is:
- A. \$910.87.
  - B. \$912.80.
  - C. \$918.75.
  - D. \$9,128.00.
18. Which of the following statements regarding Treasury bills is FALSE?
- A. They are safe, easily marketable securities.
  - B. They account for the largest amount of short-term money market instruments sold.
  - C. They are issued with three-, six-, or nine-month maturities.
  - D. They are initially sold at a discount.
19. On January 15 Perry Smith signed an agreement to take possession of 1,000 ounces of gold on March 15. The price Smith will pay for the gold is \$884 per ounce. After taking possession of the gold, Smith plans to sell 10 ounce lots to individuals at a retail price. In what kind of market is Smith operating?
- A. Dealer market.
  - B. Broker market.
  - C. Direct market.
  - D. Spot market.

20. The value of an index based on the geometric average of five stocks, where the returns on the five stocks during the given period were 15%, 5%, -10%, -8%, and 23% is *closest* to:
- A. 4%.
  - B. 5%.
  - C. 23%.
  - D. 25%.
21. An investor has a portfolio with a market value of \$1,000,000 at the end of December. The market value of the portfolio is \$893,000 at the end of January. The holding period yield on the investor's portfolio for January is *closest* to:
- A. -10.7%.
  - B. -8.9%.
  - C. -2.3%.
  - D. 12.0%.
22. Which of the following indices maintains an equal dollar in each stock in the index (equal-weighted price index)?
- A. Dow Jones Industrial Average (DJIA).
  - B. Standard and Poor's 500 Index.
  - C. Wilshire 5000 Equity Index.
  - D. Value Line Composite Average.



**PRACTICE QUESTION ANSWERS: SECURITIES MARKETS**

1. B Private placements are transactions where securities are issued to a small group of large investors. They do not require any prospectus or SEC approval, and therefore the flotation costs are less. The third market refers to the trading of exchange-listed securities on over-the-counter markets.
2. A The S&P 500, a market value-weighted index, will be most affected by the stock with the largest market capitalization (price  $\times$  shares). Stock A has a market cap of  $\$8 \times 100,000,000 = \$800$  million. The other stocks' market caps are lower.
3. D Brokers do not take possession of the product, but dealers do.
4. C Since Russell will not physically have the wheat for six months, she cannot sell in the spot market. She must therefore use the forward market to lock in a price today for her future harvest.
5. D Firms sell new securities in primary markets and actually receive the net proceeds from the sale. Seasoned securities (previously issued) are traded in secondary markets by investors.
6. D Flotation costs are all the costs associated with selling securities. This includes printing costs, legal and accounting fees, underwriter's spread, and any other cost the firm might experience.
7. C T-bills have maturities of 90 to 360 days.
8. B Money market securities are very liquid, very low risk securities with original lives of up to one year.
9. A When a new issue of securities is fully underwritten, the underwriter guarantees both the price and number of shares to be sold. The underwriter accepts all the risk: price and selling risk.
10. D Use the asked discount to determine the price you would have to pay.

$$P = \$1,000 - 0.037 \left( \frac{140}{360} \right) 11000 = \$985.57$$

11. A Use the bid discount to determine the price you would receive.

$$P = \$1,000 - 0.037 \left( \frac{140}{360} \right) 31000 = \$985.49$$

12. C Use the ask price to determine the ask yield.

$$\begin{aligned} \text{ask yield (BEY)} &= \frac{\$ \text{discount}}{\text{price}} \left( \frac{365}{n} \right) \\ &= \frac{\$14.43}{\$985.57} \left( \frac{365}{140} \right) = 0.0382 = 3.82\% \end{aligned}$$

13. **B** A market order to buy should be executed at the lowest ask price available. A market order to sell should be executed at the highest bid price available.
14. **C** Subordinated debentures do not require collateral to be pledged specifically to the issue that meets the first of HMB's financing requirements. Subordinated debentures do, however, require that regular interest payments be met in order to avoid defaulting on the bonds. Since HMB is concerned about its future ability to pay interest payments, this is not an attractive financing alternative. The other choices meet both of the company's requirements.
15. **B** Because the underlying asset for an S&P 500 Index future is the U.S. equity market, this security will increase the investor's exposure to the U.S. equity market. All of the other choices are either non-domestic or non-equity investments that would decrease the investor's exposure to U.S. equities.
16. **B** In the securities markets, federal regulators normally set margin requirements, with maintenance margin requirements set by the securities exchanges and the NASDAQ. The clearinghouse conducts the daily settlement of margin accounts. The specialist performs five duties: act as an agent for floor brokers by holding limit orders; act as a catalyst by bringing buyers and sellers together; act as auctioneer by providing a location for trading by the open outcry method; stabilize prices by preventing large jumps in trading prices; provide capital by acting as a market maker to take the other side of orders during temporary imbalances in trading.
17. **C** U.S. government bonds are quoted in 32nds. 91.28 would be equivalent to  $91\frac{28}{32}$ , which is  $91\frac{7}{8}$ . This equivalent to 91.875% of the par value of \$1,000, which is \$918.75.
18. **C** Treasury bills mature in three months, six months, and one year. They do not have 9-month maturities. All other statements are true.
19. **A** Since Smith is taking possession of the commodity and bearing the risk of reselling the gold, he is functioning as a gold dealer and operating in a dealer market.
20. **A**  $R_G = [(1 + R_1)(1 + R_2)(1 + R_3)(1 + R_4)\dots(1 + R_N)]^{1/N} - 1$

where:

$R_G$  = time-weighted return

$R_i$  = price relative for stock  $i$ ,  $i = 1$  to  $N$

$N$  = number of stocks included

Therefore:

$$\begin{aligned} R_G &= [(1 + 0.15)(1 + 0.05)(1 - 0.1)(1 - 0.08)(1 + 0.23)]^{1/5} - 1 \\ &= [(1.15)(1.05)(0.9)(0.92)(1.23)]^{1/5} - 1 \\ &= 4.22\% \end{aligned}$$

## Securities Markets

21. A Holding period return =  $\frac{V_1 - V_0}{V_0}$

$$\text{HPR} = \frac{893,000 - 1,000,000}{1,000,000}$$

$$\text{HPR} = -10.7\%$$

22. D Dow Jones Industrial Average (DJIA) is a price-weighted index. Standard and Poor's 500 Index is a market value-weighted index. Wilshire 5000 Equity Index is a market value-weighted index. Value Line Composite Average is an equal-weighted index.

# ASSET VALUATION

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The analysis of equity, debt, and derivative securities is a primary function of securities analysts and portfolio managers and is part of the core curriculum for the CFA program. Understanding the features of these securities and the methods used to analyze and value them is essential to success as a CFA candidate.

## EQUITY SECURITIES

Companies finance their operations primarily by issuing debt or by selling ownership (equity) shares to the public. This section focuses on the analysis and valuation of the ownership equity issued by public corporations. The most common form of equity issued by corporations is common stock, which is a security that represents part ownership in a company.

We begin with an overview of the equity valuation framework. We then discuss key financial ratios that characterize the performance of the company. We conclude this section with accepted methods for calculating the **intrinsic value** of a share of common stock. The intrinsic value is also known as the **fair value** of the stock. Mathematically the intrinsic value equals the present value of the cash flows that are expected to materialize in the future.

## THE EQUITY VALUATION FRAMEWORK

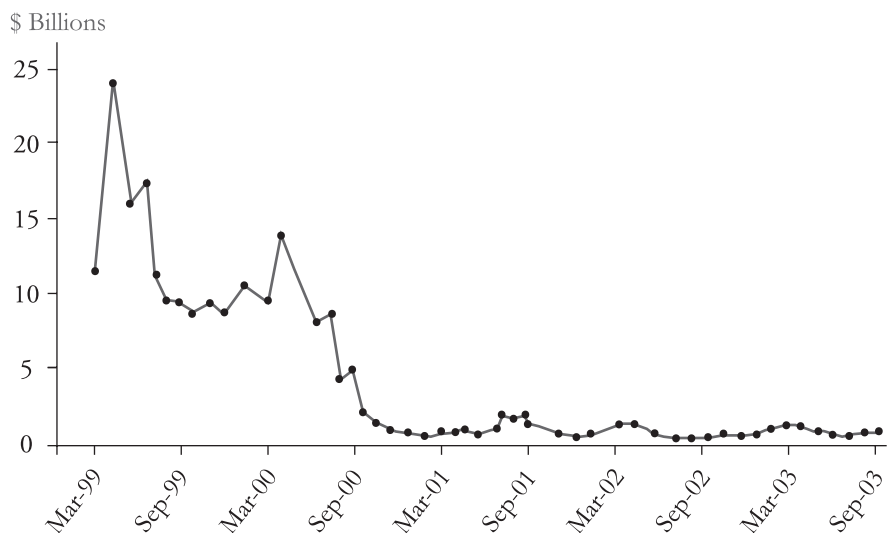
Before proceeding, we should note that stock prices sometimes deviate significantly from their intrinsic value. Consider the case of Priceline.com Inc., an Internet services company. The company first issued shares of stock in 1999 at a price of \$69 per share, for a market value of approximately \$9.8 billion, larger than most successful companies that had been in business for decades. By the end of the first month of trading, the stock price soared to \$162, and Priceline's market value jumped to \$23 billion. Within a month of issuing stock, Priceline.com became ranked among the giant companies in the world.

Did Priceline.com deserve a lofty ranking? Consider that the company's earnings (net income) for 1998 through 2002 were *negative* \$65 million, *negative* \$79 million, *negative* \$329 million, *negative* \$18 million, and *negative* 20 million, respectively. The company did not turn a profit until 2003. Eventually, Priceline.com's stock price sank to \$1.13 on December 26, 2000, and its market value

## Asset Valuation

plunged to \$190 million. A chart of Priceline.com's market value through 2003 is provided in Figure 1.

Figure 1: Priceline.com Market Value



Investors in Priceline.com and similar dot-com companies got caught up in a classic **speculative bubble**, in which stock prices are driven more by emotion and fads than by discipline and facts. As we know, bubbles can grow to a nice large size and are great fun to watch, but eventually the bubble bursts. The bigger the bubble, the bigger the messy aftermath. Stock prices reached untenable levels in the late 1990s and then came crashing down with a resounding thud. For instance, the NASDAQ index of primarily smaller technology companies hit its peak in March 2000. By the end of 2000, the tech bubble burst and the index had plunged 40% from its high.

Sound methods for valuing companies and for avoiding speculative bubbles such as the dot-com craze were provided nearly 70 years earlier by Benjamin Graham and David Dodd in their seminal work *Security Analysis*.<sup>1</sup> Graham and Dodd described a systematic and disciplined approach to equity analysis. The two key components are **financial statement analysis** and **equity valuation**.

Financial statement analysis refers to extensive, unbiased examination of the company's financial statements and key financial ratios. Analysts also must exercise care and due diligence with the numbers they examine. In particular, analysts should always perform checks on the **earnings quality** of the firm. Low-quality earnings result from aggressive accounting policies, such as recording revenues too quickly and recording expenses too slowly.

1 Benjamin Graham and David Dodd, *Security Analysis*, McGraw-Hill: New York, 1934.

After performing a clear and extensive examination of the financial statements, an analyst proceeds to valuation. The centerpiece of valuation is the **capitalization of income method**, which is used to calculate the intrinsic value of the company's stock.

Steps in the *capitalization of income method* are:

1. Estimate the amount and timing of all future cash flows.
2. Estimate the risk (uncertainty) associated with cash flows.
3. Assign a required return based upon risk.
4. Find the total present value (today) of all the expected future cash flows.

## FINANCIAL STATEMENT ANALYSIS USING RATIOS

**Financial statement ratios** can be used to evaluate various different facets of a company's performance and condition, such as: (1) working capital management, (2) operating performance, (3) risk profile, and (4) growth potential.

### Working Capital Management

Most working capital management ratios are concerned with liquidity, meaning how quickly or even if the firm can meet its short-term obligations, such as accounts payable. Generally, more liquidity is preferred to less.

- The **current ratio**, calculated as the ratio of current assets (CA) to current liabilities (CL), is the best-known measure of liquidity:

$$\text{current ratio, CR} = \frac{\text{CA}}{\text{CL}}$$

The higher the current ratio (i.e., the more current assets as compared to current liabilities), the more likely it is that the company will be able to pay its short-term bills. A current ratio less than one means the company has negative net working capital (defined as current assets minus current liabilities), and could be facing a liquidity crisis. Generally, since current assets are used to pay current liabilities, the greater the current ratio the better the liquidity position of the firm.

Some firms have very specialized inventory that is not readily utilized by many other firms. Also, the firm might have inventory that is undesirable, such as

## Asset Valuation

soon-to-be outdated semiconductors. In these cases, liquidating inventory to meet current obligations is not a viable alternative.

- The **quick ratio** is a more stringent measure of liquidity than the current ratio because it does not include inventories, which might not be very liquid. That is, the more specialized the firm's inventory (Inv), the less likely it can be liquidated quickly to meet current obligations.

$$\text{quick ratio} = \frac{\text{CA} - \text{Inv}}{\text{CL}}$$

The higher the quick ratio, sometimes called the acid test ratio, the more likely the company will be able to pay its short-term bills.

- The **receivables turnover ratio** measures how quickly the company collects aggregate accounts receivable (A/R). In other words, it measures the rate of sales compared to the collection of sales revenues.

$$\text{receivables turnover} = \frac{\text{sales}}{\text{average receivables}}$$

It is typically considered desirable to have a receivables turnover figure close to the average receivables turnover of the other companies in the same industry.

- The **average collection period** is the inverse of the receivables turnover multiplied by 365. It measures the average number of days it takes for the company's credit customers (accounts receivable) to pay their bills:

$$\text{average collection period, ACP} = \frac{365}{\text{receivables turnover}}$$

Notice that if the company's average collection period is 60 days, its receivables turnover will be  $365/60$ , or approximately 6. Using the same logic, if its receivables turnover is approximately 6, the average collection period will be 60 days. It is considered desirable to have a collection period (and receivables turnover) close to that of the average company in the same industry.

- The **inventory turnover ratio** measures the firm's efficiency in processing and managing its inventory:

$$\text{inventory turnover} = \frac{\text{COGS}}{\text{average inventory}}$$

The inventory turnover equals the number of times the company's inventory is sold (turned over) during the year. Generally, the higher the ratio, the better. If the ratio is low, it may indicate that inventory is not being sold quickly, which might suggest it includes items that are obsolete.

- The **payables turnover ratio** measures generally how quickly the firm meets its accounts payable:

$$\text{payables turnover} = \frac{\text{purchases}}{\text{average trade payables}}$$

If the ratio is high, the time between purchase and payment by the company is short (generally considered good). A low ratio may indicate that the company does not have sufficient cash to pay its bills and is postponing payment.

- The **payables payment period** is the inverse of the payables turnover ratio multiplied by 365. It measures the average amount of time it takes the company to pay its bills:

$$\text{payables payment period} = \frac{365}{\text{payables turnover ratio}}$$

Notice that in the turnover ratios we use average values in their denominators. This is because these ratios compare income statement items to balance sheet items. Recall from the chapter on Financial Reporting and Analysis that the income statement describes a reporting period, while the balance sheet is a snapshot of a single point in time. When using ratios that compare income statement items to balance sheet items, best practice is to use the average of the beginning and ending balance sheet values for the period described by the income statement.

## Operating Performance

Performance ratios can help to determine how well management operates the business. They can be divided into two categories: efficiency ratios, which deal with management's efficient utilization of the firm's assets, and profitability ratios, which measure how efficiently management manages costs in generating profits.

### *Efficiency Ratios*

- The **total asset turnover ratio** measures how efficiently management is utilizing all the firm's assets considered together (i.e., total assets):

$$\text{total asset turnover} = \frac{\text{sales}}{\text{average total assets}}$$



## Asset Valuation

Different types of industries might have considerably different turnover ratios. Manufacturing businesses that are capital intensive might have total asset turnover ratios near one, while retail businesses might have total asset turnover ratios near ten. Again, it is typically desirable for the firm to have a total asset turnover ratio close to the average for the industry. A total asset turnover ratio less than the industry average might mean the company has too many assets for its level of production (i.e., it is not fully utilizing its assets). A total asset turnover ratio higher than the industry average could imply the firm has outdated (fully depreciated) and inefficient capital assets. Naturally, if the assets are not outdated, a high asset turnover is good.

- The **fixed asset turnover ratio** measures how efficiently management is utilizing just the fixed assets. Current assets are not included in the denominator:

$$\text{fixed asset turnover} = \frac{\text{sales}}{\text{average fixed assets}}$$

Again, it is desirable to have a ratio close to the industry norm. A ratio lower than the industry average could mean the company is not fully utilizing its fixed assets. A ratio higher than the industry average might imply the firm has obsolete equipment (fully depreciated). Once again, if the fixed assets are not obsolete, a high ratio is good.

## *Profitability Ratios*

Profitability ratios measure how good management is at turning its efforts into profits. Basically, profitability ratios compare the top of the income statement (sales) to profits and measure how efficiently management is managing expenses. The different ratios are designed to isolate specific costs. Before we proceed to the

profitability ratios, here are some important terms and their relation to the income statement:

**Important terms:**

Gross profits = net sales – COGS

Operating profits = earnings before interest and taxes (EBIT)

Net income = earnings after taxes (EAT)

Capital = long-term debt + short-term debt + equity

**How they relate in the income statement:**

Sales

– COGS

Gross profit

– Operating expenses

Operating profit (EBIT)

– Interest

Earnings before taxes (EBT)

– Taxes

Earnings after taxes (EAT) = net income

**Also note that from the balance sheet:**

Liabilities + equity = assets, or

Equity = assets – liabilities

- The **gross profit margin** is the ratio of gross profit (sales less cost of goods sold) to sales:

$$\text{gross profit margin} = \frac{\text{gross profit}}{\text{sales}}$$

Obviously management wants this ratio, as well as any other profitability ratio, to be at least as great as the industry average. The gross profit margin indicates how well the company controls their cost of goods sold.

- The **operating profit margin** is the ratio of operating profit (gross profit less expenses) to sales. Operating profit is also referred to as earnings before interest and taxes (EBIT):

$$\text{operating profit margin} = \frac{\text{operating profit}}{\text{sales}} \text{ or } \frac{\text{EBIT}}{\text{sales}}$$

## Asset Valuation

The operating profit margin indicates how well the company controls its selling and production costs (rents, salaries, utilities expenses, advertising, depreciation, etc.). In other words, the operating margin indicates if the company runs a tight ship.

- The **net profit margin** is the ratio of net income (NI, also called “earnings”) to sales:

$$\text{net profit margin} = \frac{\text{NI}}{\text{sales}}$$

The net profit margin indicates how well the company controls all its expenses (including interest and taxes). A high net profit margin is desirable.

- The **return on assets** (ROA) is the ratio of net income to total assets:

$$\text{return on assets} = \frac{\text{net income}}{\text{average total assets}}$$

The analyst should be concerned if the ROA is too low relative to the company’s peers.

- The **return on equity** (ROE) is the ratio of net income to shareholders’ equity:

$$\text{return on equity} = \frac{\text{net income}}{\text{average shareholders' equity}}$$

Analysts should be concerned if ROE is too low relative to the company’s peers.

## Risk Profile

Risk analysis calculations measure the *uncertainty* of the firm’s income flows. They can be divided into two groups, those that measure *business risk* and those that measure *financial risk*.

**Business risk** is the uncertainty regarding the operating income of a company (EBIT) and is a result of the variability of sales and production costs. The three calculations that measure business risk are business risk, sales volatility, and operating leverage.

**Financial risk** is the additional volatility of equity returns (ROE) caused by the firm’s use of debt. Financial risk can be measured using balance sheet ratios, which include the debt/equity ratio, the long-term debt/total capital ratio, and the total debt ratio; or earnings and cash flow ratios, which include the interest coverage ratio, the fixed financial charge ratio, the total fixed charge coverage ratio, the cash

flow/interest expense ratio, the cash flow coverage ratio, the cash flow/long-term debt ratio, and the cash flow/total debt ratio.

- The coefficient of variation is a general way of measuring risk of any data series. It is calculated as the standard deviation of the data series divided by its mean. *Business risk* is often measured as the coefficient of variation of a company's operating income (EBIT) over several years:

$$\text{business risk} = \frac{\text{standard deviation of EBIT}}{\text{average EBIT}}$$

Between five and ten years of data should be used to calculate the coefficient of variation, because using less data calls into question the statistical reliability, and data more than ten years old is likely not to be relevant to the company's present situation. Analysts will be concerned if this ratio is too high (relative to the company's peers).

- One of the contributing sources of earnings variability is **sales volatility**. Sales volatility is the coefficient of variation of sales over several years:

$$\text{sales volatility} = \frac{\text{standard deviation of sales}}{\text{average sales}}$$

As was the case for business risk, between five and ten years of data should be used in this calculation. Analysts should be concerned if this ratio is too high relative to the company's peers.

- **Operating leverage** is a measure of the variability of operating earnings. A simple definition is:

$$\text{operating leverage} = \frac{\text{fixed costs}}{\text{fixed costs} + \text{variable costs}}$$

The ratio measures how much of the company's production costs are fixed (as opposed to variable). The greater the use of fixed costs, the more operating income (EBIT) will change when sales change. A high operating leverage indicates that the company's EBIT will be ultra sensitive to changing business cycle conditions. EBIT will rise more than other companies during up periods and will fall more than other companies during down periods.

- The **debt-equity ratio** measures management's use of fixed-cost financing as opposed to equity in financing the firm's assets:

$$\text{debt-equity ratio} = \frac{\text{long-term debt}}{\text{total equity}}$$

Higher or lower values for this ratio suggest a greater or lesser reliance on debt as a source of financing.

## Asset Valuation

- The **debt to assets**, or simply the **debt ratio**, equals the ratio of total debt to total assets and is another way of measuring management's use of debt to finance the firm's assets:

$$\text{debt to assets} = \frac{\text{total debt}}{\text{total assets}}$$

Higher or lower values for this ratio suggest a greater or lesser reliance on debt as a source of financing.

- The **interest coverage ratio**, also known as the *times-interest-earned ratio*, measures the firm's ability to meet its interest (debt) obligations from operating income (EBIT):

$$\text{interest coverage} = \frac{\text{EBIT}}{\text{interest expense}}$$

The higher this ratio, the more likely it is that the firm will be able to meet its interest payments.

## Growth Potential

Both owners and creditors are interested in the firm's growth potential. Owners pay attention to growth because stock valuation is dependent on the future growth rate of the firm's cash flows. The analysis of growth potential is important to creditors because the firm's future prospects are crucial to its ability to pay existing debt obligations. If the company does not grow, it stands a much greater chance of defaulting on its loans. In theory, the growth rate of a firm is a function of the rate of return earned on its resources and the amount of resources (profits) retained and reinvested.

For many reasons, all firms go through periods of below- or above-average growth. Analysts are therefore interested in the firm's long-run, *sustainable* growth rate. This is the rate of growth (increase in assets) the firm can maintain without having to sell new common stock. To calculate the sustainable growth rate for a firm, we must know its return on equity, ROE, and the proportion of earnings reinvested in the firm [i.e., the retention rate (RR)].

- The long-term **sustainable growth rate**,  $g$ , is calculated as:

$$g = \text{RR} \times \text{ROE}$$

where:

RR = retention rate

ROE = return on equity

- The **retention rate** is the percentage of net income that is retained by the firm (retained earnings) and reinvested in the company. The dividend payout ratio is the percentage of net income paid out to the common stockholders as dividends. The sum of the two must equal 100%.

$$\text{dividend payout ratio} = \frac{\text{dividends paid}}{\text{net income}}$$

and

$$\text{retention rate} = (1 - \text{dividend payout ratio})$$

$$= \left( 1 - \frac{\text{dividends}}{\text{net income}} \right)$$

### Example: Sustainable growth rate

Calculate the *sustainable growth rate* for the following three firms:

<i>Company</i>	<i>A</i>	<i>B</i>	<i>C</i>
Earnings per share	3.00	4.00	5.00
Dividends per share	1.50	1.00	2.00
Return on equity	14%	12%	10%

**Answer:**

$$\text{RR} = 1 - (\text{dividends} / \text{earnings})$$

$$\text{Company A: RR} = 1 - (1.50 / 3.00) = 0.500$$

$$\text{Company B: RR} = 1 - (1.00 / 4.00) = 0.750$$

$$\text{Company C: RR} = 1 - (2.00 / 5.00) = 0.600$$

$$g = \text{RR} \times \text{ROE}$$

$$\text{Company A: } g = 0.500 \times 14\% = 7.0\%$$

$$\text{Company B: } g = 0.750 \times 12\% = 9.0\%$$

$$\text{Company C: } g = 0.600 \times 10\% = 6.0\%$$

Alternatively, consider a \$100 million equity investment in Company C. The return on the equity investment is 10%, or \$10 million. Of the \$10 million, assume you consume \$4 million (a 40% dividend payout). Therefore, you retain or plowback \$6 million into the original investment, which is now worth \$106 million (the original \$100 million plus the \$6 million retained earnings). This investment grew 6%, which equaled the return on equity times the retention rate:  $0.10 \times 0.60 - 0.04 = 6\%$ .

## Relative Ratio Analysis

The value of a single financial ratio is not meaningful by itself but must be interpreted relative to industry norms, overall economy norms, and the company's own past performance.

- Comparison to the industry norm (average) is the most common type of comparison. Industry comparisons are particularly valid when the products generated by all the firms in the industry are similar.
- Comparing a company to the overall economy is particularly important when overall business conditions are changing. For example, a stable profit margin might be considered good if the economy is in recession and the economy-wide average profit margin is declining. On the other hand, it might be considered problematic if a stable profit margin occurs during an economic expansion, and overall average profit margins are increasing.
- Comparing a firm with its history is very common. Typically, the analyst will look at the current level of a ratio and then look to see if it has been worsening over time, stable over time, or improving over time.

In most ratio comparisons it is considered desirable to be near the industry (or economy) average. For example, in all turnover ratios, a value could be considered too high or too low if it differs widely from the industry average. However, for some ratios, simply being high is considered good, even if it deviates from the industry average. This is true for most ratios involving income or cash flow. For example, most analysts would agree that having a high return on assets or high profit margin is good. An analyst would not suggest that a company with a return on assets of 15% when the industry average was 10% had an ROA that was *too high*.

Sometimes the “goodness” of a ratio depends on the context. A high ROE that results from high profit margins or asset turnover is typically looked upon favorably. However, high ROEs that result from the overuse of debt (leverage) are typically met with skepticism.

**COMPREHENSIVE EXAMPLE: RATIO ANALYSIS OF A FIRM**

The following table provides a company's balance sheet and income statement for 20X7.

Balance Sheet December 31, 20X7	
Cash	\$105
Receivables	205
Inventory	310
Total current assets	620
Gross property, plant, and equipment	\$1,800
Accumulated depreciation	360
Net property, plant, and equipment	1,440
<b>Total assets</b>	<b>\$2,060</b>
Payables	\$110
Short-term debt	160
Current portion of long-term debt	55
Total current liabilities	\$325
Deferred taxes	105
Long-term debt	\$610
<b>Total Liabilities</b>	<b>\$1040</b>
Common stock	300
Additional paid in capital	400
Retained earnings	320
Common shareholders equity	\$1,020
<b>Total liabilities and equity</b>	<b>\$2,060</b>



## Asset Valuation

Income Statement	
Year Ending December 31, 20X7	
Sales	\$4,000
Cost of goods sold	3,000
Gross profit	1,000
Operating expenses	650
EBIT	350
Interest expense	50
Earnings before taxes	300
Taxes	100
Net income	200
Common dividends	60

Ratios for the industry and the year-earlier 20X6 ratios for the company are reported in the following table:

	20X6	Industry
Current ratio	2.1	1.5
Quick ratio	1.00	0.90
Average collection period	18.9	18.0
Inventory turnover	10.7	12.0
Total asset turnover	2.30	2.40
Gross profit margin	27.4%	29.3%
Net profit margin	5.8%	6.5%
Return on assets	13.3%	15.6%
Return on equity	24.1%	19.8%
Debt-to-equity	78.4%	35.7%
Interest coverage	5.9	9.2
Retention rate	50.0%	43.6%
Sustainable growth rate	12.0%	8.6%

Using the company information, calculate the 20X7 ratios, and discuss how these ratios compare with the company's performance last year and with the industry's performance.

Answer:<sup>2</sup>

- **Current ratio** = current assets / current liabilities

$$\text{Current ratio} = 620 / 325 = 1.9$$

The current ratio indicates lower liquidity levels when compared to last year, but more liquidity than the industry average.

- **Quick ratio** = (cash + receivables) / current liabilities

$$\text{Quick ratio} = (105 + 205) / 325 = 0.95$$

The quick ratio is lower than last year and is above the industry average.

- **Average collection period** = 365 / (sales / receivables)

$$\text{Average collection period} = 365 / (4,000 / 205) = 18.7$$

The average collection period is a bit lower relative to the company's past performance but slightly higher than the industry average.

- **Inventory turnover** = cost of goods sold / inventory

$$\text{Inventory turnover} = 3,000 / 310 = 9.7$$

Inventory turnover is much lower than last year and the industry average. This suggests that the company is not managing inventory efficiently and may even have obsolete inventory.

- **Total asset turnover** = sales / assets

$$\text{Total asset turnover} = 4,000 / 2,060 = 1.94$$

Total asset turnover is lower than last year and the industry average. This might indicate the management is less efficient in the use of its assets, or perhaps there has been a broad market downturn negatively affecting sales.

---

<sup>2</sup> Earlier we noted that a best practice is to use averages of balance sheet items in ratios that combine income statement and balance sheet items. However, this is not an absolute rule. Here we are only given the ending 20X7 balance sheet values, so we will use these to calculate the ratios. Different analysts may calculate the same ratios differently. When comparing ratios, make sure they have been calculated in a comparable way.

## Asset Valuation

- **Gross profit margin** = gross profit / sales

$$\text{Gross profit margin} = 1,000 / 4,000 = 25.0\%$$

The gross profit margin is lower than last year and much lower than the industry average. This might indicate that the cost of sales is not being controlled well.

- **Net profit margin** = net income / sales

$$\text{Net profit margin} = 200 / 4,000 = 5.0\%$$

The net profit margin is lower than last year and much lower than the industry average. This might indicate that the cost of sales and production is not being controlled well.

- **Debt/equity ratio**<sup>3</sup> = long-term debt / total equity

$$\text{Debt/equity ratio} = 610 / 1020 = 59.8\%$$

The debt/equity ratio is lower than last year but still much higher than the industry average. This suggests the company is trying to get its debt level more in line with the industry.

- **Interest coverage** = EBIT / interest expense

$$\text{Interest coverage} = 350 / 50 = 7.0$$

The interest coverage is better than last year but still worse than the industry average. This, along with the slip in profit margin and return on assets, might cause some concern.

- **Retention rate** =  $1 - (\text{dividends} / \text{earnings})$

$$\text{Retention rate} = 1 - (60 / 200) = 70\%$$

The retention rate is much higher than last year and much higher than the industry. This might suggest that the company is aware of its cash flow and earnings issues and is reinvesting cash into the company to improve the ratios. This is a wise decision as long as their profitability and performance ratios remain high.

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3 Note that deferred taxes were not included in the long-term debt calculation. Deferred taxes represent a long-term liability, but in this particular ratio we are more interested in the actual debt financing obtained by the company through the issuance of long-term debt securities.

- **ROA** = net income / assets

$$\text{ROA} = 200 / 2,060 = 0.097 = 9.7\%$$

The return on assets is lower than last year and far below the industry average. While an ROA close to 10% generally is quite good, the company's ROA is lagging its industry competitors. Also, notice that the ROA dropped considerably from 20X6 (when the ROA was 15.5%). The ROA also equals the product of the net profit margin and asset turnover ( $0.05 \times 1.94 = 0.097$ )

- **ROE** = net income / equity

$$\text{ROE} = 200 / 1,020 = 0.196, \text{ or } 19.6\%$$

The return on equity is lower than last year and slightly below the industry average.

- **Sustainable growth rate, g** = retention rate  $\times$  ROE

$$\text{Sustainable growth rate} = 0.70 \times 0.196 = 0.137, \text{ or } 13.7\%$$

With a high retention rate and good ROE, the company is positioned to grow at a faster rate than last year and faster than the rest of the industry. But note that actual growth (in earnings) can be above or below the sustainable growth. Actual growth cannot exceed sustainable growth for long, however, because the company will run out of funds to finance its rapid growth. If actual growth consistently lags the sustainable growth rate, management is doing something badly and the company might get taken over, especially if the stock price is low.

**Summary:** The company's liquidity, as measured by its current and quick ratios, is a little better than the industry as a whole. However, performance figures suggest that earnings continue to trail the industry, and turnover has worsened. The latter probably indicates the firm is employing too many assets to generate its level of sales. The interest coverage ratio has improved but still trails the industry. Although improving, it still might cause some concern for lenders.

The company's debt-equity ratio is very high, indicating management is utilizing too much debt. Coupled with the very high retention rate (low dividend payout), this would indicate too much cash is being burned. That is, with the high retention of earnings, the company should have adequate cash for operations and investments, but it is borrowing heavily.

## DU PONT DECOMPOSITION OF ROE

The **Du Pont system** breaks down the components or drivers of the return on equity. Specifically, the ROE equals:

$$\text{ROE} = \left( \frac{\text{net income}}{\text{sales}} \right) \left( \frac{\text{sales}}{\text{assets}} \right) \left( \frac{\text{assets}}{\text{equity}} \right)$$

$$\text{ROE} = (\text{net profit margin})(\text{total asset turnover})(\text{equity multiplier})$$

The genius of breaking ROE down into its component parts is that it indicates the firm's weakness or strength in three separate areas:

1. **Operating efficiency.** The net profit margin measures the efficiencies and inefficiencies of operating the firm. Poor efficiency produces low profit margins.
2. **Asset utilization.** Total asset turnover measures management's efficient utilization of assets (i.e., higher or lower utilization than the industry average). Under-utilization produces lower net income and return on assets due to excessive fixed costs relative to sales.
3. **Leverage.** The equity multiplier captures management's utilization of debt financing. Too much debt can mean too much risk, and too little signifies "debt capacity," the ability to issue more debt without increasing risk or cost of capital. In other words, the company can improve profitability by borrowing when its performance more than offsets the interest costs of borrowing.

For our company:

$$\text{ROE} = \left( \frac{\text{net income}}{\text{sales}} \right) \left( \frac{\text{sales}}{\text{assets}} \right) \left( \frac{\text{assets}}{\text{equity}} \right)$$

$$\text{ROE} = \left( \frac{200}{4,000} \right) \left( \frac{4,000}{2,060} \right) \left( \frac{2,060}{1,020} \right)$$

$$\text{ROE} = (0.05)(1.9417)(2.0196) = 0.196$$

We again see that the company's ROE is about the same as the industry, so management's first reaction may be to accept the number and carry on. Indeed, it should check the individual ratios in the Du Pont system to be sure that an average ROE isn't masking some very poor and/or very good ratios.

By using the Du Pont system, we can observe that both the net profit margin (net income/sales) and total asset turnover (sales/assets) are substandard. The result is

that ROA, which is equal to the product of these ratios, is far below the industry average. This suggests that operating efficiency and asset utilization should be evaluated for potential improvement. Since its ROE appears to be acceptable while its ROA is low, the equity multiplier (assets/equity) must be high.<sup>4</sup> Without even calculating the industry equity multiplier, we know that management is employing more debt than the industry average. This is confirmed by looking at the firm's debt-to-equity ratio of 59.8% compared to 35.7% for the industry. By using DuPont analysis, we can conclude that although the firm's ROE appears OK on the surface, the excessive use of debt is masking a low profit margin and an under-utilization of the firm's assets.

## LIMITATIONS OF FINANCIAL RATIOS

Although calculating and interpreting financial ratios might seem straightforward, an analyst must remember a few rules:

- Financial ratios are not useful when viewed in isolation. They are only informative when compared to other similar firms and the company's historical performance over an adequate period of time.
- Comparisons with other companies can be difficult because of different accounting treatments (e.g., inventory). This is particularly important when comparing U.S. firms to non-U.S. firms.
- It is difficult to find comparable industry ratios when analyzing companies that operate in multiple industries.
- Determining the target or comparison value for a ratio is difficult, as the industry is often difficult to identify properly.

Ratios are used for internal analysis and comparisons and across firms. They are often most useful in identifying questions that need to be answered rather than answering questions directly. In conducting your analysis, you must always be aware of the limitations of ratios. Ask yourself these questions:

- Do the firms being compared have compatible accounting practices?
- Do the ratios give consistent readings? For example, if the debt ratio is too high, is the interest coverage ratio low?
- Do the ratios yield a reasonable figure for the industry?

## EQUITY VALUATION

### Intrinsic Value

As with any other cash flow producing asset, stocks can be valued using the capitalization of income method.

<sup>4</sup> Note that the ROE equals the ROA times the equity multiplier:  $0.097 \times 2.0196 = 19.6\%$ .

## Asset Valuation

*Step 1:* Forecast the amount and timing of the cash flows that the investor expects to receive in the future. Ownership in common stock gives the owner the right to receive two forms of cash flows.

- Income distributions in the form of dividends.
- Selling price of the stock.

Dividends are declared by the board of directors of the company. Even though the exact amounts of the dividends are not known in advance, corporate boards usually favor a fairly consistent dividend pattern. So the chore of forecasting dividends might not be as laden with uncertainty as it seems. Therefore, most of the uncertainty surrounds the forecast of the future price of the stock.

*Step 2:* Estimate the risk (degree of uncertainty) associated with the cash flows. A common statistic used to summarize the company's risk is its beta. The stock's beta measures the variability of the company's stock return relative to a broad market index (such as the S&P 500 or the Wilshire 5000 stock market index). A beta equal to 1.0 indicates a stock with average risk. Presumably, companies with highly volatile cash flows will have high betas.

*Step 3:* Assign a required return based on the risk of the stock. The required return can be determined using the capital asset pricing model (CAPM), a model that provides an estimate of the required return for any asset, and is described in detail in the Portfolio Management chapter.<sup>5</sup> The required return is positively related to the stock's beta, and is used to discount the predicted cash flows when finding the present value.

*Step 4:* Find the total present value of all the cash flows.

A simple example: assume you are assigned to determine the intrinsic value (fair price) for Amazing Glass, Inc., a glass manufacturing firm, which is expected to pay the following dividend stream over the next four years.

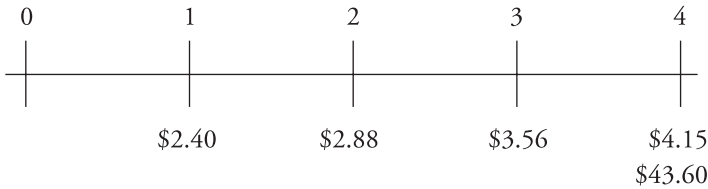
<i>Year</i>	<i>Dividend</i>
1	\$2.40
2	\$2.88
3	\$3.46
4	\$4.15

<sup>5</sup> The CAPM states that the required return equals the risk-free rate plus a risk premium required by investors as compensation for risk. The equation equals  $k_i = r_f + \beta(k_m - r_f)$ , where  $k_i$  is the required return for the stock,  $r_f$  is the risk-free rate (e.g., Treasury bill rate), and  $k_m$  is the required return for the broad market.

Moreover, assume you think the stock price four years from now will be \$43.60. Your required return is 15%.

Solution: This is a simple *present value* problem as we described in the Quantitative Methods chapter. To assist in the valuation of the stock under this scenario, we will illustrate our expected cash flows on a timeline:

**Figure 2: Expected Cash Flows**



The fair price or intrinsic value (today) equals the present value of the predicted cash flows:

$$\frac{\$2.40}{1.15} + \frac{\$2.88}{1.15^2} + \frac{\$3.46}{1.15^3} + \frac{\$4.15 + \$43.60}{1.15^4}$$

$$\frac{\$2.40}{1.15} + \frac{\$2.88}{1.3225} + \frac{\$3.46}{1.5209} + \frac{\$4.15 + \$43.60}{1.7490}$$

$$\$2.09 + \$2.18 + \$2.28 + 27.31 = \$33.86$$

Therefore, the fair or intrinsic value for the stock is \$33.86. In other words, given your assumptions about the future cash flows and riskiness associated with them, you think the stock is worth \$33.86 per share.

A stock is deemed **overvalued** if the actual stock price exceeds the stock’s intrinsic value, in which case a “sell” recommendation is made. A stock is deemed **undervalued** if the actual stock price is less than the stock’s intrinsic value, in which case a “buy” recommendation is made. For instance, if Amazing Glass stock is trading at \$30, you would conclude that the stock is undervalued. You think the stock is worth \$33.86, but it’s trading for only \$30 in the marketplace. You would recommend a purchase.

### The Dividend Discount Model

A firm is considered a going concern with an indefinite life. The value of its common stock can be thought of as the present value of an endless (infinite) stream of dividends.



## Asset Valuation

The **dividend discount model** states that the intrinsic value for a stock equals the present value of all its expected future dividends:

$$\text{intrinsic value} = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \frac{D_4}{(1+k)^4} \dots + \frac{D_\infty}{(1+k)^\infty} \quad (1)$$

where:

$D_t$  = the dividend expected at year  $t$

$k$  = the required return (derived from the CAPM) for the stock<sup>6</sup>

**Single-stage dividend discount model.** Estimating all future dividends and their present values could be a very time consuming process. One simplification is to assume that dividends grow at a constant rate forever (e.g., 5% growth forever). The model that assumes dividends will grow at a constant rate forever is called the **Gordon growth model**, named after Professor Myron Gordon who developed it in 1962.

For example, assume last year's dividend,  $D_0$ , was \$2 and that dividends are assumed to increase 5% per year forever. Therefore, the expected yearly dividends equal:

$$\text{Year 1: } D_1 = \$2(1.05) = \$2.10$$

$$\text{Year 2: } D_2 = \$2.10(1.05) = \$2.21$$

$$\text{Year 3: } D_3 = \$2.21(1.05) = \$2.32$$

$$\text{Year } n: D_n = D_{n-1} = (1.05)$$

In other words, each dividend is the previous dividend multiplied by one plus the expected rate of growth. In this manner we estimate all future dividends. Note that all of the previously listed dividends are *expected* or *predicted* values. None of the future dividends are known with certainty at time 0.

Professor Gordon showed that if we assume *dividends grow at a constant rate forever*, the intrinsic value or fair price at time  $t$  can be calculated with the simple formula:

$$\text{fair price at time } t = \frac{D_{t+1}}{k - g} \quad (2)$$

<sup>6</sup> Even though dividends are typically paid quarterly, the valuation methods used assume the four quarterly dividends are paid in one annual payment. The rounding error that results from this simplification is typically small.

**Example: Single-stage dividend discount model**

Assume last year's dividend was \$2 and that you think dividends will increase 5% per year forever. Using the CAPM, you determine that the required return for the stock is 15%. Find the stock's intrinsic value and make a buy or sell recommendation.

**Answer:**

Next year's predicted dividend ( $D_1$ ) is \$2.10 (which equals last year's dividend, \$2, grown at 5%). The required return for the stock is 15%, and the expected growth rate in dividends is 5%. Therefore the intrinsic value equals:

$$\text{intrinsic value} = \frac{\$2.10}{0.15 - 0.05} = \frac{\$2.10}{0.10} = \$21$$

If the current market price of the stock is less than \$21, you will make a buy recommendation. If the current market price is greater than \$21, you will make a sell recommendation.

**Multi-stage dividend discount model.** Notice that the constant growth dividend model does not work when the expected dividend growth rate equals or exceeds the stock's required return. For instance, if the expected growth rate in the example above equals 15%, then the "intrinsic value" becomes infinity. Also, if the growth rate exceeds the required return, the "intrinsic value" turns negative. Neither of these outcomes is sensible because the growth rate cannot possibly equal or exceed the stock's required return forever. Economic theory tells us that when companies experience high growth rates, competition will enter the company's market, which will eventually erode profitability and cause the growth rate to decline. The growth rate can exceed the required return temporarily, but not permanently. Thus the company cannot possibly increase dividends forever at a supernormal rate. The solution to this problem lies in forecasting multiple growth stages for the firm.

Growth in dividends is typically tied directly to the growth of the firm.<sup>7</sup> Although most firms are assumed to maintain consistency in growth, some firms might be in industries where new innovations result in exceptional growth over varying periods of time. Some might be able to maintain higher-than-normal growth rates for many

<sup>7</sup> Growth in a firm is usually defined as the percentage growth in earnings per share (EPS), the change in the value of the firm's assets, or the change in its stock price. As long as management pays out a constant percentage of earnings as dividends, the dividend growth rate will be the same as the firm's overall growth rate.

## Asset Valuation

years, while others might enjoy higher-than-normal growth for only a relatively brief period.

The period when firms are expected to perform well above (or below) average is known as the *supernormal growth period*. The period when the firm performs at a more sustainable, long-term rate of growth is called the *constant growth period*. During this period we assume the firm will perform as most other firms in the industry, and its dividends will grow at the industry average. Of course, not all firms (e.g., those in mature industries) will experience explicit growth periods.

Let's now consider that Amazing Glass, Inc. develops a new heat-reflective glass for automobiles. This innovation could give Amazing Glass a decided edge over competitors until the competition develops its own glass. During this period, Amazing may enjoy a higher-than-normal growth rate as sales increase dramatically. When the competition starts producing equivalent (or even better) glass, Amazing will resume a more sustainable growth rate.<sup>8</sup>

You estimate that the explicit growth period will last for four years, and that Amazing's assets, earnings, and dividends will grow at an accelerated rate of 20% during this time. Thereafter, you expect the growth rate to revert to a normal rate of 5%.<sup>9</sup> Based on risk estimates, the required return for Amazing Glass is estimated to be 15%. Find the intrinsic value for the stock and make a buy or sell recommendation. The stock currently is trading at \$30.

To find the intrinsic value for this 2-step problem, we will complete the following tasks:

1. Calculate the predicted dividends during the explicit growth, years 1 through 4.
2. Calculate the intrinsic value for the implicit growth stage, years 5 through infinity.
3. Calculate the present value of all the predicted cash flows.

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8 It could be said there are normal rates of growth for firms based on their industries. All firms in mature industries might have reached a point of zero or very low "normal" growth. Some industries, such as computer software and communications, are prone to rapid innovation and growth.

9 The normal rate of growth for the firm's dividends is the average long-term growth for firms in the industry.

*Step 1:* Calculate the predicted dividends during the explicit (supernormal) growth years.

The growth rate for the first four years is 20%, meaning dividends one through four are expected to be 20% greater than each preceding dividend. We estimate them as:

$$\begin{aligned} D_1 &= D_0 (1 + g) = \$2.00 (1.20) = \$2.40 \\ D_2 &= D_1 (1 + g) = \$2.40 (1.20) = \$2.88 \\ D_3 &= D_2 (1 + g) = \$2.88 (1.20) = \$3.46 \\ D_4 + D_3 (1 + g) &= \$3.46 (1.20) = \$4.15 \end{aligned}$$

*Step 2:* Calculate the intrinsic value for the normal growth years.

Recall that as long as the dividend growth is constant forever, we can utilize Equation 2 to find the present value. Our example fits this requirement because, starting with dividend five, dividends are expected to grow at a rate of 5% indefinitely. Using Equation 2, the intrinsic value of Amazing Glass *as of year 4* equals:

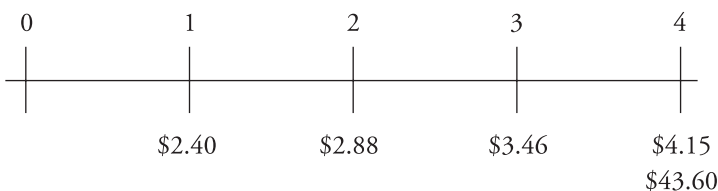
$$\text{Year 4 intrinsic value} = \frac{D_5}{k - g} \quad (3)$$

where  $D_5$  is the predicted dividend for year 5. Our assumption is that dividends will grow at 5% after year 4. Therefore,  $D_5$  equals the year 4 dividend grown at 5%:  $D_5 = \$4.15(1.05) = \$4.36$ . Therefore, the year 4 intrinsic value equals  $4.36 / (0.15 - 0.05) = \$43.60$ . Note that the year 4 intrinsic value is our prediction for the price at which we think Amazing Glass will sell at the end of year 4.

*Step 3:* Find the present value of the predicted cash flows.

Our timeline now can be illustrated as a series of four dividends (from Step 1) and a predicted selling price for year 4 (from Step 2).

**Figure 3: Dividends and Selling Price**



## Asset Valuation

We are now in a position to calculate the present value of all future cash flows to find our intrinsic value for the stock:

$$\frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \frac{D_4}{(1+k)^4} + \frac{P_4}{(1+k)^4}$$

$$\frac{\$2.40}{(1.15)} + \frac{\$2.88}{(1.15)^2} + \frac{\$3.46}{(1.15)^3} + \frac{\$4.15}{(1.15)^4} + \frac{\$43.60}{(1.15)^4}$$

$$\frac{\$2.40}{1.15} + \frac{\$2.88}{1.3225} + \frac{\$3.46}{1.5209} + \frac{\$4.15}{1.7490} + \frac{\$43.60}{1.7490}$$

$$\$2.09 + \$2.18 + \$2.28 + 2.38 + 24.93 = \$33.86$$

Given our estimates of the stock's future dividends, we estimate the price of Amazing Glass to be \$33.86. Since the actual stock price is \$30, you might issue a "buy" recommendation. In other words, you think the stock is worth \$3.86 more than its actual price. You think the stock is *undervalued* by \$3.86. Notice that these are the same dividend and predicted year 4 price that were used in the original example in this section. This example also shows how the year 4 (projected sales) price can be predicted.

## Earnings Multiplier Model

An alternative to the dividend discount model is the **earnings multiplier model**. The earnings multiplier refers to the price-to-earnings ratio, simply called the P/E ratio. Earnings (E) refers to the net income per share of common stock that was earned by the company. The P/E ratio can be calculated in two ways:

1. Trailing P/E: current price divided by prior year (latest) earnings.
2. Leading P/E: current price divided by predicted next year earnings.

We can use Equation 2 to identify the major factors affecting a company's fair P/E ratio. For instance, if we divide both sides of Equation 2 by leading earnings,  $E_{t+1}$ , we find that the fair or intrinsic P/E equals:

$$\frac{P_t}{E_{t+1}} = \frac{D_{t+1}/E_{t+1}}{k - g} \quad (4)$$

Equation 4 shows how the P/E ratio is determined by four factors:

1. The dividend payout ratio,  $D/E$ . As the dividend payout ratio increases, the P/E ratio increases (e.g., the numerator,  $D/E$ , increases), assuming risk and growth remain unchanged. In other words, if investors predict that their cash flows will rise, they will be willing to pay a higher price relative to the net income earned by the company.
2. The predicted growth rate,  $g$ . As predicted growth increases, the P/E ratio increases (e.g., the denominator,  $k - g$ , decreases), assuming risk and dividend payout remain unchanged. As investors raise their predictions of future growth, they become willing to pay a higher price for the stock (P/E increases). For example, most technology companies have high P/E ratios because investors expect earnings to grow at a high rate for the next few years. Companies in a mature, slowing industry stage (transportation, utilities, and financial sector companies) have lower P/E ratios.
3. The perceived riskiness of the company. As investors perceive that the company riskiness increases, the beta will increase, which in turn causes the required return,  $k$ , to increase. As  $k$  increases, the P/E ratio decreases (the denominator,  $k - g$ , increases), assuming dividend payout and growth remain unchanged.
4. Similarly, as investors become more risk averse, their required risk premiums rise, causing  $k$  to rise. As  $k$  rises, the P/E will fall, assuming dividend payout and growth remain unchanged.

The earnings multiplier model can be used to calculate the fair value of the stock. For instance, suppose we think Amazing Glass Inc. should have a P/E ratio that's 20% above the Industry average of 10. Therefore, we think Amazing Glass deserves a P/E of 12. Further, suppose we predict that next year's earnings per share will be \$3. We can now use the following mathematics to find the intrinsic or fair value for Amazing Glass using the earnings multiplier model. Defining P to be the fair price for Amazing Glass stock:

$$\frac{P}{E} \times E = P \quad (5)$$

$$12 \times \$3 = \$36$$

Notice that this method does not require that the company pay dividends. As long as we can determine the fair P/E and can predict next year's earnings, we can find the fair value for the company using the earnings multiplier model.

## Asset Valuation

### Preferred Stock

As with common stock, preferred stock prices are calculated by taking the present value of all expected future dividends.

Let's assume you are valuing a preferred stock with a face value of \$100 and a 6% dividend,  $D_p$ . The required return on the preferred stock,  $k_p$ , is 6%, and it will never be retired. That is, the preferred stock is *perpetual*, which means that the \$6.00 annual dividend is never expected to stop or change.

Equation 6 is the general equation used to value any dividend-paying (equity) security. If dividends are never expected to change, the growth in dividends is zero, and Equation 6 reduces to the *constant growth dividend discount model*, Equation 7.

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + k_e)^t} \quad (6)$$

$$P_0 = \frac{D_1}{k_e - \bar{g}} \quad (7)$$

Substituting the appropriate data into Equation 7, we find our preferred stock is selling at par, \$100. Note that we insert a zero for  $\bar{g}$  in the equation since the preferred dividend is constant and has no growth.

$$P_p = \frac{D_p}{k_p - \bar{g}} = \frac{D_p}{k_p - 0} = \frac{D_p}{k_p} = \frac{\$6.00}{0.06} = \$100.00$$

Because of the fixed dividend payment, preferred stock prices fluctuate in the same manner as fixed income securities, meaning that the price is inversely related to changes in interest rates. Figure 4 shows the price of our preferred stock assuming various required returns. In each case the price is found as above using  $P_p = \frac{D_p}{k_p}$ , the formula for the price of perpetual preferred stock.

As the required return increases, the price decreases. As the required return decreases, the price of the preferred stock increases. And as long as the required return is equal to the dividend rate, the preferred stock sells at its par value.

**Figure 4: Perpetual Preferred Stock Price Reactions to Changes in Required Return [for a stock with face value = \$100, dividend = \$6.00 (6%)]**

$k$	$P_p$
5.0%	120.00
5.5%	109.09
6.0%	100.00
6.5%	92.31
7.0%	85.71

## DEBT SECURITIES

Corporations and other entities often raise capital by issuing (or selling) bonds to investors willing to lend money. In essence, the corporation is borrowing money today and promising to repay the borrowed funds with interest. The bond functions as a loan from investors.

Bond issuers are able to complete such a transaction by creating a legal document (the **bond indenture**) that obligates them to repay the borrowed funds. This legal contract specifies all of the features of the bond, including its face value, coupon rate, maturity date, interest payment dates, any collateral, its rank with respect to other bonds previously issued, whether it is callable, whether it is convertible, the name of the **trustee**, any positive or negative **covenants**, the procedures followed in case of bankruptcy, and other features of the bond that are important for the lender to know in advance.

The typical corporate bond has a **face value** of \$1,000, also called the **principal** or **par value**. Unlike common stock, bonds almost always have a stated maturity, which can range anywhere from a few weeks or months to thirty or more years. On the **maturity date**, the final day that the bond is outstanding, the firm repays the original \$1,000 borrowed to the bondholder, and the debt is no longer outstanding. A bond with no maturity date is called a **perpetuity** or **perpetual bond**, but these are rare.

The firm typically makes semiannual interest-only payments until the bond matures, at which time the full principal amount is paid in one lump sum. The semiannual payments are called **coupon payments** and are determined by the coupon rate. The **coupon rate** is an annual interest rate stated as a percentage of par or face value. To find the expected annual interest payment for any bond, simply multiply the coupon rate by the face value of the bond. If the bond pays interest semiannually, divide the total annual payment by two to find the semiannual payments.



## Asset Valuation

Coupon rates can be either fixed or floating rates. Fixed rate coupons pay the same interest rate on every coupon regardless of interest rate movements in the general economy. Floating rate coupons are tied to some interest rate benchmark which adjusts the coupon rate and interest payment of each coupon.

When a firm issues bonds, there are often many investors (often tens of thousands) who will purchase portions of the issue. Therefore it is necessary to establish a *trustee* to ensure that the issuing firm acts in the best interest of the bondholders and follows all provisions as detailed in the bond indenture. The individual or institution designated as the trustee will regularly monitor the issuing firm to ensure there are no violations of the agreement between the firm and its bondholders. Trustees are responsible for monitoring all aspects of the indenture but will most likely spend a great deal of time making sure the issuer does not violate the *covenants* listed in the indenture. These can be **negative covenants** that prescribe what management cannot do, including restrictions on issuing additional debt without meeting certain conditions or without the approval of the current bondholders. They can also be **affirmative covenants** that prescribe what management must do, including maintaining certain levels of financial ratios, maintaining the firm's equipment in good working condition, and providing the trustee with periodic reports.

Bond indentures often contain a provision to retire part of the issue prior to maturity. One such provision is known as a **sinking fund**. Under this provision, management makes annual deposits with the trustee to ensure funds are available to retire the issue upon its maturity. A sinking fund can be used to retire all the bonds in the issue at once, or it can be used to retire the bonds over time, as is the case with **serial bonds**. Serial bonds are retired in portions according to their serial numbers (their registration numbers). For example, if 20% of the bonds are retired every four years, the entire issue will be retired in 20 years. In either case, a sinking fund makes investing in bonds less risky because it helps ensure timely retirement of the debt.

**Callable bonds** may also be retired early (before the stated maturity date) as a result of call options embedded in the structure of the bond.<sup>10</sup> To force the early retirement of the bonds, management announces the call publicly, and whoever owns the bonds must surrender them. Since calling the bonds requires the firm to pay the outstanding principal earlier than expected, issuers are hesitant to do so unless there is a substantial benefit. The benefit lies in being able to take advantage of lower interest rates by replacing an outstanding callable issue with another issue that carries a lower coupon rate.

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<sup>10</sup> Firms typically issue callable bonds during times of historically high interest rates. Fewer bonds are issued with call features when interest rates are historically low.

To guarantee bondholders at least a certain amount of time at the high coupon rate, some of these bonds are not callable for the first few years. This is known as *call protection* because during this period management is legally prohibited from calling the bonds. Bonds may be callable at par value, but some specify a *call premium* such as 102% of par.

In contrast to callable bonds, for which the option may be exercised by the issuer, some bonds contain options that may be exercised the bondholder. When a corporate bond is **convertible**, the owner of the bond may exchange it for another type of security, usually common or preferred stock. The bond will be convertible into a set number of shares, determined by the **conversion ratio**. For example, if the bond has a conversion ratio of 25:1, this means it is exchangeable for 25 shares of stock at any time at the discretion of the owner. If the bond sells for \$1,000, this implies a **conversion price** of  $\$1,000 / 25 = \$40.00$ . Thus, if the owner converted immediately after purchasing the bond, he would effectively pay \$40.00 per share of stock.<sup>11</sup>

Since the ability to convert to common or preferred stock is usually considered a valuable option, convertible bondholders are willing to accept a lower rate of interest. The coupon rate on convertible bonds averages about two-thirds of the coupon rate on an otherwise identical nonconvertible bond.

## Bond Sectors and Instruments

The U.S. bond market can be separated into four major segments: (1) Treasury, (2) agency, (3) state and local (or municipal), and (4) corporate. Each sector has developed its own features and issue characteristics, as well as its own trading mechanisms. Treasury securities are issued by, and are general obligations of, the federal government and are considered to be “riskless” in that they are assumed to carry no risk of default. Agency bonds are issued by agencies of the federal government and organizations affiliated with the federal government. Municipal bonds are issued by governmental units below the federal level, such as cities, counties, and states. Corporate bonds are issued by business enterprises and can be issued via private placement or public offering.

**Treasury securities.** U.S. Treasury securities (or “Treasuries”) are issued by the U.S. Treasury Department and, because they are backed by the full faith and credit of the U.S. government, are considered to be free from risk of default though they are not free from interest-rate/price risk (we will discuss risk in greater detail later in this section). The Treasury issues four different types of securities: bills, notes, bonds, and inflation-protection securities.

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<sup>11</sup> Since he paid \$1,000 for the bond and then exchanged it for 25 shares, the effective price is \$40.00, as if he had purchased the stock instead of the bond with his \$1,000.

## Asset Valuation

**Treasury bills (or T-bills)** denote the Treasury securities with the shortest maturity. All T-bills have an original maturity of less than one year. They are always sold as zero-coupon securities (i.e., the price paid for the T-bill, both at issuance and anytime prior to maturity, is always less than face value). The Treasury issues 4-week, 13-week, and 26-week T-bills at weekly auctions. Occasionally, the Treasury will issue *cash management bills*, with maturities ranging from a few days to six months, to meet short-term cash requirements.

**Treasury notes (or T-notes)** are medium-maturity Treasuries. They are currently issued with original maturities of two, three, five, or ten years. T-notes, unlike T-bills, are coupon-bearing instruments. T-notes are issued with coupon rates that are set according to market rates at the time of issuance. On quotation sheets, T-notes are denoted with an *n* to distinguish them from T-bonds.

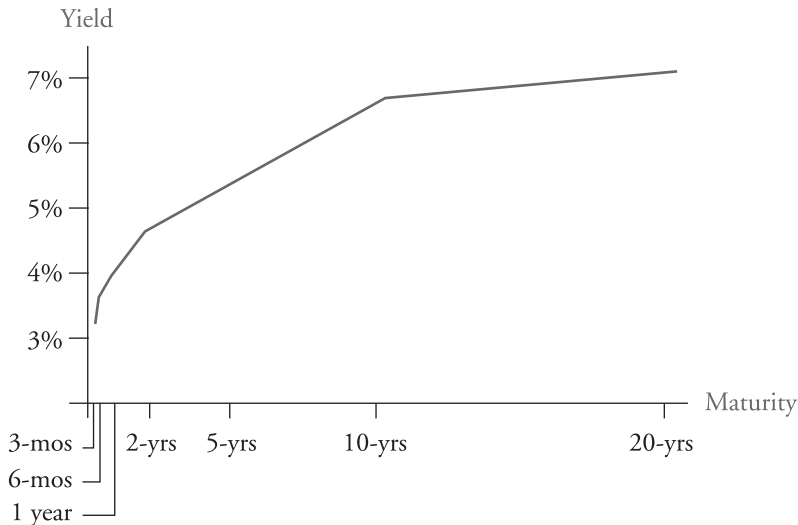
**Treasury bonds (or T-bonds)** are the longest-maturity securities issued by the U.S. Treasury. T-bonds have historically been issued with maturities from ten years up to 30 years. The required yield on the longest maturity T-bond is considered by market participants to be a bellwether for long-term interest rates.

In 1997, the U.S. Treasury started selling *Treasury inflation protection securities* (TIPS), or *inflation indexed notes* (IIN) with face values that adjust periodically with changes in the rate of inflation. The coupon rate is fixed for the life of the issue and represents the rate an investor earns, net of inflation. This rate is also known as the *real rate*.

Inflation is measured by changes in the Consumer Price Index (CPI) for All Urban Consumers (CPI-U). Changes in the rate of inflation are reflected through a series of semiannual adjustments to the principal value of the bond. Once the par value of the bond has been increased to what is known as the *inflation-adjusted principal*, the fixed coupon rate is used to determine the interest payments to investors.

When Treasury security yields are plotted against their maturities, the resulting relationship is known as the **Treasury yield curve**. Figure 5 provides a representative sample of what the Treasury yield curve might look like. Notice that this particular yield curve is upward sloping, which is normally the case. This indicates that investors are demanding higher interest rates for longer-term securities.

Figure 5: The Treasury Yield Curve



**Agency securities.** In addition to the U.S. Treasury, some agencies of the federal government and some organizations affiliated with the federal government also issue fixed income securities. Securities issued by federally related institutions, much like Treasuries, are backed by the U.S. government and, as such, bear little default risk. Securities issued by government-sponsored entities do not have the same backing that the Treasury and federally related institutions have but are generally considered low-credit-risk securities. Both types of “agency” securities, however, will have a somewhat higher rate of interest than that of a Treasury security due to increased credit or liquidity risk or both.

**Federally related institutions**, such as the Government National Mortgage Association (Ginnie Mae), are institutions owned by the U.S. government. These securities are implicitly backed by the full faith and credit of the U.S. government and are generally considered to be free from credit risk.

**Government sponsored entities (GSEs)** include the Federal Farm Credit System, the Federal Home Loan Bank System, the Federal National Mortgage Association (Fannie Mae), the Federal Home Loan Mortgage Corporation (Freddie Mac), and the Student Loan Marketing Association (Sallie Mae). These are privately owned but publicly chartered organizations created by the U.S. Congress. They issue their securities directly in the marketplace and expose investors to credit risk.

Agency securities vary widely in their features and include various forms of debentures and securities known as mortgage-backed or asset-backed securities. Mortgage-backed and asset-backed securities are created through the **securitization** of a pool of loans or other assets. The agency sponsoring the issue takes control

## Asset Valuation

of the collateral pool and issues a fixed income security to a group of investors. Basically, the loans or assets serve as collateral to repay the mortgaged-backed or asset-backed security.

**Municipal securities.** Debt securities issued by state and local governments in the United States are known as **municipal bonds** (or *munis* for short). Munis are referred to as *tax-exempts* since the coupon interest is generally exempt from federal income taxes. While interest income may be tax-free, any capital gains on these munis are subject to capital gains tax. The coupon interest on municipals, except for some *double-exempt* bonds<sup>12</sup>, is taxable at the state level. Because municipalities have a greater risk of default than the U.S. Treasury, munis typically carry higher rates of interest (on a tax-adjusted basis) than Treasuries.

Munis are often issued as *serial obligations*, which means that the issue is broken into a series of smaller issues, each with its own maturity date and coupon. Most municipal bonds are brought to the market as either general obligation or revenue bonds. *General obligation* (GO) bonds are backed by the full faith, credit, and taxing power of the issuer. In contrast, *revenue bonds* are serviced with the income generated from a specific revenue-generating project funded with the issue (e.g., bonds issued to fund a toll road).

**Corporate securities.** Because private corporations cannot rely on taxing power, debt issued by corporations is of higher risk than Treasury, agency, and most municipal debt (though *some* corporations have lower credit risk than *some* municipalities). Corporate debt may be secured or unsecured and may be sold to the public or to a private party. *Privately placed* debt is sold by a corporation to a large financial institution, such as an insurance company, that typically holds the bond issue until maturity. Rule D of the Securities and Exchange Act allows corporations to issue debt privately to qualified buyers without having to register the debt with the SEC. However, the public is not allowed to buy these unregistered securities. An institution that buys privately placed debt may not sell the issue unless it is pursuant to *Rule 144A*. Rule 144A allows for secondary market trading of privately placed debt between qualified institutional buyers.

Corporate debt securities that are going to be sold to the public must generally be registered with the SEC. Bonds sold to the public are usually “underwritten” by an investment bank (i.e., a price guarantee has been given to the borrower for the bonds sold). Securities placed privately typically have no price guarantee and are sold on a “best-efforts” basis (i.e., the entire price risk is borne by the issuer). Securities issued by private corporations include bonds, notes, commercial paper, and asset-backed securities.

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<sup>12</sup> Most U.S. states do not tax their own residents for interest on munis issued by the state or municipalities within the state. Interest on out-of-state munis is usually taxable.

Bonds issued by corporations can either be secured mortgage bonds or unsecured debentures. Corporate bonds range in maturity from one to thirty years and can be structured with embedded derivatives to meet the needs of a niche market. Corporations also issue notes which may have similar maturities and features as bonds but are sold on a continuous basis rather than all at once as with a bond issuance. Some firms also issue **commercial paper**, which is a short-term discount security similar to a Treasury bill. Only large firms with high credit ratings are able to issue commercial paper for short-term borrowing.

## Risks Associated with Fixed Income Investments

Like any other investment vehicle, bonds should be evaluated on the basis of the return offered and the risk involved. Returns on bonds come in the form of interest (coupons) and/or capital gains (i.e., increases in the market price of the bond). The size of the expected return is determined by the perceived risk of the bonds. The risks to bond investors are a function of both macroeconomic and microeconomic factors. Following is a list and brief description of the most important risks to bond investors. While this is not an exhaustive list, the key factors affecting bond values are detailed. To varying degrees (based on their maturity, coupon rate, and other characteristics), bonds are subject to the following types of risks.

**Interest rate risk (price risk)** is the risk that interest rates will increase and the value of an investor's bond holdings will decrease. Prices move in the opposite direction of interest rates (i.e., increases in rates will decrease bond prices, and decreases in rates will increase bond prices). Generally, the longer the maturity of a bond and the lower its coupon rate, the more sensitive its price is to changes in interest rates.

**Reinvestment risk** is the risk that cash flows received from a bond will have to be reinvested at a lower rate of return than that expected at the time of the original investment. Reinvestment risk is often thought of as being the opposite of interest rate risk. The required yield on a bond investment will be realized only if the cash flows during the life of the bond can be reinvested at the required yield (i.e., the yield to maturity prevailing at the time of purchase). If these cash flows are reinvested at rates that are less than the required yield, the actual return on the investment will be less than the required yield.

**Credit risk** is the risk that the issuer will not be able to pay interest or principal when due. Investors in fixed income securities are lending money to the issuer of the securities. Consequently, there is the risk that the borrower will not be able to repay the principal and interest as promised. Bond rating agencies assess this risk by rating the creditworthiness of borrowers and their outstanding bond issues. For a given borrower, each bond issue is analyzed individually, and a single borrower can have multiple ratings (e.g., one rating for senior debt, one for subordinated debt).

## Asset Valuation

The rating assigned by the agency is based on their assessment of the probability of repayment, as well as other factors such as the economic and political environment.

**Liquidity risk** is the risk that the investor will not be able to sell the bond quickly at a fair price should the need arise. Liquidity risk means an investor would either have to wait a long time before a sell order can be filled or would have to significantly reduce the price in order to sell the bond quickly. Liquidity risk can be estimated by looking at the size of the *bid-ask spread*, or the difference between the price at which a dealer would buy the bond (the bid price) and the price at which the dealer would sell the bond (the ask price).

**Inflation risk** is the risk that actual inflation will exceed expected inflation, thereby leading to a loss of purchasing power. Since fixed-coupon bonds pay a constant stream of interest income, increasing prices erode the buying power associated with bond payments. Increasing inflation results in an unexpected reduction of the purchasing power of future bond payments for existing bondholders.

**Call risk** is the risk the bond will be retired early by the issuer and that the funds received will need to be reinvested at a lower yield. Investors in callable bonds face the risk that their bonds may be redeemed early at the option of the issuer, usually after a significant decline in interest rates, and the subsequent reinvestment of the proceeds must be done at lower yields.

**Event risk** is the risk of an unforeseen event that has a negative impact on the financial condition of the issuer. If the impact is severe, this can call into question the issuer's ability to meet its obligations and can decrease the value of the issuer's bonds.

## BOND VALUATION AND YIELD MEASURES

When a corporation or other entity issues (sells) bonds, it promises to pay the holder (buyer) a series of interest payments, known as coupons, and repay the face value (i.e., the principal) at maturity. For U.S. corporations, coupon payments are paid semiannually and resemble the ordinary annuities we discussed in the chapter on Quantitative Methods. The principal repayment is a lump sum payment on the maturity date. Using the capitalization of income method, the market value of the bond can be determined by calculating the sum of the present values of those cash flows.

The bond valuation process has three fundamental steps.

*Step 1:* Determine the timing and amount of cash flows for both coupons and principal.

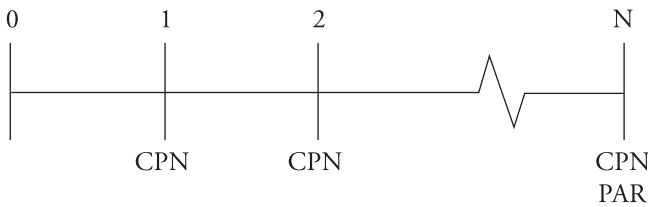
*Step 2:* Estimate the appropriate discount rate.

Step 3: Calculate the sum of the present values of the estimated cash flows.

### Step 1: Cash Flows

The typical cash flow structure for an option-free bond (i.e., one that is not callable, puttable, or convertible) is depicted by the timeline shown in Figure 6.

Figure 6: Typical Structure of Bond Cash Flows



where:

PAR = maturity value or, as it is more commonly known, the par value of the bond

CPN = coupon payment, calculated as:  $(\text{par} \times \text{coupon rate}) / (\text{number of coupons per year})$

N = life of the issue, defined as:  $(\text{years to maturity}) \times (\text{number of coupons per year})$

### Step 2: Discount Rate

The appropriate discount rate (or required yield) for a given bond is the risk-free rate plus a risk premium.

$$\text{discount rate for a risky bond} = \left( \text{disc. rate for a similar default-free bond} \right) + \left( \text{risk premium} \right) \quad (8)$$

The benchmark risk-free rate is the yield on a Treasury security of comparable maturity. The more difficult problem is to determine the appropriate risk premium that correctly reflects the differential in risk between the bond in question and the benchmark. The size of the risk premium is a function of a variety of risk factors which we discussed previously. All else equal, the riskier the security, the greater the risk premium.

### Step 3: Valuing the Bond

Once the bond's expected future cash flows have been determined and the appropriate discount rate has been estimated, the bond's value as the present value



## Asset Valuation

of the expected future cash flows discounted at the appropriate discount rate can be calculated as:

$$\text{bond value} = \frac{\text{CPN}_1}{\left(1 + \frac{i}{m}\right)^1} + \frac{\text{CPN}_2}{\left(1 + \frac{i}{m}\right)^2} + \dots + \frac{\text{CPN}_{n \times m} + \text{PAR}}{\left(1 + \frac{i}{m}\right)^{n \times m}} \quad (9)$$

where:

- i = interest (discount) rate per year
- m = number of coupons per year
- n = number of years to maturity
- $n \times m = N$ , the life of the bond

In order to illustrate the general valuation method we have just described, we will value a 10-year, 8% semiannual coupon bond, yielding 8%. But before we begin, let's recap each of the bond's key characteristics.

- "10-year" indicates that the bond matures in 10 years. This should not be confused with the bond's original length of issue, which could be up to 30 years or even longer.
- "8 percent" refers to the annual coupon rate on the bond. The bond pays 8% of its face value each year, in two equal semiannual payments. Assuming the bond has a \$1,000 face value, it will pay  $\$80/2 = \$40$  every six months.
- "Yielding 8 percent" indicates that the bond is priced to yield 8%. This means that based upon the risk of the firm and the bond, its required return or discount rate is 8%.

Since the bond pays coupons semiannually, there are 20 coupons paid in ten years. Also, since the annual required return is 8%, the semiannual required return is 4%.

The pattern of the coupon payments is nothing more than an annuity. Each payment is \$40, and the payments come exactly six months apart. In order to determine the value of the bond, we must find the present value of each coupon and sum them to find the total present value of all the coupons we will receive. This results in an annuity of twenty payments of \$40 each, discounted at 4%.

We could find all the individual present values and sum them but we can also use the ordinary annuity formula:

$$PV_A = PMT \left[ \frac{1}{\frac{i}{m}} - \frac{1}{\frac{i}{m} \left( 1 + \frac{i}{m} \right)^{m \cdot n}} \right] \quad (10)$$

where:

- $PV_A$  = the present value of an annuity (the annuity in this case is the series of coupons we will receive)
- $PMT$  = the amount of each payment (the payments are the \$40 coupons)
- $i$  = the annual interest (discount) rate
- $n$  = the number of years
- $m$  = the number of coupons per year

\$543.61 represents the present value of the stream of coupon payments expected, if we buy the bond. Of course we would also expect the repayment of the principal in ten years. The relationship between present and future values of a lump sum can be expressed in the following manner:

$$FV_n = PV \left( 1 + \frac{i}{m} \right)^{m \cdot n} \quad (11)$$

Rearranging the equation above to solve for  $PV$  yields:

$$PV = \frac{FV_n}{\left( 1 + \frac{i}{m} \right)^{m \cdot n}} \quad (12)$$

where:

- $FV$  = the future value (in this case, the lump sum return of principal or face value of the bond)
- $i$  = the annual interest (discount) rate
- $m$  = the compounding periods per year (in this case, coupons are paid semiannually)
- $n$  = the number of years

In this example,

$$PV = \frac{\$1000}{\left( 1 + \frac{0.08}{2} \right)^{(2)(10)}} = \frac{\$1000}{(1.04)^{20}} = \frac{\$1000}{2.191123} = \$456.39$$

## Asset Valuation

Thus, the present value of \$1,000 to be received in 20 semiannual periods (10 years) at 4% is \$456.39. Now that we have the present value of both the series of coupons and the lump sum face value, we find the total price of our bond is  $\$543.61 + \$456.39 = \$1,000$ .

Two important points: (1) When the coupon rate and required return on a bond are the same, the bond will always be quoted at (sell at) par value; (2) The model assumes you purchase the bond on a coupon date. This means both methods assume the next coupon will be received in exactly six months (180 days). In practice, this is obviously not the typical case.

**Accrued interest.** When you purchase a bond between interest payment dates, you will have to pay accrued interest because even though coupons are paid semiannually, interest accrues daily. The bond in the above example pays \$80 in interest every 360 days<sup>13</sup> (in two payments of \$40), or 22.22 cents per day. The holder of the bond (the seller) is entitled to the interest accrued since the last coupon payment.

Let's assume the bond last paid interest 115 days ago (65 days before the next interest payment). Since you must pay the seller the interest accrued since the last coupon, you must pay him  $\$0.2222/\text{day} \times 115 \text{ days} = \$25.55$  in addition to the bond's quoted price. If a bond price is quoted without accrued interest added in, the quote is called the *clean price*. Quoting the clean price is the standard market practice. A price quote that includes the accrued interest is known as the *dirty price*.

**Price vs. yield.** In the following examples, we will illustrate the inverse relationship between bond yield and price.

A bond has these characteristics:

- Coupon rate = 8%. (It pays 8% of its face value in two equal, semiannual \$40 payments).
- Ten years remaining until maturity.
- Face value = \$1,000.

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<sup>13</sup> A 360-day year is assumed for some classes of bonds, the actual number of days for others. The number of days used for a given bond is known as the *day count convention*.

When the required return is 8%, the bond's price is \$1,000. This is because its required return equals its coupon rate. Using the TI Business Analyst II Plus<sup>®</sup> calculator, we'll find the value of our bond at various required returns. The TI BA II Plus keystrokes are as follows:

Required Return = 8%  
 \$40 [PMT]  
 4% [I/Y]  
 20 [N]  
 \$1,000 [FV]  
 [CPT] [PV] = -\$1,000

Let the required return fall to 7% and 6% respectively:

<u>Required Return = 7%</u>	<u>Required Return = 6%</u>
\$40 [PMT]	\$40 [PMT]
3.5% [I/Y]	3% [I/Y]
20 [N]	20 [N]
\$1,000 [FV]	\$1,000 [FV]
CPT [PV] = -\$1,071.06	CPT [PV] = -\$1,148.77

Let the required return increase to 9% and 10% respectively:

<u>Required Return = 9%</u>	<u>Required Return = 10%</u>
\$40 [PMT]	\$40 [PMT]
4.5% [I/Y]	5% [I/Y]
20 [n]	20 [n]
\$1,000 [FV]	\$1,000 [FV]
CPT [PV] = -\$934.96	CPT [PV] = -\$875.38

As shown in the illustration, the price of the bond moves in the opposite direction from the change in required return, giving us three very important relationships that will be true for all bonds that do not contain options:

- If the required return equals the coupon rate, the bond sells at par (i.e., face value).
- If the required return is less than the coupon rate, the bond sells at a premium (i.e., greater than face value).
- If the required return is greater than the coupon rate, the bond sells at a discount (i.e., less than face value).

**Price vs. maturity.** Another important bond characteristic is the relationship between maturity and the change in price for a given change in required return. We already know that there is an inverse relationship between the bond's price and its required return. Now we'll show that the greater the time until maturity of the bond, the greater the change in value associated with an interest rate change. Using

## Asset Valuation

the same bond illustrated above, let's assume that the maturity of the bond is 20 years instead of 10 (i.e., there will be 40 semiannual periods instead of 20).

$$\begin{aligned}
 &\underline{\text{Required Return} = 8\%} \\
 &\$40 \quad \text{[PMT]} \\
 &4\% \quad \text{[I/Y]} \\
 &40 \quad \text{[n]} \\
 &\$1,000 \quad \text{[FV]} \\
 &\text{[CPT] [PV]} = -\$1,000
 \end{aligned}$$

Again, as long as the required return and coupon rate are equal, the bond sells at par.

Let the required return fall to 7% and 6% respectively:

$  \begin{aligned}  &\underline{\text{Required Return} = 7\%} \\  &\$40 \quad \text{[PMT]} \\  &3.5\% \quad \text{[I/Y]} \\  &40 \quad \text{[n]} \\  &\$1,000 \quad \text{[FV]} \\  &\text{[CPT] [PV]} = -\$1,106.77  \end{aligned}  $	$  \begin{aligned}  &\underline{\text{Required Return} = 6\%} \\  &\$40 \quad \text{[PMT]} \\  &3\% \quad \text{[I/Y]} \\  &40 \quad \text{[n]} \\  &\$1,000 \quad \text{[FV]} \\  &\text{[CPT] [PV]} = -\$1,231.15  \end{aligned}  $
---	---

Let the required return increase to 9% and 10% respectively:

$  \begin{aligned}  &\underline{\text{Required Return} = 9\%} \\  &\$40 \quad \text{[PMT]} \\  &4.5\% \quad \text{[I/Y]} \\  &40 \quad \text{[n]} \\  &\$1,000 \quad \text{[FV]} \\  &\text{[CPT] [PV]} = -\$907.99  \end{aligned}  $	$  \begin{aligned}  &\underline{\text{Required Return} = 10\%} \\  &\$40 \quad \text{[PMT]} \\  &5\% \quad \text{[I/Y]} \\  &40 \quad \text{[n]} \\  &\$1,000 \quad \text{[FV]} \\  &\text{[CPT] [PV]} = -\$828.41  \end{aligned}  $
---	--

In Figure 7, we have summarized the relationship between a bond's price and changes in the required return and the relationship between price and maturity for the same changes in required return. Notice that for both bonds, the prices increase (decrease) as the required return decreases (increases). However, the magnitude of the price changes is greater for the longer maturity bond (i.e., the 20-year bond).

**Figure 7: Bond Price Reactions to Changes in Required Yield and Maturity**

<i>Required Return</i>	<i>Maturity (Years)</i>	
	<i>10</i>	<i>20</i>
6%	1,148.77	1,231.15
7%	1,071.06	1,106.77
8%	1,000.00	1,000.00
9%	934.96	907.99
10%	875.38	828.41

**Price vs. coupon rate.** In addition to the time until maturity, the change in a bond's price is also affected by its coupon rate. In general, *the higher the coupon rate, the smaller the change in price* for a given change in required return. Let's revisit our 20-year bond again, but let's now assume it has a 12% coupon rate instead of 8% (i.e., there will be \$60 semiannual payments instead of \$40) and that the initial required yield on the bond is 12% (i.e., the bond trades at par value). Using a TI Business Analyst II calculator:

Required Return = 12%  
 \$60 [PMT]  
 6% [I/Y]  
 40 [n]  
 \$1,000 [FV]  
 [CPT] [PV] = -\$1,000

Let the required return fall to 11% and 10% respectively:

Required Return = 11%  
 \$60 [PMT]  
 5.5% [I/Y]  
 40 [n]  
 \$1,000 [FV]  
 [CPT] [PV] = -\$1,080.23

Required Return = 10%  
 \$60 [PMT]  
 5% [I/Y]  
 40 [n]  
 \$1,000 [FV]  
 [CPT] [PV] = -\$1,171.59

Let the required return increase to 13% and 14% respectively:

Required Return = 13%  
 \$60 [PMT]  
 6.5% [I/Y]  
 40 [n]  
 \$1,000 [FV]  
 [CPT] [PV] = -\$929.27

Required Return = 14%  
 \$60 [PMT]  
 7% [I/Y]  
 40 [n]  
 \$1,000 [FV]  
 [CPT] [PV] = -\$866.68

Let's sum up the results of our findings in Figure 8. The first column shows the change in required return from the original coupon rate. Remember, the bond will sell at par value as long as the coupon rate and required return are equal. The second and third columns demonstrate that changes in price are less severe (for both increases and decreases) for the bond with the shorter maturity. The third and fourth columns demonstrate that changes in price are less severe for bonds with larger coupons. We can summarize the results of our examples in two general rules for all fixed income securities.

**Rule 1:** The longer the maturity of any fixed income instrument, the more sensitive its price is to changes in interest rates.

**Rule 2:** The greater the interim cash flows associated with a fixed income security, the less sensitive its price is to changes in interest rates.

## Asset Valuation

**Figure 8: Bond Price Reactions to Changes in Required Return (Par = \$1,000)**

<i>Change in Required Return</i>	<i>Price Change From Par</i>		
	<i>10-year, 8%</i>	<i>20-year, 8%</i>	<i>20-year, 12%</i>
-2%	+148.77	+231.15	+171.59
-1%	+71.06	+106.77	+80.23
No change	0	0	0
+1%	-65.04	-92.01	-70.73
+2%	-124.62	-171.59	-133.32

These rules give us guidance in establishing investing strategies related to bonds. If you expect interest rates to rise in the near future, you want to hold short-term bonds to minimize the subsequent drop in prices. However, if you expect interest rates to fall in the near future, you want to hold long-term bonds to take advantage of the price increases.

## YIELD MEASURES

In the bond market, investment decisions are typically made on the basis of a bond's yield rather than its price. A bond's yield affects the price at which it trades and serves as an important measure of potential or expected return. One well-known yield measure, the yield to maturity, is found by reversing the bond valuation process (described previously in this section) and solving for yield rather than price. In this section we explore various yield measures used in the market. Some yield measures provide little information and suffer from significant limitations, while others provide more reliable information and are more indicative of the true return on a bond investment.

### Nominal Yield

The **nominal yield** is the stated coupon rate on the bond. This is stated as a percentage of the bond's par value. The nominal yield will equal the required return on the bond only when the bond sells at par value.

For example, a bond with a 6% coupon rate that is selling at par has a nominal yield and required return (yield to maturity) of 6%. If the required return on the bond changes to 8%, the bond will sell at a discount, but the nominal yield will stay at 6%. Thus the nominal yield may be misleading under certain circumstances.

## Current Yield

The **current yield** is the annual coupon payment divided by the bond's current price. It is easy to see that this measure looks at just one source of return: *the bond's annual interest income*. That is, the current yield does not consider capital gains/losses or reinvestment income. The formula for the current yield is:

$$\text{current yield} = \frac{\text{annual cash coupon payment}}{\text{bond price}} \quad (13)$$

Note that the nominal yield and the current yield are only useful measures of the bond's expected return when the bond is trading at or very close to par value. As the discount or premium on a bond increases, the nominal and current yields become less relevant.

## Yield to Maturity

**Yield to maturity (YTM)**, also referred to as the bond's promised yield, is the most widely used bond yield measure. YTM considers both interest income and capital gains/losses, and also the timing of the cash flows received over the life of an issue. However, the actual return to an investor will equal the YTM only if all interim cash flows are reinvested at an interest rate equal to the YTM.

The basic bond valuation model (introduced previously) is used to find YTM. In most cases, the price of the bond is known since this can be observed in the market. The required yield is the unknown. Therefore, the problem becomes solving for the discount rate that equates the present value of the bond's cash flows to its current market price. You should be aware that the formulas used for bond valuation in this chapter are conceptual aids only; in practice, bond calculations are almost always done with the help of a financial calculator.



## Asset Valuation

The formula for computing YTM, which can be used with annual ( $m = 1$ ), semiannual ( $m = 2$ ), or any other frequency of coupons, is as follows:

$$\text{bond value} = \frac{\text{CPN}_1}{\left(1 + \frac{\text{YTM}}{m}\right)^1} + \frac{\text{CPN}_2}{\left(1 + \frac{\text{YTM}}{m}\right)^2} + \dots + \frac{\text{CPN}_{n \times m} + \text{PAR}}{\left(1 + \frac{\text{YTM}}{m}\right)^{n \times m}} \quad (14)$$

where :

bond value = current market price

$\text{CPN}_t$  = coupon payment received at time  $t$

PAR = maturity, par, or face value of the bond

$m$  = number of coupon payments per year

$n$  = number of years to maturity

YTM = yield to maturity

The yield to maturity is the interest rate that, when plugged into the bond value equation, makes the discounted value of the bond's cash flows equal to the current market price. In order to find the YTM, the analyst (or the analyst's calculator) chooses a value for the YTM, discounts the bond's cash flows to their present value using the YTM, adds the present values, and compares the sum of the present values to the market price of the bond. If the sum of the present values is equal to the bond's market price, the YTM has been found and the process can stop. If they are not equal, the process begins again with a higher or lower YTM. Let's calculate the YTM on the following bond.

Consider a 10-year, \$1,000 par value, 9%, semiannual coupon bond, trading at a price of \$1,103.57.

On the TI BA II calculator:

- The present value (PV) is the bond's price, \$1,103.57, with a negative sign inserted to symbolize the fact that if you buy the bond, the purchase price is a negative cash flow.
- Since the bond pays semiannually,  $N$  is two times the number of years,  $2 \times 10 = 20$ .
- The future value (FV) is the maturity or face value of the bond, \$1,000, a positive cash flow to the bondholder.
- Since the bond pays coupons semiannually, the payment is half the annual coupon,  $90/2 = \$45$ , a series of positive cash flows to the bondholder.

-\$1,103.57 [PV]  
\$45 [PMT]  
20 [n]  
\$1,000 [FV]  
[CPT] [I/Y] = 3.7544%

Since the bond pays semiannual coupons, the computed value of 3.7544% is the *semiannual* measure of yield. It is the required yield per half year, but market convention dictates that YTM be quoted in nominal (or un compounded) percent per year. Therefore, *double the semiannual yield* of 3.7544% to find the YTM for the semiannual-pay bond (7.51%). This yield measure (the *uncompounded* YTM) is also known as the *bond equivalent yield* (BEY).

The relationship between the different yield measures we have discussed depends on whether a bond is trading at par, at a discount, or at a premium. These relationships are shown in Figure 9. The relationships demonstrated in Figure 9 will hold in all cases.

**Figure 9: Price-Yield Relationships**

<i>Bond Selling at:</i>	<i>Relationship</i>
Premium	Coupon rate > current yield > yield to maturity
Par	Coupon rate = current yield = yield to maturity
Discount	Coupon rate < current yield < yield to maturity

**Assumptions of YTM.** YTM is a summary measure of yield. Whether an investor actually realizes a return equal to the YTM depends on several assumptions that underlie the calculation of YTM. Some of these key assumptions are as follows.

- **All coupons can be reinvested at the YTM.** Reinvesting coupons at a rate other than the YTM will result in a realized yield that is more or less than the YTM. The only way to actually generate a rate of return equal to the bond's promised yield at the time of purchase is to reinvest all coupons at the YTM. This is the *reinvestment assumption* that is part of any present value-based measure of yield and is the source of *reinvestment risk*.
- **The bond can be held to maturity.** This usually means that it cannot be called or redeemed prior to maturity. When a bond is called, the maturity will be shortened, and the call price is often different from par value. Both of these factors make it unlikely that the realized yield will equal the YTM at the time of purchase.
- **All coupon payments are received in a prompt and timely fashion.** YTM is often referred to as a *promised yield* because it assumes that principal and interest payments will be made as promised.

As a final consideration, we must mention that not all yields are directly comparable. For instance, when choosing to invest between two different bonds, you should never compare the nominal yield on one bond to the current yield on another bond. Nominal yields are only comparable to other nominal yields. The same is true for current yields and yields to maturity. In addition, YTM on bonds are not directly comparable if the coupon payment frequencies differ. The YTM on

## Asset Valuation

a semiannual pay bond is an uncompounded YTM, and is not comparable to the YTM on an annual pay bond without an adjustment. In order to convert the YTM on a semiannual pay bond to a comparable annual pay YTM, use the following formula:

$$\text{YTM}_{\text{annual basis}} = \left( 1 + \frac{\text{YTM}_{\text{semiannual basis}}}{2} \right)^2 - 1 \quad (15)$$

After adjusting the bond-equivalent yield on the semiannual pay bond to an effective annual yield, the investment decision can be made.

## DERIVATIVES

A **derivative** is a security that *derives* its value from the value or return of another asset or security. The creation and growth of derivatives markets is largely attributable to both the need of certain market participants to control the risk (uncertainty) associated with certain assets and the desire of other market participants to speculate on the future value of the same assets. Thus it is possible to use derivative instruments to either decrease or increase the risk associated with an investor's portfolio. In this section we discuss three of the most well-known derivatives: options, forwards, and futures. Some of these derivatives trade "over the counter" and others trade on established exchanges, such as the Chicago Board of Trade (CBOT), the Chicago Board Options Exchange (CBOE), the Chicago Mercantile Exchange (CME), and the New York Futures Exchange (NYFE). While it is possible to structure any of the derivatives covered in this section as a customized over-the-counter transaction, we will discuss each of the derivatives in their most common settings (i.e., standardized exchanges are generally used for options and futures, while the over-the-counter market is generally used for forwards). We begin our discussion with options.

## OPTIONS

Imagine an investor believes that the price of XYZ Company's stock will increase above its current price of \$20 per share some time in the next six months but is unsure of how much or exactly when the stock price is going to increase. The investor is hesitant to purchase the stock since there is still a risk that the stock will decline in value, but he wants to take advantage of the price increase if and when it actually occurs. This is where the derivatives market comes in. The investor in this scenario needs the flexibility to participate in the stock price increase but avoid any losses that might occur should it decrease. For a price, the investor can obtain such flexibility through the purchase of a **call option**. Likewise, an investor can obtain the flexibility to participate in an expected price decline of a particular stock through the purchase of a **put option**.

An option gives its owner the right, but not the legal obligation, to buy or sell an underlying asset on or before a predetermined future date (the exercise date) at a predetermined price (the *exercise* or *strike price*). Options give the buyer the right to decide whether or not the trade will eventually take place, but the seller of the option must transact if the buyer desires it. Although there are many different types of assets covered by options, we will focus our attention on stock options.

- The owner of a call option has the right to purchase the underlying stock at a specific price for a specified time period.
- The owner of a put option has the right to sell the underlying stock at a specific price for a specified time period.

The *writer* of an option takes a short position in the option, while the owner takes the long position. Long and short positions in call and put options give us four possible options positions:

- **Long call:** The buyer of the call option. A long call position has the *right* to buy the stock at the specified price. An investor will hold a long call position when he expects the price of the stock to increase.
- **Short call:** The writer (seller) of the call option. A short call position has the *obligation* to sell the stock at the specified price if the owner of the option decides to exercise (buy the asset). An investor will take a short position in a call when he expects the price of the stock to remain unchanged or decrease.
- **Long put:** The buyer of a put option. A long put position has the *right* to sell the stock at the specified price. An investor will hold a long put position when he expects the price of the stock to decrease.
- **Short put:** The writer (seller) of a put option. A short put position has the *obligation* to buy the stock if the owner of the option decides to exercise (sell the asset). An investor will take a short position in a put when he expects the price of the stock to remain unchanged or increase.

## OPTIONS TERMINOLOGY

Before we go any further into the reasons to hold options and their expected payoffs, we must introduce the terminology essential to understanding the features of options contracts. We have already described some of the differences between call and put options. Now we introduce features that are common to all options.

Options on stocks and other assets are standardized legal contracts that entitle the owner to either purchase or sell an asset at a pre-specified price. The price at which the option holder is entitled to buy or sell the asset is known as the **exercise price** or **strike price**. The exercise price will remain constant throughout the life of the option.

## Asset Valuation

In general, options have a finite life. The final day that the option is still alive or exercisable is known as the **expiration date**. Therefore the **time to expiration** is the number of days until the option reaches the expiration date. Most exchange-traded options can be purchased with anywhere from a few days to several months until the expiration date. After the expiration date, however, it is no longer exercisable.

Owners of options (the long positions) buy them by paying the price of the option, which is called the **option premium**, to the seller of the option. Listed stock option contracts trade on exchanges and are normally traded in bundles of 100. Each option contract, therefore, contains 100 options, and each individual option gives the owner the right to buy or sell one share of stock.

Options, both call and put, can be **American** or **European-style**. The distinction between American and European options is when the owner can exercise. It has nothing to do with where they are traded.

- **American-style options** may be exercised at any time up to and including the expiration date.
- **European-style options** may be exercised only on the expiration date.

If two options are identical (maturity, underlying stock, strike price, etc.) in all ways, except that one is a European option and the other is an American option, the value of the American option will equal or exceed the value of the European option. This is because the early exercise feature of the American option gives it more flexibility, so it should be worth at least as much as, and possibly more than, a comparable European option. The increased flexibility stems from the fact that the American option gives the holder many more opportunities to exercise. That is, the European option has only one day that it can be exercised, while the American option can be exercised at any time before it expires.

To see how an option contract works, consider the stock of ABC Company. ABC shares sell in the market for \$55 and have an American call option available on them that sells for a premium of \$10. This call option has an exercise price of \$50 and has an expiration date in five months.

Recall that earlier in this section it was stated that options contracts generally trade in bundles of 100 options for 100 shares of the underlying stock. Price quotations, however, generally are stated on a per share or per option basis. For the ABC Company call option, the option premium was quoted as \$10 per option, but the total price paid for the bundle of options would be  $\$10 \times 100 = \$1,000$ . Likewise, the total amount paid for the underlying stock if the call option were to be exercised would be  $\$50 \times 100 = \$5,000$ .

If one ABC call option is purchased for \$10, the buyer can purchase one share of ABC stock from the option seller anytime over the next five months for \$50. Note

that if the holder exercises the option immediately, he will effectively pay \$60 for the stock (the \$50 exercise price plus the \$10 option premium). Since the stock is currently selling for \$55, the holder will not profit unless the stock rises to at least \$60. However, the holder will not let the option expire unexercised unless the stock price is less than \$50 at the expiration date.

The writer of the option gets to keep the \$10 premium received from the holder no matter what the stock does during this time period (i.e., whether the buyer exercises the option or not). If the buyer exercises the option, the seller will receive the \$50 strike price and must deliver one share of ABC stock to the buyer for each option sold.

A single put option on ABC stock with the same exercise price gives the buyer the right to *sell* one share of ABC for \$50 at any time during the next five months. The put writer has the obligation to buy the ABC stock from the owner at the exercise price if the buyer chooses to exercise.

## PAYOFFS AND MONEYNES

In order to understand option moneyness and profits, it is first necessary to understand the structure of option payoffs at expiration. The payoff of a call or put option at expiration to the option holder is:

$$C_{\text{buyer}} = \text{Max}(0, S - X)$$

$$P_{\text{buyer}} = \text{Max}(0, X - S)$$

where:

S = market price of the stock

X = exercise price of the option

$C_{\text{buyer}}$  = call option payoff to option buyer

$P_{\text{buyer}}$  = put option payoff to option buyer

Notice that we have written the payoff functions as the maximum (Max) of zero or  $S - X$  (for call options) and the maximum of zero or  $X - S$  (for put options). If the difference between the exercise price and the stock price is ever unfavorable (i.e., stock price is less than exercise price for a call, or stock price is greater than exercise price for a put), the option would go unexercised since the payoff to the option holder would be negative. Therefore the option payoff to the buyer is limited by zero. The writer of the contract faces the opposite payoff function at expiration. Options, therefore, are a zero sum game in the sense that the benefit to one party to the transaction is a cost to the opposite party.

Asset Valuation

Payoff diagrams for call and put options are shown in Figures 10 and 11, respectively:

Figure 10: Expiration Payoff Diagram for a Call

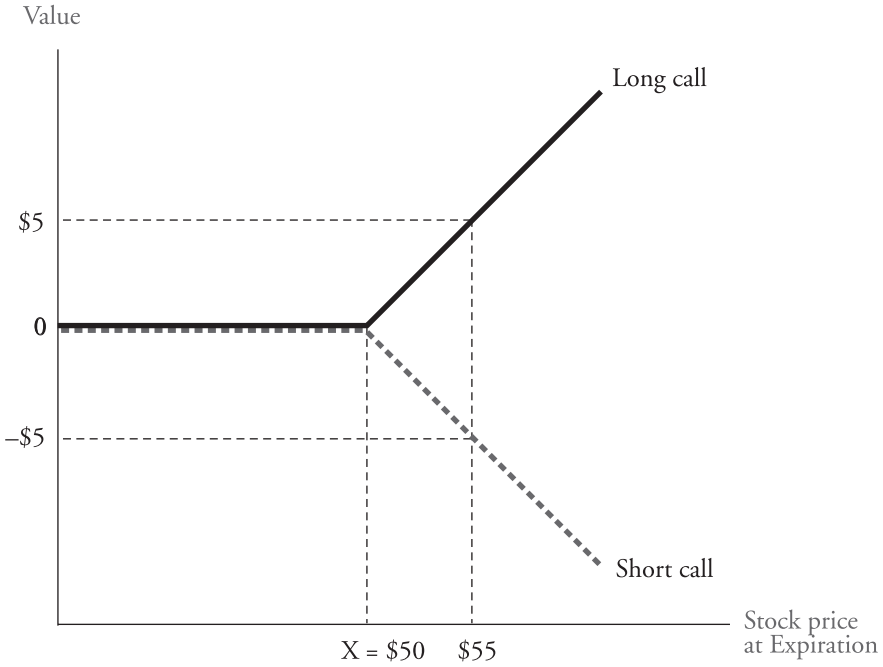
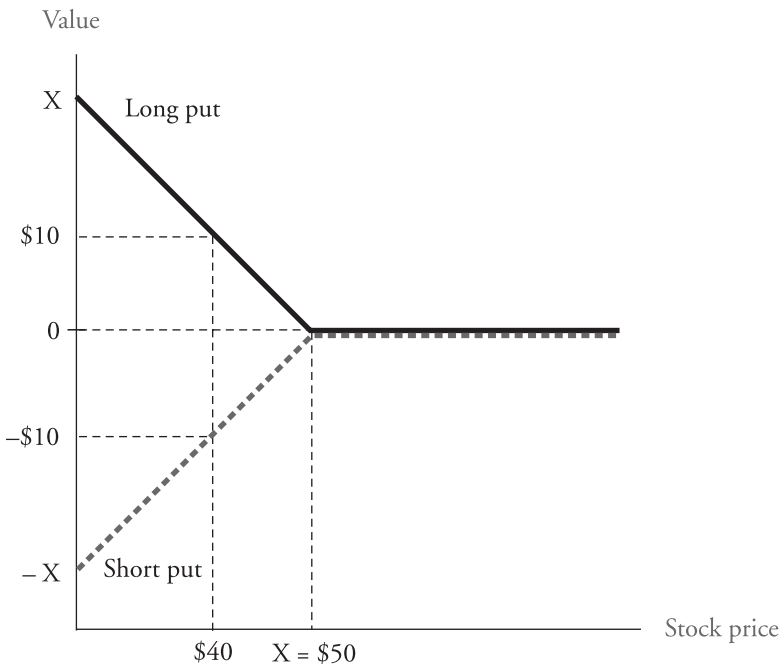


Figure 11: Expiration Payoff Diagram for a Put



**Moneyness** refers to the relationship between the strike price and the stock price. If the relationship is such that the payoff to the option holder at expiration (or exercise) is greater than zero, the option is said to be **in-the-money**. For a call option, this occurs when the market price of the stock is greater than the option's exercise price. If the stock price equals the exercise price, the option is said to be **at-the-money**. If the stock price is less than the exercise price, the call option is **out-of-the-money**. For put options, the moneyness relationships are the opposite.

We have summarized the relationships between the stock price, exercise price, and moneyness for calls and puts in Figure 12.

**Figure 12: Option Moneyness**

<i>Option</i>	$S < X$	$S = X$	$S > X$
Call	Out-of-the-money	At-the-money	In-the-money
Put	In-the-money	At-the-money	Out-of-the-money

Let's look at an example. We'll assume that the exercise price for an option on a share of ABC stock is \$50, and the stock price is \$55. If this is a call option, the option is in the money because it allows the owner to buy a \$55 stock for \$50. If it is a put option, it is out of the money because it allows the owner to sell a \$55 stock for \$50.

The amount by which any option is in the money is known as the option's **intrinsic value**. This is the exercise value of the option today, and as we know, the option payoff is limited by zero (i.e., cannot be negative). Therefore, the intrinsic value of an out-of-the-money option is zero. How do we then interpret an out-of-the-money option with a premium greater than zero? For this to be the case, there must be some time remaining until the option expires. Since there is always some chance that the option can move into the money during the remaining time to expiration, there is some potential value. The value lies in the time to expiration and the probability that the option will move into the money during this time. Thus, every option has a value based on two components—the intrinsic value and the **time value**. The amount of the time value is the difference between the market price (premium) of the option and its intrinsic value.

A question that is probably bothering you at this point relates to the premium paid for the option. That is, if an option is in the money, is exercising it profitable? The answer, unfortunately, is "it depends." This is because moneyness is determined solely by the strike and stock prices. To calculate profit, you must take the premium into consideration. Returning to our preceding example, we'll assume the option premium paid was \$10.



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For the call option,  $S = \$55$ ,  $X = \$50$ , and the premium paid, which we denote as  $C = \$10$ . Let's assume the date is the day the option matures (expires). Would the holder exercise the option? The answer is yes, even though there is a loss. Let's look at the reason.

If the holder does not exercise, she is still out the original premium paid of \$10. If she exercises the option, she gains \$5 from buying and reselling the stock, which helps to offset the \$10 cost of the option. Let's specify the profit of a call option as:<sup>14</sup>

$$\text{profit}_{\text{call}} = (S - X) - C \quad (16)$$

where:

$S$  = stock price

$X$  = exercise price

$C$  = call premium

If the holder exercises the call,  $\text{profit} = (\$55 - \$50) - \$10 = -\$5$ . If the holder does not exercise, the profit is  $-\$10$ , the cost of the option. In this case, the holder will exercise, even though the overall transaction is a loss. Exercising the option does not generate a profit, but it reduces the loss.

Let's now assume  $S = \$50$ ,  $X = \$55$ , and  $C = \$10$ :

$$\text{profit}_{\text{call}} = (S - X) - C = (\$50 - \$55) - \$10 = -\$15$$

In this case, the holder will not exercise the option because doing so would only increase the loss from \$10 to \$15. Since  $X > S$ , the call is out of the money. Any time an option is out of the money at maturity, the holder is better off simply letting the option expire.

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<sup>14</sup> The examples assume we are at the maturity of the option contract, so the values are intrinsic or exercise values. Since the vast majority of options will have time value in addition to the intrinsic value, an option is "worth more alive than dead" as long as it has some remaining life. Thus it is likely that the option would be sold for more than its intrinsic value, rather than exercised prior to maturity.

Turning to the put option, assume  $S = \$50$ ,  $X = \$55$ , and  $P = \$10$ . The profit for a put option can be expressed by the following equation:

$$\text{profit}_{\text{put}} = (X - S) - P \quad (17)$$

where:

$S$  = stock price

$X$  = exercise price

$P$  = put premium

$$\text{profit}_{\text{put}} = (\$55 - \$50) - \$10 = -\$5$$

Again, the holder will exercise, even though there is a net loss on the transaction, because the put is in the money; exercising the put decreases the loss. In this example, the investor would have suffered a loss of \$10 without exercising the option, but by exercising, the loss is reduced to \$5. Of course, if the put is out of the money ( $X < S$ ) at maturity, it will not be exercised.

This brings us to an important point about the risk of options. A buyer of an option has the risk that the option will expire out of the money, and he will lose the entire premium paid for the option. The buyer also has the upside potential of participating in stock price increases (in the case of calls) or stock price decreases (in the case of puts). The writer of an option has significantly more risk. A call option writer has the benefit of receiving the option premium but has the risk that the stock price will rise. Therefore, the call writer could theoretically experience an unlimited loss. Similarly, a put writer could watch as the price of the underlying stock falls to zero. The loss to the put writer could be very large indeed but is limited somewhat by the lower boundary of stock prices. That is, a stock's price cannot fall below zero.

## FACTORS THAT AFFECT OPTION VALUE

As you can infer from this discussion, the value of an option is determined by whether it is *expected* to become an in-the-money option. Since the vast majority of options are issued out of the money, the investor can use several factors to estimate whether the option will get in the money: current stock price, exercise price, maturity, and the volatility of the underlying stock (i.e., its standard deviation).

**Stock price.** Generally, for a given exercise price, the higher the stock price, the higher the value of a call option and the lower the value of a put. Rather than resort to mathematics in proving this relationship, let's use simple logic. As the price of a stock increases toward the exercise price of a call option, the probability that the market price will surpass the exercise price and the option will move into the money increases. Thus, the value of the call option increases as the stock price

## Asset Valuation

increases. Conversely, as the price of a stock decreases towards the exercise price of a put option, the probability that the market price will fall below the exercise price increases. Therefore, the value of the put option increases as the stock price decreases. Remember, the exercise price is fixed in the option contract, so the probability of getting in the money is determined by whether the stock price is expected to move favorably for the option holder.

**Exercise price.** Given a certain stock price, it is reasonable to assume that a call option with a low exercise price should be more valuable than one on the same stock with a higher exercise price, simply because the former allows the option holder to purchase the stock at a lower price and a higher payoff. At the same time, the higher the exercise price, the higher the value of a put option because it allows the option holder to sell the stock at a higher price and a higher payoff.

**Time until expiration.** The time until expiration of most put and call options on common stock is fairly short (e.g., six months). There are long-term options [e.g., Long-term Equity Anticipation Securities (LEAPS), which have lives of up to 2.5 years], but ordinary stock options are more prevalent.

The relationship between the maturity of an option and its value is positive. That is, the longer the remaining life, the greater its value. Again, logic would dictate that the longer the remaining life of the option, the higher the probability it will get in the money (i.e., there is more time for the stock price to move favorably for the option holder).

**Stock volatility.** In the risk discussion, we used standard deviation as a measure of risk because it provides a standardized measure of the volatility of the investment's returns. The greater the volatility of the stock's price, the higher the probability it will get in the money.

The focus of our discussion on options has been on stock options, but the options market encompasses much more than just stock options. Options are traded on a number of assets, including bonds, bond indexes, stock indexes, foreign currencies, interest rates, commodities, futures, swaps, and even other options. While we do not get into the specifics of each type of option available, most of the same principles detailed in this section apply to other types of options as well.

## FORWARDS AND FUTURES

Much like the options market, the forwards and futures markets developed to manage and speculate on the uncertainty associated with the price of an asset. Unlike the options market, however, forwards and futures represent a contractual obligation to transact with the counterparty. In other words, each side to a forward

or futures contract must buy or sell the underlying asset or settle the contract in cash. There is no option to perform on the contract; it must be done upon the expiration of the contract. Forward and futures contracts exist on agricultural and other commodities, stocks, bonds, interest rates, and currencies.

Before we discuss the features of forwards and futures, it is useful to briefly review the spot and forward markets.

Recall that the spot market is a market for the immediate delivery of an asset. Buyers and sellers who desire to exchange an asset today will go to the spot market to find a counterparty to their proposed transaction. In some instances, however, a buyer or seller may not wish to exchange an asset today but knows that she will need to exchange one in the future. Unfortunately, there is uncertainty associated with waiting until the future date to enter the spot market and complete the transaction. This is where the forward market comes in. In the forward market, contracts between buyers and sellers are written based on an agreed-upon price and quantity today, but delivery does not occur until some pre-specified date in the future.

Being able to secure a known price today for an exchange that will occur in the future has a definite benefit. However, forward contracts have a downside resulting from the lack of a liquid market in which holders can buy and sell contracts quickly. If it were possible, a corn farmer could buy and sell forward contracts as the harvest date neared in order to hedge<sup>15</sup> most of his risk. Since such a forward market doesn't exist, futures markets were developed. Futures are essentially standardized forward contracts traded on organized exchanges. In the case of a corn contract, standardization would concern issues such as the grade of corn, the number of bushels in one contract, the delivery price per bushel, and the delivery location.

## Forwards

A **forward contract** is a privately negotiated contract that obliges one party to buy and the other to sell a specific quantity of an asset at a set price on a specific date in the future. Typically, neither party pays anything to initiate the contract.

Investors can enter into forward contracts as a speculation on the future price of the underlying asset, but more often they enter into forward contracts to hedge a risk they already have. The forward contract is used to eliminate uncertainty about the price of an asset they plan to buy or sell at a later date. Forward contracts on physical assets, such as agricultural products, have existed for centuries.

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<sup>15</sup> Hedging is the reduction or elimination of risk.

## Asset Valuation

The party to the forward contract who agrees to buy the financial or physical asset is said to have a *long forward position* and is referred to as *the long*. The party to the forward contract that agrees to sell (deliver) the asset has a *short forward position* and is referred to as *the short*.

When the contract expires, most contracts allow for two methods of settlement. *Cash settlement* is frequently used and consists of the payment of the net difference between the contract price and the spot price on the settlement date. *Physical delivery* of the actual underlying asset can also occur when the short counterparty actually delivers the underlying asset to the long counterparty at the agreed-upon price.

With cash settlement, one of the counterparties will pay the other an amount equal to the difference between the spot price and forward price times the contract size. Usually, the contract size is the value of a certain amount of an asset such as a foreign currency or a quantity of a commodity such as an agricultural product.

We say the payoffs to a forward contract are *symmetric*. Because the transaction price is set in the contract, if the price of the specified asset increases above the contract price, the buyer (long position) gains value. She has agreed to pay a lower price for the asset. At the same time, the seller (short position) loses the same value because she has agreed to accept the lower price.

If the forward contract has no provisions for early settlement, the counterparties can still, by mutual agreement, settle the contract early if they choose to do so.

## Futures

Futures are similar to forward contracts in several ways and are often used for many of the same reasons that forward contracts are used. There are some key differences, however, that make futures a distinct financial instrument.

Futures are similar to forwards in that both:

1. Have similar terminology. The purchaser of a futures contract is said to have gone long or taken a *long position*, while the seller of a futures contract is said to have gone short or taken a *short position*. For each contract traded, there is a buyer and a seller.
2. Obligate the long to buy and the short to sell a set quantity of an asset for a fixed price on a specified future date.
3. May be settled in cash or by delivery. The required method is specified in the futures contract.

Futures differ from forwards in the following ways:

1. Futures contracts trade on organized exchanges and require the payment of a margin deposit at inception. Forwards are private contracts that usually do not trade once the agreement is signed, and they rarely require a margin payment.
2. Futures are standardized. Forwards are customized contracts satisfying the needs of the parties involved.
3. A **clearinghouse** is the counterparty to all futures contracts. Forwards are contracts between the originating counterparties.
4. The government regulates futures markets. Forward contracts are usually not regulated.

As previously stated, a major difference between forwards and futures is that futures contracts have *standardized contract terms*. Futures contracts specify the quality and quantity of the underlying asset that can be delivered, the delivery time, and the manner of delivery. The exchange also sets the minimum price fluctuation, which is called the **tick size**.

It might seem like such strict standards would restrict trading activity, but in fact they encourage it. Standardization tells traders exactly what is being traded and the conditions of the transaction. This uniformity promotes *liquidity*, which is a primary driver of securities trading.

Each exchange has a *clearinghouse*. The clearinghouse guarantees that traders in the futures market will honor their obligations by acting as the guarantor for each position. The clearinghouse does not actually buy or sell contracts, but it matches every buyer to a seller. Since the clearinghouse acts as the middleman, buyers and sellers need not even know each other. The clearinghouse also ensures each side performs as required under the contract, effectively eliminating any risk of non-performance. By doing this, the clearinghouse provides the means for either side of the trade to exit positions at any time before the expiration date without having to contact the other side of the initial trade. Thus the clearinghouse ensures a highly liquid, smooth-flowing futures market.

The closing price of a futures contract on a given day is the **settlement price**. Every trading day, all futures contracts are **marked to market** based on the settlement price. Marking to market is adjusting the **margin balance** (i.e., the money deposited with the clearinghouse) in a futures account each day for the change in value of the contract from the previous trading day.

## Asset Valuation

If the futures contract has lost value, those who bought the contract have lost the same amount. Their margin account will be reduced by the difference. If a margin account falls below a certain level, called the **maintenance margin**, the investor receives a **margin call** and must either close the position or deposit additional margin. Those who were short the contract have made that amount and may withdraw it. In this fashion, profits and losses on futures contracts are settled every trading day.

### Example: Computing margin balances

Consider a long position on a July wheat contract, which covers 5,000 bushels. Assume that the contract requires an initial margin deposit of \$150 and a maintenance margin of \$100. Compute the margin balance for this position after a two-tick decrease in price followed by a one-tick increase, then a one-tick decrease. Note that on this contract one tick is one cent, and the price at which the position was opened is \$2.

### Answer:

Each \$0.01 change (tick) in the price of wheat for this contract represents a gain or loss of \$50 ( $\$0.01 \times 5,000 = \$50.00$ ). Figure 13 illustrates the change in the margin balance as the price of this contract changes each day. Note that the required deposit is based upon the previous day's price change.

Figure 13: Daily Margin Balance (Day 0 is the date of entering the contract)

<i>Day</i>	<i>Required Deposit</i>	<i>Settle Price</i>	<i>Change in Price</i>	<i>Gain/Loss</i>	<i>Margin Balance</i>
Beg 1	\$150				\$150
End 1	0	\$1.98	−\$0.02	−\$100	\$50
2	\$100	\$1.99	+\$0.01	+\$50	\$200
3	0	\$1.98	−\$0.01	−\$50	\$150

### Discussion:

**Beginning of Day 1:** The long investor pays a deposit of \$150 to enter into the futures contract at a price of \$2.00 per bushel.

**End of Day 1:** The price in the spot market has dropped to \$1.98. This means the long position has agreed to pay \$2.00 for a product worth \$1.98, a loss of \$0.02 per bushel. On a contract for 5,000 bushels, this means a loss of \$100, which is deducted from the investor's margin and drops the margin to \$50.

**End of Day 2:** Since the investor's margin fell to \$50 the previous day, the investor is required to bring the margin back to its original level of \$150 with a \$100 deposit. Then, at the end of the day, the contract is settled at \$1.99.

Settling futures contracts through marking to market everyday means profits and losses are calculated and collected/paid daily. This effectively closes the futures contract and opens a new one each day at the last settle price. In this case the contract was settled at \$1.98 on the previous day, so when the contract is settled at \$1.99, the long position experiences a \$0.01 per bushel (\$50.00) gain. This is added to the investor's margin, which ends the day at \$200.

**End of Day 3:** The contract is again settled at \$1.98, so the long position experiences another \$0.01 per bushel loss (\$50.00). The amount is deducted from the margin, which ends Day 3 at \$150.

The concept of **convergence** is important to futures contracts. The futures price at any point should be an estimate of the spot price on the maturity date of the futures contract. This means that as the maturity date approaches, the futures and spot prices must *converge* (i.e., become equal).

There are four ways to terminate a futures contract:

1. A short can satisfy the contract by delivering the goods, a long by accepting delivery and paying the contract price to the short. This is called *delivery*. The location for delivery (for physical assets), terms of delivery, and specification of what is to be delivered are all specified in the contract. Deliveries represent less than 1% of all settlements.
2. In a *cash-settlement* contract, the futures account is marked to market based on the settlement price on the last day of trading, and the contract is closed with a payment or receipt of cash.
3. A position can be closed by making an *offsetting trade*. Since the other side of your position is held by the clearinghouse, if you make an exact opposite trade (maturity, quantity, and good) to your current position, the clearinghouse will net your positions out, leaving you with no obligations. This is how most futures positions are settled.



## Asset Valuation

4. A position may also be settled through an *exchange for physicals*. Here you find a trader with an opposite position to your own, deliver the goods, and settle between yourselves off the floor of the exchange (called an *ex-pit* transaction). This is the sole exception to the federal law that requires that all trades take place on the floor of the exchange. You must then contact the clearinghouse and tell them what happened. An exchange for physicals differs from a delivery in that the contract is not closed on the floor of the exchange and the two traders privately negotiate the terms of the transaction.

## SUMMARY

### EQUITY SECURITIES

- A. The equity valuation framework
1. The market value of a company equals its stock price times shares outstanding. The market value is also called the company's market capitalization.
  2. In a speculative bubble, the market value of stocks are driven more by emotion and fads than by discipline and facts.
  3. Benjamin Graham and David Dodd provided a systematic and disciplined approach to equity analysis. The two key components of this method are financial statement analysis and equity valuation.
    - a. Financial statement analysis refers to extensive, unbiased examination of the company's financial statements and key financial ratios.
    - b. Valuation encompasses various methods used to calculate the intrinsic value of the company's stock.
  4. To value any security we use the capitalization of income method.
    - a. Estimate all future cash flows from the security.
    - b. Estimate the risk of the cash flows.
    - c. Based upon the risk, determine a required return (discount rate).
    - d. Find the total present value (today) of the cash flows.
- B. Financial statement analysis using ratios
1. Working capital management
    - a. Most working capital ratios are concerned with a firm's liquidity, meaning the firm's ability to meet short-term obligations.
    - b. The current ratio is the ratio of total current assets to total current liabilities.
      - (1) Generally, a higher CR is desired.
      - (2) A CR that is considerably higher than the industry average could mean the firm has too much invested in short-term assets, such as inventory or accounts receivable.

$$CR = \frac{\text{total current assets}}{\text{total current liabilities}}$$

- c. The quick ratio eliminates inventories from the numerator of the CR because inventories are not always easily liquidated.
  - (1) Again, larger is usually better.
  - (2) Too high could mean accounts receivable are too high.

$$QR = \frac{\text{total current assets} - \text{inventory}}{\text{total current liabilities}}$$

## Asset Valuation

- d. The receivables turnover ratio shows how quickly the firm is collecting accounts receivable.
- (1) Measures how many times, on average, the firm collects its accounts receivable in one year.
  - (2) Should be near the industry average.
  - (3) Too low means the firm is not collecting quickly enough, but too high could mean they have set their credit terms too strictly and are losing business.
  - (4) Customers will go to the firm offering the best credit terms.

$$\text{receivables turnover} = \frac{\text{sales}}{\text{average receivables}}$$

- e. The average collection period is another measure of collection of accounts receivable.
- (1) Measures the average number of days it takes the firm to collect an account receivable.
  - (2) Should be near the industry average.
  - (3) Too long (too many days) means the firm is not collecting quickly enough, but too short (too few days) could mean its has set its credit terms too strictly and is losing business.
  - (4) Customers will go to the firm offering the best credit terms.

$$\text{ACP} = \frac{365}{\text{receivables turnover}}$$

- f. The inventory turnover ratio measures how many times inventory is “cycled” during the year.
- (1) How many times the inventory is depleted and refilled.
  - (2) Should be near the industry average.
  - (3) Too high could mean the firm is not holding enough inventory for the level of sales and risks outages.
  - (4) Too low could mean the firm is holding too much inventory.
    - (a) Risk of inventory obsolescence.
    - (b) Too much cash invested (tied up) in inventory.

$$\text{inventory turnover} = \frac{\text{COGS}}{\text{average inventory}}$$

- g. The payables turnover ratio measures how many times during the year accounts payable (trade credit) is paid off and “refilled.”
- (1) Should be near the industry average.
  - (2) Too high could mean the firm is paying too quickly. The firm should take as long as possible without risking good credit rating.
  - (3) Too low could mean the firm is not paying quickly enough.
    - (a) Bad reputation.

- (b) Possible loss of trade credit.

$$\text{payables turnover} = \frac{\text{purchases}}{\text{average trade payables}}$$

- h. The payables payment period measures the average number of days the firm takes to meet trade credit obligations.
- (1) Should be near the industry average.
  - (2) Too short (too few days) could mean the firm is paying too quickly. Should take as long as possible without risking good credit rating.
  - (3) Too long (too many days) could mean the firm is not paying quickly enough.
    - (a) Bad reputation.
    - (b) Possible loss of trade credit.

$$\text{payables payment period} = \frac{365}{\text{payables turnover}}$$

2. Operating performance ratios comprise two categories, efficiency and profitability, that measure how well management is running the firm.

- a. Efficiency ratios

- (1) The total asset turnover ratio measures how efficiently management is utilizing all the firm's assets.
  - (a) Should be near the industry average.
  - (b) Too low could mean the firm is underutilizing its assets.
  - (c) Too many short-term assets and/or too many fixed assets.
  - (d) Too high could mean outdated assets (old and fully depreciated).

$$\text{total asset turnover} = \frac{\text{sales}}{\text{average total assets}}$$

- (2) The fixed asset turnover ratio measures the utilization of just the firm's fixed assets.
  - (a) Should be near the industry average.
  - (b) Too low could mean the firm is underutilizing its fixed assets.
  - (c) Too many fixed assets.
  - (d) Too high could mean outdated assets (old and fully depreciated).

$$\text{fixed asset turnover} = \frac{\text{sales}}{\text{average fixed assets}}$$

## Asset Valuation

### b. Profitability ratios

- (1) Gross profit margin measures the operating profitability of the firm before the effects of all fixed obligations and taxes (i.e., sales less cost of goods sold).

$$\text{gross profit margin} = \frac{\text{gross profit}}{\text{sales}}$$

- (2) Operating profit margin measures the operating profitability of the firm after all operating expenses but before interest and taxes.

$$\text{operating profit margin} = \frac{\text{operating profit}}{\text{sales}} = \frac{\text{EBIT}}{\text{sales}}$$

- (3) Net profit margin measures the profitability of the firm after all expenses (before dividends).

$$\text{net profit margin} = \frac{\text{NI}}{\text{sales}}$$

- (4) Return on equity (ROE) measures the return to common equity capital providers.

$$\text{return on equity} = \frac{\text{net income}}{\text{average shareholders' equity}}$$

### 3. Risk profile

- a. Business risk is the uncertainty regarding the operating income of a company (EBIT) and is a result of the variability of sales and production costs.
- b. Financial risk is the additional volatility of equity returns (ROE) caused by the firm's use of debt.
- c. Business risk is often measured as the coefficient of variation of a company's operating income (EBIT) over several years:

$$\text{business risk} = \frac{\sigma_{\text{EBIT}}}{\text{average EBIT}}$$

- d. Sales volatility is the coefficient of variation of sales over several years:

$$\text{sales volatility} = \frac{\sigma_{\text{sales}}}{\text{average sales}}$$

- e. The debt-equity ratio measures management's use of fixed-cost financing as opposed to equity in financing the firm's assets.

$$\text{debt-equity ratio} = \frac{\text{long-term debt}}{\text{total equity}}$$

- f. The debt/assets ratio, or total debt ratio, is another way of measuring management's use of debt to finance the firm's assets. All debts, including short-term sources such as accounts payable, are included.

$$\text{debt/assets} = \frac{\text{total debt}}{\text{assets}} = \frac{\text{total debt}}{\text{total capital}}$$

- g. The interest coverage ratio also known as the *times-interest-earned ratio*, measures the firm's ability to meet its interest (debt) obligations with operating income (EBIT).

$$\text{interest coverage} = \frac{\text{EBIT}}{\text{interest expense}}$$

#### 4. Growth potential

- a. The sustainable growth rate is the rate of growth (increase in assets) the firm can maintain without having to sell new common stock.

$$g = \text{RR} \times \text{ROE}$$

where:

g = sustainable growth rate

RR = retention rate

ROE = return on (common) equity

- b. The retention rate is the percentage of net income that is held within the business as retained earnings. The dividend payout ratio is the percentage of net income paid out to the common stockholders as dividends. The sum of the two must equal 100%.

$$\text{dividend payout ratio} = \frac{\text{dividends paid}}{\text{net income}}$$

$$\text{retention rate} = (1 - \text{dividend payout ratio})$$

$$= \left( 1 - \frac{\text{dividends}}{\text{net income}} \right)$$

5. The Du Pont System is an approach that can be used to analyze return on equity (ROE) by looking at combinations of various ratios. It uses basic algebra to break down ROE into a function of different ratios, so an analyst can see the impact of leverage, profit margins, and turnover on shareholder returns.

## Asset Valuation

- a. The traditional Du Pont approach starts with ROE:

$$\text{return on equity} = \left( \frac{\text{net income}}{\text{equity}} \right)$$

- b. Through algebraic manipulation we arrive at the Du Pont equation:

$$\text{ROE} = \left( \frac{\text{net income}}{\text{sales}} \right) \left( \frac{\text{sales}}{\text{assets}} \right) \left( \frac{\text{assets}}{\text{equity}} \right)$$

or

$$\text{ROE} = \left( \frac{\text{net profit}}{\text{margin}} \right) \left( \frac{\text{asset}}{\text{turnover}} \right) \left( \frac{\text{equity}}{\text{multiplier}} \right)$$

- c. This is the traditional Du Pont equation. It breaks down a ROE into three key components. If ROE is low, it must be that at least one of the following is true:
- (1) The company has a poor profit margin.
  - (2) The company has poor asset turnover.
  - (3) The company is not properly leveraged.
6. Relative ratio analysis
- a. The value of a single financial ratio is not meaningful by itself. It must be interpreted relative to one of three factors: industry norms, overall economy norms, and the company's own historical (past) performance.
- (1) Comparison to the industry norm (average) is the most common type of comparison. Industry comparisons are particularly valid when the products generated by all the firms in the industry are similar.
  - (2) Comparing a company to the overall economy is particularly important when overall business conditions are changing. For example, a stable profit margin might be considered good if the economy is in recession and the economy-wide average profit margin is declining. On the other hand, it might be considered problematic if a stable profit margin occurs during an economic expansion and overall average profit margins are increasing.
  - (3) Comparing a firm with its history is very common. Typically the analyst will look at the current level of the ratio and then look to see if it has been declining over time, stable over time, or improving over time.

7. Limitations of financial ratios
  - a. Ratios can be very misleading, and the analyst must remember a few rules:
    - (1) Financial ratios are not useful when viewed in isolation. They are only valid when compared to other firms or the companies' historical performance.
    - (2) Comparisons with other companies can be difficult because of different accounting treatments.
    - (3) It is difficult to find comparable industry ratios when analyzing companies that operate in multiple industries.
    - (4) Sound conclusions usually cannot be made from viewing one set of ratios. All ratios must be viewed relative to one another over time.
    - (5) Determining the target or comparison value for a ratio can present problems when the industry is difficult to identify.
  - b. Ratios are used for internal analysis and comparisons and across firms. They are often most useful in identifying questions that need to be answered rather than answering questions directly. In conducting your analysis, you must always be aware of the limitations of ratios.
    - (1) Do the firms being compared have compatible accounting practices?
    - (2) Do the ratios give consistent readings? For example, if the debt ratio is too high, is the interest coverage ratio low?
    - (3) Do the ratios yield a reasonable figure for the industry?

C. Equity valuation

1. Common stock is the best-known equity security.
  - a. Cash flows associated with common stock include dividends and the eventual selling price of the stock (received when the investor sells the stock).
  - b. The intrinsic value for a common stock equals the present value of the cash flows the investor expects to receive.
2. The constant growth dividend discount model assumes dividends will grow at a constant rate forever.
  - a. Using the constant growth model, the intrinsic value (fair price,  $P_t$ ) equals:
 
$$P_t = \frac{D_{t+1}}{k - g}$$
  - b. The constant growth model does not work for a company that is growing at a rate equal to or above the stock's required return.



## Asset Valuation

3. The multi-stage dividend discount model permits the company to grow at different rates over different periods of time.
  - a. Some companies experience an explicit above average growth period.
    - (1) Dividends grow at a rate above the normal industry growth rate.
    - (2) This *supernormal* growth is due to a *temporary* competitive advantage.
    - (3) Each dividend during this period is estimated using an explicit growth rate.
  - b. Following the explicit growth period, the firm will return to its normal growth rate implied by its position in the industry. This is known as the implicit growth period.
  - c. The estimated price of the stock is the present value of all future dividends.
    - (1) We find the present value of all dividends to be received during the explicit growth period.
    - (2) We find the present value of all dividends to be received during the implicit growth period.
    - (3) Price = (Present value of dividends during the explicit growth period) + (Present value of dividends during the implicit or constant growth period).

$$P = \sum_{t=1}^n \frac{D_t}{(1+k)^t} + \frac{\left(\frac{D_{n+1}}{k-g}\right)}{(1+k)^n}$$

4. The earnings multiplier model shows the relationship between the company's P/E ratio and its dividend payout, growth, and required return.
  - a. The earnings multiplier equation is:
$$\frac{P_t}{E_{t+1}} = \frac{D_{t+1}/E_{t+1}}{k-g}$$
  - b. P/Es rise as the company's dividend payout ratio,  $D/E$ , rises, assuming growth prospects and risk are unchanged.
  - c. P/Es rise as the company's growth rate,  $g$ , rises, assuming dividend payout and risk are unchanged.
  - d. P/Es fall as the company's required return,  $k$ , rises, assuming dividend payout and growth remain unchanged.
  - e. P/Es fall as investor risk aversion rises (which cause  $k$  to rise), assuming dividend payout and growth remain unchanged.

- f. Intrinsic value or the fair price,  $P$ , can be calculated by multiplying the fair P/E times predicted earnings:

$$\frac{P}{E} \times E = P$$

5. Preferred stock pays contractual dividends, stated as percentage of face value.
- Preferred stock can be perpetual, which means that it has no maturity.
  - Its price is the present value of all future dividends discounted at its required return.
  - The present value of a perpetuity is the cash flow (preferred dividend) divided by the required return,  $k_p$ .

$$P_p = \frac{D_p}{k_p}$$

## DEBT SECURITIES

- A. Bond features, sectors, and risks
- Bonds are debt securities and represent borrowing by the issuing firm.
    - The indenture contains a legal description and all characteristics of the bond including maturity, coupon dates, any collateral, covenants, et cetera.
    - The trustee monitors the actions of the firm to ensure adherence to all the conditions of the indenture.
    - Bonds typically have maturities ranging from 5 to 30 years.
    - Corporate bonds are typically sold with face values of \$1,000.
      - U.S. corporate bonds usually pay semiannual coupon payments, while Eurobonds usually pay annually.
      - The coupon rate is stated as a percentage of face value.
      - Each semiannual coupon is half of the annual interest payment.
      - The par value (face value) of the bond represents the amount of funds borrowed and gets repaid at maturity.
    - Holder of bonds are not entitled to vote on corporate matters.
    - Bonds can be backed by (secured by) collateral or, in the case of debentures, have no assets directly supporting the issue.
    - Bonds are ranked for payoff in case of bankruptcy.
      - Bonds are ranked by collateral (i.e., mortgage bonds have seniority over debentures).
      - Bonds are ranked by order of seniority (i.e., earlier issues have seniority over newer issues).
    - Covenants prescribe or prohibit certain actions for the issuer.
      - Affirmative covenants dictate what management must do, such as file periodic reports with the trustee.

## Asset Valuation

- (2) Negative covenants describe what management cannot do, such as issue more debt without approval of the trustee.
  - i. Bonds sometimes have a sinking fund.
    - (1) Annual deposits the firm must make with the trustee to ensure sufficient funds are available to retire the issue as scheduled.
    - (2) Serial bonds are often retired from a sinking fund.
  - j. Callable bonds give the firm the opportunity to retire the bond issue early and replace it with another issue at a lower rate of interest.
  - k. Bonds can be convertible (i.e., exchanged at the discretion of the holder for common stock). The conversion ratio describes how many shares of common stock can be obtained by surrendering the bond.
2. The U.S. bond market can be separated into four major segments: Treasury, agency, state and local (or municipal), and corporate.
- a. U.S. Treasury securities (or “Treasuries”) are issued by the U.S. Treasury Department.
    - (1) Treasuries are backed by the full faith and credit of the U.S. government and are considered to be free from risk of default.
    - (2) They are not free from interest rate/price risk.
    - (3) T-bills have a maturity of less than one year and are issued at a discount to their face value.
    - (4) T-notes have maturities ranging from two to ten years and are coupon-bearing securities.
    - (5) T-bonds have maturities of greater than ten years and, like T-notes, are coupon-bearing securities.
    - (6) The Treasury yield curve is constructed by graphing the yields of outstanding Treasury securities of various maturities.
  - b. Agencies of the federal government and some organizations affiliated with the federal government also issue fixed income securities.
    - (1) Federally related institutions are owned by the U.S. government. Their securities are implicitly backed by the full faith and credit of the U.S. government.
    - (2) Government-sponsored entities are privately owned but publicly chartered organizations created by the U.S. Congress. Their securities are not backed by the full faith and credit of the U.S. government.
    - (3) Agency securities vary widely in their design but include debentures as well as mortgage-backed and asset-backed securities.

- c. Debt securities issued by state and local governments in the United States are known as municipal bonds.
  - (1) Interest on munis is generally exempt from federal income taxes, but any capital gains are subject to capital gains tax. The interest is generally taxable at the state level.
  - (2) Munis have higher risk of default than Treasury securities.
  - (3) Munis are issued as general-obligation (backed by tax revenue) or revenue bonds (backed by funds from a public works project).
- d. Public corporations also issue fixed income securities.
  - (1) Corporate debt securities are sold through public and private placements.
  - (2) Publicly sold corporate debt must be registered with the SEC while privately placed corporate debt can go unregistered.
  - (3) Corporate debt can be either secured or unsecured.
  - (4) Maturities for corporate debt vary from less than one year for commercial paper to thirty years or more for corporate bonds. Corporate notes generally have a maturity greater than one year but less than thirty years.
- 3. The risks to bond investors are a function of both macroeconomic and microeconomic factors.
  - a. Interest rate risk (price risk) is the risk that interest rates will increase and the value of an investor's bond holdings will decrease. Generally, the longer the maturity of the bond and the lower the coupon rate, the more sensitive it is to changes in interest rates.
  - b. Reinvestment risk is the risk that cash flows received from a bond holding will have to be reinvested at a lower rate of return than that expected at the time of the original investment. If cash flows are reinvested at rates that are less than the required yield, the actual return will be less than the required yield.
  - c. Credit risk is the risk that the issuer will not be able to pay interest and/or principal when due. Bond rating agencies assess this risk by rating the creditworthiness of borrowers and their bond issues outstanding.
  - d. Liquidity risk is the risk that the investor will not be able to sell the bond quickly at a fair price should the need arise. Liquidity risk can be estimated by looking at the size of the bid-ask spread.
  - e. Inflation risk is the risk that actual inflation will exceed expected inflation, thereby leading to a loss of purchasing power.
  - f. Call (prepayment) risk is the risk the bond will be retired early by the issuer and that the funds received will need to be reinvested at a lower yield.
  - g. Event risk is the risk of an unforeseen event that has a negative impact on the financial condition of the issuer.

## BOND VALUATION AND YIELD MEASURES

- A. To value any security we use the capitalization of income method.
1. Estimate all future cash flows from the security.
  2. Estimate the risk of the cash flows.
  3. Based upon the risk, determine a required return (discount rate).
  4. Find the total present value (today) of the cash flows.
  5. The total present value of all coupons and the face value is our estimate of the value of the bond.
  6. When bonds are purchased between coupon payment dates, the buyer must pay accrued interest to the seller.
  7. Bond prices are sensitive to changes in interest rates (required return).
    - a. When the coupon rate equals the required return, the bond will sell at par (face value).
    - b. When the required return increases (decreases), the price of the bond will fall (rise).
    - c. The longer the maturity of the bond, the greater the price reaction to a given change in required return.
    - d. The greater the coupon rate on the bond, the smaller the price reaction to a given change in required return.
  8. In the bond market, investment decisions are typically made on the basis of a bond's yield rather than its price. A bond's yield affects the price at which it trades and serves as an important measure of potential or expected return.
    - a. The nominal yield is the stated coupon rate on the bond. Nominal yield is only a meaningful measure if the bond is selling at or close to par value.
    - b. The current yield is the annual coupon payment divided by the bond's current price. Current yield is only meaningful if the bond is selling at or close to par value.
    - c. The yield to maturity on a bond is the discount rate that makes the sum of the present values of a bond's cash flows equal to its current market price.
      - (1) YTM considers both interest income and capital gains/losses and also the timing of the cash flows received over the life of an issue.
      - (2) The yield to maturity makes the following three assumptions
        - (a) Interim cash flows can be reinvested at a rate equal to the yield to maturity.
        - (b) The bond is held until its maturity date.
        - (c) All coupons and principal payments are received as expected.
    - d. Not all yields are directly comparable. To compare the yield on an annual pay bond to the yield on a semiannual pay bond, first convert the semiannual bond yield to an annual basis.

## DERIVATIVES

### A. Options

1. A derivative is a security that derives its value from the value or return of another asset or security.
2. An option contract gives its owner the right, but not the legal obligation, to buy or sell an underlying asset on or before a predetermined future date (the exercise date) at a predetermined price (the exercise or strike price).
  - a. The seller of the option has the obligation to perform if the buyer decides to exercise the option.
  - b. The owner of a call option has the right to purchase the underlying stock at a specific price for a specified time period.
  - c. The owner of a put option has the right to sell the underlying stock at a specific price for a specified time period.
3. There are four options positions.
  - a. Long call: the buyer of the call option. Has the right to buy the stock at the specified price.
  - b. Short call: the writer (seller) of the call option. Has the obligation to sell the stock at the specified price if the owner of the option chooses to exercise (buy the asset).
  - c. Long put: the buyer of a put option. Has the right to sell the stock at the specified price.
  - d. Short put: the writer (seller) of a put option. Has the obligation to buy the stock if the owner of the option chooses to exercise (sell the asset).
4. Options can be either American-style or European-style.
  - a. American options may be exercised at any time up to and including the contract's expiration date.
  - b. European options may be exercised only on the contract's expiration date.
  - c. If two options are identical (maturity, underlying stock, strike price, etc.) in all ways except that one is a European option and the other is an American option, the value of the American option will equal or exceed the value of the European option.
  - d. Options contracts generally trade in bundles of 100 options for 100 shares of the underlying stock.
5. Moneyness refers to the relationship between the strike price and the stock price.
  - a. An option is in-the-money when there is a positive payoff associated with the exercise of the option. An option is out-of-the-money when there is a negative payoff. An option is at-the-money when the payoff is equal to zero.
    - (1) A call option's payoff is:  $C_{\text{buyer}} = \text{Max}(0, S - X)$ .
    - (2) A put option's payoff is:  $P_{\text{buyer}} = \text{Max}(0, X - S)$ .

## Asset Valuation

- b. The payoff to the writer of an option is the opposite of the payoff to the buyer of the option.

<i>Option</i>	$S < X$	$S = X$	$S > X$
Call	Out-of-the-money	At-the-money	In-the-money
Put	In-the-money	At-the-money	Out-of-the-money

6. An option's value is equal to its intrinsic value (payoff if exercised immediately) plus its time value (associated with probability of moving into the money).
- Intrinsic value<sub>call</sub> =  $S - X$
  - Intrinsic value<sub>put</sub> =  $X - S$
7. Profit on an option contract is determined by the stock price, the exercise price, and the premium paid for the option.
- Profit<sub>call</sub> =  $(S - X) - C$
  - Profit<sub>put</sub> =  $(X - S) - P$
8. The value of an option is either a positive or negative function of the stock price, the exercise price, its remaining maturity, and the volatility of the underlying stock.

<i>Variable</i>	<i>Call Value</i>	<i>Put Value</i>
Stock price	+	-
Exercise price	-	+
Maturity	+	+
Volatility	+	+

9. Options exist on many different assets, including bonds, bond indexes, stock indexes, foreign currencies, interest rates, commodities, futures, swaps, and even other options.

### B. Forwards and futures

- In spot markets, securities, commodities, and other goods are traded for immediate delivery.
- In forward markets, traders agree to transact at a future date at a specified price.
- A forward contract is privately negotiated and obliges one party to buy and the other to sell a specific quantity of an asset, at a set price, on a specific date in the future.
- There are two primary reasons to enter into a forward contract:
  - Speculation.
  - Hedging risk.
- The party agreeing to purchase the asset in the future is referred to as the long. The party agreeing to sell the asset in the future is referred to as the short.
- Parties to a forward contract may agree to a cash settlement rather than

- a delivery of the underlying product/good.
7. Payoffs to a forward contract are *symmetric*.
  8. Futures contracts are similar to forwards in that both are:
    - a. Similar in the terminology used to distinguish the buyer and the seller.
    - b. Deliverable contracts that obligate the long to buy and the short to sell a certain quantity of an asset for a certain price on a specified future date.
    - c. Cash settlement contracts that are settled by paying the contract value in cash on the expiration date.
  9. Futures contracts differ from forward contracts in the following ways:
    - a. Futures contracts trade on organized exchanges and require the payment of a margin (deposit) at inception. Forwards are private contracts that usually do not trade and rarely require a margin payment.
    - b. Whereas forwards are customized contracts satisfying the unique needs of the parties involved, futures contracts are highly standardized. Uniformity due to standardization promotes liquidity.
    - c. Futures contracts have a daily price change limit, which sets the maximum price movement allowed in a single day.
    - d. A single clearinghouse is the counterparty to all futures contracts. Forwards are contracts with the originating counterparty.
    - e. The government regulates futures markets. Forward contracts are usually not regulated.
  10. Each futures exchange has a clearinghouse.
    - a. The clearinghouse guarantees the transaction (removes default risk) by splitting each trade and acting as the intermediary for the trade.
    - b. The clearinghouse makes sure each side of the contract fulfills the terms of the agreement.
  11. Marking to market is the process of adjusting the margin balance in a futures account each day for the change in the value of the contract from the previous trading day, based on that day's settlement price.
    - a. Settling futures contracts through marking to market everyday means profits and losses are calculated and collected/paid daily.
    - b. Profits to one party are losses to the opposite party.
  12. The concept of convergence is very important to futures contracts. As the maturity date approaches, the futures and spot prices must *converge* (i.e., come together and become equal).
  13. There are four ways to satisfy your commitment under a futures contract:
    - a. A short can satisfy the contract by delivering the goods, a long by accepting delivery and paying the contract price to the short. This is called delivery. Deliveries represent less than one percent of all settlements.



## Asset Valuation

- b. In a cash-settlement contract, delivery is not an option. The futures account is marked to market based on the settlement price on the last day of trading.
  - c. A position can be closed by making a close-out or offsetting trade in the futures market. This is how most futures positions are settled.
  - d. A position may also be settled through an exchange for physicals in which a trader finds another trader with an opposite position to his own, delivers the goods, and settles up off the floor of the exchange.
14. Futures payoffs are calculated in the same way as forwards payoffs.
15. Forwards and futures contracts exist on many different assets, including commodities, stocks, bonds, interest rates, and foreign currencies.

**PRACTICE QUESTIONS: ASSET VALUATION**

1. At the end of 20X7, PriceCo, a low cost retailer, had an inventory balance of \$2,300 and an inventory turnover ratio for the year of 1.9. PriceCo had a receivables turnover ratio of 2.3 for 20X7 based on sales of \$7,200 for the year. If in 2006 cost of goods sold (COGS) was \$4,720, and accounts receivable (AR) was \$2,930, are COGS and AR increasing or decreasing, respectively?

	<u>COGS</u>	<u>AR</u>
A. Increasing		Increasing
B. Decreasing		Increasing
C. Decreasing		Decreasing
D. Increasing		Decreasing

Use the following table to answer Questions 2 through 4.

	<i>FRD Co.</i>	<i>Spencer</i>	<i>Henley Inc.</i>	<i>Worth</i>
Total liabilities	\$3,000	\$5,000	\$3,000	\$10,000
Owners equity	\$10,000	\$10,000	\$4,000	\$20,000
Total assets	\$13,000	\$15,000	\$7,000	\$30,000
Revenues	\$20,000	\$30,000	\$40,000	\$50,000
Net income	\$2,000	\$3,000	\$1,000	\$5,000
Operating expenses	\$6,000	\$4,000	\$28,000	\$10,000
COGS	\$10,000	\$20,000	\$10,000	\$30,000

2. Which of the companies in the table has the greatest degree of financial leverage?
- A. FRD Co.  
 B. Spencer.  
 C. Henley Inc.  
 D. Worth.
3. Which of the companies in the table has the highest return on equity?
- A. FRD Co.  
 B. Spencer.  
 C. Henley Inc.  
 D. Worth.
4. Which of the companies in the table has the highest gross profit margin?
- A. FRD Co.  
 B. Spencer.  
 C. Henley Inc.  
 D. Worth.

## Asset Valuation

5. Higgins & Company manufactures replacement vacuum cleaner parts for the U.S. and Canadian markets. The company has been able to sustain a high return on equity for each of the past ten years despite a steady decline in profitability. Which of the following scenarios is a possible explanation as to how Higgins & Company has been able to consistently provide a high return to equity shareholders over time?
- A. The company has slowly decreased its interest expense over ten years through a reduction in debt in its capital structure and has also maintained a constant level of asset utilization.
  - B. The first five years of the decade were marked with increases in efficient asset utilization, while the last five years relied on increasing amounts of debt financing.
  - C. The company has decreased the relative proportion of fixed assets on its balance sheet over the ten-year period through asset sales and has used the proceeds to decrease its reliance on debt financing.
  - D. In the first five years, the company decreased the proportion of equity financing on the balance sheet and in the following five years, increased the proportion of equity financing.
6. Janet Walker is evaluating several companies for a possible equity investment. Walker has developed a model which predicts that the best investment opportunities are within companies that have an ROE better than the industry average, a lower-than-average level of leverage, and a higher-than-average asset turnover ratio. Data on the companies that Walker is evaluating and the corresponding industry data are presented in the following table:

	<i>Rogers Inc.</i>	<i>QuickCo</i>	<i>Blair Inc.</i>	<i>Dyna Corp.</i>	<i>Industry Average</i>
Assets	100	151	200	250	162
Sales	183	273	297	372	282
Debt	31	30	64	82	50
Net income	13	22	24	23	20

Using Walker's criteria, which company should she choose for an investment?

- A. QuickCo.
- B. Dyna Corp.
- C. Blair Inc.
- D. Rogers Inc.

Use the following information to answer Questions 7 through 12.

The following income and balance sheet information is for Vincent Corporation. All financial statement numbers are in \$ millions (except per-share data).

<i>Income Statement</i>	<i>20X6</i>	<i>20X7</i>
Revenues	\$4,950	\$5,140
Cost of goods sold	2,420	2,540
Operating expenses	1,660	1,770
Interest expense	30	25
Income taxes	308	295

<i>Balance Sheet</i>	<i>20X6</i>	<i>20X7</i>
Cash	\$380	\$400
Accounts receivable	670	700
Inventories	610	600
Net property, plant and equip.	1,350	1,400
Current liabilities	590	600
Long-term debt	574	300
Stockholders' equity	1,846	2,200
Shares outstanding	260	260
Annual dividend per share	\$0.58	\$0.60
Stock price per share	\$28.00	\$30.00

7. Using year-end balance sheet values, is Vincent Corporation increasing or decreasing the profitability of equity and assets, respectively, from 20X6 to 20X7?

- | <u>Equity</u> | <u>Assets</u> |
|---------------|---------------|
| A. Decreasing | Increasing    |
| B. Increasing | Decreasing    |
| C. Increasing | Increasing    |
| D. Decreasing | Decreasing    |

8. Vincent Corporation's operating profit margin in 20X7 is:

- A. between 5% and 8%.
- B. between 8% and 11%.
- C. between 11% and 14%.
- D. between 14% and 17%.

## Asset Valuation

9. From 20X6 to 20X7, how has Vincent Corporation's debt ratio and interest coverage ratio changed?
- | <u>Debt ratio</u> | <u>Interest coverage</u> |
|-------------------|--------------------------|
| A. Increased      | Decreased                |
| B. Decreased      | Increased                |
| C. Increased      | Increased                |
| D. Decreased      | Decreased                |
10. What is Vincent Corporation's retention rate for the year 2006?
- A. Between 40% and 50%.  
B. Between 50% and 60%.  
C. Between 60% and 70%.  
D. Between 70% and 80%.
11. In 20X7, Vincent Corporation had a P/E ratio between:
- A. 4 and 8.  
B. 8 and 12.  
C. 12 and 16.  
D. 16 and 20.
12. What was Vincent Corporation's sustainable growth rate in 20X6 and 20X7, respectively?
- | <u>20X6</u> | <u>20X7</u> |
|-------------|-------------|
| A. 20.7%    | 16.1%       |
| B. 8.2%     | 16.1%       |
| C. 20.7%    | 7.1%        |
| D. 8.2%     | 7.1%        |
13. An analyst has gathered the following information about a firm:
- The last dividend was \$3.00.
  - The required return on equity is 15%.
  - Its assets, earnings, and dividends are expected to grow 5% per year indefinitely.
- The firm's stock should sell at a market price of:
- A. \$20.00.  
B. \$30.00.  
C. \$31.50.  
D. \$34.50.

14. Karen Bennet has estimated dividend growth rates for New England Fisheries Inc. (NEFI). She believes that the mature company has been experiencing abnormally low growth rates for the past two years but will return to the normal long-term growth rate. Bennet has forecasted that the current dividend of \$1.25 will grow at a rate of 2% in the coming year. Bennet then expects the growth rate to increase by 1% per year until reaching the long-term growth rate of 5%. Using Bennet's forecast, what should be the current market price of NEFI's stock assuming investors require a 12% rate of return?
- \$17.05.
  - \$17.74.
  - \$17.93.
  - \$18.02.
15. Which of the following statements about the dividend discount model is TRUE?
- Supernormal growth rates normally last 15 to 20 years.
  - If a stock's required rate of return is 12% and its dividend growth rate is a constant 5%, the formula implies that the stock's expected dividend yield is 5%.
  - The price of a stock is the present value of all expected future dividends, discounted at the dividend growth rate.
  - If the expected rate of return on a stock exceeds the required rate return, the stock is considered a good buy.
16. Which of the following is FALSE regarding the use of the constant growth dividend discount model?
- The growth rate must be greater than the required rate of return.
  - The dividend stream is assumed to be known with certainty.
  - The required rate of return must be greater than the growth rate.
  - The growth rate can be positive or negative.
17. Victory Company just reported a net income of \$10 per share. The company has a stated policy of retaining 70% of its earnings. If Victory Company's return on equity is 20%, calculate next year's *expected* dividend per share.
- \$2.00.
  - \$3.00.
  - \$3.42.
  - \$7.98.

## Asset Valuation

18. Texas Oil Company just reported earnings of \$11.00 per share, giving the company book value of \$91.50 per share. The required return on the stock is 11%. Texas Oil Company's payout ratio is 40%. Assuming dividends will grow at a constant rate forever, the company's intrinsic value is *closest to*:
- A. \$100.
  - B. \$116.
  - C. \$124.
  - D. \$133.
19. BIG Company has a new technology that allows the company to produce more advanced labeling equipment. The new product will generate overall company growth of 25% per year for the next three years, at which time the company's growth rate is expected to return to its historical 10% level. The company just reported earnings of \$8.00 per share, and its book value is \$40.00 per share. Given the company's new technology and other factors, you calculate a discount rate of 16%. BIG Company retains 65% of its earnings to support its capital-spending program. The company's intrinsic value using the supernormal form of the dividend discount model is *closest to*:
- A. \$40 per share.
  - B. \$59 per share.
  - C. \$74 per share.
  - D. \$81 per share.
20. Which of the following will cause the fair P/E to rise?
- A. Dividend payout falls.
  - B. Company risk rises.
  - C. Investor risk aversion rises.
  - D. Company growth rises.
21. Spiel Corp. has just announced it will issue \$100 million in debenture bonds in the public debt market. Which of the following pieces of information would not be included in the bond's indenture?
- A. The bonds will be subordinated to a previously outstanding bond issue.
  - B. Spiel Corp.'s manufacturing facilities will serve as collateral for the issue.
  - C. Interest on the bond will be paid semiannually based on a floating rate of interest.
  - D. Spiel Corp. will be prohibited from making any further debt issuances if its interest coverage ratio falls below 1.5.

22. Bair Enterprises has just announced that the firm will be liquidating its assets as a result of its bankruptcy status. Which of the following bondholders is least likely to receive any payment from the liquidation?
- First mortgage bonds.
  - Subordinated debentures.
  - Subordinated mortgage bonds.
  - Senior debentures.
23. Jefferson Inc. is about to issue \$10 million worth of debentures to the public. Its underwriters have informed them, however, that to achieve a reasonable coupon rate on the bond, the firm must include negative covenants in the indenture. Which of the following covenants suggested by the firm is not a negative covenant?
- The firm must periodically provide reports to the trustee.
  - The firm's level of indebtedness must not increase.
  - The firm must gain bondholder approval before declaring stock dividend.
  - The firm cannot acquire other businesses while the bonds are outstanding.
24. James Turner is analyzing bonds issued by two separate companies, HCG Corporation and Riviera Trading Company (RTC). Both companies have similar operations, balance sheets, and profitability, and the bonds are trading at par. The only difference between their bond issuances is the presence of a sinking fund in RTC's bonds. Based on the preceding information, which of the following statements is TRUE?
- The RTC bond should carry a higher coupon rate than the HCG bond.
  - HCG's bonds will have greater interest rate risk than RTC's bonds.
  - The likelihood of principal repayment will be greater for RTC's bonds.
  - The market for RTC's bonds will be more liquid than for RTC's bonds.
25. When corporate bonds are callable, which of the following statements would be TRUE?
- They may be converted to common stock at the holder's discretion.
  - They may be exchanged for another bond at the owner's discretion.
  - They can be retired early by management.
  - They cannot be held until maturity.



## Asset Valuation

26. GigantiCo is a multinational mining corporation that relies heavily on debt financing to maintain and expand its operations. Yesterday, the company issued \$520 million in serial bonds that will be retired in equal amounts over a period of twenty years. The bonds have a coupon rate of 4.5%. Using the preceding information, which of the following is CORRECT?
- A. GigantiCo will have outstanding principal at the end of year 7 equal to \$338 million and will have paid \$16.38 million in interest for the year.
  - B. GigantiCo will have outstanding principal at the end of year 7 equal to \$312 million and will have paid \$15.21 million in interest for the year.
  - C. GigantiCo will have outstanding principal at the end of year 7 equal to \$338 million and will have paid \$15.21 million in interest for the year.
  - D. GigantiCo will have outstanding principal at the end of year 7 equal to \$312 million and will have paid \$16.38 million in interest for the year.
27. Jesse Smith is considering investing in a semiannual coupon bond with a par value of \$2,000, a coupon rate of 6.5%, and a maturity of 20 years. What is the dollar amount of each coupon payment and the total amount of interest that will be paid to Smith if she invests in this bond?
- | <u>Coupon</u> | <u>Total interest</u> |
|---------------|-----------------------|
| A. \$65       | \$1,300               |
| B. \$130      | \$1,300               |
| C. \$130      | \$2,600               |
| D. \$65       | \$2,600               |
28. Charles Mendleson, the Chief Financial Officer of CBX Autoworks, is considering issuing \$275 million of subordinated debentures to fund a major capital spending project. Mendleson believes that current market interest rates will remain stable over the next three to five years but will then start declining rapidly. Which of the following bonds should Mendleson issue?
- A. Serial bonds.
  - B. Callable bonds.
  - C. Convertible bonds.
  - D. Non-callable fixed-rate bonds.

29. Mueller Corporation issued convertible bonds three years ago to raise funds for an acquisition. The convertible bonds were issued at par value with a conversion ratio of 15 to 1, a coupon rate of 5%, and a maturity of ten years. Mueller's convertible bonds currently sell for \$993, and the company's common stock has a current price of \$53 per share. Given the preceding information, which of the following statements regarding Mueller's bonds is FALSE?
- A. An otherwise comparable non-convertible bond issued by Mueller would have a coupon rate of 7.5%.
  - B. By issuing a convertible bond, Mueller has effectively purchased a call option from the investor.
  - C. If an investor purchased Mueller's convertible bonds and converted immediately, he would effectively pay \$66 per share.
  - D. Purchasers of Mueller's convertible bonds pay for the option embedded in the bonds by accepting a lower coupon rate.
30. Harwood Enterprises, a large retailer of pet supplies, is about to call its 9.5% semiannual 20-year callable bonds. The next coupon date is in 85 days. The current market price is \$1,115. The call price is \$1,050. What is the amount that investors holding the callable bonds will receive?
- A. \$1,115.
  - B. \$1,050.
  - C. \$1,075.
  - D. \$1,140.
31. Which of the following fixed income securities are explicitly backed by the full faith and credit of the U.S. government?
- A. 15-year, 5% coupon mortgage-backed federally related institution security.
  - B. 20-year, 6% coupon Treasury bonds.
  - C. 10-year, 9% coupon corporate bond.
  - D. 8-year, 6% coupon government sponsored entity debenture.
32. Jason Hall is a debt analyst for a small portfolio management firm in the United States. Hall has recently observed a change in the forecasted Treasury yield curve relative to the current yield curve. The forecasted yield curve reflects consensus expectations among securities analysts as to the level of interest rates of various maturities one year from now. The current and forecasted yield curves are presented in the following table.

<i>Treasury Yields</i>	<i>Maturity (Years)</i>					
	1	2	5	10	20	30
Current	1.0%	1.8%	2.4%	3.7%	5.1%	5.9%
Forecasted	1.5%	1.9%	2.7%	3.7%	4.0%	4.2%

## Asset Valuation

- Which of the following best describes the expectations reflected in the forecasted Treasury yield curve?
- A. In one year, investors will be less uncertain about holding long-term debt securities relative to short-term debt securities.
  - B. In one year, investors will be more uncertain about holding long-term debt securities relative to short-term debt securities.
  - C. In one year, there will be no change in uncertainty about holding long-term debt securities relative to short-term debt securities.
  - D. In one year, investors will be less unwilling to hold long-term debt securities.
33. The city of Phoenix is about to raise \$400 million by issuing long-term debt securities. An analyst for Hughes Capital Inc., Gail Roberts, is asked to write an investment summary of Phoenix's new debt issue. Which of Robert's statements regarding Phoenix's municipal bond issues is NOT specific to this type of security?
- A. The debt issue will contain a sinking fund.
  - B. Interest income on the bond is exempt from federal taxes.
  - C. Interest will be paid based on the authority of the city of Phoenix to levy taxes.
  - D. The bonds' pre-tax yield will be lower than that of an otherwise comparable Treasury bond.
34. Technology Inc. is a relatively small high-tech firm with little fixed assets and a mediocre credit rating. The company has decided to raise capital for expansion by issuing debt. Which of the following forms of corporate debt can Technology Inc. not issue?
- A. Long-term debentures.
  - B. Commercial paper.
  - C. Asset-backed securities backed with receivables.
  - D. Medium-term notes.
35. Hank Simpson is considering making an investment in fixed income securities. He is concerned, however, that the bonds he is considering will either have too much price sensitivity to changes in interest rates or will experience a widening of their respective bid/ask spreads. What two risks is Simpson concerned about?
- A. Interest rate risk and credit risk.
  - B. Reinvestment risk and liquidity risk.
  - C. Price risk and liquidity risk.
  - D. Reinvestment risk and event risk.

36. All of the following are potential risks of investing in bonds EXCEPT:
- A. loss of purchasing power as a result of higher-than-expected inflation.
  - B. issuer receives an unexpected upgrade in his credit rating.
  - C. early retirement of the bond following a decline in interest rates.
  - D. lower-than-expected reinvestment rates.
37. Consider a bond issued by Grift Corporation. The bond has a semiannual coupon payment of 5.0%, a 10-year maturity, and a par value of \$1,000. If yields on comparable default-free bonds rise by 1.0% but Grift's credit rating remains unchanged, all of the following are likely to occur EXCEPT:
- A. the bond's risk premium will decrease.
  - B. reinvestment income will increase.
  - C. the price of the bond will decrease.
  - D. the present value of the coupon payments will decrease.
38. Which of the following represents the value of a bond with a \$1,000 face value, ten years to maturity, a 10% coupon rate (semiannual coupons), and a required return of 10%?
- A. \$587.
  - B. \$924.
  - C. \$1,000.
  - D. \$1,065.
39. If an investor purchased only the principal of a bond with a \$1,000 face value, a coupon rate of 6% (semiannual coupons), ten years to maturity, and a required return of 8%, he would pay:
- A. \$408.
  - B. \$456.
  - C. \$864.
  - D. \$1,094.
40. Which of the following represents the value of a bond with a \$1,000 face value, ten years to maturity, a 10% coupon rate (semiannual coupons), and a required return of 12%?
- A. \$885.
  - B. \$984.
  - C. \$1,000.
  - D. \$1,125.

## Asset Valuation

41. Information on six different corporate bonds is given in the following table.

	<i>SFG Corp.</i>	<i>Alton Inc.</i>	<i>QED Inc.</i>	<i>Bell Corp.</i>	<i>Weld Corp.</i>	<i>MDB Inc.</i>
Par value	\$1,000	\$2,000	\$1,000	\$1,000	\$2,000	\$1,000
Coupon rate	7.2%	6.5%	5.9%	4.6%	7.7%	8.0%
Maturity (years)	10	15	5	8	15	20
Yield to maturity	7.2%	6.0%	6.5%	7.0%	7.2%	8.5%

Based on the information in the table, which bonds sell in the market at a premium, and which bonds have a current yield greater than their coupon rate?

- |              | <u>Premium</u> | <u>Current yield</u> |
|--------------|----------------|----------------------|
| A. MDB Inc.  |                | Alton Corp.          |
| B. Weld Inc. |                | QED Inc.             |
| C. Weld Inc. |                | Alton Corp.          |
| D. MBD Inc.  |                | QED Inc.             |
42. Consider four bonds issued by Bear Corporation, Servers Inc., OldCo, and Waldon LLC. Each of the bonds is approximately the same credit quality, and each has a coupon rate of 3.5%. The maturity of the bonds differs, however, with maturities of 5, 7, 9, and 12 years for Bear Corporation, Servers Inc., OldCo, and Waldon LLC, respectively. Which of the four bonds will be most affected by a 1% increase in interest rates?
- A. Waldon LLC.  
B. Servers Inc.  
C. Bear Corporation.  
D. OldCo.
43. Julia Nickel is considering investing in a 10-year, 4.0% semiannual corporate bond issued by BCS Corp. with a credit rating of AA. Nickel's brother, however, has suggested that she invest in a comparable 10-year, 6.0% semiannual corporate bond issued by FWX Corp. with a credit rating of AA. Which of the following will be true of the BCS Corp. bond relative to the FWX Corp. bond?
- A. As interest rates rise, the BCS Corp. bond will decrease by a greater amount than the FWX Corp. bond.  
B. As interest rates fall, the BCS Corp. bond will decrease by a greater amount than the FWX Corp. bond.  
C. As interest rates rise, the BCS Corp. bond will decrease by a smaller amount than the FWX Corp. bond.  
D. As interest rates fall, the BCS Corp. bond will decrease by a smaller amount than the FWX Corp. bond.

44. Larry is analyzing a discount bond currently trading for \$615. The bond will mature in ten years with proceeds of \$1,000. Assuming annual compounding, what is the required rate of return on the bond?
- A. 4%.
  - B. 5%.
  - C. 6%.
  - D. 7%.
45. Larry is evaluating a bond issued by Burke Corp and a bond issued by Ethan Corp. Both bonds are selling at par value. Each bond pays a 7% coupon rate. The Burke bond will mature in five years, while the Ethan bond matures in eight years. If each bond's yield increases from 8% to 9%, which one of the following statements is TRUE?
- A. The value of the Burke bond will decrease more than the Ethan bond.
  - B. The value of the Ethan bond will decrease more than the Burke bond.
  - C. The value of the Burke bond will increase more than the Ethan bond.
  - D. The value of the Ethan bond will increase more than the Burke bond.
46. United States Steel has a bond outstanding with a 6% coupon paid semi-annually and a maturity of 10 years. If the bond's yield is 5%, the price of the bond is closest to:
- A. \$956.
  - B. \$1,000.
  - C. \$1,078.
  - D. \$1,153.
47. A bond issued by StillWater Corporation carries a coupon rate of 8%, paid semiannually, and a yield to maturity of 6.5%. StillWater's bond has 15 years remaining until the final maturity date and a par value of \$1,000. An analyst with BondInvest Inc. has estimated the clean price of the StillWater bond to be \$1,142. Which of the following statements regarding StillWater's bond is INCORRECT?
- A. If yields rise by approximately 1.5%, StillWater's bond will trade at its par value of \$1,000, plus accrued interest.
  - B. A Treasury bond with 15 years remaining to maturity and a coupon rate of 8% will have a lower yield to maturity than StillWater's.
  - C. The analyst's model makes the implicit assumption that the valuation date is also a coupon date.
  - D. An investor looking to sell his StillWater bond will need to pay accrued interest to the buyer of the bond.

## Asset Valuation

48. Which of the following bonds will have its value most affected by a 1% increase in interest rates?
- A. Bond A with a 6% coupon and 10-year maturity.
  - B. Bond B with a 6% coupon and 20-year maturity.
  - C. Bond C with an 8% coupon and 10-year maturity.
  - D. Bond D with an 8% coupon and 20-year maturity.

Use the following information to answer Questions 49 through 51:

Brighton Company has two bond issues outstanding the details of which are as follows:

	<i>Bond Series</i>	
	<i>A</i>	<i>B</i>
Maturity (years remaining)	10	15
Coupon rate	6.5%	7.0%
Coupon frequency	Semiannual	Semiannual
Face value	\$1,000	\$1,000
Market price	\$964.47	\$913.54

49. Calculate the current yield for each bond series.
- |    | <u>Series A</u> | <u>Series B</u> |
|----|-----------------|-----------------|
| A. | 6.7%            | 7.0%            |
| B. | 6.5%            | 7.0%            |
| C. | 6.7%            | 7.7%            |
| D. | 6.5%            | 7.7%            |
50. Calculate the yield to maturity for each of Brighton Company's bond series.
- |    | <u>Series A</u> | <u>Series B</u> |
|----|-----------------|-----------------|
| A. | 6.5%            | 7.0%            |
| B. | 7.0%            | 8.0%            |
| C. | 7.0%            | 7.0%            |
| D. | 6.5%            | 8.0%            |
51. Which of the following statements regarding Brighton Company's bonds is FALSE?
- A. Nominal yield on B > YTM on B.
  - B. YTM on A < YTM on B.
  - C. Nominal yield on A < YTM on A.
  - D. YTM on B > Current yield on B.

52. Sheldon Wills is considering investing in one of four bonds. He wants to choose the investment offering the highest return. The details of each bond are listed in the following table:

	<i>Bond</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Maturity (years remaining)	20	20	20	20
Coupon rate	6.5%	7.0%	7.5%	8.0%
Coupon frequency	Annual	Semiannual	Semiannual	Annual
Face value	\$1,000	\$1,000	\$1,000	\$1,000
Market price	\$853	\$1,056	\$960	\$1,051
YTM	8.0%	6.5%	7.9%	7.5%

Which bond should Wills choose?

- A. Bond A.  
 B. Bond B.  
 C. Bond C.  
 D. Bond D.
53. Which of the following is not an assumption of the yield to maturity measure of a bond's return?
- A. The investor will be able to keep the bond in her portfolio until the final maturity date.  
 B. Interim cash flows will be reinvested in securities with returns equal to the YTM.  
 C. The bond's cash flows are received at the time and in the amount expected.  
 D. The price of the bond will increase over the life of the bond.
54. Martha Hibbert holds a long option on Gill Company, a national microchip manufacturer, with an exercise price of \$45. Hibbert can exercise the option immediately for a profit of \$5.10. Alternatively, Hibbert can wait to see if the stock price and the profit on the option increase further. What type of option does Hibbert hold?
- A. European call option.  
 B. American call option.  
 C. European put option.  
 D. American put option.



## Asset Valuation

55. A put option on Boeing Company has an exercise price of \$75. The current stock price of Boeing Company is \$63 per share. The put option is:
- A. in-the-money.
  - B. at-the-money.
  - C. out-of-the-money.
  - D. too volatile to determine.
56. Phil Harmony is an options trader for an investment management firm. Harmony recently wrote a European put option on CBA stock. The option expires in 90 days, has a strike price of \$25, and has a premium of \$2.91 per option. The price of CBA common stock is currently \$23. Which of the following statements regarding the option on CBA stock is FALSE?
- A. Harmony has written an option with positive time value.
  - B. The buyer of the option is obligated to sell CBA stock to Harmony on the option's expiration date.
  - C. The intrinsic value of the option is greater than its time value.
  - D. Harmony has the obligation to buy CBA stock if the buyer of the option chooses to exercise.
57. Jim Mitchell is certain that the price of Fashion Co. stock will decrease by at least \$8 in the near future. The stock is currently selling in the marketplace for \$35 per share. Which of the following option strategies is the most straightforward way for Jim to take advantage of the situation?
- A. Short put option with strike price of \$35.
  - B. Long put option with strike price of \$30.
  - C. Short call option with strike price of \$35.
  - D. Long call option with strike price of \$30.
58. Six months ago, Samantha Marquez bought a European put option on the common stock of Vegas Holding Company (VHC) for a premium of \$4. The put option has a strike price of \$25. The price of VHC at the time of the option purchase was \$33. The current price of VHC stock is \$23. What is the intrinsic value at the time of purchase and the current profit, assuming the option can be exercised immediately?
- |    | <u>Intrinsic value</u> | <u>Current profit</u> |
|----|------------------------|-----------------------|
| A. | \$0                    | \$2                   |
| B. | \$4                    | -\$2                  |
| C. | \$4                    | \$2                   |
| D. | \$0                    | -\$2                  |

59. A European put option exists on a broad market index. The strike price of the option is \$150, and the current market price of the underlying index is \$148. The expiration date for the option is June 15th. The value of the option will increase if all of the following occur EXCEPT:
- the price of the index decreases by 10%.
  - exercise price changes to \$148.
  - the expiration date becomes July 15th.
  - option gets changed to an American-style call option.
60. Which of the following option positions has the highest level of risk associated with the position?
- Long call.
  - Short put.
  - Short call.
  - Long put.
61. The stock of CSB Corporation is selling in the market for \$23 per share. Currently an investor can obtain a call option on CSB with a strike price of \$20 for a premium of \$6.40. A put option is also available on CSB with a strike price of \$25 and a premium of \$5.30. What are the intrinsic value of the call option and the time value of the put option?
- |    | <u>Call (intrinsic)</u> | <u>Put (time)</u> |
|----|-------------------------|-------------------|
| A. | \$3.00                  | \$3.30            |
| B. | \$3.40                  | \$3.30            |
| C. | \$3.40                  | \$2.00            |
| D. | \$3.00                  | \$2.00            |
62. A soybean farmer has entered into a futures contract to hedge his exposure to soybean prices. The futures price on the farmer's three month contract at the time of contract initiation is \$45 per bushel. The next day the price of soybeans jumps to \$50 per bushel. Which of the following statements accurately describes the soybean farmer's position at the start of the second day after the contract is initiated?
- The farmer owes \$5 to the counterparty.
  - The farmer is owed \$5 from the counterparty.
  - The farmer is owed \$50 from the counterparty.
  - The farmer does not owe anything.

## Asset Valuation

63. The following table presents the price of a futures contract on pork bellies over time as well as the spot price of pork bellies over time.

	<i>Spot &amp; Futures Prices of Pork Bellies</i>				
	Jan 15th	Feb 15th	Mar 15th	April 15th	May 15th
Spot Price	85	86	87	88	89
Futures Price	93	92	91	90	89

- What is the futures property demonstrated by the table?
- A. Mark to market.
  - B. Clearing.
  - C. Settlement.
  - D. Convergence.
64. An investor weighing the advantages and disadvantages of hedging his interest rate risk using forwards or futures on interest rates has decided to enter into a forward contract on the 90-day T-bill rate. All of the following are potential reasons for the investor's decision EXCEPT:
- A. The investor's contract needs did not coincide with a standardized expiration date.
  - B. The investor was able to disregard increasing exposure to credit risk as a result of the hedge.
  - C. The investor needed a high degree of liquidity in case the hedge suddenly needed to be reversed.
  - D. The amount of the asset to be hedged was significantly different than a standardized contract amount.
65. Michelle, a wheat farmer, believes that the price of wheat will fall significantly in the future. She is convinced the price decline will happen before she is ready to sell her harvest. George, a speculator, thinks that the price of wheat will rise in the future. He is convinced that the price increase will occur before the time of the wheat harvest. What futures position should Michelle and George, respectively, enter into?
- | <u>Michelle</u>  | <u>George</u> |
|------------------|---------------|
| A. Long futures  | Long futures  |
| B. Short futures | Short futures |
| C. Long futures  | Short futures |
| D. Short futures | Long futures  |

66. Nine months ago, Jeffrey Jones took a short position in a gold forward contract. The terms of the contract specified that the contract price (futures price) would be \$830 per ounce based on a contract size of 1,500 ounces. The tenor of the contract was to be nine months. If the current price of gold is \$845 per ounce and assuming cash settlement, what is the payoff and to what party is payment owed?
- A. The counterparty owes Jones \$22,500.
  - B. Jones owes the counterparty \$1,500.
  - C. Jones owes the counterparty \$22,500.
  - D. The counterparty owes Jones \$1,500.

Use the following information to answer Questions 67 and 68.

Bill Michaels has entered into a long futures contract on soybeans. The contract terms are as follows: the tenor is three months; the contract price is \$35; the contract size is 2,000 bushels; and the tick size is one cent. Michaels must deposit margin in the amount of \$700. The soybean spot price moves down three ticks on the first day after Michaels enters into the contract, moves down another two ticks on the second day, moves up three ticks on the third day, and moves down five ticks on the fourth day.

67. What is the margin account balance after the second day?
- A. \$660.
  - B. \$560.
  - C. \$640.
  - D. \$600.
68. What is the margin account balance after the fourth day?
- A. \$600.
  - B. \$660.
  - C. \$560.
  - D. \$640.

## Asset Valuation

69. Tim Matthews is short a futures contract for oats that will expire tomorrow. The price stated in his contract is \$54 per bushel. Matthews expects the spot price of oats will close tomorrow at a price of \$56 per bushel. If his price prediction is correct, Matthews can satisfy his commitment in the futures contract in which of the following ways?
- A. Settle the contract in cash by receiving a payment in the amount of \$2 per bushel.
  - B. Negotiate privately with a long futures holder to accept delivery of the contracted bushels of oats.
  - C. Deliver the contracted bushels of oats to the specified location, and accept payment of \$56 per bushel.
  - D. Enter into an offsetting short futures contract with a futures price equal to the spot price.
70. Jason King recently gave a presentation on the similarities and differences between options, forwards, and futures. During his presentation, King made four statements regarding the features common to all three types of derivatives. Which of his following statements CORRECTLY summarizes a feature common to options, forwards, and futures?
- A. Options, forwards, and futures positions can all be reversed quickly to eliminate the contract obligation.
  - B. Options, forwards, and futures all require both parties to the contract to transact at a future date.
  - C. Options, forwards, and futures all have payoff structures in which the gain to one party is a loss to the counterparty.
  - D. Options, forwards, and futures all have cumbersome regulations imposed by government regulatory bodies.

**PRACTICE QUESTION ANSWERS: ASSET VALUATION**

1. **B** To know if COGS and AR are increasing or decreasing over time, we must know what their values were in 20X7 (2006 is stated already in the question). We are given enough information to calculate the values indirectly as follows:

$$\text{inv turnover} = \frac{\text{COGS}}{\text{Inv}} \Rightarrow \text{COGS} = \text{inv} \times \text{inv turnover}$$

$$\text{COGS} = 2,300 \times 1.9 = 4,370$$

$$4,370 < 4,720 \Rightarrow \text{COGS is decreasing}$$

$$\text{AR turnover} = \frac{\text{sales}}{\text{AR}} \Rightarrow \text{AR} = \frac{\text{sales}}{\text{AR turnover}}$$

$$\text{AR} = \frac{7,200}{2.3} = 3,130$$

$$3,130 > 2,930 \Rightarrow \text{AR is increasing}$$

2. **C** Total assets / owners equity or total liabilities / owners equity

$$\text{FRD Co.: } 13,000/10,000 = 1.30 \text{ or } 3,000/10,000 = 0.30$$

$$\text{Spencer: } 15,000/10,000 = 1.50 \text{ or } 5,000/10,000 = 0.50$$

$$\text{Henley Inc.: } 7,000/4,000 = 1.75 \text{ or } 3,000/4,000 = 0.75, \text{ greatest leverage}$$

$$\text{Worth: } 30,000/20,000 = 1.50 \text{ or } 10,000/20,000 = 0.50$$

3. **B** Net income / owners equity

$$\text{FRD Co.: } 2,000/10,000 = 0.20$$

$$\text{Spencer: } 3,000/10,000 = 0.30, \text{ highest ROE}$$

$$\text{Henley Inc.: } 1,000/4,000 = 0.25$$

$$\text{Worth: } 5,000/20,000 = 0.25$$

4. **C** (Revenues – COGS) / revenues

$$\text{FRD Co.: } (20,000 - 10,000)/20,000 = 0.50$$

$$\text{Spencer: } (30,000 - 20,000)/30,000 = 0.33$$

$$\text{Henley Inc.: } (40,000 - 10,000)/40,000 = 0.75, \text{ highest gross profit margin}$$

$$\text{Worth: } (50,000 - 30,000)/50,000 = 0.40$$

## Asset Valuation

5. **B** Du Pont system states that  $ROE = \text{net profit margin} \times \text{total asset turnover} \times \text{equity multiplier (leverage)}$ . Thus if profitability is falling steadily over a 10-year period, the only way to maintain a high ROE is to increase total asset turnover (asset efficiency) or the amount of debt financing over the 10-year period to offset the decline in net profit margin.
6. **A** To answer this question, you must first compute the ROE using the Du Pont model. We begin with profit margin, followed by total asset turnover, equity multiplier, and return on equity:

$$\text{Rogers: } PM = \frac{13}{183} = 7.1\%; \quad TAT = \frac{183}{100} = 1.83; \quad EM = \frac{100}{(100 - 31)} = 1.45$$

$$\text{QuickCo: } PM = \frac{22}{273} = 8.1\%; \quad TAT = \frac{273}{151} = 1.81; \quad EM = \frac{151}{(151 - 30)} = 1.25$$

$$\text{Blair: } PM = \frac{24}{297} = 8.1\%; \quad TAT = \frac{297}{200} = 1.49; \quad EM = \frac{200}{(200 - 64)} = 1.47$$

$$\text{Dyna: } PM = \frac{23}{372} = 6.2\%; \quad TAT = \frac{372}{250} = 1.49; \quad EM = \frac{250}{(250 - 82)} = 1.49$$

$$\text{Industry: } PM = \frac{20}{282} = 7.1\%; \quad TAT = \frac{282}{162} = 1.74; \quad EM = \frac{162}{(162 - 50)} = 1.45$$

Next we must calculate the ROE for each company and the industry:

$$\text{Rogers: } 7.1\% \times 1.83 \times 1.45 = 18.8\%$$

$$\text{QuickCo: } 8.1\% \times 1.81 \times 1.25 = 18.2\%$$

$$\text{Blair: } 8.1\% \times 1.49 \times 1.47 = 17.6\%$$

$$\text{Dyna: } 6.2\% \times 1.49 \times 1.49 = 13.7\%$$

$$\text{Industry: } 7.1\% \times 1.74 \times 1.45 = 17.9\%$$

From these calculations, we can see that only QuickCo meets all three of Walker's criteria: higher-than-average ROE, lower-than-average use of leverage (as evidenced by the low equity multiplier), and higher-than-average asset turnover ratio.

7. D Profitability of equity and assets is reflected in the ROE and ROA ratios, respectively. To determine if ROE and ROA are increasing or decreasing, we need to calculate net income, total equity, and total assets for 20X6 and for 20X7.

	20X6	20X7
Revenue	\$4,950	\$5,140
– COGS	2,420	2,540
– Operating expenses	1,660	1,770
– Interest expense	30	25
– Taxes	<u>308</u>	<u>295</u>
Net income	\$532	\$510
<hr/>		
Assets	2006	2007
Cash	\$380	\$400
+ Accounts receivable	670	700
+ Inventories	610	600
+ Net PPE	<u>1,350</u>	<u>1,400</u>
Total assets	\$3,010	\$3,100

The ROE is declining, as shown in the following ratios:

$$ROE_{20X6} = \frac{532}{1,846} = 28.8\% \quad ROE_{20X7} = \frac{510}{2,200} = 23.2\%$$

The ROA is also declining as shown in the following ratios:

$$ROA_{20X6} = \frac{532}{3,010} = 17.7\% \quad ROA_{20X7} = \frac{510}{3,100} = 16.5\%$$

8. D Operating profit (EBIT) margin is calculated as follows:

$$\begin{aligned} \text{operating margin} &= \frac{\text{operating profit}}{\text{sales}} \\ &= \frac{(5,140 - 2,540 - 1,770)}{5,140} \\ &= \frac{830}{5,140} \\ &= 16.1\% \end{aligned}$$



## Asset Valuation

9. B Compute the debt ratio (also called the debt-to-assets ratio) as follows:

$$\text{debt ratio} = \frac{\text{total debt}}{\text{total assets}}$$

$$\text{debt ratio}_{20X6} = \frac{574}{3,010} = 19.1\% \quad \text{debt ratio}_{20X7} = \frac{300}{3,100} = 9.7\%$$

The ratio is falling.

Compute the interest coverage ratio as follows:

$$\text{int. coverage} = \frac{\text{EBIT}}{\text{Interest expense}}$$

$$\text{int. coverage}_{2003} = \frac{870}{30} = 29.0x \quad \text{int. coverage}_{2004} = \frac{830}{25} = 33.2x$$

The ratio is increasing indicating that Vincent is better able to meet its interest commitments.

10. D Compute the retention ratio as follows:

$$\text{retention ratio} = 1 - \text{payout ratio}$$

where:

$$\text{payout ratio} = \frac{\text{dividends per share}}{\text{earning per share}}$$

$$\text{earnings per share} = 532 / 260 = \$2.05$$

$$\text{payout ratio} = \frac{0.58}{2.05} = 28.3\%$$

$$\text{retention rate} = 1 - 0.283 = 71.7\%$$

11. C P/E ratio equals stock price divided by earnings per share.

$$\text{EPS} = \$510 / 260 = \$1.96$$

$$\text{P/E} = \$30 / \$1.96 = 15.31$$

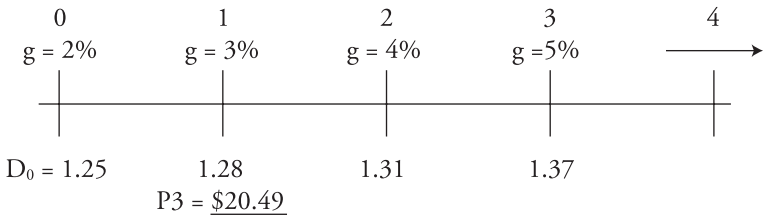
12. A Sustainable growth equals ROE times retention rate.

$$g_{20X6} = 0.288 \times 0.717 = 20.7\% \quad g_{20X7} = 0.232 \times 0.694 = 16.1\%$$

13. C The firm's dividends are expected to grow at a rate of 6% forever, so we can use the constant growth dividend valuation model.

$$P_0 = \frac{D_1}{k - g} = \frac{\$3.00(1.05)}{0.15 - 0.05} = \$31.50$$

14. B Using a timeline:



Since dividend 3 is at the beginning of the infinite period of constant growth, we can use it to find the price at time 2 by growing  $D_2$  at the assumed constant rate to find  $D_3$ .

$$P_3 = \frac{1.37(1.05)}{0.12 - 0.05} = \$20.49$$

$$\begin{aligned} P_0 &= \frac{1.28}{1.12} + \frac{1.31}{(1.12)^2} + \frac{1.37}{(1.12)^3} + \frac{20.49}{(1.12)^3} \\ &= 1.14 + 1.05 + 0.97 + 14.58 \\ &= \$17.74 \end{aligned}$$

15. D Supernormal growth can last a few years, but not normally 15 to 20 years. The dividend discount model can be rearranged as:

$$k_e = \frac{D_1}{P_0} + g$$

Thus  $12\% \neq 5\% + 5\%$ . Expected dividends are discounted by  $(k_e - g)$ . By definition, if  $k_e < \frac{D_1}{P_0} + g$ , where  $k_e$  = required return and  $\left(\frac{D_1}{P_0} + g\right)$  an investment's total return, the stock is considered a good buy (under-priced).

16. A  $k_e$  must be greater than  $g$  for the constant growth dividend valuation model to work.

## Asset Valuation

17. C  $g = \text{ROE} \times \text{retention rate}$

$$g = 0.20 \times 0.70$$

$$g = 14\%$$

$$D_0 = \text{current dividend} = \text{EPS} \times \text{payout ratio}$$

$$D_0 = \$10 \times 30\%$$

$$D_0 = \$3.00$$

$$D_1 = D_0 (1 + g)$$

$$D_1 = 3.00(1.14)$$

$$D_1 = \$3.42$$

18. C  $\text{Intrinsic value} = \frac{D_1}{k - g}$

where:

$$D_0 = \text{current dividend} = \text{EPS} \times \text{payout ratio} = \$11 \times 40\% = \$4.40$$

$$\text{ROE} = \frac{\text{EPS}}{\text{Book Value}} = \frac{\$11}{\$91.50} = 12\%$$

$$g = \text{ROE} \times (1 - \text{payout}) = 12\%(1 - 0.4) = 7.2\%$$

$$D_1 = D_0 (1 + g) = \$4.40(1 + 0.072) = \$4.72$$

$$k_e = \text{required return} = 11\%$$

$$P_0 = \frac{\$4.72}{0.11 - 0.072} = \$124.21 \approx \$124$$

19. C  $D_1 = D_0 (1 + g)$ ;  $D_2 = D_1 (1 + g)$ ;  $D_3 = D_2 (1 + g)$ ; Supernormal growth  
 fair price in year 3:  $\frac{D_4}{k - g}$   
 $D_0 = \text{EPS} \times \text{payout ratio} = 8 \times 0.35 = 2.80$   
 $D_1 = 2.80 (1 + 0.25) = 3.50$ ;  $PV = 3.50 / (1.16)^1 = 3.02$   
 $D_2 = 3.50 (1 + 0.25) = 4.38$ ;  $PV = 4.38 / (1.16)^2 = 3.25$   
 $D_3 = 4.38 (1 + 0.25) = 5.48$ ;  $PV = 5.48 / (1.16)^3 = 3.51$   
 $P_3 = \frac{5.48(1 + 0.10)}{(0.16 - 0.10)} = 100.47$ ;  $P_0 = \frac{100.47}{(1.16)^3} = 64.37$   
 price =  $64.37 + 3.51 + 3.25 + 3.02 = 74.15 \approx \$74$
20. D P/Es rise as the company's growth rate,  $g$ , rises, assuming dividend payout and risk are unchanged.  
 P/Es fall as the company's dividend payout ratio ( $D/E$ ) falls, assuming growth prospects and risk are unchanged.  
 P/Es fall as the company's required return ( $k$ ) rises, assuming dividend payout and growth remain unchanged.  
 P/Es fall as investor risk aversion rises (which causes  $k$  to rise), assuming dividend payout and growth remain unchanged.
21. B The legal contract between the firm and the bondholders is known as the indenture. A debenture is a non-collateralized bond and thus will not have specific assets pledged to the issue.
22. B Debentures are always subordinated (have lower rank than) mortgage bonds that are backed with the firm's fixed assets, so the bonds that are least likely to be paid in a bankruptcy are the subordinated debentures.
23. A A negative covenant prohibits or restricts the issuer from certain actions. An affirmative covenant requires the issuer to perform some action. Requiring Jefferson Inc. to provide periodic reports to the trustee is an example of an affirmative, not a negative covenant.
24. C Through the use of a sinking fund, an issuer makes deposits in an account with the trustee. Over time the account accumulates enough funds to retire the bonds all at once or over time. Thus the likelihood of repayment of principal increases with the use of a sinking fund.

## Asset Valuation

25. **C** Callable bonds do not have to be called. When they are, the holders of the bonds must surrender them to the firm. Management typically calls bonds when interest rates have fallen and the bonds can be replaced with a new issue with a lower coupon rate.
26. **A** GigantiCo has issued \$520 million in bonds. The bonds will be retired in equal amounts over 20 years. Therefore,  $520/20 = \$26$  million of principal will be retired each year. After 7 years, the amount of outstanding principal is  $520 - (26 \times 7) = \$338$  million. This is the amount outstanding at the end of the year. Interest is paid based on year 7's beginning balance of  $338 + 26 = \$364$  million. Interest is therefore  $364 \times 0.045 = \$16.38$  million.
27. **D** Compute the coupon payment as follows:

$$\text{semiannual coupon} = \$2,000 \left( \frac{0.065}{2} \right) = \$65$$

Compute the total interest (i.e., the sum of all coupon payments) as follows:

$$\text{total interest} = 20 \text{ years} \times 2 \text{ coupons per year} \times \$65 = \$2,600$$

28. **B** Since Mendleson expects rates to fall in the future, it would be advantageous to CBX to issue callable bonds so that the debt can be replaced at a lower interest rate in the future. All other choices are incorrect.
29. **B** Mueller has effectively sold a call option to the bond buyers. All other statements are correct.
30. **C** The bondholder receives the call price (either par or premium) plus any accrued interest earned through that date. Accrued interest is calculated as follows:

$$\begin{aligned} \text{accrued interest} &= \text{coupon payment} \times \left( \frac{\text{days accrued}}{\text{days in coupon period}} \right) \\ &= \left( \frac{\$1,000 \times 0.095}{2} \right) \times \left( \frac{180 - 85}{180} \right) \\ &= \$47.50 \times 0.52778 \\ &= \$25.07 \end{aligned}$$

Therefore the call price plus accrued interest yields a price of  $\$1,050 + \$25 = \$1,075$  to the investor.

31. **B** Only Treasury securities are explicitly backed by the full faith and credit of the U.S. government. Some agency securities (issued by federally related institutions) are implicitly but not explicitly backed by the full faith and credit of the U.S. government.

32. **A** An upward-sloping yield curve means that as the maturity of Treasury bonds increases, the yield required on the bonds increases. Therefore, investors are requiring extra compensation for taking on the uncertainty associated with long-term bonds. The forecasted yield curve shows that long-term yields are falling while short-term yields are rising, reflecting less uncertainty about long-term rates relative to short-term rates.
33. **A** Any type of bond could potentially have a sinking fund. Only municipal securities have interest exempt from federal taxes, interest paid from the tax authority of a municipality (such as the city of Phoenix), and a pre-tax yield lower than that on a comparable Treasury bond.
34. **B** Commercial paper can only be issued by corporations with the highest credit ratings, which tend to be relatively large, well-established firms.
35. **C** Interest rate risk (also called price risk) is the risk of increasing interest rates decreasing the price of the bond. Liquidity risk is the possibility of being unable to sell the bond quickly and at a fair price. Liquidity risk is measured by the bid/ask spread on the bond.
36. **B** An upgrade in the credit rating would have a favorable impact on the value of the bond and is, therefore, not a risk.
37. **A** As interest rates rise, the price of a bond (which includes principal and interest payments) will decrease. The reinvestment income generated by the reinvestment of coupon payments as they are received will increase, however. An increase in interest rates is unlikely to result in a decrease in the risk premium, especially if there has been no change in the credit quality of the issue.
38. **C** When the coupon rate and the required return are the same, the bond will sell at par.
39. **B** This is equivalent to a zero-coupon bond and is calculated just like a lump sum present value calculation. Keystrokes:

$$-1,000 \quad [FV]$$

$$10 \times 2 = [N]$$

$$8/2 = [I/Y]$$

$$[CPT] \quad [PV] = \$456$$

Please note that it is market convention to compute the value of zero-coupon bonds using a semiannual periods assumption.

## Asset Valuation

40. A Bond price without accrued interest. Keystrokes:
- 1000 [FV]
  - 100/2 = [PMT]
  - 12/2 = [I/Y]
  - 10\*2 = [N]
  - CPT [PV] = \$885
41. B When required return (YTM) is below a bond's coupon rate, the bond price rises above the face value so that it sells at a premium. Weld Corp. and Alton Inc. have YTM's that are lower than their coupon rate. When the YTM is greater than the coupon rate, the bond will sell at a discount (price is less than par value). Thus the current yield, which is the coupon payment divided by the market price, will be greater than the coupon rate, which is the coupon payment divided by the par value. QED, Bell, and MDB all have YTM's that are greater than their coupon rates.
42. A As the maturity of a fixed income investment increases, the bond's price sensitivity to changes in interest rates increases. When maturity increases, the number of periods of discounting increases, causing the present value of the cash flows to be more sensitive to the interest rate used to discount the cash flows. Since Waldon LLC has the longest-maturity bonds and the other features of the bonds are all equivalent, Waldon LLC will have the highest interest rate sensitivity.
43. A As the size of the interim cash flows decreases, the investor receives a smaller proportion of the total cash flows sooner rather than later. Therefore, the cash flows are more susceptible to the effects of discounting over time. As interest rates change, the change in bond price will be more sensitive for bonds with smaller coupon rates. The BCS Corp. bonds have a smaller coupon rate but are otherwise comparable to the FWX Corp. bonds.

44. **B** A discount bond is a lump sum to be received in the future. In this case the investor will receive \$1,000 in ten years. To receive this cash flow, the investor must pay \$615 for the bond now. Compute the required return using the present value of a lump sum formula as follows (note we are using annual discounting, not semiannual):

$$PV = \frac{FV_n}{(1+i)^n}$$

where inputs equal:

$$PV = 615$$

$$FV_{10} = 1,000 \text{ (input } -1000 \text{ in calculator)}$$

$$n = 10$$

$$615 = \frac{1,000}{(1+i)^{10}}$$

$$\text{solve for } i: \text{ hint } \left[ \sqrt[n]{a} = a^{\frac{1}{n}} \right]$$

$$(1+i)^{10} = \frac{1,000}{615}$$

$$(1+i) = 1.626^{0.1} = 1.0498$$

$$i = 0.0498 \approx 5\%$$

45. **B** A bond with a longer maturity and lower coupon will have greater price movement for any given change of interest rates. The price movement will be the inverse of the interest rate movement. In this particular problem, the higher interest rates cause the bond price to fall. Since the Ethan bonds have a longer maturity (everything else equal), their price will fall more than Burke's bonds.
46. **C** Using a business calculator, the inputs are:

$$PMT = 6\% \times \$1,000 = \frac{\$60}{2} = \$30$$

$$m = 2$$

$$n = 10 \text{ years} \times 2 = 20 \text{ years}$$

$$i = \frac{5\%}{2} = 2.5\%$$

$$FV = \$1,000$$

$$PV = \$1,077.95 \approx \$1,078$$



## Asset Valuation

Alternatively:

$$PV_A = PMT \left[ \frac{1}{\frac{i}{m}} - \frac{1}{\frac{i}{m} \left(1 + \frac{i}{m}\right)^{m \times n}} \right] = 30 \left[ \frac{1}{\frac{.05}{2}} - \frac{1}{\frac{.05}{2} \left(1 + \frac{.05}{2}\right)^{2 \times 10}} \right] = 467.70$$

$$PV = \frac{FV_n}{\left(1 + \frac{i}{m}\right)^{m \times n}} = \frac{1,000}{\left(1 + \frac{.05}{2}\right)^{2 \times 10}} = 610.27$$

$$\text{bond price} = PV_A + PV = 467.70 + 610.27 = 1,077.97 \approx 1,078$$

47. **D** Accrued interest is the interest owed an investor between interest payment dates. Any accrued interest (interest owed to the seller that hasn't been received yet) must be paid to the seller by the buyer of the bond. Statement D says the seller pays the buyer.
48. **B** The lower the coupon and the longer the maturity of a bond, the greater its price will react to a change in interest rates. The 6% coupon (lower coupon) and 20 years to maturity (longer maturity) will have the greatest impact on a bond's value in this problem.
49. **C** Calculate the current yield on each bond series as follows:

$$\text{current yield}_A = \frac{(\$1,000 \times 0.065)}{\$964.47} = \frac{\$65}{\$964.47} = 6.7\%$$

$$\text{current yield}_B = \frac{(\$1,000 \times 0.07)}{\$913.54} = \frac{\$70}{\$913.54} = 7.7\%$$

50. **B** Calculate the yield to maturity for Brighton Company's bonds as follows:

<u>Series A</u>		<u>Series B</u>	
-\$964.47	[PV]	-\$913.54	[PV]
\$32.50	[PMT]	\$35.00	[PMT]
20	[n]	30	[n]
\$1,000	[FV]	\$1,000	[FV]
[CPT] [I/Y] =	$3.5 \times 2 = 7.0\%$	[CPT] [I/Y] =	$4.0 \times 2 = 8.0\%$

51. **A** Both of Brighton Company's bond issues are selling in the market at a discount to face value. Recall the following relationship for a discount bond: Nominal yield < Current yield < Yield to maturity. Choice A violates this relationship and is therefore untrue.

52. **C** The YTM on Bonds A and D are stated on an annual basis since they have annual coupon payments. Therefore we should convert the YTM on Bonds B and C to annual yields and rank the bonds accordingly:

$$\text{YTM}_{\text{B—annual basis}} = \left(1 + \frac{0.065}{2}\right)^2 - 1 = 6.6\%$$

$$\text{YTM}_{\text{C—annual basis}} = \left(1 + \frac{0.079}{2}\right)^2 - 1 = 8.1\%$$

Once all yields are stated on an annual basis, we observe that Bond C has the highest YTM.

53. **D** The YTM does not assume that the price will increase over time. It does assume that the bond can be held until maturity, that all cash flows will be received as expected, and that cash flows can be reinvested at an interest rate equal to the YTM.
54. **B** American options may be exercised at any time up to and including the contract's expiration date, while European options may be exercised only on the contract's expiration date. If the profit from the option increases as the price of the underlying stock increases, the option must be a call option. Since Hibbert's option can be exercised immediately or in the future and has a value directly related to the price of the stock, she must be holding an American call option.
55. **A** If the exercise price of a put option is greater than the stock price, the put option is considered in-the-money. In contrast, if the exercise price of a call option is greater than the stock price, the call option is considered out-of-the-money.
56. **B** An option contract gives the owner the right, but not the legal obligation, to buy or sell an underlying asset at a predetermined future date at a predetermined price. The seller is obligated to transact if the owner desires to do so.
57. **B** If Jim expects the price of the stock to fall, he should hold a long put option with a strike price greater than \$27. The stock is expected to fall from \$35 to \$27, an \$8 decline. Therefore the put option will be in-the-money and will have a payoff of at least  $\$30 - \$27 = \$3$  at expiration.
58. **D** The intrinsic value at the time of purchase is equal to the payoff (assuming immediate exercise). Thus, at the purchase date, the intrinsic value of the option was  $\text{Max}(0, 25 - 33) = \text{Max}(0, -8) = \$0$ . The \$4 premium reflected the option's time value. The current profit is equal to the payoff:  $\$25 - \$23 = \$2$  minus the premium paid:  $\$2 - \$4 = -\$2$ .
59. **B** As the exercise price on a put option decreases, the probability that the index price will end up below the exercise price (i.e., the probability that the option will be in-the-money) decreases, and the value of the option decreases. All of the other answers would serve to increase the value of a European put option.

## Asset Valuation

60. C The writer of a call has potentially unlimited risk. The price of stock can rise infinitely high, causing the payoff of the call (which the writer will lose upon exercise) to become infinitely large.
61. A Calculate the intrinsic value of the call as follows: stock price – exercise price =  $\$23 - \$20 = \$3$ . To calculate the time value of the put option, we must first calculate the intrinsic value as: exercise price – stock price =  $\$25 - \$23 = \$2$ . Next subtract the intrinsic value from the option premium to determine the time value:  $\$5.30 - \$2.00 = \$3.30$ .
62. A At the end of the day, all futures contracts are marked to market. It is the process of settling all futures contracts each day at the settle price. The seller and buyer of the contract recognize profits and losses every day. Therefore, at the end of the first day, the farmer owes \$5 to the long position and will settle this debt because of the mark to market feature. At the beginning of the second day after the contract is initiated, neither party will owe anything to the opposite party.
63. D At maturity, the futures price must equal the spot price. This is the concept of convergence.
64. C Standardization promotes liquidity, and customization reduces liquidity. Futures contracts (which are standardized) have good liquidity, while forward contracts (which are customized) tend to be illiquid.
65. D Michelle believes the price will decline but wants to sell her wheat harvest at the higher futures price. She should therefore take a short futures position. George believes the price of wheat will increase. Therefore he will want to buy the wheat at the lower futures price (a long futures position).
66. C Calculate the payoff as  $(\$845 - \$830) \times 1,500 = \$15 \times 1,500 = \$22,500$ . Since the spot price (the current price in the market) is higher than the futures price stated in the contract, the long position has value and is owed payment from the short position (i.e., Jones owes money to the counterparty).
67. D As you can see from the following table, each tick change represents a  $0.01 \times 2,000 = \$20$  change in value of the futures position. Therefore, at the end of Day 2 the value of the position has dropped a total of five ticks, or \$100. This loss in value gets marked to market at the end of each day. Thus at the end of Day 2, the margin balance would have declined to \$600 from the original \$700 deposit.

<i>Day</i>	<i>Tick Change</i>	<i>Dollar Change</i>	<i>Settle Price</i>	<i>Gain/Loss</i>	<i>Margin Balance</i>
0			\$35.00		\$700.00
1	-3	-\$0.03	\$34.97	-\$60.00	\$640.00
2	-2	-\$0.02	\$34.95	-\$40.00	\$600.00
3	3	\$0.03	\$34.98	\$60.00	\$660.00
4	-5	-\$0.05	\$34.93	-\$100.00	\$560.00

68. C Once again referring to the table in Answer 67, the total tick change after the four days of trading is  $-3 - 2 + 3 - 5 = -7$ . This represents a total decline in the margin account of \$140. Thus, after four days of trading, the balance in the margin account declines to \$560 from the original deposit of \$700.
69. B This privately negotiated settlement is known as an exchange for physicals. It is an uncommon method to close a futures position whereby two contract holders arrange to deliver the underlying asset and close out the futures positions off the floor of the exchange. This is the only situation in which trading can take place off the exchange floor.
70. C The payoffs to options, forwards, and futures are zero sum games. In other words, when one party to the contract gains, the counterparty loses.

# PORTFOLIO MANAGEMENT

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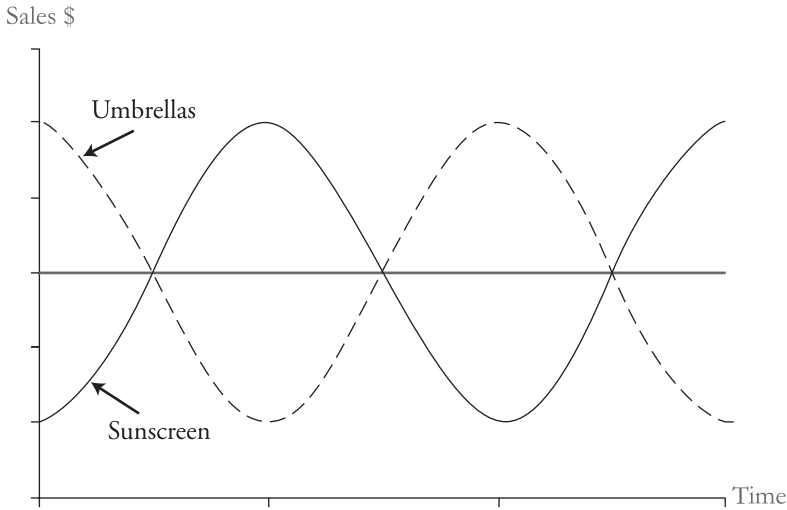
Investors must determine the best risk/return opportunities available in the marketplace. In fact, portfolio theory assumes that investors prefer higher return for a given level of risk and lower risk for a given level of return. However, it is not as simple as finding a set of “good” or even “best” individual investments that meet the investor’s criteria. The investor has to consider the relationship *between* the investments.

When choosing investments, the old saying “don’t put all your eggs into one basket” is quite appropriate. The investing equivalent of this expression is that you should not use all your money to buy the common stock of only one firm. The success or failure of this investment strategy depends upon the fortunes of only that one firm. However, if you purchase the common stock of several firms, the fate of any one company has far less impact on the value of your portfolio.

Consider the classic example of an entrepreneur, Sally Smith, living in a seaside resort. If Sally sells nothing but sunscreen, her sales and profits will experience wide swings through the seasons. In particular, when the weather is hot and sunny, she will sell great amounts of sunscreen, but when the weather is mild and cloudy, she won’t sell much at all. Sally is following a high-risk feast or famine strategy. Instead, she could expand her product line to also include umbrellas. When the weather is cloudy and rainy, her sales of sunscreen will be down, but her sales of umbrellas will be up. When the weather is hot and sunny, her sales of umbrellas will be down, but her sales of sunscreen will be up. Sally now has diversified her product portfolio by adding umbrellas to her product mix. The end result is that her sales and profits will avoid the dramatic swings that would result from a single product line. She now will have sales both when the weather is sunny and when it’s rainy.

The following graph illustrates the time trend for Sally’s sales during sunny and stormy weather. Notice the large swings in sales for the single product portfolios of either sunscreen (the solid black curve) *or* umbrellas (the dashed black curve). But, once she diversifies equally into a two product portfolio, her company sales are smoothed out (the solid blue) line.

Figure 1: Illustration of Diversification



In Figure 1, the changes in the sales for the two products exactly offset each other. Assume the time axis covers a period in which the weather starts progressively sunny, then becomes progressively cloudy, and finally becomes progressively sunny. During the first time period as the weather becomes progressively sunny, sales of sunscreen rise and sales of umbrellas fall. During the subsequent cloudier period, sunscreen sales fall and umbrella sales rise. The third time period repeats the first.

Using mathematical terminology, we say that sunscreen and umbrella sales are perfectly negatively correlated. **Correlation** is a statistic, ranging from  $-1$  to  $+1$ , that measures the strength of relationship between the movements in the values of two assets. In the graph above, the correlation between sales of sunscreen and sales of umbrellas is  $-1$ .

In his Nobel Prize-winning work of 1959, Harry Markowitz<sup>1</sup>, the father of modern portfolio theory, proved mathematically what investors had assumed for decades. Using variance to measure risk, Markowitz demonstrated that the correlation between securities is a more important component of portfolio risk than the individual security variances, especially for portfolios comprising a large number of assets. By combining securities that exhibit low correlation with each other, you can significantly reduce the risk of your portfolio. In other words, the lower the correlation between assets, the greater the diversification potential becomes.

<sup>1</sup> Harry Markowitz, *Portfolio Selection: Efficient Diversification of Investments*, John Wiley and Sons, 1959.

## Portfolio Management

We typically assume that all investors are **risk averse**. By definition, a risk-averse investor prefers:

- Higher returns to lower returns for a given level of expected risk.
- Lower risk to higher risk for a given level of expected returns.

**Portfolio theory** assumes that investors seek to maximize the utility (i.e., contentment or satisfaction) derived from holding their portfolios. However, investors differ in their degree of risk aversion. Consequently, the optimal portfolio for one investor will be different from that of another investor.

For example, consider two investors: Derek King, a young executive, and Aaron Burk, a retiree. We would expect that Derek is willing to accept large amounts of risk in exchange for a high expected return on his portfolio. Derek is willing to accept higher risk now because he won't need withdrawals from his investment portfolio for a long time. As a result, Derek is willing to live with greater ups and downs in his portfolio's performance. We say that Derek has low risk aversion (or, alternatively, high risk tolerance).

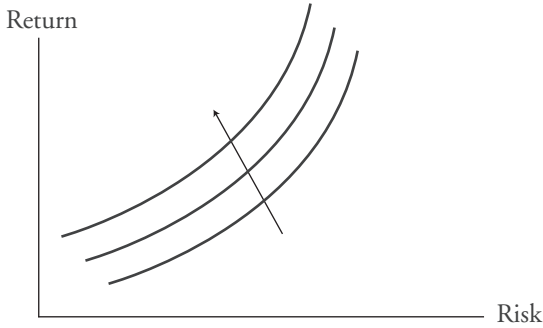
On the other hand, Aaron is not willing to accept high levels of risk because he needs withdrawals from the portfolio immediately. For instance, Aaron might need \$50,000 from his portfolio every year simply to meet his basic needs and expenses. He is retired, so he has no income from work. Therefore, any drop in his portfolio value might jeopardize the amount he can withdraw. We say that Aaron has high risk aversion (or, alternatively, low risk tolerance).

As a result of their differing attitudes toward risk, Derek and Aaron will select different portfolios. Derek's portfolio will be tilted toward less-liquid, higher-risk/higher-return assets, while Aaron's portfolio will be tilted toward more-liquid, safe assets.

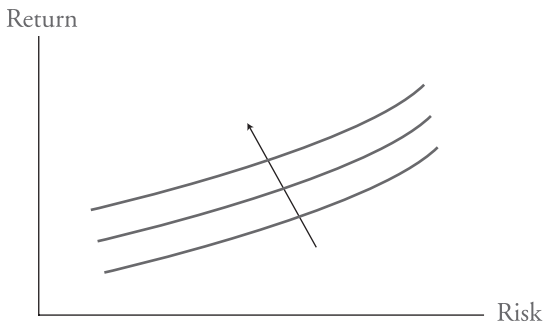
It makes sense that an investor will want a higher return in exchange for higher risk. We assume that risk is bad and return is good. In order to accept more of what is considered bad (i.e., risk), the investor requires more of what is considered good (i.e., rate of return). The additional rate of return can be viewed as the compensation that the investor requires as payment for the additional risk that he is willing to accept. An investor with low risk aversion (e.g., Derek King) will require only a small compensation in exchange for additional risk. In contrast, the investor with high risk aversion (Aaron Burk) requires a large compensation in exchange for additional risk. The additional return that an investor requires is related to his degree of risk aversion.

Visual representation of our two investors is provided below. The curves in the graphs are called indifference curves. Each indifference curve represents the set of investments, or risk-return combinations, over which the investor is indifferent or equally satisfied.

**Figure 2: High Risk Aversion**



**Figure 3: Low Risk Aversion**



An investor will find investments that lie along each indifference curve equally attractive. However, as the investor moves to higher indifference curves, towards the upper left corner, his welfare is increased (i.e., more return for the same or less risk, or less risk for the same or greater return).

Let's first look at the lowest indifference curve for Aaron Burk, our highly risk-averse investor. Aaron will be equally happy with any return/risk combination on this curve. Notice that as risk increases, Aaron demands an increasingly higher rate of return as compensation. Notice also that required return increases quickly for small increases in expected risk, so the indifference curves are quite steep for the highly risk-averse investor. Steep indifference curves indicate a high level of risk aversion (i.e., a conservative investor who will demand a much higher return for a small increase in risk). On the other hand, note that the indifference curves for our



## Portfolio Management

less risk-averse investor are less steep (i.e., an aggressive investor who will demand relatively little extra return for a small increase in risk). For our more aggressive investor, Derek King, required return increases more slowly as expected risk increases. Flatter indifference curves indicate a less risk-averse investor.

Graphically, **risk aversion** is depicted by an indifference curve that slopes upward and to the right. This implies that additional return is required whenever risk rises in order to keep the investor as satisfied as he was prior to the change in risk. It is important to recall that *both* Derek and Aaron are risk averse. They merely differ in degree of risk aversion.

As a final comment on indifference curves, note that either investor prefers investments that lie on higher indifference curves. For example, investments lying along higher indifference curves represent investments that have higher return at the same risk level as investments lying on lower indifference curves. Therefore, indifference curves lying farthest to the upper left are preferred. It is the responsibility of the portfolio manager to find investment combinations that are most preferred by the client.

The major implication of risk aversion is that investors demand higher return in exchange for higher risk. For example, BB rated bonds (i.e., below-investment grade or “junk” bonds) offer higher expected yields than BBB and higher rated bonds. BB rated bonds are riskier than BBB rated bonds. Also, investors will require higher expected returns from higher-risk stocks. This is the major rationale underlying asset pricing models such as the capital asset pricing model, which will be discussed later in this chapter.

## THE EFFICIENT FRONTIER

Markowitz developed mathematical procedures that would produce the set of theoretically best portfolios. While the mathematics needed to explain Markowitz’ theory are beyond the scope of this book, simple intuition will illustrate the big picture. Assume we could line up on a table all the portfolios that have the same level of risk. While the portfolio risks are identical, the expected returns are not. Since all the portfolios have the same level of risk, the choice of best portfolio is simple: The rational investor will select the portfolio that has the highest expected return. Conversely, assume we line up all the portfolios that have the same expected return, but different levels of risk. Once again, the choice of best portfolio is simple: The rational investor would select the portfolio that has the lowest risk.

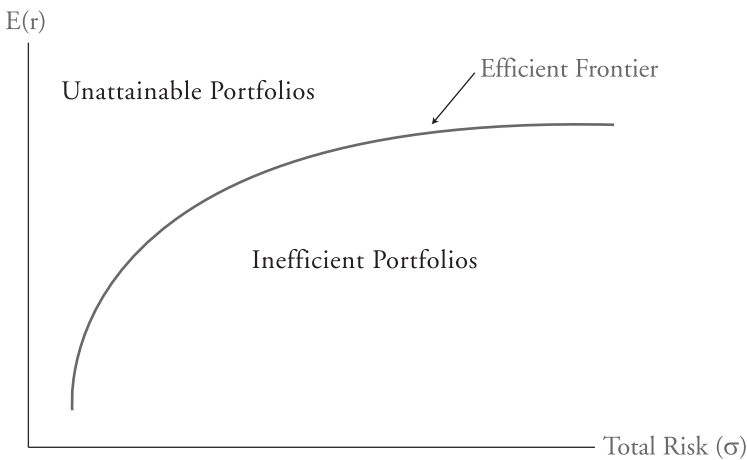
Markowitz translated the actions of a rational investor into mathematical formulas that are used to derive the best set of portfolios. Using optimization theory, he derived formulas for the set of weightings that would produce portfolios that

satisfied two important conditions. The theoretical best portfolio will have both of the following:

1. The least risk for a given expected return level.
2. The highest expected return for a given risk level.

Each portfolio that satisfies these two conditions is called an **efficient portfolio**. No other portfolio has a higher return and the same or lower risk, and no other portfolio has less risk and the same or higher return. A plot of the risk-return combinations of all efficient portfolios is called the **efficient frontier**.

**Figure 4: The Efficient Frontier**



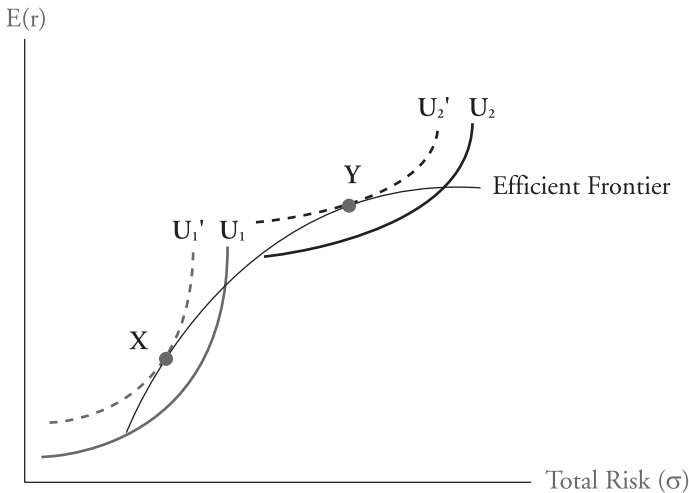
As illustrated in Figure 4, there are no portfolios that lie above the efficient frontier, and all portfolios that lie beneath the efficient frontier are inferior to those that lie on the frontier. Each different point along the efficient frontier represents a different yet still efficient portfolio. As we move from bottom left to upper right, the portfolio expected returns [denoted  $E(r)$ ] and risks rise.

A natural question then becomes, “Which efficient portfolio is best for a particular client?” The answer lies in finding the portfolio that best matches the client’s risk/return preferences. If our client has high risk aversion (i.e., low risk tolerance), we would recommend a portfolio toward the lower end of the efficient frontier. If our client has low risk aversion (i.e., high risk tolerance), we would recommend a portfolio toward the higher end of the efficient frontier. By using optimization methods, we can find the appropriate portfolio for the client.

## Portfolio Management

A graphical illustration of the optimization principle is provided in Figure 5. First recall that an indifference curve represents the risk/return combinations over which the investor is indifferent. Also recall that the investor is better off with investments that lie on indifference curves that lie farther to the upper left in risk/return space (the dashed curves below).

**Figure 5: Optimal Portfolios**



Based on Figure 5, you should note two conclusions regarding the portfolio management process:

1. The optimal portfolio for an investor is the one that lies on the highest possible indifference curve that is just tangent to the efficient frontier.
2. The more risk averse an investor is, the lower the optimal portfolio on the efficient frontier.

Our highly risk-averse investor (the one with steeper indifference curves) will select portfolio X, while our less risk-averse investor (the one with flatter indifference curves) will select portfolio Y. The investors' respective choices make sense—the highly risk-averse investor selects an efficient portfolio with relatively low risk (and relatively low return), while the less risk-averse investor selects an efficient portfolio with relatively high risk (and relatively high return).

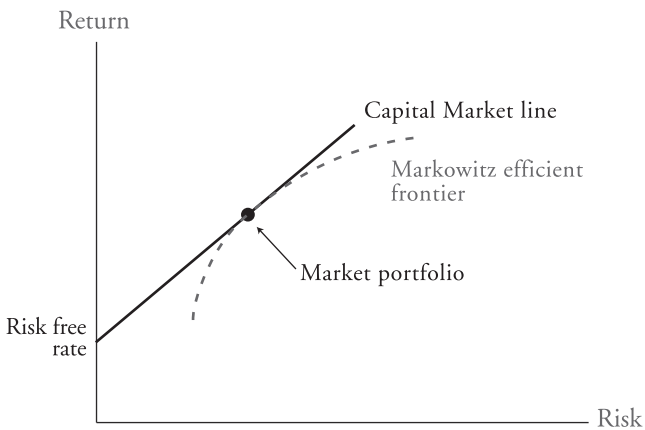
## THE CAPITAL MARKET LINE

Suppose you serve as an advisor for Jane Jones, who needs help allocating her money across three major asset classes: stocks, bonds, and cash. We know what stocks and bonds are, but what is meant by the third asset class, cash? A *cash*

investment is a risk-free, short-term security such as 30-day Treasury bills. Treasury bills or “T-bills” are short-term debt obligations issued by the U.S. Treasury and are considered to be as close to default free as any debt obligation gets. Therefore, a **risk-free asset** exists in which there is no uncertainty about its future value. This risk-free asset will have a standard deviation of returns equal to zero, and the correlation of its returns to those of any other asset will also equal zero.

Markowitz’ efficient frontier discussed in the previous section did not consider the existence of a risk-free asset. The introduction of a risk-free asset changes the efficient frontier from a curve into a straight line called the **capital market line** (also referred to as the CML). As illustrated in Figure 6, the capital market line begins at the risk-free rate and extends on a line tangent to the Markowitz frontier. It represents all risk/return combinations of portfolios that combine risky and risk-free assets in an optimal manner.

**Figure 6: The Capital Market Line**



Notice that the risk/return combinations lying on the capital market line dominate all risk/return combinations lying on the Markowitz efficient frontier (the dashed curve). There is only one point in which the capital market line and the Markowitz frontier meet. This point of tangency is very important in portfolio theory. It

## Portfolio Management

represents the portfolio, called the **market portfolio**, that has the best risk/return tradeoff. Formally, the market portfolio maximizes the **Sharpe ratio**<sup>2</sup>:

$$\text{Sharpe ratio} = \frac{E(R_p) - r_f}{\sigma_p} \quad (1)$$

where :

$E(R_p)$  = expected return for the portfolio

$r_f$  = risk-free rate of return

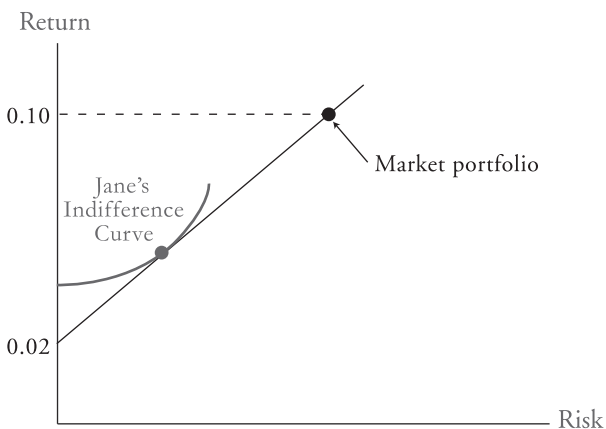
$\sigma_p$  = expected standard deviation for portfolio p

Sometimes, the numerator in Equation 1 is called the mean **excess return** for the portfolio. Therefore, the Sharpe ratio is a measure of the excess return per unit of risk for the portfolio. No other portfolio on the Markowitz curve or on the capital market line has a higher Sharpe ratio than the market portfolio.

**Bringing it all together.** Let's now revisit our client, Jane Jones. Our task is to find the appropriate allocations to stocks, bonds, and cash for Jane. For simplicity, let's assume that the market portfolio contains just stocks and bonds in a 70/30 mix (e.g., 70% stocks, 30% bonds). Further, assume that the market portfolio has an expected annual mean return of 10% and that the risk-free rate is expected to be 2%.

Let's also assume that Jane is close to retirement and, therefore, is a conservative investor. Using our theory from the prior sections, this would mean that her indifference curve is rather steep as illustrated in the following.

**Figure 7: Optimal Asset Allocation**



2 The Sharpe ratio is named after Professor William Sharpe who introduced the concept.

As discussed in the previous section, the most desired or optimal investment is located at the point of tangency between the indifference curve and the efficient frontier, which in this case is the capital market line. The tangency point represents the investment that maximizes Jane's risk/return preference. Notice that the tangency point is approximately halfway between the risk-free asset and the market portfolio. Therefore, Jane can maximize her risk/return preferences by allocating approximately 50% to the market portfolio and 50% to cash.

Now that we know how much to allocate to cash, how much of Jane's money should be allocated to stocks and how much to bonds? The answer lies in knowing that the market portfolio is allocated 70% to stocks and 30% to bonds.

Therefore, since 50% is allocated to the market portfolio, Jane's final allocation becomes as follows:

$$\begin{aligned} \text{Allocation to stocks: } & 0.50 \times 0.70 = 0.35 \\ \text{Allocation to bonds: } & 0.50 \times 0.30 = 0.15 \\ \text{Allocation to cash: } & 0.50 \end{aligned}$$

As her advisor, you should recommend that Jane allocate 35% to stocks, 15% to bonds, and 50% to cash. Notice that most of Jane's investment is in safer investments (bonds and t-bills).

This illustration can be generalized easily to a market portfolio containing additional asset classes such as real estate, venture capital, or commodities. Allocating a client's total portfolio across major asset classes is called **asset allocation**, an important determinant of the performance of the overall portfolio.

## EXPECTED RETURN AND RISK FOR AN INDIVIDUAL SECURITY

We will continue with our discussion of portfolio theory by first concentrating on expected returns for individual assets. The expected return for any investment depends upon the price you initially paid for it and the promised cash flow(s) in the future. For instance, assume you pay \$100 for an investment that promises to pay \$110 in one year. Your *expected* return is 10%.

$$\hat{R} = \frac{V_1 - V_0}{V_0} = \frac{\$110 - \$100}{\$100} = 0.10 = 10\% \quad (2)$$

where :

$\hat{R}$  = your expected return

$V_0$  = your initial investment

$V_1$  = the expected value of your investment in one year

## Portfolio Management

10% is the *expected* return because the \$110 cash flow is due in one year, and many things could happen to interfere with receiving the cash flow *exactly as expected*. A change in how the \$110 is received, such as receiving it sooner or later than promised, receiving it in installments, or receiving a larger or smaller amount, may make your actual return greater or less than the expected return.

Let's define in greater detail the differences between the expected and the actual return. The **expected return** is *estimated* at the beginning of the investment period (i.e., looking into the future) and is based upon the price you *paid*, the cash flow(s) you *expect* to receive (i.e., dividends), and any price appreciation (i.e., capital gain) that is expected to occur. Although you might be able to predict with some accuracy the amount of dividends you will receive, the amount of price appreciation is subject to the effects of many different factors. Depending on the events that occur between the day you purchase the stock and the day you sell it, the price could increase, decrease, or stay the same.

In contrast to the expected return, the **actual return**, often referred to as the *historical return*, is the calculated return on the investment (i.e., looking back from the end of the investment period) based upon the price you paid and the cash flow(s) you actually received. The actual return on an investment can only be determined after the investment period has concluded. If the actual return is equal to or greater than the expected return estimated at the beginning of the investment period, the investor will be satisfied.

Figure 8 shows a **probability distribution** of *possible* 1-year returns for Meyer's Manufacturing (MM) common stock. We assume MM will operate without any unexpected firm-specific events, but we will allow different states of the economy over the period: boom, normal, and recession.<sup>3</sup> In calculating the expected return for MM stock, we must estimate the probability of each state of the economy as well as the stock's performance if that state occurs.

**Figure 8: Probability Distribution of Returns for MM**

<i>State of the Economy</i>	<i>Probability of This State Occurring</i>	<i>Expected Return in This State</i>
Boom	0.30	20%
Normal	0.40	15%
Recession	0.30	10%

<sup>3</sup> Of course, there are many degrees of boom and recession, creating infinite possible states of the economy. For simplicity, however, we assume only three possible, distinct states of the economy.

If a “normal” economy persists over the next year, we expect MM stock to earn 15%. If the economy goes into a recession, the expected return for MM drops to 10%. A boom economy will result in a 20% return for MM.

The formula for the **expected return** for any asset is:

$$E(R) = \sum_{t=1}^T P_t R_t \quad (3)$$

where:

$E(R)$  = expected return for the asset

$R_t$  = our estimate of the return on the asset if economic state  $t$  occurs

$P_t$  = the probability that economic state  $t$  will occur

$T$  = the number of possible states of the economy

Applying the formula to MM:

$$E(R) = 0.30(0.10) + 0.40(0.15) + 0.30(0.20) = 15\%$$

We have estimated the *expected* return for MM based on our estimates of possible states of the economy and the return on MM for each state that could possibly occur. Now we will estimate the accompanying risk using the standard deviation of possible returns for MM.

$$\sigma_{MM} = \sqrt{\sum_{t=1}^T P_t (R_t - E(R))^2} \quad (4)$$

where:

$\sigma_{MM}$  = standard deviation of the possible returns for MM stock<sup>4</sup>

$$\sigma_{MM} = (0.30(0.20 - 0.15)^2 + 0.4(0.15 - 0.15)^2 + 0.30(0.10 - 0.15)^2)^{1/2} = (0.0015)^{1/2} = 0.0387 = 3.87\%$$

We now have MM stock's expected return and standard deviation. What does this tell us? From the empirical rule we presented in the chapter on Quantitative Methods, we know that approximately 68% of all possible returns lie within one standard deviation of the mean (i.e., expected return), and 95% lie within two standard deviations. In addition, 99% of all possible returns will lie within three standard deviations of the mean. Since one standard deviation for MM equals 3.87%, we calculate the following ranges:

$$\begin{aligned} \pm \text{one standard deviation} &= 15\% \pm 1(3.87\%) = 11.13\% \text{ to } 18.87\% \\ \pm \text{two standard deviations} &= 15\% \pm 2(3.87\%) = 7.26\% \text{ to } 22.74\% \\ \pm \text{three standard deviations} &= 15\% \pm 3(3.87\%) = 3.39\% \text{ to } 26.61\% \end{aligned}$$

<sup>4</sup> This is the formula for the *ex ante* (expected) standard deviation. In the Quantitative Methods chapter we calculated the *ex post* (historical) standard deviation.



## Portfolio Management

Given the ranges we calculated using the expected return and standard deviation, we can estimate the following probability distribution of possible returns for MM. At the beginning of the year we *expect* our actual return to be 15% for the year, but due to its risk (i.e., variability), we estimate there is:

1. A **68% probability** that the actual return for MM will be between 11.13% and 18.87%.
2. A **95% probability** that the actual return for MM will be between 7.26% and 22.74%.
3. A **99% probability** that the actual return for MM will be between 3.39% and 26.61%.

Based on projections for MM, your expected return for the coming year is 15%, and you are 99% certain that the actual return will not be below 3.39% or above 26.61%.

## EXPECTED RETURN AND RISK FOR A PORTFOLIO

Let's now assume MM is one of six different stocks you hold in a portfolio with a total value of \$50,000. We'll assume you have already estimated the expected return for all six of your stocks as shown in Figure 9. In a manner similar to calculating the expected return for an individual stock, the expected return for your portfolio is a weighted average of the expected returns of the individual stocks in the portfolio:

$$E(R_p) = \sum_{i=1}^n w_i E(R_i) \quad (5)$$

where:

$E(R_p)$  = the expected return for the portfolio

$E(R_i)$  = the expected return for stock  $i$

$w_i$  = the "weight" of stock  $i$  in the portfolio

$n$  = the number of stocks in the portfolio

Figure 9: Composition of a Portfolio of Six Stocks

<i>Stock</i>	$\hat{R}$	<i>Investment</i>	<i>Weight</i>
A	12%	\$9,000	0.18
B	11%	6,000	0.12
C	10%	8,000	0.16
D	14%	9,000	0.18
E	13%	8,000	0.16
MM	15%	10,000	0.20
		\$50,000	1.00 = 100%

$$E(R_p) = 0.18(0.12) + 0.12(0.11) + 0.16(0.10) + 0.18(0.14) + 0.16(0.13) + 0.20(0.15) = 12.68\%$$

The expected return on your portfolio of six stocks is 12.68%.

## Risk

We know that estimating the expected return on any investment does not give us complete information. We must also estimate the associated risk. The risk (i.e., standard deviation) associated with the expected return of a portfolio is *not* simply a weighted average of the risk of the individual stocks.

For simplicity, we will now assume a portfolio contains only two stocks, a and b.<sup>5</sup> The expected return and standard deviations for the individual stocks are shown in Figure 10, as is the portfolio expected return.

5 To calculate the variance of the 6-stock portfolio, the equation would have to consider the weight of each stock in the portfolio, its variance, plus the covariance between each pair of stocks:

$$\begin{aligned} \sigma_p^2 = & w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + w_{MM}^2 \sigma_{MM}^2 + 2w_A w_B \text{Cov}_{A,B} + \\ & 2w_A w_C \text{Cov}_{A,C} + 2w_A w_D \text{Cov}_{A,D} + 2w_A w_E \text{Cov}_{A,E} + 2w_A w_{MM} \text{Cov}_{A,MM} + 2w_B w_C \text{Cov}_{A,C} \\ & + 2w_B w_D \text{Cov}_{B,D} + 2w_B w_E \text{Cov}_{B,E} + 2w_B w_{MM} \text{Cov}_{B,MM} + 2w_C w_D \text{Cov}_{C,D} + \\ & 2w_C w_E \text{Cov}_{C,E} + 2w_C w_{MM} \text{Cov}_{C,MM} + 2w_D w_E \text{Cov}_{D,E} + 2w_D w_{MM} \text{Cov}_{D,MM} + \\ & 2w_E w_{MM} \text{Cov}_{E,MM} \end{aligned}$$

**Figure 10: Expected Returns, Standard Deviations, and Weights for a 2-Stock Portfolio**

<i>Stock</i>	<i>Expected Return</i>	<i>Standard Deviation</i>	<i>Weight</i>
a	0.12	0.040	0.40
b	0.14	0.055	0.60
$E(R_p) = 0.4(0.12) + 0.6(0.14) = 0.048 + 0.084 = 0.1320 = 13.2\%$			

As shown in Figure 10, the expected return for the portfolio, 13.2%, is the weighted average of the expected returns for the two stocks. In contrast, the standard deviation is based on the standard deviations of each stock, the weight of each stock in the portfolio, and the covariance of the returns between the two stocks.

The variance for the two asset portfolio is calculated with the following formula:

$$\sigma_p^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \text{Cov}_{a,b}$$

Covariance and correlation are mathematically related as follows:

$$\rho_{a,b} = \frac{\text{Cov}_{a,b}}{\sigma_a \sigma_b} \Rightarrow \text{Cov}_{a,b} = \rho_{a,b} \sigma_a \sigma_b$$

Therefore the formula for the variance (and standard deviation) of a two asset portfolio can be rewritten using correlation (often identified by the Greek letter rho,  $\rho$ ) as follows:

$$\sigma_p^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \sigma_a \sigma_b \rho_{a,b} \tag{6}$$

$$\sigma_p^2 = (0.4)^2 (0.04)^2 + (0.6)^2 (0.055)^2 + 2(0.4)(0.6)(0.04)(0.055)\rho_{a,b}$$

$$\sigma_p = [0.001345 + 0.001056 (\rho_{a,b})]^{1/2}$$

Recall that the correlation between assets a and b is the degree to which their returns move together and that the correlation ranges from a possible low of  $-1$  (for perfectly negatively related assets) to a possible high of  $+1$  (for perfectly positively related assets).

To demonstrate the effects of changes in the degree of correlation on the standard deviation of the portfolio, we will substitute different values for  $\rho_{a,b}$  into Equation 6, and observe how the standard deviation of the portfolio is affected. Figure 11 shows the variance and standard deviation for the portfolio, given differing values for the correlation of returns.

It is apparent from Figure 11 that the variance and standard deviation (our measure for total risk) decrease as correlation decreases. This is a mathematical demonstration of **diversification**. By combining stocks with less-than-perfectly positive (+1.0) correlation, we can reduce the risk (standard deviation) of the portfolio below the weighted average of the individual stocks' standard deviations.<sup>6</sup>

**Figure 11: Variance and Standard Deviation of Portfolio a,b Assuming Different Values of Correlation,  $\rho_{a,b}$**

$\rho_{a,b}$	$\sigma_p^2$	$\sigma_p$
-1.0	0.000289	0.0170
-0.5	0.000817	0.0286
0.0	0.001345	0.0367
+0.5	0.001873	0.0433
+1.0	0.002401	0.0490

For example, when we assume the maximum correlation (+1.0), the standard deviation of the portfolio is a simple weighted average of the standard deviations of the two stocks (i.e.,  $0.4(0.04) + 0.6(0.055) = 0.049$ ). When we begin reducing the correlation, the portfolio standard deviation falls until, when correlation is  $-1.0$ , it reaches its minimum value. At the weights specified, 40% in stock *a* and 60% in stock *b*, the minimum standard deviation for our portfolio is 0.0170, which is considerably less than the standard deviation of either stock on an individual basis.

<sup>6</sup> Note that if the correlation between the two assets equal +1, then the portfolio standard deviation is simply the weighted average of the standard deviations of the two assets. Mathematically, if correlation equals +1, then  $\sigma_p^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \sigma_a \sigma_b$  which equals  $\sigma_p^2 = (w_a \sigma_a + w_b \sigma_b)^2$ . Therefore, the standard deviation equals  $\sigma_p = (w_a \sigma_a + w_b \sigma_b)$ , which is the weighted average of the standard deviations of the two assets that comprise the portfolio. Therefore, the portfolio standard deviation is highest when the portfolio standard deviation equals the weighted average of the individual asset risks. This is the case of no diversification benefit. If the correlation is less than +1, the portfolio standard deviation will be *less* than the weighted average of the individual asset risks.

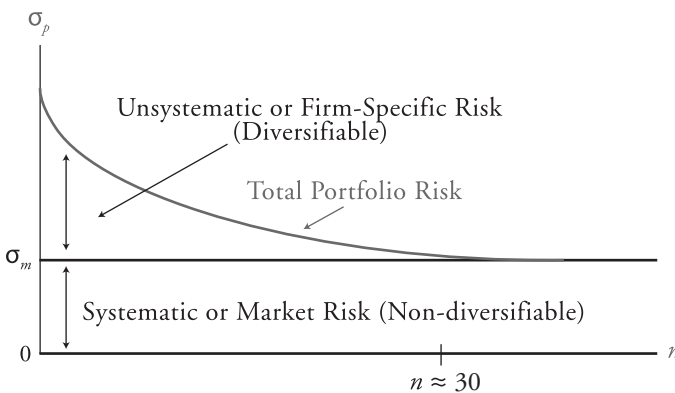
## Systematic and Unsystematic Risk

The total risk for any asset consists of two components: **systematic risk** and **unsystematic risk**. Systematic risk (also called market risk) is that part of the asset's total risk that cannot be reduced through diversification. Sources of systematic risk are macroeconomic factors such as interest rates and inflation. Unsystematic risk (also called diversifiable risk) is that part of the asset's total risk that can be virtually eliminated through diversification. Examples of the sources of unsystematic risk are company-specific events such as strikes, product recalls, and management resignations.

Figure 12 shows the relationship of portfolio risk, measured by standard deviation, and the number of different stocks in the portfolio. It demonstrates that adding each stock decreases the portfolio standard deviation, but the amount of the decrease in risk is less with each additional stock. It is clear, however, that when a large number of stocks are combined in a portfolio, the risk of the portfolio is dramatically reduced.

You will also notice there is a value below which we cannot drive the portfolio standard deviation, no matter how many different stocks we hold. At this point, virtually all the unsystematic risk has been eliminated, and we are left with only systematic risk,  $\sigma_m$ , or market risk. The exact number of different stocks at that point is uncertain, but after about 30 stocks, the diversification gain from adding more stocks is very small.

**Figure 12: Reduction of Portfolio Standard Deviation (risk),  $\sigma_p$ , From Increasing the Number of Stocks,  $n$ , in the Portfolio.**



## THE CAPITAL ASSET PRICING MODEL

Portfolios often are ranked based on risk-adjusted performance. A portfolio is highly ranked if it produces a superior rate of return within its risk class. Therefore, to be highly ranked, the portfolio must outperform a benchmark representing its risk class. Two questions arise: (1) how do we construct the appropriate risk-adjusted benchmark for a portfolio, and (2) what must the portfolio manager do to outperform the benchmark? This section addresses these questions by presenting and using the **capital asset pricing model (CAPM)**. The CAPM was developed by Professor William Sharpe in the 1960s. Because of his ground breaking research, Sharpe was later awarded the Nobel Prize in Economics in 1990.

According to the CAPM, the rate of return that investors require for any risky asset depends on two factors:

1. The general level of interest rates (i.e., the risk-free rate).
2. Compensation for risk.

As discussed earlier, the yield on the T-bill can be used for the risk-free rate. The second factor, compensation for risk, equals the additional rate of return that investors require as compensation for the risk associated with the asset. The higher the risk, the higher the required compensation. The compensation for risk is the required return in excess of the risk-free rate and is also known as the **risk premium** for the asset.

### RISK PREMIUM FOR AN INDIVIDUAL STOCK

The CAPM is based on the principle that only systematic risk matters when determining the fair price of an asset. From our previous discussions we know that as long as stock returns are not perfectly positively correlated, we can diversify away most, if not all, of the stock's unsystematic risk. Therefore, unsystematic risk of a stock is irrelevant in a diversified portfolio. As a result, CAPM assumes that investors receive no compensation for bearing unsystematic risk because it can be easily eliminated in a portfolio.

The CAPM defines systematic risk of an asset relative to all assets in the market. This measure is called the asset's **beta**. Beta has a simple interpretation: the degree to which returns on a stock and returns on the broad market move together. A stock with average systematic risk will have a beta equal to 1.<sup>7</sup> A stock with higher-than-average systematic risk will have a beta greater than 1, and a stock with lower-

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<sup>7</sup> Because the returns on the market portfolio are perfectly correlated with themselves (i.e., they are one and the same), CAPM defines the beta for the market portfolio as being equal to 1.

## Portfolio Management

than-average systematic risk will have a beta less than 1. The beta for a risk-free asset equals zero.

Recall that a **risk premium** equals the difference between the required return and the risk-free rate. For example, the risk premium for the broad market portfolio (the portfolio of all assets) equals:

$$\text{market risk premium} = k_m - r_f$$

where:

$k_m$  = the required return on the market portfolio,  $m$

$r_f$  = the risk-free rate

If an individual stock has beta equal to one, its systematic risk equals that of the broad market and, according to the CAPM, its required return will equal that of the broad market. If, however, the beta is different from 1, the required return for the stock will differ from that of the broad market. The CAPM makes this adjustment as follows:

$$k_i = r_f + \beta_i(k_m - r_f) \tag{7}$$

where:

$k_i$  = the required return for the individual stock,  $i$

Therefore, the risk premium for an individual stock,  $k_i - r_f$  is:

$$k_i - r_f = \beta_i(k_m - r_f) \tag{8}$$

## ESTIMATING BETA: THE CHARACTERISTIC LINE

To estimate historical beta for a stock, we run a simple **linear regression** of the stock returns against the broad market index. Linear regression, which is described in the Level II curriculum, is a technique for estimating the straight line that best describes the relationship between dependent and independent variables.

$$R_i = \alpha + \beta R_m + \varepsilon \tag{9}$$

where:

$R_i$  = dependent variable (i.e., the returns on stock  $i$ )

$R_m$  = independent variable (i.e., the returns on the market)

$\alpha$  = intercept of the regression line

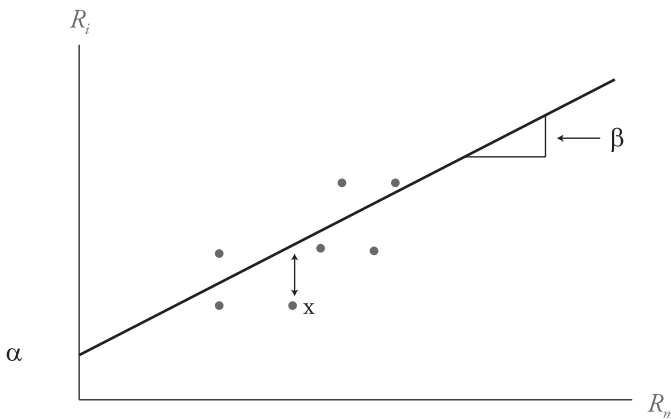
$\beta$  = regression slope

$\varepsilon$  = random error term

Since we assume the return on the stock market is the independent variable, the regression line estimates the movement in the individual stock that is related to market movements. This regression equation is vital to portfolio theory and is called the **characteristic line**.

To show how this regression works, we will assume that we have observed seven weekly returns for our stock and for the stock market.<sup>8</sup> We line up the returns, week one for both, week two for both, week three for both, and so on, and run the regression. Figure 13 shows the observations (the weekly returns) plotted on a graph and the regression line that best fits them.

**Figure 13: Typical Regression Output Using the Characteristic Line.**



Point  $x$  on the graph is an individual observation, a combination of the returns on the market and on our stock for one of the seven weeks. The distance between it and the regression line is its *error*. A computer-based statistical application squares the error for each observation and sums (adds) them together. When it finds the line that results in the smallest sum of the squared errors, this result becomes the “best fit” line identifying the relationship between the variables.

Let’s assume the following equation from our regression analysis:

$$R_i = 0.6 + 1.2R_m$$

The intercept for our estimated line is 0.6, and the slope coefficient is 1.2. The slope coefficient, referred to simply as beta, is by far the most important outcome from our regression. The beta tells us our stock return changes 1.2 percentage points for every one percentage point change in the market return. Another way

<sup>8</sup> We use past or historical returns in the regression.



## Portfolio Management

of saying this is our stock is 1.2 times or 20% more volatile than the market. As explained above, since the beta coefficient measures the influence of the market on our stock's returns, it measures our stock's *systematic risk*.

Equation 10 demonstrates the calculation for the regression coefficient,  $\beta$ . The equation can be used to find the beta for any asset by substituting in values for asset  $i$  in the equation.

$$\beta_i = \frac{\text{Cov}_{i,m}}{\sigma_m^2} \quad (10)$$

$$\beta_i = \frac{\sigma_i \sigma_m \rho_{i,m}}{\sigma_m^2} = \frac{\sigma_i}{\sigma_m} \rho_{i,m}$$

where:

$\beta_i$  = the beta coefficient from regressing the returns from stock  $i$  against the market

$\sigma_i$  and  $\sigma_m$  = the standard deviations of stock  $i$  and the market

$\rho_{i,m}$  = the correlation of stock  $i$  with the market

Thus, the beta of an asset is the ratio of its standard deviation to the standard deviation of the market, multiplied by the correlation between the asset and the market. If we substitute values for a risk-free asset and for the overall stock market, we obtain the following:

$$\beta_{\text{risk-free asset}} = 0.0$$

$$\beta_{\text{market}} = 1.0$$

Beta for the risk-free asset must be zero because the standard deviation for the risk-free asset is zero, and its correlation with the market is zero. Beta for the market is defined as being 1.0 because the correlation of the market with itself is +1.0, and the ratio of its standard deviation to itself must also be one.

## THE SECURITY MARKET LINE

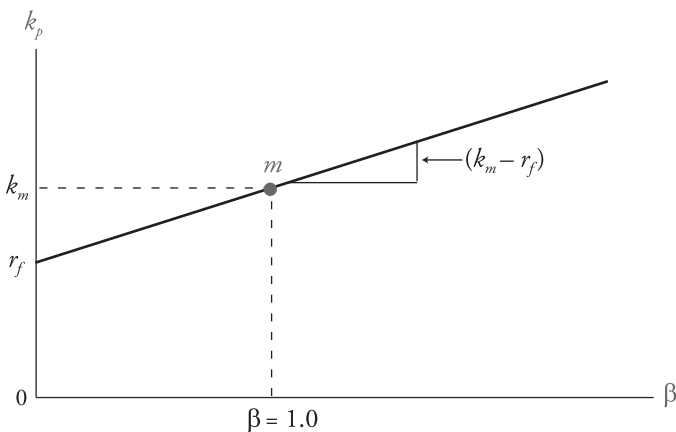
Once we estimate the beta for a stock, we can derive its **required return** using the CAPM equation. We could repeat this process for any number of stocks and could tabulate the required returns and betas as follows:

<i>stock</i>	<i>required return, k</i>	<i>beta</i>
1	$k_1$	$\beta_1$
2	$k_2$	$\beta_2$
.	.	.
.	.	.
.	.	.
N	$k_N$	$\beta_N$

Each  $k$  is derived using the CAPM:  $k_i = r_f + \beta_i(k_m - r_f)$ . If we were to plot these paired observations ( $k$  and  $\beta$  from the table), we would derive a line called the **security market line (SML)**, which is merely a graph of the CAPM. Notice that the security market line starts at the risk-free rate and extends through the market portfolio. The slope (rise/run) of the SML equals the market risk premium,  $k_m - r_f$ .

Even without risk (i.e., where  $\beta = 0$ ) investors will require the risk-free rate of return, which includes the real rate of return (RR) and a premium for inflation (IP). Since the risk-free asset has a beta of 0.0, we know  $r_f$  the rate of return on the risk-free asset, must be the intercept term of the CAPM graph in Figure 14.

**Figure 14: The Security Market Line**



**Changes in inflation.** Both the independent variable and the intercept term in the CAPM include a premium for expected inflation. If inflation expectations should

## Portfolio Management

increase, we see that both  $r_f$  and  $k_m$  will increase. In the CAPM, the risk-free rate is a nominal rate defined as the real risk-free rate plus a premium for expected inflation:

$$r_f = RR + IP \quad (11)$$

$$k_m = r_f + RP_m \quad (12)$$

where:

RR = the real risk-free rate

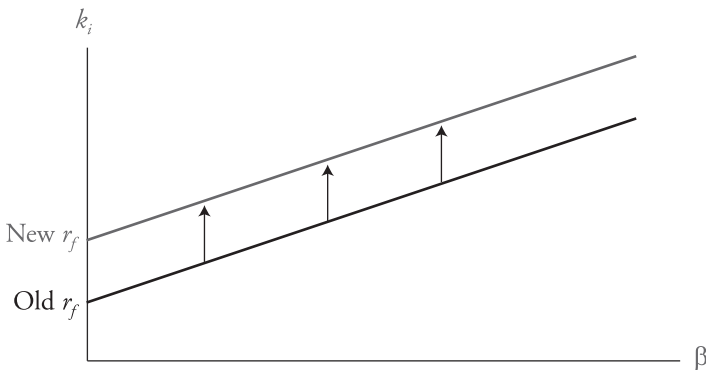
IP = the inflation premium

$RP_m$  = the market risk premium

For example, if inflation expectations increase one percentage point, both  $r_f$  and  $R_m$  would increase by the same percentage point. Since  $(k_m - r_f)$  is the slope of the security market line, and both variables increase the same amount, the slope remains unchanged.

However, the intercept will increase (i.e., move up) by one percentage point, causing an upward shift in the CAPM as illustrated in Figure 15. With an increase in inflation expectations, investors will require higher returns on all stocks. Of course, a decrease in inflation expectations would lead to a downward shift in the CAPM, as investors require lower returns on all stocks.

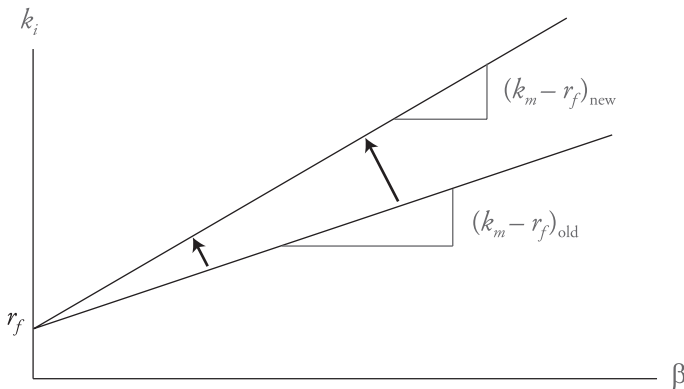
**Figure 15: An Upward Shift in the CAPM Caused by an Increase in Inflation Expectations**



**Changing risk aversion.** The slope of the CAPM,  $(k_m - r_f)$ , is referred to as the *market risk premium*. It measures the average degree of risk aversion in the market. If investors suddenly required more return per unit of risk, the risk premium would

increase. The increased risk premium causes the slope of the CAPM to increase as illustrated in Figure 16.

**Figure 16: An Upward Rotation of the CAPM Caused by an Increase in Risk Aversion**



The CAPM required return is often used as the discount rate when finding the fair value for a stock. Therefore, changes in inflation expectations and risk aversion have important implications for stock prices. As inflation expectations rise, required returns (discount rates) rise, and therefore, stock prices fall. Likewise, as risk aversion rises, required returns (discount rates) rise, and therefore, stock prices fall. This is a direct result of the inverse relationship between present values and discount rates.

## AN APPLICATION OF THE CAPM

Assume you are assigned the task to make a buy or sell recommendation on XYZ, Inc., which has a beta equal to 1.20. Assume the required market premium ( $k_m - r_f$ ) equals 5% and that the risk free rate,  $r_f$ , equals 2%. Further assume you have predicted the future cash flows for XYZ, and predict that its stock will return 10% next year. Should you recommend a purchase or sale for XYZ stock?

To answer the question, first find the required return for XYZ using the CAPM:

$$k_i = 0.02 + 1.2(0.05) = 8\%$$

Based on the output from the CAPM, you will recommend a purchase of XYZ only if you think the future return on the stock will equal or exceed 8%. Your predicted return (10%) exceeds the required return (8%). Since your predicted return exceeds the stock's required return, you should recommend a purchase. In other words, you

## Portfolio Management

think the stock is undervalued at today's prices. You think the stock price for XYZ is *under* its fair value.

The difference between the investor's expected return and required return for an asset is called the asset's **alpha**. Alpha measures the degree to which an asset is over- or undervalued. For XYZ:

$$\text{alpha} = 0.10 - 0.08 = 2\%$$

Therefore, you think XYZ's stock price is undervalued by 2%.

Our estimation of the expected return on XYZ stock (or any other stock) could just have easily been equal to or below the required return, in which case the stock would be considered fairly valued or overvalued, respectively.

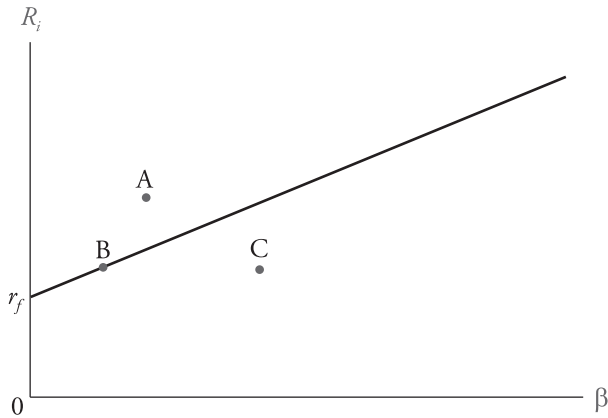
We can also use graphical analysis to determine whether individual stocks (or portfolios) are priced correctly. For example, if a stock is priced correctly, such that its expected return is appropriate for its risk, it will fall directly on the SML. If not, the plot will fall above or below the SML. Let's take a look at a few examples.

In Figure 17, three stocks are plotted against the SML.<sup>9</sup> Since stock B falls directly on the SML, its required return is equal to its expected return, and the stock is priced correctly. Stock A, however, lands above the SML. Since the plots represent the combination of expected return and risk (measured by beta), we see that the expected return for stock A is greater than its required return, as denoted by the SML. Since its expected return is greater than its required return, Stock A must be underpriced. The exact opposite could be said for Stock C. Stock C plots below the SML, indicating that the stock is expected to realize a return less than its required return. Stock C must be overpriced.

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<sup>9</sup> Note that the plots could represent portfolios as well as individual stocks.

Figure 17: Over- and Under-Priced Securities and the Security Market Line



## PORTFOLIO BETAS

The beta for a portfolio is calculated much the same way as the beta for an individual stock. To find the beta for an individual stock, we employed the characteristic line in Equation 9. Recall that we regressed the returns of an individual stock against those of the market for the same period of time to obtain the beta value for a particular company. To use the characteristic line to estimate a portfolio beta, we use the same procedure. In this case, we regress the portfolio's returns against market returns. (As with the estimation of an individual stock beta, we use historical returns.)

An alternative method of estimating a portfolio beta is to find the weighted average of the betas of the individual stocks using Equation 13.

$$\beta_p = \sum_{i=1}^n w_i \beta_i \quad (13)$$

where:

$\beta_p$  = the portfolio beta

$\beta_i$  = the beta for stock  $i$

$w_i$  = the weight of stock  $i$  in the portfolio

$n$  = the number of stocks in the portfolio

Assume you hold a portfolio comprised of three stocks as follows:

<i>Stock</i>	<i>Beta</i>	<i>\$ Invested</i>
A	1.3	1,000
B	0.8	2,000
C	1.7	2,000

## Portfolio Management

$$\beta_p = w_a\beta_a + w_b\beta_b + w_c\beta_c$$

$$\beta_p = 0.2(1.3) + 0.4(0.8) + 0.4(1.7)$$

$$\beta_p = 0.26 + 0.32 + 0.68 = 1.26$$

If the risk-free rate is 6% and the market risk premium is 4%, we can use the CAPM to find the expected return on your portfolio.

$$\hat{R}_p = r_f + \beta(k_m - r_f) \quad (14)$$

$$\hat{R}_p = 0.06 + 1.26(0.04) = 0.1104 = 11.04\%$$

## THE INVESTMENT POLICY STATEMENT

The goal of portfolio management is to combine securities and other assets into portfolios that fit an investor's needs and to manage those portfolios to achieve investment goals. The investor's needs are defined in terms of portfolio return and risk, and the portfolio manager seeks to enhance return based on the level of risk the investor is willing to assume.

The first step in the portfolio management process requires a portfolio manager to complete a detailed analysis of the investor whose funds are being managed.

**Individual investors** structure their financial plans based on their age, financial status, and personal needs. The needs of individuals can be determined using many different frameworks. The life cycle framework utilizes the investor's stage of life to determine the general needs of the portfolio. For instance, a relatively young investor will be able to tolerate more risk and will tend not to rely heavily on the portfolio to provide income. In contrast, an older investor who is close to retirement has few years to recover from losses in the portfolio and will have a much lower risk tolerance. An investor who is well into his retirement years will also have a low tolerance for risk in the portfolio but will also rely heavily on the portfolio for income. As you can see, the stage of life that an investor occupies could significantly influence his investment style. However, not every investor within a life cycle category is the same.

An alternative to the life cycle model is the psychological profile framework. A portfolio manager who uses a psychological profile will attempt to determine the

relative risk-seeking or risk-avoidance behavior of the investor regardless of the investor's age. For example, some relatively young investors may have an extreme aversion to risk. In this case, the portfolio manager should allocate a higher-than-average proportion of the investor's portfolio to lower-risk assets. In addition, an older investor with substantial wealth may enjoy the thrill of investing in the high-technology market. Assuming that the investor has been educated on the implications of this investment style, it may be appropriate for the portfolio manager to allocate a higher-than-average proportion of the portfolio to risky securities.

The key point to remember is that a careful accounting of the investor's risk tolerance and return requirements is essential to properly manage the portfolio.

**Institutional investors** include entities such as endowments, pension funds, insurance companies, banks, investment companies, and government agencies. The needs of institutional investors are very different depending on that entity's reasons for existence. For example, a large pension fund will formulate investment goals and risk tolerance around the relative age of the workforce and the promised level of pension benefits. An endowment, on the other hand, does not need to support a retiring workforce but will have stated objectives that may include providing long-term or short-term funding for an arts program at a university. The important thing for a portfolio manager working with an institutional investor to remember is that, similar to individual investors, each institutional investor must be considered in isolation, and the portfolio should be designed to meet the investor's needs and goals.

No matter whether a portfolio manager is working with an individual or institutional investor, it is essential that he or she create an investment policy statement to govern the portfolio.

The **investment policy statement (IPS)** is the framework that provides structure to the investment process. It forces investors to understand their own needs and limitations and to articulate them within the construct of realistic goals. The policy statement helps investors understand the risks and costs of investing and guides the actions of the portfolio manager. The purpose of the policy statement is to impose investment discipline on the client and the portfolio manager. Every investment policy statement should be a formal written document that explicitly details the risk tolerance and return requirements as well as the limitations that the individual or institutional investor faces.

The policy statement should also state the standards by which the portfolio's performance will be judged and specify an appropriate **benchmark** that is consistent with the investor's risk preferences. The portfolio should be measured against the stated benchmark. Also, the manager should be evaluated on how closely he or she adhered to the portfolio's limitations related to attaining the required return.



### RETURN REQUIREMENTS AND RISK TOLERANCE

Return requirements are often stated initially in absolute terms (dollar amounts). However, specifying investment goals only in terms of return may expose an investor to inappropriate high-risk investment strategies. Also, return-only objectives can lead to unacceptable behavior on the part of investment managers, such as excessive trading (churning) to generate abnormally large commissions.

The risk tolerance of the investor will play a role in setting the return requirements since a high return requirement will necessitate a high risk tolerance, and a low risk tolerance will necessitate a low return requirement.

### LIMITATIONS TO INVESTMENTS

The investor's return requirements and risk tolerance will be restricted by and in some cases determined by the limitations unique to each investor. An investor's portfolio is constrained by various factors which include cash flow requirements, the investor's time horizon, tax concerns, and laws governing investment activity.

*Liquidity* requirements are the investor's need for ready cash. Thus, if the investor will depend on the portfolio to meet certain cash requirements, there will need to be a greater emphasis on assets that produce income. High liquidity requirements may limit the ability to invest in assets such as real estate that cannot be sold quickly on favorable terms. This may affect the portfolio's overall risk such that the return requirement may need to be reduced.

*Time horizon* refers to the time between making an investment and needing the funds generated by the investment. Since losses are harder to overcome in a short time frame, investors with shorter time horizons usually prefer lower-risk investments.

*Tax concerns* play an important role in investment planning since taxes will reduce the gross returns on an investor's portfolio. Tax codes in most countries are complex, and tax considerations will be widely different for each individual and institution.

*Legal and regulatory factors* are more of a concern to institutional investors than individuals, but the investment strategies of both may be restricted due to these constraints. For example, certain institutions are prohibited by law from investing in derivative securities.

*Unique circumstances* are considerations specific to the investor that do not fall into the other categories. For example, an individual might be unwilling for religious reasons to invest in companies that produce tobacco or alcohol.

## SUMMARY

### DIVERSIFICATION

- A. Diversification refers to the reduction of risk resulting from adding assets to the portfolio.
- B. Correlation is a statistic measuring the strength of relationship between two assets.
  - 1. The correlation ranges from  $-1$  to  $+1$ .
  - 2. The lower the correlation between assets, the greater the potential diversification benefits.

### RISK AVERSION

- A. Risk aversion refers to the degree to which the investor dislikes risk.
  - 1. All rational investors are risk averse.
  - 2. Rational investors will demand a risk premium to compensate for risk.
  - 3. Rational investors will always consider both risk and return in evaluating investments.
    - a. They will minimize risk for a given level of return.
    - b. They will maximize return for a given level of risk.
- B. An indifference curve maps out the set of risk/return combinations over which the investor is indifferent.
  - 1. Indifference curves portray the degree of risk aversion for an investor.
  - 2. Indifference curves slope upward and to the right in risk/return space.
  - 3. Indifference curves for a highly risk-averse investor have a very steep slope.
  - 4. Indifference curves for a less risk-averse investor have a less steep slope.

### THE EFFICIENT FRONTIER

- A. An efficient portfolio is one that has the smallest possible risk for a given expected return and has highest possible expected return for a given risk level.
- B. The efficient frontier is a plot of the risk-return combinations for the set of efficient portfolios.
- C. The optimal portfolio lies at the tangency point between an investor's highest indifference curve and the efficient frontier.

### THE CAPITAL MARKET LINE

- A. The capital market line is the efficient frontier given the existence of a risk-free asset.
  - 1. A risk-free asset is one in which there is no uncertainty about its future value.
  - 2. The Treasury bill serves as a proxy for the risk-free asset.

## Portfolio Management

3. The standard deviation of the risk-free asset equals zero, and its correlation with any other asset also equals zero.
- B. The capital market line is a plot of the risk-return values for portfolios that combine the risk-free asset with risky assets in an optimal manner.
  1. The capital market line starts at the risk-free rate and lies tangent to the Markowitz efficient frontier.
  2. The tangency point is risk/return coordinate for the market portfolio.
  3. The market portfolio is the portfolio that maximizes the excess return to risk ratio.
- C. The point of tangency between the an investor's highest indifference curve and the capital market line identifies the appropriate allocation to the risk-free asset and to the market portfolio.

## EXPECTED RETURN AND RISK FOR AN INDIVIDUAL SECURITY

- A. The expected return is based upon the purchase price and expected cash flows.
  1. The expected return on any risky investment is uncertain.
  2. Any change in the amount or timing of the expected cash flow(s) will make the actual (historical) return different from the expected return.
- B. Whenever there is uncertainty in expected returns, there is a probability distribution of the different possible returns.
  1. The expected return is the weighted average of the possible returns.
  2. Using the Empirical Rule we know that:
    - a. A 68% probability exists that the actual return will fall within  $\pm 1$  standard deviation of the expected return.
    - b. A 95% probability exists that the actual return will fall within  $\pm 2$  standard deviations from the expected return.
    - c. A 99% probability exists that the actual return will fall within  $\pm 3$  standard deviations from the expected

## EXPECTED RETURN AND RISK FOR A PORTFOLIO

- A. The expected return for a portfolio equals the weighted average of the expected returns for the individual assets comprising the portfolio.
  1. Portfolio weights equal the percentages allocated to each asset.
  2. Portfolio weights sum to 100%.
- B. Portfolio risk is measured by the standard deviation of the portfolio's returns.
- C. The standard deviation for a portfolio is not simply a weighted average of the individual stocks' standard deviations. Portfolio standard deviation depends upon:
  1. The standard deviations of the individual stocks.
  2. The weights of the stocks in the portfolio.
  3. The correlations among the stocks.

- D. The correlation between two stocks is a standardized measure of the tendency for two stocks to move together.
  1. Perfect positive correlation,  $+1.0$ , means they always move in the same direction to the same degree.
  2. Perfect negative correlation,  $-1.0$ , means they always move in the opposite direction to the same degree.
  3. A correlation of  $0.0$  means there is no statistical relationship between the movements of the stocks.
- E. All else being equal, the lower the correlations, the lower the portfolio standard deviation.
- F. When the correlation between two assets equals  $+1$ , no diversification benefit is derived, and the portfolio standard deviation will equal the weighted average of the standard deviations of the individual assets.
- G. If the correlation is less than  $1$ , the portfolio standard deviation will be less than the weighted average of the standard deviations of the individual assets.
- H. The total risk for any asset consists of two components: systematic risk and unsystematic risk.
- I. Systematic risk is that part of the asset's total risk that cannot be reduced through diversification.
- J. Unsystematic risk is that part of the asset's total risk that can be eliminated through diversification.
- K. Portfolio risk falls at a decreasing rate as more stocks are added to the portfolio.

### THE CAPITAL ASSET PRICING MODEL

- A. The capital asset pricing model (CAPM) provides the means for determining the required return for any risky asset.
  1. Required return is based upon the risk of the investment.
  2. The greater the risk associated with an investment, the higher the required return.
- B. The CAPM states that the required return equals the risk-free rate plus appropriate compensation for risk.
  1. The only risk that matters in the CAPM is systematic risk.
  2. Investors get compensated for systematic risk because it cannot be diversified away.
  3. Investors do not get compensated for unsystematic risk because it can be diversified away.
  4. An individual stock's risk premium is the stock's beta multiplied by the market risk premium,  $(k_m - r_f)$ .
  5. The equation for the CAPM is  $k_i = r_f + \beta_i (k_m - r_f)$ .
- C. The beta measures the degree to which the returns on a stock and on the broad market move together.
  1. A stock with average systematic risk has a beta equal to  $1$ .

## Portfolio Management

2. A stock with higher-than-average systematic risk has a beta greater than 1.
  3. A stock with less-than-average systematic risk has a beta less than 1.
  4. A stock with no systematic risk has a beta equal to zero.
- D. The historical beta for any stock is estimated as the slope of a linear regression of the stock returns against the market index returns.
1. The steeper the slope, the higher the beta.
  2. The regression line used to derive the beta is called the characteristic line.
  3. Beta is calculated as follows: 
$$\beta_i = \frac{\text{COV}_{i,m}}{\sigma_m} = \frac{\sigma_i}{\sigma_m} \rho_{i,m}$$
- E. The security market line is a graph of the capital asset pricing model.
1. The slope of the security market line equals the market risk premium.
  2. The starting point for the security market line is the risk-free rate.
- F. The security market line uses beta as the relevant measure of risk.
- G. The risk-free rate increases when inflation expectations rise, causing a parallel shift upward in the security market line. Required returns will rise and prices will fall.
- H. The risk-free rate decreases when inflation expectations fall, causing a parallel shift downward in the security market line. Required returns will fall and prices will rise.
- I. The slope of the security market line increases as risk aversion among investors rises. Required returns will rise and prices will fall.
- J. The slope of the security market line decreases as risk aversion among investors falls. Required returns will fall and prices will rise.
- K. The required return for a stock can be determined by knowing the stock's beta, the risk-free rate, and the market risk premium.
- L. Undervalued stocks (or portfolios) have an expected return greater than the required return and plot above the SML. Overvalued stocks (or portfolios) have an expected return less than the required return and plot below the SML.
- M. The beta for a portfolio is a weighted average of the betas of the individual stocks in the portfolio. It can also be estimated through the use of regression analysis.

## THE INVESTMENT POLICY STATEMENT

- A. The life cycle framework utilizes the investor's stage of life to determine the general needs of the portfolio.
1. A relatively young investor will usually not rely heavily on the portfolio to provide income and be able to tolerate more risk.
  2. An older investor who is close to retirement has few years to recover from losses in the portfolio and will have a much lower risk tolerance.

3. An investor who is well into her retirement years will usually rely heavily on the portfolio for income, and thus have a low tolerance for risk in the portfolio.
- B. The psychological profile framework attempts to determine the general risk-seeking or risk-avoidance behavior of the client. The psychological profile is then used to determine which assets are appropriate for the investor's portfolio.
  - C. Institutional investors encompass a wide variety of entities and include banks, endowments, insurance companies, and others. Institutional investment needs will vary widely depending on the reason for the institution's existence.
  - D. The *investment policy statement* (IPS) is the framework that provides structure to the investment process.
    1. The purpose of the IPS is to impose investment discipline on the client and the portfolio manager.
    2. Every IPS should:
      - a. Be a formal written document.
      - b. Detail the risk tolerance and return requirements as well as the limitations (time horizon, tax considerations, liquidity needs, legal requirements, unique circumstances) that the individual or institutional investor faces.

**PRACTICE QUESTIONS: PORTFOLIO THEORY**

1. Consider the common stocks of two independent companies, DFI Construction (DFI) and BCS Financial Services (BCS). Every time DFI's common stock posts a 2% gain, BCS's common stock realizes a 2% loss. What, if any correlation exists between the two stocks?
  - A. +1
  - B. -1.
  - C. +2.
  - D. 0.
  
2. A rational (i.e., risk-averse) investor would choose which of the following investments?

<i>Investment</i>	<i>Expected Return</i>	<i>Standard Deviation</i>
SpendCo	8%	22%
Thrift, Inc.	10%	20%
Money Corp.	11%	50%
SaveCo	9%	22%

- A. Thrift, Inc.
- B. SaveCo.
- C. Money Corp.
- D. SpendCo.

Use the following information for NorthCoast Oil Company to answer Questions 3 and 4.

<i>State of the Economy</i>	<i>Probability of State Occurring</i>	<i>Expected Return</i>
Rapid Expansion	20%	23%
Moderate Expansion	40%	11%
Moderate Recession	30%	-8%
Harsh Recession	10%	-12%

3. What is the expected return for NorthCoast Oil Company?
  - A. 4.8%.
  - B. 5.4%.
  - C. 11.0%.
  - D. 14.1%.

4. Which of the following is NorthCoast Oil Company's expected standard deviation?
- A. 7.2%.  
 B. 10.9%.  
 C. 11.2%.  
 D. 12.6%.
5. Find the standard deviation for a portfolio comprised of stocks  $a$  and  $b$  in the specified proportions.  $\rho_{a,b} = 0.40$

<i>Stock</i>	<i>Expected Return</i>	<i>Standard Deviation</i>	<i>Weight</i>
a	0.12	0.040	0.40
b	0.14	0.055	0.60

- A. 0.024.  
 B. 0.039.  
 C. 0.042.  
 D. 0.051.
6. Harvey Cutler is estimating a characteristic line for the stock of Orion Auto Group (OAG). To facilitate his analysis, Cutler has performed a linear regression. The results of the regression for OAG indicate slope and intercept parameters of 1.7 and 0.02, respectively. Which of the following conclusions can be drawn from Cutler's estimation of the characteristic line?
- A. OAG has significantly above-average systematic risk.  
 B. OAG has 70% more unsystematic risk than the market.  
 C. OAG has a slightly positive correlation with the risk-free asset.  
 D. OAG's total risk is 1.7 times greater than that of the aggregate market.



## Portfolio Management

7. The correlations between four different stocks are presented in the following correlation matrix:

<i>Correlation Matrix</i>				
	<i>Stock A</i>	<i>Stock B</i>	<i>Stock C</i>	<i>Stock D</i>
<i>Stock A</i>	1.00			
<i>Stock B</i>	0.10	1.00		
<i>Stock C</i>	-0.80	0.05	1.00	
<i>Stock D</i>	-0.20	0.95	0.00	1.00

Which combination of stocks would achieve the *most* diversification for an investor, and which combination would achieve the *least* diversification?

Most diversification

Least diversification

- A. Stock A and Stock C                      Stock B and Stock D  
B. Stock C and Stock D                      Stock B and Stock D  
C. Stock A and Stock C                      Stock A and Stock D  
D. Stock C and Stock D                      Stock A and Stock D
8. Jill Willshire is creating an equally weighted portfolio consisting of two assets. The portfolio has a beta of 0.5, and one of the assets has a beta of zero. Which of the following is TRUE of Willshire's portfolio?
- A. The systematic risk of the portfolio is greater than that of the market as a whole.  
B. One of the assets has a beta equal to that for the market portfolio.  
C. The correlation coefficient between the two assets in the portfolio is equal to  $-1$ .  
D. The expected return on the portfolio should be greater than the expected return on the aggregate market.
9. Estimate the beta for the following portfolio:

<i>Stock</i>	<i>Beta</i>	<i>\$ Invested</i>
A	0.90	2,500
B	1.50	3,000
C	0.70	3,500
D	0.50	1,000

- A. 0.73.  
B. 0.85.  
C. 0.97.  
D. 1.10.

10. An investor's portfolio has a beta of 1.1, financial analysts predict the market return for the coming year to be 10%, and U.S. Treasury Bills were just auctioned to yield 3%. What is the market risk premium?
- 3%.
  - 7%.
  - 19%.
  - 13%.
11. An investor's portfolio beta is 1.5 and has a required return of 14%. If the market risk premium is 6% and inflation expectations for the next year are increased from 2% to 4%, what will happen to the investor's required return? It will:
- remain the same because inflation impacts all assets equally.
  - increase 2 percentage points to 16%.
  - increase  $1.5(2) = 3$  percentage points to 17%.
  - be indeterminable—without knowing the risk-free rate, the impact on my portfolio cannot be determined.
12. Investors often gain confidence in the stock market when economic activity improves after a long recession. This increase in confidence is usually accompanied by a decrease in investors' aversion to risk. As investors' risk aversion falls:
- their required return will decrease.
  - a company's beta will rise.
  - a company's beta will fall.
  - their required return will increase.
13. Gretchen Freedman is explaining the attributes of the capital market line to a group of her firm's institutional clients. Which of Freedman's following comments made during the conference is INCORRECT?
- The capital market line is the efficient frontier when the risk-free asset exists.
  - Portfolios on the capital market line will dominate portfolios on the efficient frontier for risky assets.
  - The capital market line demonstrates the tradeoff between expected return and beta for a set of efficient portfolios.
  - An investor's optimal portfolio will be the tangency point between her indifference curve and the capital market line.

## Portfolio Management

Use the following data to answer Questions 14 and 15.

The market portfolio is represented by three assets: stocks, bonds, and real estate. Data on the percentage represented by each asset class in the market portfolio as well as the expected return on each asset class is presented in the following table:

<i>Market Portfolio</i>		
<i>Asset</i>	<i>%</i>	<i>E(R)</i>
Stocks	60%	15%
Bonds	30%	7%
Real Estate	10%	8%

14. If an investor finds it optimal to invest 60% of his assets in the market portfolio and 40% of his assets in the risk-free asset, what percentage of the investor's assets would be invested in each asset class?
- |    | <u>Stocks</u> | <u>Bonds</u> | <u>Real Estate</u> |
|----|---------------|--------------|--------------------|
| A. | 36%           | 18%          | 6%                 |
| B. | 60%           | 40%          | 0%                 |
| C. | 40%           | 20%          | 15%                |
| D. | 24%           | 12%          | 4%                 |
15. If an investor finds it optimal to invest 70% of his assets in the market portfolio and 30% of his assets in the risk-free asset, what is the expected return on the portfolio if the risk-free asset has an expected return of 5%?
- A. 7.8%.  
B. 8.9%.  
C. 9.2%.  
D. 9.8%.
16. Which of the following statements about the security market line is FALSE?
- A. The relevant risk measure in the security market line is the standard deviation.  
B. The security market line is a graph of the CAPM.  
C. The slope of the security market line equals the market risk premium.  
D. The security market line can be used to identify mispriced securities.
17. Assume your portfolio earned 10% with a standard deviation of 10%. Also assume the risk-free rate was 5%. What was your portfolio's Sharpe ratio?
- A. 0.  
B. 0.20.  
C. 0.50.  
D. 2.0.

18. Using the CAPM, what is the required return for a stock with a beta of 2 if the market risk premium is 8% and if the risk-free rate is 3%?
- 8%.
  - 13%.
  - 19%.
  - 22%.
19. Timothy McAllen is evaluating the common stock of Hillard Technology Group (HTG). McAllen has noted that the expected return on HTG is estimated by equity analysts to be 13% for the next year. McAllen further observes that HTG's sensitivity to the aggregate market is approximately 1.2. Forecasts of the market return and the risk-free rate for the next year are 9% and 2.5%, respectively. Given these estimates, which of the following is CORRECT?
- HTG stock is undervalued and has an alpha of positive 2.7%.
  - HTG stock is overvalued and has an alpha of positive 2.7%.
  - HTG stock is undervalued and has an alpha of negative 2.7%.
  - HTG stock is overvalued and has an alpha of negative 2.7%.
20. A portfolio manager is responsible for ensuring that his portfolio of 47 U.S. stocks mimics the returns of a broad market index. The manager has just hired an assistant who quickly recommends that twenty more stocks be added to the portfolio. The manager, however, disagrees with the assistant's recommendation and adds no further stocks. Which of the following reasons *best* summarizes why the portfolio manager made no change to the portfolio?
- The level of risk aversion for the current portfolio's investors had recently decreased.
  - The current portfolio has a negligible amount of unsystematic risk and thus no need for further diversification.
  - The current portfolio has an above-average level of systematic risk that will allow it to generate positive alpha.
  - The marginal utility of the current portfolio has already reached its theoretical minimum.
21. The Capital Asset Pricing Model (CAPM) states the risk premium for an individual security consists of:
- the product of the beta and the return on the market.
  - the product of the risk-free rate and beta.
  - the sum of the risk-free rate and the beta.
  - the product of the beta and the market risk premium.

## Portfolio Management

22. RFG Capital manages a \$100 million portfolio invested equally in 60 different stocks. Half of the securities have a beta of 1.2 and a standard deviation of 20%. The other half of the securities have a beta of 0.8 and a standard deviation of 25%. Which of the following statements *best* describes RFG Capital's portfolio?
- A. RFG Capital's portfolio is not diversified.
  - B. RFG Capital's portfolio has a standard deviation of 22.5%.
  - C. RFG Capital's portfolio has an expected return of 22.5%.
  - D. RFG Capital's portfolio has a beta equal to 1.0.
23. Peterson Tractors, Inc. has an expected return of 15% over the upcoming 12 months. The company has a beta of approximately 1.4. Analysts estimate that the risk-free rate is currently at a level of 4.5%, and the expected return on the market as a whole is equal to 12%. Which of the following statements regarding Peterson is INCORRECT?
- A. A sudden increase in investor risk aversion would result in a market risk premium of less than 7.5%.
  - B. A 1% increase in inflation expectations would lead to a 1% increase in Peterson's required return.
  - C. An increase in the market risk premium of 1.5% would lead to an increase in Peterson's required return of more than 1.5%.
  - D. Peterson's stock has an expected alpha of zero.
24. Sarah manages a \$1,000,000 diversified portfolio consisting of 20 equally weighted common stocks. The current portfolio has a beta of 0.8. She decides to sell a security that has a beta of 1.5. Sarah uses the proceeds to purchase another stock that has a beta of 0.5. Based on Sarah's changes, calculate the new beta of the portfolio:
- A. 0.65.
  - B. 0.75.
  - C. 0.85.
  - D. 0.95.
25. As a senior manager at your investment management firm, you are expected to provide regular training to new employees. This month you are expected to discuss the slope of the characteristic line and the security market line. What should you tell the new employees that the slopes of these two lines equal?
- | <u>Characteristic line</u> | <u>Security market line</u> |
|----------------------------|-----------------------------|
| A. Standard deviation      | Market risk premium         |
| B. Beta                    | Market return               |
| C. Standard deviation      | Market return               |
| D. Beta                    | Market risk premium         |

Use the following information to answer Questions 26 and 27:

Suppose you invest \$9,000 in the S&P 500 stock market index and \$1,000 in Treasury bills. Assume the S&P 500 index has an expected return of 10% and a standard deviation of 20%. The T-bill rate is 2%.

26. What is the expected return on your portfolio?
  - A. 6.0%.
  - B. 9.0%.
  - C. 9.2%.
  - D. 10.0%.
  
27. What is the standard deviation on your portfolio?
  - A. 9%.
  - B. 12%.
  - C. 15%.
  - D. 18%.
  
28. A portfolio manager at 22nd Street Capital has just added 200 shares of EBN stock to his portfolio. The portfolio manager added EBN after considering the company and three of its primary competitors all offering the same expected return. Which of the following is *most likely* the reason the portfolio manager chose EBN over its competitors?
  - A. EBN offered the largest level of alpha of the four stocks under consideration.
  - B. EBN exhibited the lowest level of correlation with the manager's existing portfolio of assets.
  - C. EBN had the greatest increase in quantity of sales over the last six months.
  - D. Of the four stocks under consideration, EBN was trading at the lowest price per share in the stock market.
  
29. If the investor's risk aversion increases:
  - A. the slope of the capital market line will fall.
  - B. the tangency point between the investor's indifference curve and the capital market line slides down the capital market line.
  - C. the slope of the security market line rises.
  - D. the investor's indifference curve becomes less steep.
  
30. Another name for unsystematic risk is:
  - A. non-diversifiable risk.
  - B. diversifiable risk.
  - C. market risk.
  - D. beta.

## Portfolio Management

31. According to the CAPM, the required return for an asset with a zero beta equals:
- A. zero.
  - B. the market return.
  - C. the market premium.
  - D. the risk-free rate.
32. According to the life cycle approach to investing, for which of the following scenarios would a large allocation of the portfolio to risky assets be the *most appropriate*?
- A. For a client who is newly retired, age 63.
  - B. For a client who is in the later stages of life, age 90.
  - C. For a client who is in earlier stage of his career, age 30.
  - D. For a client who is in the later stage of his career, age 55.
33. Jimmy Daniels has just been promoted to managing director of his asset management firm. Daniels will no longer manage individual accounts but will oversee all of the portfolio managers working for his firm. The manager taking over Daniels' accounts should be able to meet Daniels' former investors' goals by reading their associated investment policy statements if Daniels has done which of the following?
- A. Carefully defined the return requirements according to the national average for investors of the same age.
  - B. Carefully defined the risk tolerance and return requirements in light of the portfolio limitations.
  - C. Tracked the portfolio investment style using notes in the investors' electronic accounts.
  - D. Maintained an investment style that tracks the benchmark stated in the investment policy statement.
34. Your high net-worth client states she is investing now for her daughter's college education. Your client is 35 years old, and her daughter will attend college in 10 years. Select the *most appropriate* statement.
- A. Your client has high short-term cash flow limitations.
  - B. Your client has high legal restrictions and low tax planning needs.
  - C. Your client's most important limitations are time horizon and taxes.
  - D. Your client is highly risk averse.

## PRACTICE QUESTION ANSWERS: PORTFOLIO THEORY

1. **B** When stocks move in exactly the opposite direction they are said to be perfectly negatively correlated and have a correlation coefficient of negative one.
2. **A** A rational investor will always maximize return for a given level of risk. The standard deviations of SaveCo and SpendCo are equal, so the investor should choose SaveCo since the expected return is higher for the same risk level. Thrift, Inc., should be preferred to SaveCo since it has both a higher expected return and a lower standard deviation. Money Corp. has a risk level (standard deviation) that is disproportionately higher than its higher level of expected return. By investing in Money Corp., the investor would need to more than double standard deviation in order to gain only one percentage point in expected return, which would not be a rational choice.
3. **B**  $E(R) = 0.2(23) + 0.4(11) + 0.3(-8) + 0.1(-12) = 5.4\%$
4. **D** 
$$\sigma_p = \left[ \begin{aligned} &(0.23 - 0.054)^2(0.2) + (0.11 - 0.054)^2(0.4) \\ &+ (-0.08 - 0.054)^2(0.3) + (-0.12 - 0.054)^2(0.1) \end{aligned} \right]^{1/2} = 12.6\%$$
5. **C** 
$$\begin{aligned} \sigma_p^2 &= (0.4)^2(0.04)^2 + (0.6)^2(0.055)^2 + 2(0.4)(0.6)(0.04)(0.055)(0.4) \\ &= 0.0017674 \\ \sigma_p &= (\sigma_p^2)^{1/2} = (0.0017674)^{1/2} = 0.042 \end{aligned}$$
6. **A** The characteristic line is used to estimate a stock's or a portfolio's beta, which is a measure of systematic risk. The market by definition has a beta of one. Therefore a stock with a beta greater than one has above-average systematic risk, and a stock with a beta less than one has below-average systematic risk.
7. **A** To achieve the most diversification, you want to combine stocks with the least correlation. Stock A and Stock C have a correlation of  $-0.80$ , the lowest in the table. To achieve the least diversification, combine stocks with the highest correlation. Stock B and Stock D have the greatest level of correlation.
8. **B** Portfolio beta is the weighted average of the betas within the portfolio. If an equally weighted portfolio of two assets has a beta of 0.5 and one asset in the portfolio has a beta of zero, the following equation must be true:

$$0.5 = 0.5(0) + 0.5(\beta) \Rightarrow \beta = \frac{0.5 - 0.5(0)}{0.5} = \frac{0.5}{0.5} = 1$$

Therefore, the second asset in the portfolio has a beta of 1. Since the market portfolio also has a beta of 1, the asset in the portfolio has the same beta as the market portfolio. In fact, the second asset could even be the market portfolio, as suggested by CAPM.



## Portfolio Management

9. C  $\beta_p \sum_{i=1}^n w_i \beta_i = \frac{2.5}{10.0}(0.9) + \frac{3.0}{10.0}(1.5) + \frac{3.5}{10.0}(0.7) + \frac{1.0}{10.0}(0.5) = 0.97$
10. B The market risk premium is the slope of the CAPM,  $(k_m - r_f)$ . Since  $r_f = 3\%$  and  $k_m = 10\%$ , the market premium is 7%.
11. B When inflation expectations increase, the SML shifts up. The required return on all assets will increase two percentage points, so your portfolio's required return will increase to 16%.
12. A Modern Portfolio Theory (MPT) suggests that as risk aversion falls, investors will demand a smaller risk premium per unit of risk assumed. A smaller risk premium will cause the overall required rates of return on risky assets to decrease.
13. C The capital market line uses total risk (standard deviation) as its measure of risk, not beta.
14. A 60% of the total available funds will be allocated to the market portfolio which consists of three asset classes. The available funds would be allocated to the three asset classes in the following percentages:
- $$W_{\text{stocks}} = 0.60 \times 0.60 = 0.36 = 36\%$$
- $$W_{\text{bonds}} = 0.60 \times 0.30 = 0.18 = 18\%$$
- $$W_{\text{real estate}} = 0.60 \times 0.10 = 0.06 = 6\%$$
15. D To answer this question, we must first find the expected return on the market portfolio (which is the weighted average of the expected returns on the portfolio assets):
- $$E(R_p) = 0.6(15\%) + 0.3(7\%) + 0.1(8\%) = 11.9\%$$
- Next we find the weighted average of the expected returns on the market portfolio and the risk-free asset (70% is allocated to the market portfolio and 30% to the risk-free asset):
- $$E(R_p) = 0.7(11.9\%) + 0.3(5\%) = 9.8\%$$
16. A The security market line is a graph of the CAPM and as such uses the beta as the relevant measure of risk.
17. C The Sharpe ratio equals the excess return (portfolio return minus the risk free rate) divided by the standard deviation of the portfolio:
- $$(0.10 - 0.05) / 0.10 = 0.05 / 0.10 = 0.50$$
18. C  $k = r_f + \beta \times \text{market risk premium} = 0.03 + 2(0.08) = 0.19$

19. **A** Alpha equals the difference between the expected (predicted) return and the required return. HTG has an expected return of 13% over the coming year. HTG's required return can be calculated using the CAPM:

$$R_{\text{HTG}} = 2.5 + 1.2(9.0 - 2.5) = 10.3\%$$

This means that the stock is attractive (undervalued today) by  $13.0 - 10.3 = 2.7$  percentage points.

20. **B** Diversification reduces or eliminates only unsystematic risk. A portfolio of 47 stocks most likely contains little unsystematic risk. Therefore, adding more stocks to the portfolio will not significantly increase the diversification benefits of the portfolio. Therefore the cost to add more stocks to the portfolio may not be worthwhile.
21. **D**  $\beta(R_m - r_f) =$  company risk premium;  $(R_m - r_f) =$  market risk premium

22. **D** 
$$\beta_p \sum_{i=1}^n w_i \beta_i$$

$$\beta_p = 0.5(1.2) + 0.5(0.8) = 1.0$$

It is statistically incorrect to take the weighted-average standard deviation to create a portfolio standard deviation because of correlation of the individual assets.

23. **A** An increase in investor risk aversion would tend to drive the market risk premium up. Since the current market risk premium is equal to  $12\% - 4.5\% = 7.5\%$ , an increase in risk aversion would lead to a market risk premium of greater than, not less than, 7.5%.

24. **B** 
$$\beta_p \sum_{i=1}^n w_i \beta_i$$

$$w_i = \frac{1}{20} = 0.05$$

$$\text{portfolio beta} = 0.8 - 0.05(1.5) + 0.05(0.5) = 0.75$$

25. **D** The slope of the characteristic line is beta. It is an estimate of the sensitivity of a security's returns to the returns on the aggregate market. The security market line starts at the risk-free rate and extends through the market portfolio, which has a beta equal to one. Slope equals rise over run: Rise = market return less risk free rate, and run = market beta = 1, so rise/run = market return less risk free rate, which is the market risk premium.

26. **C** The portfolio expected return equals the weighted average of the two asset returns. You invest 90% in the market index and 10% in T-bills. The portfolio expected return therefore equals:

$$0.90(0.10) + 0.10(0.02) = 0.092$$

## Portfolio Management

27. **D** Note that the standard deviation for the risk-free asset equals zero. Also, the correlation of the risk-free asset with any risky asset equals zero. So the portfolio standard deviation formula simplifies to:

$$w_m \sigma_m = 0.90(0.20) = 0.18$$

28. **B** If all of the stocks under consideration are otherwise comparable in terms of expected return and risk, the stock with the lowest correlation with the existing portfolio will offer the greatest amount of diversification benefits.
29. **B** As the investor's risk aversion increases, his indifference curves will become steeper, causing the point of tangency to move closer to the risk-free point (slide down the capital market line).
30. **B** Other names are diversifiable, firm-unique, or company-specific risk.
31. **D** The equation for the CAPM is  $k = r_f + \beta(k_m - r_f)$ . If beta equals zero, the required return equals the risk-free rate.
32. **C** Investors in the earlier stages of their career have higher risk tolerance and higher return requirements. They have a long time horizon and can withstand portfolio volatility. A higher allocation to stocks is more appropriate in this stage of life.
33. **B** An investment policy statement should state clearly and carefully the risk tolerance, return requirements, and portfolio limitations so that the policy is portable to another manager should the need arise. These items should be defined according to the investor's unique circumstances, not according to national averages or other rules of thumb.
34. **C** Investor limitations typically include cash flow (liquidity) needs, investment time horizon, legal requirements, and taxes. Your client has a clear time horizon limitation of 10 years and will pay for her daughter's education with after-tax dollars. As her advisor, you must emphasize these limitations. It does not appear that your client will need any withdrawals from her investment portfolio until 10 years from now (low cash flow limitations), and there's nothing in the information to suggest that your client is highly risk averse.

# CFA INSTITUTE CODE OF ETHICS AND STANDARDS OF PROFESSIONAL CONDUCT

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When you register for the Level I exam, you will become a Candidate for the CFA designation. Candidates are required to abide by the **CFA Institute Code of Ethics and Standards of Professional Conduct** (the “Code and Standards”). The Code and Standards are described in CFA Institute’s *Standards of Practice Handbook*.

The *Standards of Practice Handbook* is available from the CFA Institute Web site and is reprinted in the CFA Curriculum volumes you will receive when you register for each level of the CFA Exams. The *Handbook* includes guidance for each of the Standards, recommended procedures for compliance, and many examples of how the Standards should be applied in practice.

In addition to the Code and Standards, with which all Members and Candidates must comply, CFA Institute has developed several sets of voluntary standards that relate to particular segments of the financial services industry. These include:

- Global Investment Performance Standards (GIPS®) for calculating and presenting investment performance.
- Asset Manager Code of Professional Conduct.
- CFA Institute Research Objectivity Standards.
- CFA Institute Soft Dollar Standards. “Soft dollar” arrangements are benefits a broker might provide to investment managers that direct clients’ trades to the broker. Soft dollars must be used in ways that benefit the clients, not just the investment manager.

The Code of Ethics<sup>1</sup> is as follows:

“Members of CFA Institute (including Chartered Financial Analyst® [CFA®] charterholders) and candidates for the CFA designation (“members and candidates”) must:

- act with integrity, competence, dignity, respect, and in an ethical manner with the public, clients, prospective clients, employers, employees, colleagues in the investment profession, and other participants in the global capital markets.

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## CFA Institute Code of Ethics and Standards of Professional Conduct

- place the integrity of the investment profession and the interests of clients above their own personal interests.
- use reasonable care and exercise independent professional judgment when conducting investment analysis, making investment recommendations, taking investment actions, and engaging in other professional activities.
- practice and encourage others to practice in a professional and ethical manner that will reflect credit on themselves and the profession.
- promote the integrity of, and uphold the rules governing, capital markets.
- maintain and improve their professional competence and strive to maintain and improve the competence of other investment professionals.”

The Standards of Professional Conduct<sup>2</sup> are as follows:

### I. PROFESSIONALISM

- A. Knowledge of the Law.** Members and Candidates must understand and comply with all applicable laws, rules, and regulations (including the CFA Institute *Code of Ethics and Standards of Professional Conduct*) of any government, regulatory organization, licensing agency, or professional association governing their professional activities. In the event of conflict, Members and Candidates must comply with the more strict law, rule, or regulation. Members and Candidates must not knowingly participate or assist in any violation of laws, rules, or regulations and must disassociate themselves from any such violation.
- B. Independence and Objectivity.** Members and Candidates must use reasonable care and judgment to achieve and maintain independence and objectivity in their professional activities. Members and Candidates must not offer, solicit, or accept any gift, benefit, compensation, or consideration that reasonably could be expected to compromise their own or another’s independence and objectivity.
- C. Misrepresentation.** Members and Candidates must not knowingly make any misrepresentations relating to investment analysis, recommendations, actions, or other professional activities.
- D. Misconduct.** Members and Candidates must not engage in any professional conduct involving dishonesty, fraud, or deceit or commit any act that reflects adversely on their professional reputation, integrity, or competence.

### II. INTEGRITY OF CAPITAL MARKETS

- A. Material Nonpublic Information.** Members and Candidates who possess material nonpublic information that could affect the value of an investment must not act or cause others to act on the information.
- B. Market Manipulation.** Members and Candidates must not engage in practices that distort prices or artificially inflate trading volume with the intent to mislead market participants.

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<sup>2</sup> Ibid.

### III. DUTIES TO CLIENTS

- A. **Loyalty, Prudence, and Care.** Members and Candidates have a duty of loyalty to their clients and must act with reasonable care and exercise prudent judgment. Members and Candidates must act for the benefit of their clients and place their clients' interests before their employer's or their own interests.
- B. **Fair Dealing.** Members and Candidates must deal fairly and objectively with all clients when providing investment analysis, making investment recommendations, taking investment action, or engaging in other professional activities.
- C. **Suitability.**
  - 1. When Members and Candidates are in an advisory relationship with a client, they must:
    - a. Make a reasonable inquiry into a client's or prospective clients' investment experience, risk and return objectives, and financial constraints prior to making any investment recommendation or taking investment action and must reassess and update this information regularly.
    - b. Determine that an investment is suitable to the client's financial situation and consistent with the client's written objectives, mandates, and constraints before making an investment recommendation or taking investment action.
    - c. Judge the suitability of investments in the context of the client's total portfolio.
  - 2. When Members and Candidates are responsible for managing a portfolio to a specific mandate, strategy, or style, they must make only investment recommendations or take investment actions that are consistent with the stated objectives and constraints of the portfolio.
- D. **Performance Presentation.** When communicating investment performance information, Members or Candidates must make reasonable efforts to ensure that it is fair, accurate, and complete.
- E. **Preservation of Confidentiality.** Members and Candidates must keep information about current, former, and prospective clients confidential unless:
  - 1. The information concerns illegal activities on the part of the client or prospective client,
  - 2. Disclosure is required by law, or
  - 3. The client or prospective client permits disclosure of the information.

### IV. DUTIES TO EMPLOYERS

- A. **Loyalty.** In matters related to their employment, Members and Candidates must act for the benefit of their employer and not deprive their employer of the advantage of their skills and abilities, divulge confidential information, or otherwise cause harm to their employer.

**B. Additional Compensation Arrangements.** Members and Candidates must not accept gifts, benefits, compensation, or consideration that competes with, or might reasonably be expected to create a conflict of interest with, their employer's interest unless they obtain written consent from all parties involved.

**C. Responsibilities of Supervisors.** Members and Candidates must make reasonable efforts to detect and prevent violations of applicable laws, rules, regulations, and the Code and Standards by anyone subject to their supervision or authority.

## V. INVESTMENT ANALYSIS, RECOMMENDATIONS, AND ACTIONS

**A. Diligence and Reasonable Basis.** Members and Candidates must:

1. Exercise diligence, independence, and thoroughness in analyzing investments, making investment recommendations, and taking investment actions.
2. Have a reasonable and adequate basis, supported by appropriate research and investigation, for any investment analysis, recommendation, or action.

**B. Communication with Clients and Prospective Clients.** Members and Candidates must:

1. Disclose to clients and prospective clients the basic format and general principles of the investment processes used to analyze investments, select securities, and construct portfolios and must promptly disclose any changes that might materially affect those processes.
2. Use reasonable judgment in identifying which factors are important to their investment analyses, recommendations, or actions and include those factors in communications with clients and prospective clients.
3. Distinguish between fact and opinion in the presentation of investment analysis and recommendations.

**C. Record Retention.** Members and Candidates must develop and maintain appropriate records to support their investment analysis, recommendations, actions, and other investment-related communications with clients and prospective clients.

## VI. CONFLICTS OF INTEREST

**A. Disclosure of Conflicts.** Members and Candidates must make full and fair disclosure of all matters that could reasonably be expected to impair their independence and objectivity or interfere with respective duties to their clients, prospective clients, and employer. Members and Candidates must ensure that such disclosures are prominent, are delivered in plain language, and communicate the relevant information effectively.

**B. Priority of Transactions.** Investment transactions for clients and employers must have priority over investment transactions in which a Member or Candidate is the beneficial owner.

- C. **Referral Fees.** Members and Candidates must disclose to their employer, clients, and prospective clients, as appropriate, any compensation, consideration, or benefit received from, or paid to, others for the recommendation of products or services.

VII. **RESPONSIBILITIES AS A CFA INSTITUTE MEMBER OR CFA CANDIDATE**

- A. **Conduct as Members and Candidates in the CFA Program.** Members and Candidates must not engage in any conduct that compromises the reputation or integrity of CFA Institute or the CFA designation or the integrity, validity, or security of the CFA examinations.
- B. **Reference to CFA Institute, the CFA Designation, and the CFA Program.** When referring to CFA Institute, CFA Institute membership, the CFA designation, or candidacy in the CFA Program, Members and Candidates must not misrepresent or exaggerate the meaning or implications of membership in CFA Institute, holding the CFA designation, or candidacy in the CFA Program.



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