Mathematical Aspects of Dynamic Systems: Uniform Oscillator and Beat Phenomenon

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Introduction

Greetings, Professor Darren. My name is Cheng Hao Min from Xiamen University Malaysia, Student ID MAT2004144, and today I will be presenting a study focused on the mathematical aspects of dynamic systems, particularly discussing the Uniform Oscillator and the Beat Phenomenon.

1 Structure of the Presentation

Let's look at the structure of this presentation. We will initially focus on the basics of vector fields on a line and then shift our attention toward vector fields on a circle, followed by an introduction to a uniform oscillator. In the end, we will discuss a fascinating concept known as the Beat Phenomenon.

2 Vector Fields on a Circle

In previous studies, we have been solving the equation $\dot{x} = f(x)$, which is visualized as a vector field on the line. Today, we will introduce a new differential equation that corresponds to the vector field on a circle and explores a simple model of a uniform oscillator. We will also discuss the Beat Phenomenon.

3 Uniform Oscillator

Here, I'll talk about a concept called vector fields on a circle. By labeling each point on the circle according to its angle from the origin, we can define a function that tells us the rate of change of this angle, also known as velocity. The circle introduces some unique characteristics due to its cyclic nature. For example, the trajectories will return to their initial place after a shift of exactly 2π, allowing for the existence of periodic solutions for the first time.

4 Uniform Oscillator Model

We now introduce a basic dynamic system model known as the uniform oscillator. This model follows the simple differential equation $\dot{\theta} = \omega$. Solving this equation gives us $\theta = \omega t + \theta(0)$, where θ is the point on a circle corresponding to the origin that varies with time, $\dot{\theta}$ is the velocity vector of that point, and ω is a constant representing the rate of change of θ with respect to time.

5 Example: Speedy and Pokey

Consider an example where two joggers, Speedy and Pokey, are running at steady paces around a circular track. Speedy completes one lap in T_1 seconds, while Pokey takes longer, T_2 seconds. Speedy will periodically overtake Pokey. How long will it take for Speedy to lap Pokey once, assuming they start together?

6 Solution: Speedy and Pokey

By using the formula derived from our uniform oscillator model, we can find that Speedy will lap Pokey after a time of $\frac{1}{\frac{1}{T_1}-\frac{1}{T_2}}$. This interaction between two entities moving at different frequencies leads us to the fascinating concept of the Beat Phenomenon.

7 Beat Phenomenon

This brings us to the Beat Phenomenon. Although both joggers are running independently with no direct interaction, they still show a "periodic interactive" behavior. This Beat Phenomenon refers to the periodic phase synchronization and desynchronization seen between two independent oscillators with different frequencies.

8 Example: Church Bells

Consider the case of two church bells ringing every 3 and 4 seconds, respectively. How long will it be until the next time they ring together? Utilizing the formula we discussed earlier, we find that it takes 12 seconds before the bells ring together again.

9 Experimental Demonstration

To better understand the Beat Phenomenon, let's conduct a little experiment using a multiple-tone generator.