

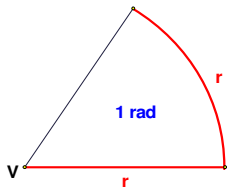
## TRIGONOMETRIJA (m@h)

### mjerne jedinice

veličinu kuta izražavamo u

- stupnjevima ( $^{\circ}$ ),  $1^{\circ} = 60'$  (minuta),  $1' = 60''$  (sekundi),  $1^{\circ} = 3600''$
- radijanima (rad)

### definicija radijana



veličina kuta je 1 rad ako je duljina pridruženog luka jednaka duljini polumjera kružnice

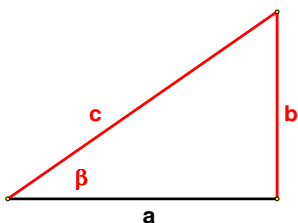
### veza stupnjeva i radijana

$$180^{\circ} = \pi \text{ rad} \Rightarrow 1^{\circ} = \frac{\pi}{180} \text{ rad} \Rightarrow \alpha^{\circ} = \frac{180^{\circ}}{\pi} \cdot \alpha(\text{rad})$$

### veza radijana i stupnjeva

$$\pi \text{ rad} = 180^{\circ} \Rightarrow 1 \text{ rad} = \frac{180^{\circ}}{\pi} \approx 57^{\circ}17'45'' \Rightarrow \alpha(\text{rad}) = \frac{\pi}{180^{\circ}} \cdot \alpha^{\circ}$$

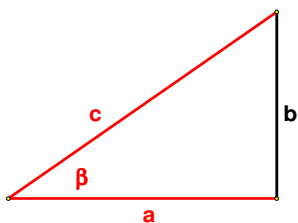
### definicija trigonometrijskih funkcija na pravokutnom trokutu



#### sinus

sinus šiljastog kuta je omjer nasuprotne katete i hipotenuze

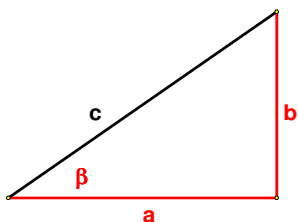
$$\sin \beta = \frac{b}{c}$$



#### kosinus

kosinus šiljastog kuta je omjer priležeće katete i hipotenuze

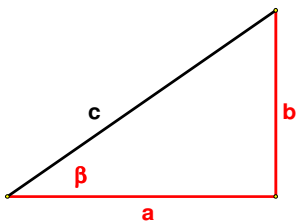
$$\cos \beta = \frac{a}{c}$$



#### tangens

tangens šiljastog kuta je omjer nasuprotne i priležeće katete

$$\text{tg } \beta = \frac{b}{a}$$

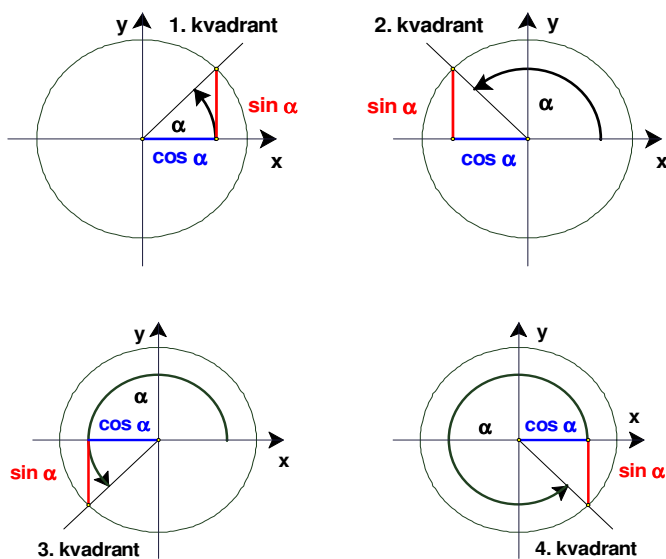


#### kotangens

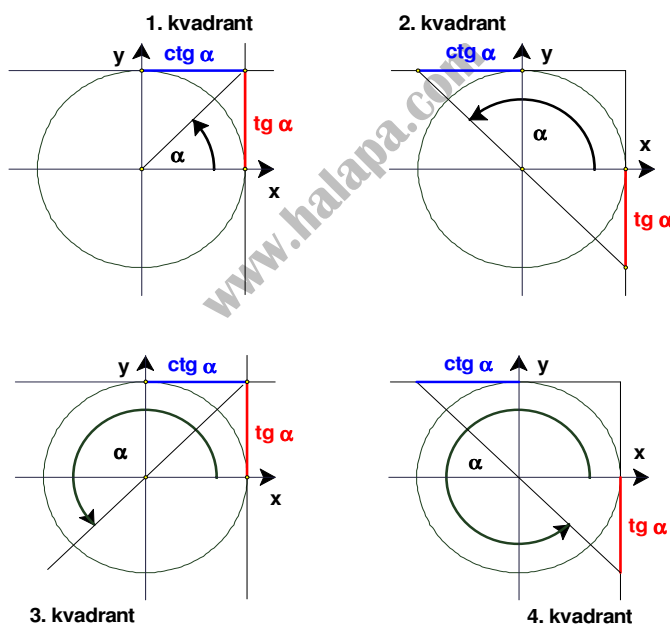
kotangens šiljastog kuta je omjer priležeće i nasuprotne katete

$$\text{ctg } \beta = \frac{a}{b}$$

definicije trigonometrijskih funkcija na trigonometrijskoj (jediničnoj, brojevnoj) kružnici  
sinus i kosinus



tangens i kotangens



funkcije i kofunkcije

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x, \quad \sin(90^\circ - x) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x, \quad \cos(90^\circ - x) = \sin x$$

$$\operatorname{tg}\left(\frac{\pi}{2} - x\right) = \operatorname{ctg} x, \quad \operatorname{tg}(90^\circ - x) = \operatorname{ctg} x$$

$$\operatorname{ctg}\left(\frac{\pi}{2} - x\right) = \operatorname{tg} x, \quad \operatorname{ctg}(90^\circ - x) = \operatorname{tg} x$$

### periodičnost funkcija

$$\sin(\alpha + k \cdot 360^\circ) = \sin \alpha, \quad \sin(\alpha + k \cdot 2\pi) = \sin \alpha$$

$$\cos(\alpha + k \cdot 360^\circ) = \cos \alpha, \quad \cos(\alpha + k \cdot 2\pi) = \cos \alpha$$

$$\operatorname{tg}(\alpha + k \cdot 180^\circ) = \operatorname{tg} \alpha, \quad \operatorname{tg}(\alpha + k \cdot \pi) = \operatorname{tg} \alpha$$

$$\operatorname{ctg}(\alpha + k \cdot 180^\circ) = \operatorname{ctg} \alpha, \quad \operatorname{ctg}(\alpha + k \cdot \pi) = \operatorname{ctg} \alpha$$

### svodenje na prvi kvadrant

- iz drugog u prvi

$$\sin(180^\circ - \alpha) = \sin \alpha, \quad \sin(\pi - \alpha) = \sin \alpha$$

$$\cos(180^\circ - \alpha) = -\cos \alpha, \quad \cos(\pi - \alpha) = -\cos \alpha$$

$$\operatorname{tg}(180^\circ - \alpha) = -\operatorname{tg} \alpha, \quad \operatorname{tg}(\pi - \alpha) = -\operatorname{tg} \alpha$$

$$\operatorname{ctg}(180^\circ - \alpha) = -\operatorname{ctg} \alpha, \quad \operatorname{ctg}(\pi - \alpha) = -\operatorname{ctg} \alpha$$

- iz trećeg u prvi

$$\sin(180^\circ + \alpha) = -\sin \alpha, \quad \sin(\pi + \alpha) = -\sin \alpha$$

$$\cos(180^\circ + \alpha) = -\cos \alpha, \quad \cos(\pi + \alpha) = -\cos \alpha$$

$$\operatorname{tg}(180^\circ + \alpha) = \operatorname{tg} \alpha, \quad \operatorname{tg}(\pi + \alpha) = \operatorname{tg} \alpha$$

$$\operatorname{ctg}(180^\circ + \alpha) = \operatorname{ctg} \alpha, \quad \operatorname{ctg}(\pi + \alpha) = \operatorname{ctg} \alpha$$

- iz četvrtog u prvi

$$\sin(360^\circ - \alpha) = -\sin \alpha, \quad \sin(2\pi - \alpha) = -\sin \alpha$$

$$\cos(360^\circ - \alpha) = \cos \alpha, \quad \cos(2\pi - \alpha) = \cos \alpha$$

$$\operatorname{tg}(360^\circ - \alpha) = -\operatorname{tg} \alpha, \quad \operatorname{tg}(2\pi - \alpha) = -\operatorname{tg} \alpha$$

$$\operatorname{ctg}(360^\circ - \alpha) = -\operatorname{ctg} \alpha, \quad \operatorname{ctg}(2\pi - \alpha) = -\operatorname{ctg} \alpha$$

### (ne)parnost

$$\sin(-\alpha) = -\sin \alpha, \quad \cos(-\alpha) = \cos \alpha, \quad \operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha, \quad \operatorname{ctg}(-\alpha) = -\operatorname{ctg} \alpha$$

### vrijednosti trigonometrijskih funkcija za neke kutove

$\alpha$	0 ili $0^\circ$	$\frac{\pi}{6}$ ili $30^\circ$	$\frac{\pi}{4}$ ili $45^\circ$	$\frac{\pi}{3}$ ili $60^\circ$	$\frac{\pi}{2}$ ili $90^\circ$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\operatorname{tg} \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$
$\operatorname{ctg} \alpha$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

## osnovni trigonometrijski identiteti

$$\cos^2 \alpha + \sin^2 \alpha = 1, \quad \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}, \quad \operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}, \quad \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$$

$$1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}, \quad 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$$

## predznaci trigonometrijskih funkcija

kvadrant		sin	cos	tg	ctg
I.	$0^\circ - 90^\circ$	+	+	+	+
II.	$90^\circ - 180^\circ$	+	-	-	-
III.	$180^\circ - 270^\circ$	-	-	+	+
IV.	$270^\circ - 360^\circ$	-	+	-	-

## izračunavanje jedne trigonometrijske funkcije pomoću ostalih

$$\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha} = \frac{\operatorname{tg} \alpha}{\pm \sqrt{1 + \operatorname{tg}^2 \alpha}} = \frac{1}{\pm \sqrt{1 + \operatorname{ctg}^2 \alpha}}$$

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha} = \frac{1}{\pm \sqrt{1 + \operatorname{tg}^2 \alpha}} = \frac{\operatorname{ctg} \alpha}{\pm \sqrt{1 + \operatorname{ctg}^2 \alpha}}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\pm \sqrt{1 - \sin^2 \alpha}} = \frac{\pm \sqrt{1 - \cos^2 \alpha}}{\cos \alpha} = \frac{1}{\operatorname{ctg} \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\pm \sqrt{1 - \sin^2 \alpha}}{\sin \alpha} = \frac{\cos \alpha}{\pm \sqrt{1 - \cos^2 \alpha}} = \frac{1}{\operatorname{tg} \alpha}$$

## funkcije zbroja i razlike

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$\operatorname{ctg}(\alpha + \beta) = \frac{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}$$

$$\operatorname{ctg}(\alpha - \beta) = \frac{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta + 1}{\operatorname{ctg} \alpha - \operatorname{ctg} \beta}$$

$$\sin(\alpha + \beta + \gamma) = \sin \alpha \cdot \cos \beta \cdot \cos \gamma + \cos \alpha \cdot \sin \beta \cdot \cos \gamma + \cos \alpha \cdot \cos \beta \cdot \sin \gamma - \sin \alpha \cdot \sin \beta \cdot \sin \gamma$$

$$\cos(\alpha + \beta + \gamma) = \cos \alpha \cdot \cos \beta \cdot \cos \gamma - \sin \alpha \cdot \sin \beta \cdot \cos \gamma - \sin \alpha \cdot \cos \beta \cdot \sin \gamma - \cos \alpha \cdot \sin \beta \cdot \sin \gamma$$

### funkcije višestrukih kutova

$$\sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha, \quad \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cdot \cos^2 \alpha - 1 = 1 - 2 \cdot \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \cdot \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}, \quad \operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2 \cdot \operatorname{ctg} \alpha}$$

$$\sin 3\alpha = 3 \cdot \sin \alpha - 4 \cdot \sin^3 \alpha, \quad \cos 3\alpha = 4 \cdot \cos^3 \alpha - 3 \cdot \cos \alpha$$

$$\operatorname{tg} 3\alpha = \frac{3 \cdot \operatorname{tg} \alpha - \operatorname{tg}^3 \alpha}{1 - 3 \cdot \operatorname{tg}^2 \alpha}, \quad \operatorname{ctg} 3\alpha = \frac{\operatorname{ctg}^3 \alpha - 3 \cdot \operatorname{ctg} \alpha}{3 \cdot \operatorname{ctg}^2 \alpha - 1}$$

### funkcije polukutova

$$\sin \alpha = 2 \cdot \sin \frac{\alpha}{2} \cdot \cos \frac{\alpha}{2}, \quad \cos \alpha = \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}$$

$$\operatorname{tg} \alpha = \frac{2 \cdot \operatorname{tg} \frac{\alpha}{2}}{1 - \operatorname{tg}^2 \frac{\alpha}{2}}, \quad \operatorname{ctg} \alpha = \frac{\operatorname{ctg}^2 \frac{\alpha}{2} - 1}{2 \cdot \operatorname{ctg} \frac{\alpha}{2}}$$

$$1 - \cos \alpha = 2 \cdot \sin^2 \frac{\alpha}{2}, \quad 1 + \cos \alpha = 2 \cdot \cos^2 \frac{\alpha}{2}$$

$$\frac{1 - \cos \alpha}{1 + \cos \alpha} = \operatorname{tg}^2 \frac{\alpha}{2}, \quad \frac{1 + \cos \alpha}{1 - \cos \alpha} = \operatorname{ctg}^2 \frac{\alpha}{2}$$

$$\frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha} = \operatorname{tg} \frac{\alpha}{2}, \quad \frac{1 + \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 - \cos \alpha} = \operatorname{ctg} \frac{\alpha}{2}$$

### formule pretvorbe

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} \cdot [\cos(\alpha - \beta) - \cos(\alpha + \beta)], \quad \cos \alpha \cdot \cos \beta = \frac{1}{2} \cdot [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \cdot [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

$$\operatorname{tg} \alpha \cdot \operatorname{tg} \beta = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}, \quad \operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta = \frac{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}{\operatorname{tg} \alpha + \operatorname{tg} \beta}, \quad \operatorname{tg} \alpha \cdot \operatorname{ctg} \beta = \frac{\operatorname{tg} \alpha + \operatorname{ctg} \beta}{\operatorname{tg} \beta + \operatorname{ctg} \alpha}$$

$$\sin \alpha + \sin \beta = 2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}, \quad \sin \alpha - \sin \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}, \quad \cos \alpha - \cos \beta = -2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \sin \alpha = \sqrt{2} \cdot \cos \left( \frac{\pi}{4} - \alpha \right), \quad \cos \alpha - \sin \alpha = \sqrt{2} \cdot \sin \left( \frac{\pi}{4} - \alpha \right)$$

$$\sin \alpha - \cos \alpha = \sqrt{2} \cdot \sin \left( \alpha - \frac{\pi}{4} \right)$$

$$\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta}, \quad \operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cdot \cos \beta}, \quad \operatorname{ctg} \alpha + \operatorname{ctg} \beta = \frac{\sin(\alpha + \beta)}{\sin \alpha \cdot \sin \beta}$$

$$\operatorname{ctg} \alpha - \operatorname{ctg} \beta = \frac{\sin(\beta - \alpha)}{\sin \alpha \cdot \sin \beta}, \quad \operatorname{tg} \alpha + \operatorname{ctg} \beta = \frac{\cos(\alpha - \beta)}{\sin \alpha \cdot \cos \beta}, \quad \operatorname{ctg} \alpha - \operatorname{tg} \beta = \frac{\cos(\alpha + \beta)}{\sin \alpha \cdot \cos \beta}$$

odnosi između trigonometrijskih funkcija u trokutu ( $\alpha + \beta + \gamma = 180^\circ$ )

$$\sin \alpha + \sin \beta + \sin \gamma = 4 \cdot \cos \frac{\alpha}{2} \cdot \cos \frac{\beta}{2} \cdot \cos \frac{\gamma}{2}$$

$$\cos \frac{\alpha}{2} + \cos \frac{\beta}{2} + \cos \frac{\gamma}{2} = 4 \cdot \cos \frac{\alpha + \beta}{4} \cdot \cos \frac{\alpha + \gamma}{4} \cdot \cos \frac{\beta + \gamma}{4}$$

$$\operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg} \gamma = \operatorname{tg} \alpha \cdot \operatorname{tg} \beta \cdot \operatorname{tg} \gamma$$

$$\operatorname{ctg} \frac{\alpha}{2} + \operatorname{ctg} \frac{\beta}{2} + \operatorname{ctg} \frac{\gamma}{2} = \operatorname{ctg} \frac{\alpha}{2} \cdot \operatorname{ctg} \frac{\beta}{2} \cdot \operatorname{ctg} \frac{\gamma}{2}$$

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$$\operatorname{tg} \frac{\alpha}{2} = t \Rightarrow \sin \alpha = \frac{2t}{1+t^2}, \quad \cos \alpha = \frac{1-t^2}{1+t^2}, \quad \operatorname{tg} \alpha = \frac{2t}{1-t^2}$$

potencije trigonometrijskih funkcija

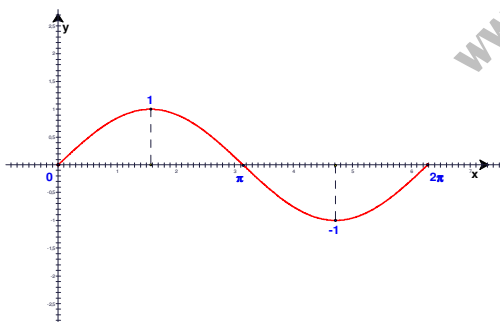
$$\sin^2 \alpha = \frac{1}{2} \cdot [1 - \cos 2\alpha], \quad \cos^2 \alpha = \frac{1}{2} \cdot [1 + \cos 2\alpha]$$

$$\sin^3 \alpha = \frac{1}{4} \cdot [3 \cdot \sin \alpha - \sin 3\alpha], \quad \cos^3 \alpha = \frac{1}{4} \cdot [\cos 3\alpha + 3 \cdot \cos \alpha]$$

$$\sin^4 \alpha = \frac{1}{8} \cdot [\cos 4\alpha - 4 \cdot \cos 2\alpha + 3], \quad \cos^4 \alpha = \frac{1}{8} \cdot [\cos 4\alpha + 4 \cdot \cos 2\alpha + 3]$$

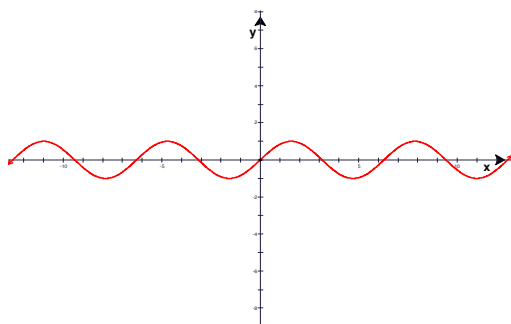
sinusoida

graf funkcije sinus,  $f(x) = \sin x$ ,  $\sin : \mathbb{R} \rightarrow [-1, 1]$



sinusoida na  $[0, 2\pi]$

- temeljni period je  $2\pi$
- nultočke su  $0, \pi, 2\pi$
- maksimum je 1 u točki  $\frac{\pi}{2}$
- minimum je  $-1$  u točki  $\frac{3\pi}{2}$

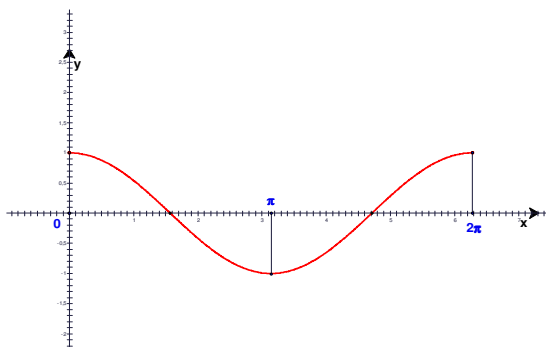


sinusoida na  $\langle -\infty, +\infty \rangle$

- temeljni period je  $2\pi$
- nultočke su  $k \cdot \pi$ ,  $k \in \mathbb{Z}$
- maksimum je 1 u  $\frac{\pi}{2} + k \cdot 2\pi$ ,  $k \in \mathbb{Z}$
- minimum je  $-1$  u  $-\frac{\pi}{2} + k \cdot 2\pi$ ,  $k \in \mathbb{Z}$
- centralno je simetrična s obzirom na ishodište

## kosinusoida

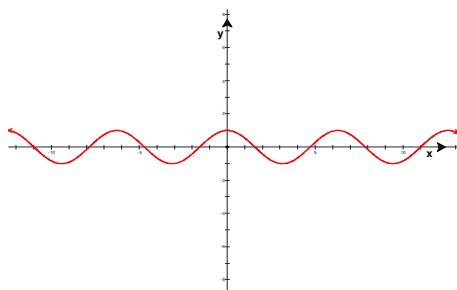
graf funkcije kosinus,  $f(x) = \cos x$ ,  $\sin : \mathbb{R} \rightarrow [-1, 1]$



### kosinusoida na $[0, 2\pi]$

- temeljni period je  $2\pi$
- nultočke su  $\frac{\pi}{2}, \frac{3\pi}{2}$
- maksimum je 1 u točkama 0,  $2\pi$
- minimum je  $-1$  u točki  $\pi$

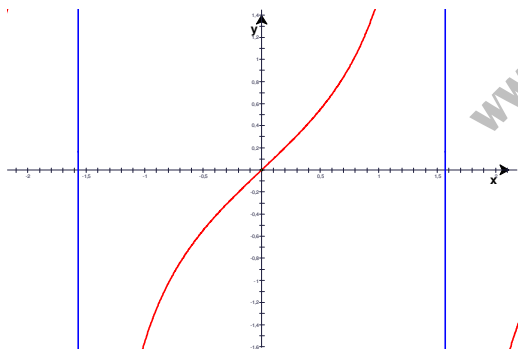
### kosinusoida na $\langle -\infty, +\infty \rangle$



- temeljni period je  $2\pi$
- nultočke su  $\frac{\pi}{2} + k \cdot \pi$ ,  $k \in \mathbb{Z}$
- maksimum je 1 u  $k \cdot 2\pi$ ,  $k \in \mathbb{Z}$
- minimum je  $-1$  u  $\pi + k \cdot 2\pi$ ,  $k \in \mathbb{Z}$
- kosinusoida je simetrična obzirom na y-os

## tangensoida

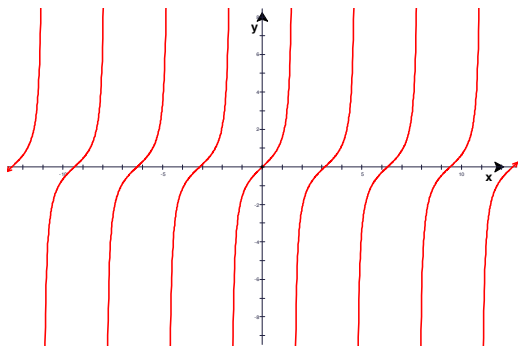
graf funkcije tangens,  $f(x) = \operatorname{tg} x$ ,  $\operatorname{tg} : \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k \cdot \pi : k \in \mathbb{Z} \right\} \rightarrow \mathbb{R}$



### tangensoida na $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$

- temeljni period je  $\pi$
- nultočka u 0
- maksimum i minimum ne postoje
- funkcija je rastuća
- asimptote su pravci  $x = -\frac{\pi}{2}$ ,  $x = \frac{\pi}{2}$

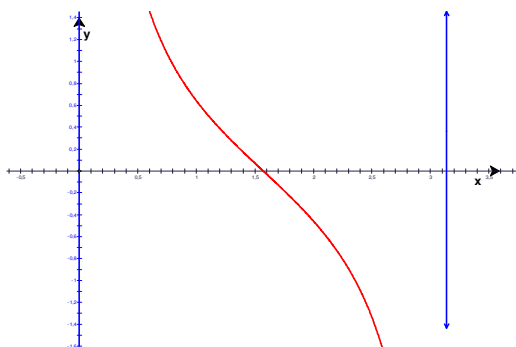
### tangensoida na $\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k \cdot \pi : k \in \mathbb{Z} \right\}$



- temeljni period je  $\pi$
- nultočke su  $k \cdot \pi$ ,  $k \in \mathbb{Z}$
- maksimum i minimum ne postoje
- funkcija je rastuća na  $\left\langle -\frac{\pi}{2} + k \cdot \pi, \frac{\pi}{2} + k \cdot \pi \right\rangle$
- centralno je simetrična s obzirom na ishodište
- asimptote su pravci  $x = \frac{\pi}{2} + k \cdot \pi$ ,  $k \in \mathbb{Z}$

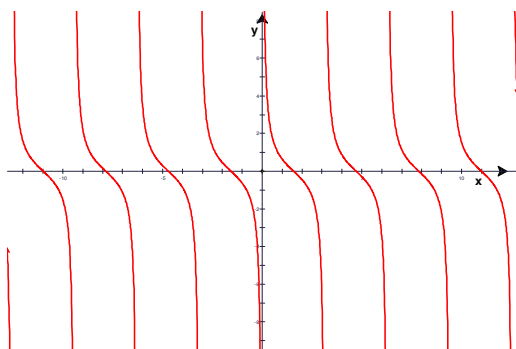
## kotangensoida

graf funkcije kotangens,  $f(x) = \text{ctg } x$ ,  $\text{ctg} : \mathbb{R} \setminus \{k \cdot \pi : k \in \mathbb{Z}\} \rightarrow \mathbb{R}$



### kotangensoida na $\langle 0, \pi \rangle$

- temeljni period je  $\pi$
- nultočka je  $\frac{\pi}{2}$
- maksimum i minimum ne postoje
- funkcija je padajuća
- asimptote su pravci  $x = 0$ ,  $x = \pi$



### kotangensoida na $\mathbb{R} \setminus \{k \cdot \pi : k \in \mathbb{Z}\}$

- temeljni period je  $\pi$
- nultočke su  $\frac{\pi}{2} + k \cdot \pi$ ,  $k \in \mathbb{Z}$
- maksimum i minimum ne postoje
- funkcija je padajuća na  $\langle k \cdot 2\pi, \pi + k \cdot 2\pi \rangle$
- centralno je simetrična s obzirom na ishodište
- asimptote su pravci  $x = k \cdot \pi$ ,  $k \in \mathbb{Z}$

graf funkcije  $f(x) = a \cdot \sin(b \cdot x + c)$

**amplituda** je broj  $|a|$ , maksimum funkcije je  $|a|$ , minimum je  $-|a|$

**periodičnost** temeljni period je  $\frac{2\pi}{|b|}$

**fazni pomak** sinusoidu počinjemo crtati u točki  $-\frac{c}{b}$ ,

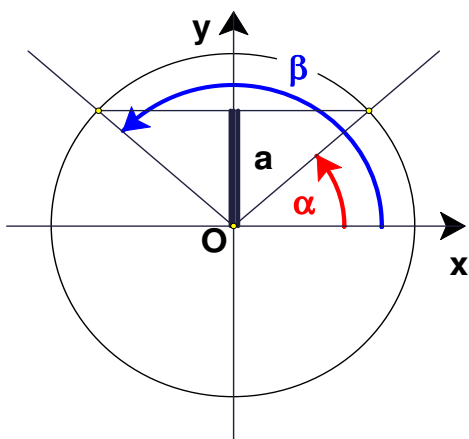
graf funkcije  $f(x) = a \cdot \sin(b \cdot x + c)$  dobivamo translacijom grafa funkcije  $g(x) = \sin(b \cdot x)$

trigonometrijska jednačzba

$$\sin x = a, |a| \leq 1$$

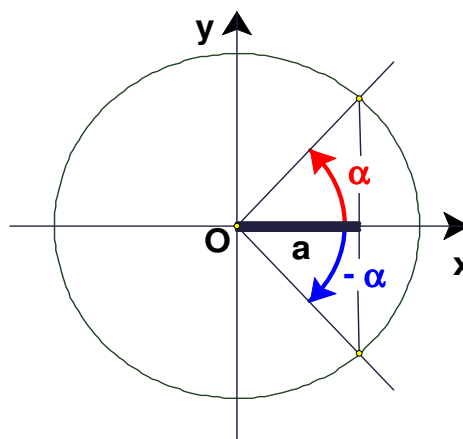
trigonometrijska jednačzba

$$\cos x = a, |a| \leq 1$$



$$x_1 = \alpha + k \cdot 2\pi, k \in \mathbb{Z}$$

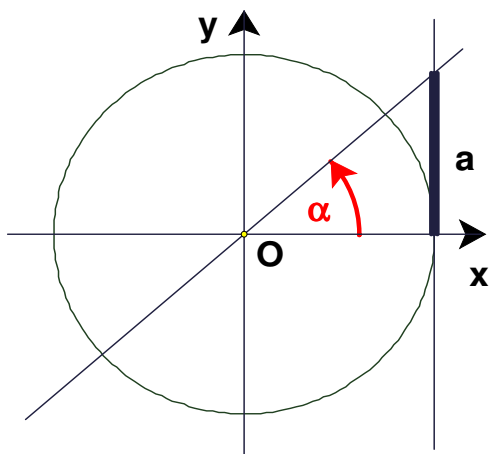
$$x_2 = \beta + k \cdot 2\pi = \pi - \alpha + k \cdot 2\pi, k \in \mathbb{Z}$$



$$x_{1,2} = \pm \alpha + k \cdot 2\pi, k \in \mathbb{Z}$$

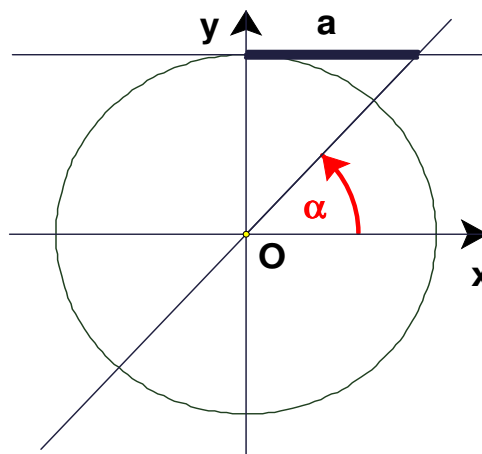


trigonometrijska jednađba  
 $\operatorname{tg} x = a, a \in \mathbb{R}$



$$x = \alpha + k \cdot \pi, k \in \mathbb{Z}$$

trigonometrijska jednađba  
 $\operatorname{ctg} x = a, a \in \mathbb{R}$



$$x = \alpha + k \cdot \pi, k \in \mathbb{Z}$$

linearne trigonometrijske jednađbe homogene s obzirom na  $\sin x$  i  $\cos x$

$$a \cdot \sin x + b \cdot \cos x = 0, a \neq 0 \text{ i } b \neq 0$$

postupak rješavanja

$$a \cdot \sin x + b \cdot \cos x = 0 \quad /: \cos x \Rightarrow a \cdot \operatorname{tg} x + b = 0$$

kvadratne trigonometrijske jednađbe homogene s obzirom na  $\sin x$  i  $\cos x$

$$a \cdot \sin^2 x + b \cdot \sin x \cdot \cos x + c \cdot \cos^2 x = 0, a \neq 0 \text{ i } c \neq 0$$

postupak rješavanja

$$a \cdot \sin^2 x + b \cdot \sin x \cdot \cos x + c \cdot \cos^2 x = 0 \quad /: \cos^2 x \Rightarrow a \cdot \operatorname{tg}^2 x + b \cdot \operatorname{tg} x + c = 0 \Rightarrow \\ \Rightarrow [t = \operatorname{tg} x] \Rightarrow a \cdot t^2 + b \cdot t + c = 0$$

$$a \cdot \sin^2 x + b \cdot \sin x \cdot \cos x + c \cdot \cos^2 x = d, a \neq 0 \text{ ili } c \neq 0, d \neq 0$$

postupak rješavanja

$$a \cdot \sin^2 x + b \cdot \sin x \cdot \cos x + c \cdot \cos^2 x = d \Rightarrow \text{zamjenom } d = d \cdot (\sin^2 x + \cos^2)$$

nakon sređivanja dobije se homogena kvadratna jednađba

linearna nehomogena s obzirom na  $\sin x$  i  $\cos x$  ili projekcijska

$$a \cdot \sin x + b \cdot \cos x = c \quad (a, b, c \neq 0)$$

postupak rješavanja

1. inačica: univerzalna supstitucija

$$t = \operatorname{tg} \frac{x}{2} \Rightarrow \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, \operatorname{tg} x = \frac{2t}{1-t^2}$$

$$2. \text{ inačica: } a \cdot \sin x + b \cdot \cos x = c \quad /: a \Rightarrow \sin x + \frac{b}{a} \cdot \cos x = \frac{c}{a} \Rightarrow \left[ \text{supstitucija, } \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \right] \Rightarrow$$

$$\Rightarrow \sin x + \frac{\sin \alpha}{\cos \alpha} \cdot \cos x = \frac{c}{a} \quad /: \cos \alpha \Rightarrow \sin x \cdot \cos \alpha + \cos x \cdot \sin \alpha = \frac{c}{a} \cdot \cos \alpha \Rightarrow$$

$$\Rightarrow \sin(x + \alpha) = \frac{c}{a} \cdot \cos \alpha, \text{ osnovna trigonometrijska jednađba rješiva za } \left| \frac{c}{a} \cdot \cos \alpha \right| \leq 1$$