Parachute Flight Time Optimization in PUBG

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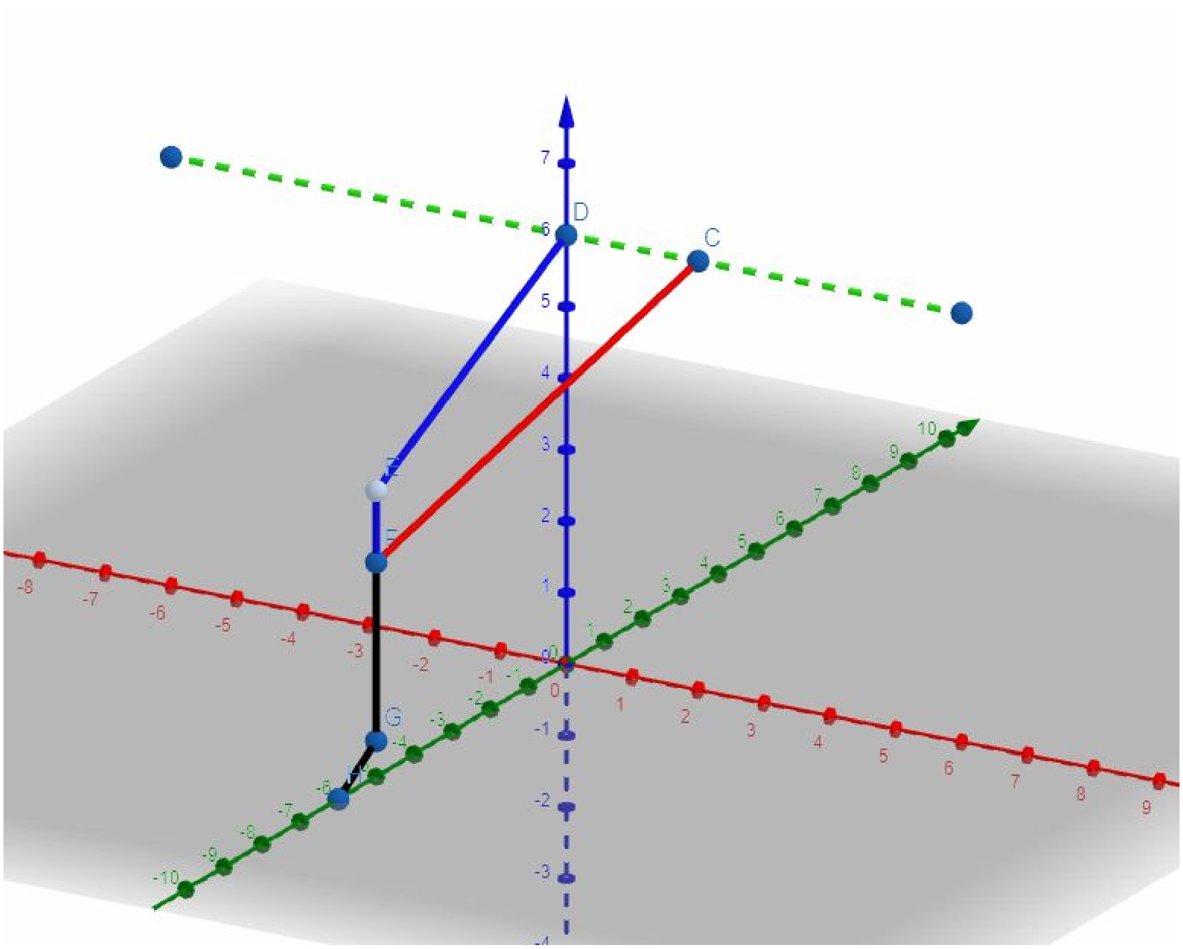
Let's set up our problem first. The figure below will give an exaggerated flight path description. Here is a legend.

Green line – flight path of the plane.

Blue line – flight path of Player 1. Perpendicular to the flight path.

Red line – flight path of Player 2. At some unknown angle to the flight path.

Black lines – parachuting path followed by both Player 1 and Player 2.



Now let's describe the problem a little more in depth. Player 2 will exit the plane at point C before Player 1 exits at point D. He, therefore, is out of the plane longer than Player 1 and will be lower by the time he reaches the plane containing points E, F, and G that is parallel to the red and blue axes. His total flight time consists of the time to traverse segments CF, FG, and GH. Player 1 will wait on the plane until point D, fly to point E, to point F, G, and H. We will assume, for the sake of math, that both players fly at the minimum vertical speed (maximum horizontal speed) until reach the plane containing point E, F, and G that is parallel to the red and blue axes. Both players will then point downward with 0 horizontal velocity. Their chutes will automatically deploy at point G, and while holding the forward (W) key, both players will end at point H. We will also assume that we can neglect acceleration times between velocities. This stems from the fact that all velocity changes are experienced equally between the two players and will therefore cancel out.

It can quickly been seen that both players traverse segments FG and GH at the same velocity. This simplifies the math because this amount of time in the air is cancelled out between the two players. So we only need to consider up to point F from the plane. Furthermore, our proof utilizes this specific scenario, but if Player 2 were to come in at a different angle than Player 1, it would only shorten the angled flight path and would still be shorter (we will allow intuition to prove this). Because of this, our proof will be true for all cases, except when the target is directly under the flight path. An angled jump (within certain limitations) would still be shorter in the case of a target in the flight path, however, a different proof is required to show this factually.

Before the proof. Let's define a function and some variables here to make typing easier. TOF() will represent the time of flight for a specific segment. Example: TOF(CE) is the time it takes to flight from point C to E. Vcs is the parachuting velocity at the maximum vertical velocity (slowest horizontal velocity, 0). Vcf is the parachuting velocity at the minimum vertical velocity (maximum horizontal velocity). Vp is the velocity of the plane.

The premise of our proof is simple. Let's have an imaginary segment between points C and E. If we can prove that TOF(CD) + TOF(DE) > TOF(CE), then we can define a minimum angle for which this is true simply by settings both sides of the inequality equal to each other. At this boundary, segment CF will be equal to segment CE. Player 2 will get to the ground first, find the S686 and headshot Player 1. Bummer.

The time it takes to traverse any segment can easily be found by multiplying the inverse of the player's velocity by the length of the segment. The segment length can be represented by the distance between two endpoints which we will define as D(P1, P2). Example: the length of segment CD is D(C, D).

Oh yeah… I also stopwatched the plane and a player a maximum horizontal parachute velocity. I only did it once, so hopefully the numbers are right. The plane traveled one km in roughly 7 seconds, and the player at max horizontal velocity travels one km in 25ish seconds, this translates to a plane speed of 514 km/h and a max horizontal player speed of 144 km/h. This means we can write Vcf as 3.569*Vp since 514 divided 144 is 3.569. This will allow us to eliminate a constant later and simplify the math. Also, velocity and speed are different things, I know… I will use them interchangeably except for in one portion of the proof, but ultimately, we are only concerned with the magnitude of the velocity, which is speed, unless explicitly stated otherwise.

With all this info, we can write an inequality…

$$\text{TOF(CD)} + \text{TOF(DE)} > \text{TOF(CE)}$$

$$\frac{1}{3.569V\,cf}D(C,D) + \frac{1}{V\,cf}D(D,E) > \frac{1}{V\,cf}D(C,E)$$

Since segments CD and DE are at a right angle, we can use the Pythagorean theorem to substitute D(C,E).

$$\frac{1}{3.569V\,cf}D(C,D) + \frac{1}{V\,cf}D(D,E) > \frac{1}{V\,cf}\sqrt{D(C,D)^2 + D(D,E)^2}$$

The next few steps will show simplification through algebra.

$$\frac{1}{Vcf^2}\{0.28D(C,D)+D(D,E)\}^2 > \frac{1}{Vcf^2}\{D(C,D)^2+D(D,E)^2\}$$

$$0.078D(C,D)^2+0.56D(C,D)D(D,E)+D(D,E)^2 > D(C,D)^2+D(D,E)^2$$

$$1.646 > \frac{D(C,D)}{D(D,E)}$$

Now we can define an angle θ between segments DE and CE. Let's set the inequality sides equal to each other to find our boundary for which the statement holds.

$$1.646 = \frac{D(C,D)}{D(D,E)}$$

Remember that opposite over adjacent is the tangent of the angle, so let's write that.

$$tan\ (\theta) = \frac{D(C,D)}{D(D,E)} = 1.646$$

So we take the inverse tangent of both sides.

$$\theta = tan^{-1}(1.646) = 58.72°$$

Well, this is our angle between DE and CE, so to find the angle from the flight path to CE, we simply recall that there are 180 degrees in a triangle.

$$180° - 90° - 58.72° = 31.28°$$

So as long as the angle from the flight path to our imaginary segment CE is greater than 31.28 degrees, then are inequality holds true. Let's write that mathematically.

$$1.646 > tan(\theta)$$

Interpreting this isn't too bad. If the tangent of our angle remains less the 1.646, then the angle stays below 58.72 degrees, meaning the angle between CE and the flight path will be will go up, bringing it closer to the 90 degree jump. Try plugging some numbers into a calculator, it's pretty cool how it works out.

TOF(CD) + TOF(DE) > TOF(CE) ∎

Now, I'm certain of some fringe cases that won't hold true. And when player's pull chutes early, it only complicates things, but just in general, our proof stands. It is only meant to show that a perpendicular jump is intuitive, but not necessarily correct. Anyone reading, please feel free to add or correct anything on here.

Finally… By reading this, you agree that if you get a kill as a direct result from this proof, you owe me a beer.

REV2: Simplified proof with boundary conditions and updated chute timings due to improper measurement.