The 4th Tetration of π

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$$\pi^{\pi^{\pi^{\pi}}} = \left(3 \times \frac{\pi}{3}\right)^{\pi^{\pi^{\pi}}}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\pi^{\pi^{\pi}}} \left(\frac{\pi}{3}\right)^{\pi^{\pi^{\pi}}}$$

Make a new definition for $\left(\frac{\pi}{3}\right)^{\pi^{\pi^{\pi}}}$

$$3^{b} = \left(\frac{\pi}{3}\right)^{\pi^{\pi^{\pi}}}$$

$$\ln\left(3^{b}\right) = \ln\left(\left(\frac{\pi}{3}\right)^{\pi^{\pi^{\pi}}}\right)$$

$$\mathrm{b} * \ln(3) = \pi^{\pi^{\pi}} \ln\left(\frac{\pi}{3}\right)$$

$$\mathrm{b} = \frac{\pi^{\pi^{\pi}} \ln\left(\frac{\pi}{3}\right)}{\ln(3)}$$

$$\therefore 3^{\frac{\pi^{\pi^{\pi}} \ln\left(\frac{\pi}{3}\right)}{\ln(3)}} = \left(\frac{\pi}{3}\right)^{\pi^{\pi^{\pi}}}$$

Substituting that in,

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\pi^{\pi^{\pi}}} \left(\frac{\pi}{3}\right)^{\pi^{\pi^{\pi}}}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\pi^{\pi^{\pi}}} 3^{\frac{\pi^{\pi^{\pi}} \ln\left(\frac{\pi}{3}\right)}{\ln(3)}}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\pi^{\pi^{\pi}}} 3^{\frac{\pi^{\pi^{\pi}} (\ln(\pi) - \ln(3))}{\ln(3)}}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\pi^{\pi^{\pi}}} 3^{\pi^{\pi^{\pi}} \frac{\ln(\pi) - \ln(3)}{\ln(3)}}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\pi^{\pi^{\pi}}} 3^{\pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)}$$

ω is an integer that makes $\left\lfloor \pi^{\pi^{\pi}} \right\rfloor + \left\lfloor \pi^{\pi^{\pi}} \left(\frac{\ln(\pi)}{\ln(3)} - 1\right) \right\rfloor - \omega + 1$ prime this will be used later with Fermat's Little Theorem

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\omega} 3^{\left\{\pi^{\pi^{\pi}}\right\}} 3^{\left\lfloor \pi^{\pi^{\pi}} \right\rfloor - \omega} 3^{\pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\omega} 3^{\{\pi^{\pi^{\pi}}\}} 3^{\lfloor \pi^{\pi^{\pi}} \rfloor - \omega} 3^{\left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor + \left\{\pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\}}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\omega} 3^{\{\pi^{\pi^{\pi}}\}} 3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega} 3^{\left\{\pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\}}$$

$$\pi^{\pi^{\pi^{\pi}}} = 3^{\omega} 3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega} 3^{\{\pi^{\pi^{\pi}}\} + \left\{\pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\}}$$

Approximating $3^{\{\pi^{\pi^{\pi}}\} + \left\{\pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\}}$ with a fraction where a and b are integer will help us later,

$$\frac{a}{b} \approx 3^{\{\pi^{\pi^{\pi}}\} + \left\{\pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\}}$$

Subbing our approximation in

$$\pi^{\pi^{\pi^{\pi}}} \approx 3^{\omega} 3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega} \frac{a}{b}$$

$$\pi^{\pi^{\pi^{\pi}}} \approx 3^{\omega} a \frac{3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega}}{b}$$

$$\pi^{\pi^{\pi^{\pi}}} \approx \sum_{n=1}^{3^{\omega}} a \frac{3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega}}{b}$$

Now let us take the fractional part of both sides

$$\left\{\pi^{\pi^{\pi^{\pi}}}\right\} \approx \left\{\sum_{n=1}^{3^{\omega}} a \frac{3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega}}{b}\right\}$$

Because we are summing, we can take the fraction part of the inside of the sum kind of like this

$$\{1.5 + 1.5 + 1.5\} = \{\{1.5\} + \{1.5\} + \{1.5\}\} = \{0.5 + 0.5 + 0.5\}$$

$$\left\{\pi^{\pi^{\pi^{\pi}}}\right\} \approx \left\{\sum_{n=1}^{3^{\omega}} \left\{a \frac{3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega}}{b}\right\}\right\}$$

Expanding the fraction

$$\approx \left\{\sum_{n=1}^{3^{\omega}} \left\{a \frac{3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega}}{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega + 1} \frac{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega + 1}{b}\right\}\right\}$$

$$\approx \left\{\sum_{n=1}^{3^{\omega}} \left\{\frac{3^{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega}}{\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega + 1} \frac{a\left(\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}}\left(\frac{\ln(\pi)}{\ln(3)} - 1\right)\right\rfloor - \omega + 1\right)}{b}\right\}\right\}$$

If $\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b} \in \mathbb{N}_1$

$$\approx \left\{\sum_{n=1}^{3^\omega}\left\{\sum_{n=1}^{\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b}} \frac{3^{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega}}{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1}\right\}\right\}$$

$$\approx \left\{\sum_{n=1}^{3^\omega}\left\{\sum_{n=1}^{\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b}} \left\{\frac{3^{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega}}{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1}\right\}\right\}\right\}$$

Now if we consider Fermat's Little Theorem which is

$$a^{p-1} \equiv 1\ (mod\ p)$$

p being a prime number and a being an integer. That like saying

$$\left\{\frac{a^{p-1}}{p}\right\} = \frac{1}{p}$$

Applying that we get

$$\left\{\pi^{\pi^{\pi^\pi}}\right\} \approx \left\{\sum_{n=1}^{3^\omega}\left\{\sum_{n=1}^{\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b}} \left\{\frac{3^{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega}}{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1}\right\}\right\}\right\}$$

$$\approx \left\{\sum_{n=1}^{3^\omega}\left\{\sum_{n=1}^{\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b}} \frac{1}{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1}\right\}\right\}$$

$$\approx \left\{\sum_{n=1}^{3^\omega}\left\{\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b} \frac{1}{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1}\right\}\right\}$$

$$\approx \left\{3^\omega\left\{\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b} \frac{1}{\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1}\right\}\right\}$$

$$\approx \left\{\sum_{n=1}^{3^\omega}\left\{\frac{a\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}{b\left(\lfloor \pi^{\pi^\pi} \rfloor + \left\lfloor \pi^{\pi^\pi}\left(\frac{\ln(\pi)}{\ln(3)}-1\right)\right\rfloor - \omega + 1\right)}\right\}\right\}$$

$$\approx \left\{ \sum_{n=1}^{3^{\omega}} \left\{ \frac{a}{b} \right\} \right\}$$

$$\approx \left\{ 3^{\omega} \left\{ \frac{a}{b} \right\} \right\}$$

For this to work $\left(\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}} \left(\frac{\ln(\pi)}{\ln(3)} - 1 \right) \right\rfloor - \omega + 1 \right)$ must be prime and $\frac{a\left(\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}} \left(\frac{\ln(\pi)}{\ln(3)} - 1 \right) \right\rfloor - \omega + 1 \right)}{b}$ must be an integer.

In the table below $\left(\lfloor \pi^{\pi^{\pi}} \rfloor + \left\lfloor \pi^{\pi^{\pi}} \left(\frac{\ln(\pi)}{\ln(3)} - 1 \right) \right\rfloor - \omega + 1 \right) = 1396421656717678499$ which means $\omega = 3$

$$1.41256907853377\ldots = 3^{\left\{ \pi^{\pi^{\pi}} \right\} + \left\{ \pi^{\pi^{\pi}} \left(\frac{\ln(\pi)}{\ln(3)} - 1 \right) \right\}} \approx \frac{a}{b}$$

a	b	$^{a}/_{b}$	$\frac{a * 1396421656717678499}{b}$	$3^{3}\left\{ \frac{a}{b} \right\}$
7	5	1.4	1954990319404749824	10.8
141	100	1.41	1968954535971926784	11.07
1413	1000	1.413	1973143800942079744	11.151
7063	5000	1.4126	1972585232279392768	11.1402
141257	100000	1.41257	1972543339629691136	11.13939
1412569	1000000	1.412569	1972541943208034304	11.139363
14125691	10000000	1.41256910	1972542082850200064	11.1393657
35314227	25000000	1.41256908	1972542054921766912	11.13936516
1412569079	1000000000	1.4125690790	1972542053525345280	11.139365133
2825138157	2000000000	1.41256907850	1972542052827134464	11.1393651195
141256907853	100000000000	1.412569078530	1972542052869027072	11.13936512031
706284539267	500000000000	1.4125690785340	1972542052874612736	11.139365120418
7062845392669	5000000000000	1.41256907853380	1972542052874333440	11.1393651204126

Largest a and b I've computed

a =14125690785337739326584946318544187247150072812421472319974726554957176172116
57041575512524933438105049675063220254847491056045087070451045421235892366387561
1028364513773159148573391434915836864776087

b =10000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000

$\frac{a*1396421656717678499}{b} = 1972542052874297088$

$3^3 \left\{\frac{a}{b}\right\} =$
11.139365120411896181779355060069305567305196593537975263931761698384375664714740
12253883817320282883634122670694688088225851321735090217822637336909389246414977
658418718752970114815687427275953489543571