Modeling the growth of the Great Rabbit

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Abstract

In this work we use the available information about the Great Rabbit to model it's population growth. The model assumes the population growth is limited only by food availability and divides it into two regimes: external feeding regime (in which rabbits eat animals in their surrounding area) and internal feeding regime (in which rabbits multiply and eat themselves). The model predicts a maximum number of rabbits, which depends on the relation between three measurable numbers and sheds some light which kind of countermeasures could be taken to confront them. This is a novel approach that opens the way for further studies and eventually being able to exterminate this mabeast.

1 Introduction

The Great Rabbit is one of the Three Great Mabeasts that plague this world. They consist of a near limitless horde of fist sized white rabbits that travel together and seem to act in perfect coordination [1].

The rabbits are carnivorous and eat any living animal in their way, not leaving any traces. This, in combination to their staggering numbers, is why this rabbits are considered a unstoppable calamity.

Reports of the Great Rabbit date as far back as 350 years ago. The tales are always similar, with the Rabbit appearing seemingly out of nowhere and wiping whole villages out of existence. Its activity does not seem to be decreasing over time, with the recent record we have being from just two years ago, in Karsten territory [2].

Little has been achieved in regards to facing the Great Rabbit. Usually when a population finds out the horde is coming for them, they evacuate and hope for the best. There have been successful attempts at driving them away from populated areas [2], but trying to exterminate them with our current methods has proved to be futile.

We believe that the first step towards a more permanent solution about the Great Rabbit is understanding it. With this in mind, we have used the knowledge available about it to model its growth. In the next section we detail the hypothesis used for the model, which is described in detail in section 3. Then, in section 4 we see the model at work with some examples we deemed insightful.

Figure 1: *The Great Rabbit*

2 What we know about the Great Rabbit

Little is known about the Great Rabbit, as collecting field data about them can prove unusually dangerous. Therefore, a lot of the information we have is gathered through folklore or the study of incidents involving this particular mabeast. That being said, we will use what we do know about it to form a set of hypothesis for our model.

As stated before, Great Rabbit is a horde of rabbits which eat whatever living animal they find in their way. It has been reported that they leave the plant life in their way intact, suggesting that they are exclusively carnivorous. There are also reports of rabbits eating each other [3].

A defining characteristic of the Great Rabbit is the ability to multiply themselves in a very short span of time [1,2,3,4]. This makes them particularly hard to get rid of. However, we know for a fact that they can't multiply indefinitely and that there is some limit to their number. Some sources report numbers going up to around 80.000 seen at the same time [4].

With this information in mind, we can start forming our hypothesis. We will list them as follows.

1. The rabbits need some kind of energy to survive. They get this energy by eating animals.
2. Their multiplication ability is not tied to their need for food. In other words, they can multiply themselves even if they are starving.
3. Since they need to eat and food is limited, their number is limited by external factors (like food availability).
4. They do not waste any energy. All energy available from their food source is taken by the Great Rabbit, without loss.

They key hypothesis (and the one we are assuming with the least empirical backup) is the first one. As they are no ordinary beasts, we are not sure they need to eat in order to survive. But for the purposes of this model, we will assume they do.

The second hypothesis is based on the fact that the rabbits eat themselves. If they need to eat to survive and they create copies of themselves to consume them, it follows that multiplying does not cost them any of the energy they get from eating. Otherwise, the amount of energy would remain constant throughout the whole multiplication and consumption process and the rabbits would gain nothing from it.

The third hypothesis is based on the fact that they can't multiply indefinitely. We assume that this limit is based on their need for food or other environmental factors, rather than a magically imposed limit.

Finally, the fourth hypothesis is taken to make the math easier. But it is not key to the model and can be modified if needed.

3 Model

3.1 External feeding regime

Let us have an N number of rabbits. Following hypothesis 1, we will propose that during a certain time interval Δt each individual rabbit will need an intake of a certain amount of energy to survive (E_R). Also, during the same time interval they will be able to move to any point on a certain area (A_R). Now, this area has to have some amount of edible animals for the rabbits; to simplify the model we will use hypothesis 4 and think of them as energy the rabbits can consume, which allows us to define an energy density ρ_E for the area A_R.

Figure 2: *Diagram of the area covered by a single rabbit in a given Δt time. Said area (A_R) is shown in red, while the area the rabbit can't reach yet is shown in a striped pattern.*

Next, it is important to recognize that the ammount of rabbits should change over time. We will incorporate this into our model by thinking of N iteratively. So, suppose we start with a certain number N_0 of rabbits. After a time interval Δt (the same we mentioned before) these rabbits will have multiplied themselves and we will have a different amount of them, which we'll call N_1. We can continue this process indefinitely: after two time intervals we will have N_2 rabbits, after three we'll have N_3, and so on. Generalizing, after i time intervals we will have N_i rabbits.

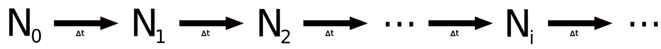

Figure 3: *How the iteration works. We start with N_0 rabbits. After a Δt time, we go to N_1 rabbits, then to N_2 and so on. Repeat this process i times and we get N_i rabbits.*

Now we can start assembling our model. If the energy needed by one rabbit to survive a Δt time is E_R, it follows that the energy needed for N_i rabbits to survive that same time is

$$E_N = E_R N_i \tag{1}$$

Now we have to consider how much energy the rabbits are able to consume in that span of time. We know that a single rabbit can cover an area A_R and that that area has an energy density ρ_E. Assuming the rabbits do not overlap their search areas (meaning they cover the biggest area possible) the total energy available for consumption by N_i rabbits is given by $\rho_E A_R N_i$. But we have to take into account the energy already consumed by the rabbit horde (and so, not available for intake); we will do this by subtracting $\rho_E A_R N_{i-1}$ from our total available energy. So, in the end, the amount of energy N_i rabbits can consume in a Δt span of time is given by

$$E_A = \rho_E A_R N_i - \rho_E A_R N_{i-1}$$

which can be reorganized into the expression

$$E_A = \rho_E A_R (N_i - N_{i-1}) \tag{2}$$

We can see that the energy available depends on how many rabbits are alive at the current iteration (N_i) and how many there were in the past iteration (N_{i-1}). Figure 4 shows gives a visual representation of the first iteration as an example of how this works.

Now we impose that the energy available for consumption has to be greater or equal to the amount of energy needed by the rabbits to survive

$$E_N \leq E_A$$

We can use this condition with equations 1 and 2 (and assume ρ_E and A_R are not zero) to reach the following expression

$$\frac{E_R}{\rho_E A_R} \leq 1 - \frac{N_{i-1}}{N_i} \tag{3}$$

Finally, we have to take into account that the presence of the Great Rabbit implies changes on its environment. Specifically, while we can assume E_R and A_R constant, we can't do the same for ρ_E; the growing size of a carnivore

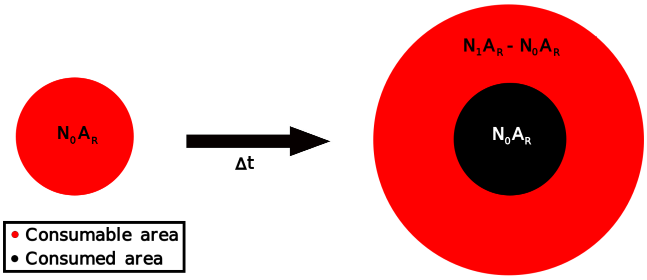

Figure 4: *First iteration of how the area available for consumption evolves. We start with an N_0A_R area; after a time Δt passes, the rabbits can cover an N_1A_R area. But the area they covered before has already been depleted of energy, so there is nothing for the Great Rabbit to consume there.*

rabbit horde is sure to scare animals nearby, reducing the energy available for consumption. For this reason we propose that the energy density is inversely proportional to the number of rabbits: $\rho_E = \frac{\rho_{E0}}{N_i}$, where ρ_{E0} is the density with no rabbits present. Inserting this into ecuation 3 we get

$$\frac{E_R}{\rho_{E0}A_R} \leq \frac{1}{N_i} - \frac{N_{i-1}}{N_i^2} \tag{4}$$

which we will call from now on the "Witch's condition", which has to be true for every amount N_i of rabbits in order for the Great Rabbit to keep on growing. This relation is characterized by two factors: the way the rabbits multiply (how we go from N_i to N_{i+1}) and the number $\frac{E_R}{\rho_{E0}A_c}$. Thus, we will define the "Witch's number" as

$$W \stackrel{\text{def}}{=} \frac{E_R}{\rho_{E0}A_R} \tag{5}$$

Note that W depends on the interval Δt we are taking, because each of the parameters that composes it has that dependency. For example, if Δt is a year, the rabbits will need more energy (E_R) to survive that time and will be able to travel longer distances (A_R) than if Δt was a few seconds, giving us different Witch's numbers for different time intervals.

It is also important to notice that we discarded the cases of $\rho_{E0} = 0$ and $A_R = 0$ in order to reach equation 3. However, these cases are trivial: if either ρ_{E0} or A_R is zero, then it follows from equation 2 that $E_A = 0$. Since $E_N > 0$ (because we are assuming the rabbits need energy to survive) the inequality $E_N \leq E_A$ is always false and thus the rabbit population is unable to grow.

We have deduced a condition for the Great Rabbit to keep on growing, but we have yet to address how that growth specifically happens. Several research attempts have been made to study this phenomenon, leading to many unfortunate losses [5] and no reliable information about it. So for the purposes of this research we will assume each rabbit creates C copies of itself during a Δt interval of time. So, if we start with a single rabbit ($N_0 = 1$), after a Δt time we will have $N_1 = (C+1)$ rabbits and after two Δt intervals we will have $N_2 = (C+1)^2$ of them. We can generalize and say that after i time intervals we will have $N_i = (C+1)^i$ rabbits. This is what is commonly known as "exponential growth".

3.2 Internal feeding regime

We have already established that the Witch's condition (equation 4) has to be fulfilled for the Great Rabbit to grow. Then, we can ask ourselves: What happens when that relation is not fulfilled?

As mentioned in section 2, the Great Rabbit will start eating itself if no other food source is available. We can quantify this phenomenon in terms of two factors: the number of rabbits each individual rabbit has to eat to survive a Δt time (K) and the number C we defined before.

During this regime, two things will happen in sequence. First, a certain number of rabbits will be created. After that, a different number a rabbits will be eaten. If each rabbit creates C copies of itself, then the first step of the sequence will leave us with $(C+1)N_i$ rabbits. Then, if each rabbits eats a K number of its kind, we will end up with a $\frac{(C+1)N_i}{(K+1)}$ number of rabbits. Figure 5 shows a brief example of this mechanism at work.

We can use the iterative notation introduced before and say that on the i-th step we will have

$$N_i = \frac{C+1}{K+1}N_{i-1} \tag{6}$$

$\frac{C+1}{K+1}$ is also an important number, so we will give it a name. The "Starvation number" is defined as

$$S \stackrel{\text{def}}{=} \frac{C+1}{K+1} \tag{7}$$

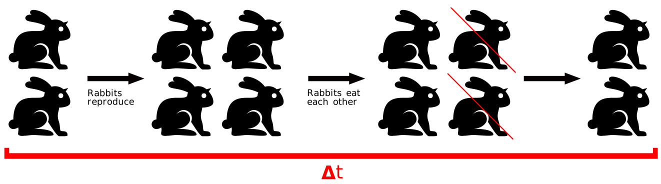

Figure 5: *Example of the internal feeding regime. In this particular case, we have $C = 1$ and $K = 1$. This way, each rabbit creates a copy of itself, and then proceeds to eat a single rabbit in order to avoid starvation.*

Note that there are three possible scenarios regarding S: it could be equal, greater or lower than one. If $S > 1$ we see from equation 6 that the Great Rabbit keeps on growing (because $N_i > N_{i-1}$. If $S = 1$ then $N_i = N_{i-1}$ and so the population does not grow nor decrease. If $S < 1$, it follows that $N_i < N_{i-1}$ and we get a decreasing population.

Once again, we highlight that both C and K (and, in consequence, S) are in reality dependant on the Δt interval we are talking about.

4 Using the model

With the two regimes understood, we can start plugging in numbers and seeing how this model works. We will focus our work on the W, S and C numbers, as they characterize the behaviour of the system.

4.1 Does the Great Rabbit ever stop growing?

As we did before, let us imagine we start with a single rabbit which starts creating copies of itself. Let also W be so that the Witch's condition is met for that single rabbit. This way, we are on the external feeding regime and the Great Rabbit starts growing.

For the growth to stop, two things have to happen in sequence:

1. The Witch's condition has to stop being fulfilled for some N_i amount of rabbits, so that the Great Rabbit is forced change from external feeding to internal feeding.

2. The internal feeding regime has to be so that the rabbit population decreases.

To analyze the first condition, we can take a look at equation 4. It is easy to see that the right side of the inequality tends to zero as N_i grows. Since W is necessarily greater than zero (because E_D, ρ_{E0} and A_R are all positive numbers) this means that the Witch's condition will not be met starting from a certain N_i number of rabbits. Which specific N_i is the limit number depends on C, as shown in Figure 6.

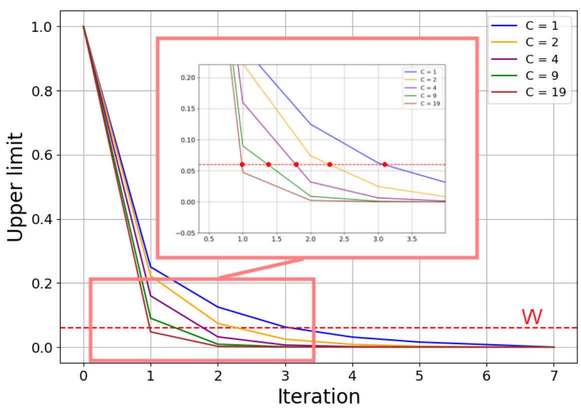

Figure 6: *Right side of the Witch's condition (Upper limit) as a function of the iteration for different C numbers. An arbitrary number W (dotted line) was drawn to show when the Witch's condition stops being true. The red dots in the inset show that precise moment.*

We see then that C and the Witch's number are what determines when the feeding regime changes. W is the threshold starting from which we get a change in the feeding regime and C determines when we reach that threshold. A bigger C implies an earlier change in regimes: this makes sense, since more rabbits per iteration mean less food for each one.

Now that we have changed regimes let us direct our attention to equation 6, which gives us the evolution of the rabbit quantity. We can see that S is what determines how this evolution goes. As stated before, there are three possible situations: $S > 1$, $S = 1$ and $S < 1$.

Figure 7 shows five curves with the same C and W, but different S numbers. Because C and W are the equal for all curves, all five of them stay the same until the regime change happens. In that moment we see the effect of S: if $S > 1$ the growth continues (with faster growth corresponding to bigger S numbers), if $S = 1$ the curve stays as a constant, and if $S < 1$ the rabbit amount start decreasing (with faster decrease belonging to lower S).

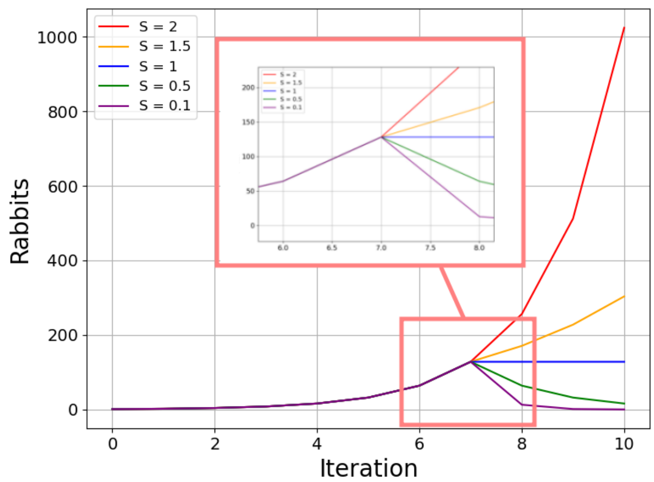

Figure 7: *Rabbit growth for different S values. C and W are the same for all the curves. In the inset the point where the regime change happens is shown.*

However, as stated in section 2, we know that there is a limit to the number of rabbits alive at the same time. What this means is that the Great Rabbit can't grow during the internal feeding regime (if it could, the Rabbit could grow forever without the need of an external food source). For that reason we can discard the $S > 1$ scenario and keep just the other two as possibilities. Note that discarding this case makes it so the maximum rabbit amount is reached at the point of regime change.

In summary, we have characterized the general behaviour of the rabbit population according to this model. We start with a growing population on the external feeding regime which lasts until the Witch's condition (which depends on W and C) is no longer met. When that happens we enter the internal feeding regime and have three possible scenarios: the population can continue increasing, stay constant or start decreasing. Which scenario we get depends on S, but empirical evidence allows us to discard an $S > 1$ case.

4.2 Can we predict the Great Rabbit's size?

We have determined that the Great Rabbit reaches its maximum size when the regime change happens. Said moment is determined by W and C. Furthermore, if we know C, we know the size of the Great Rabbit for any iteration. That brings us to the question: Can we predict this maximum size knowing only W and C?

To answer this question, we direct our attention to equation 4. We wrote the Witch's condition for any N_i, but after that we took $N_i = (C+1)^i$. Replacing this N_i into equation 4 leaves us with

$$W \leq \frac{1}{(C+1)^i} - \frac{1}{(C+1)^{i+1}}$$

From this inequality we can get a condition for i

$$i \leq \frac{\log(\frac{C}{(C+1)W})}{\log(C+1)} \tag{8}$$

which is what we were looking for. The moment i becomes greater than $\frac{\log(\frac{C}{(C+1)W})}{\log(C+1)}$, the Witch's condition stops being met and the growth regime changes.

A detail to have in mind is that this is a discrete model, meaning that i is necessarily a natural number. $\frac{\log(\frac{C}{(C+1)W})}{\log(C+1)}$, on the other hand, can be any real number greater than zero. What this means is that if the condition is fulfilled for some i but not for $i+1$, we are going to see the regime change on the $i+1$ step.

4.3 An application example

We can now try to put this model to use. The four factors we have to determine in order to do that are C, W, S and Δt.

Unfortunately, are many unknown variables and determining those three numbers for the real Great Rabbit is something we can't do due to the lack of empirical evidence. However, we can do some good guesses. We know that the Great Rabbit's maximum size ever recorded is around 80.000 rabbits. We also know exponentiation, which allows us to notice that $5^7 = (4+1)^7 = 78.125$ gives us a pretty close match to those 80.000 rabbits. We will take $C = 4$ then: each rabbit creates four copies of itself in a Δt time.

As for W, we can use equation 4 with $i = 7$ and $C = 4$ to notice that $W = 0.00001024$ is the threshold we are looking for.

We also need S. As discussed in section 4.1, there are two possible scenarios: either the Great Rabbit's size stays constant or it starts decreasing. Either one is possible. The rabbit horde could just stay the same size and that is it ($S = 1$), or it could start decreasing in population ($S < 1$) and be forced to move and find a new food source. For the sake of argument we will take both situations into account. Taking $S = 1$ would mean that each rabbit has to eat four of its brothers to survive a Δt time, and $S < 1$ would mean that they have to eat more than that.

Finally, we need to turn iterations into time: we need to decide on a Δt. Once again we only have guesswork to rely on here. We know that a standard rabbit weights around two kilograms, so we will assume each one of the Great Rabbit's members weights that much. We also have evidence to suggest that they can eat (roughly) one kilogram of food per minute [6]. This would mean that each rabbit would need about eight minutes to eat four other rabbits in the internal feeding regime. With that reasoning we will settle on $\Delta t = 4$ minutes.

The resulting simulation is shown in Figure 8. We see that it takes 28 minutes for the Great Rabbit to reach its maximum size. In the case of $S < 1$ we can also predict a time before they starve, defined as the time it will take for the population to fall under one rabbit. This time depends on the specific Starvation number. It can range from a few minutes (S very close to 0) to hours (S close to but lower than 1).

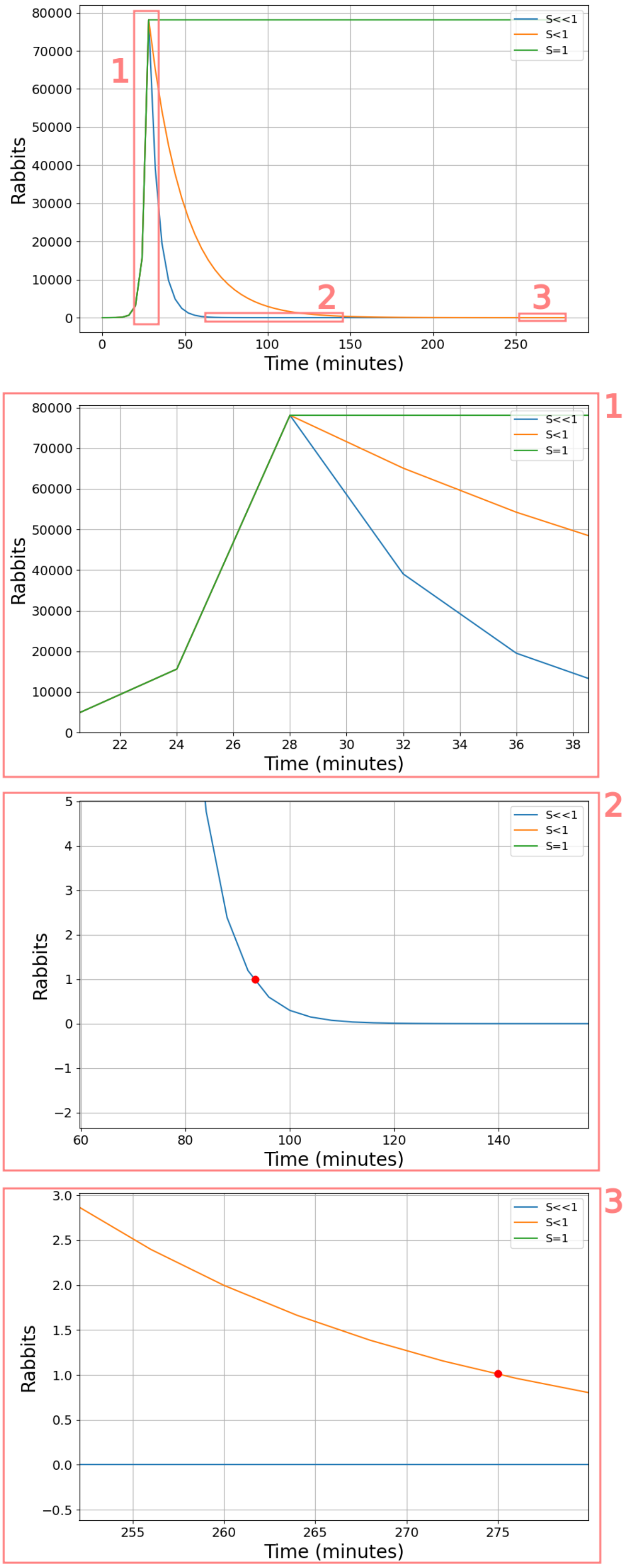

Figure 8: *Rabbit population evolution for the parameters mentioned earlier, using three different S numbers. Zoom 1 shows that the population reaches its peak at 28 minutes. Zooms 2 and 3 show when the population falls under one rabbit for each of the S used.*

Here we took a known number (the Great Rabbit's maximum recorded size) and induced a few things from there. But it is easy to see that we can to the opposite: if we measure C, S, W for a certain Δt then we can predict the

Great Rabbit's size, how long it would take for them to starve (if that even happens) and such.

5 Conclusions

Over the course of this work we were able to create a model for the growth of the Great Rabbit. With the input of three measurable parameters (W, C and S) and deciding on a Δt interval, this model is able to make quantitative predictions about the size of the Great Rabbit in a certain area. It can also predict if (and when) will the rabbits go extinct, given there are no significant changes in their environment.

It is also important to note the qualitative information we can extract from the model. We know that there are two regimes: one in which the Great Rabbit grows by consuming the animals in its way (external feeding regime), and one where its size stays constant or decreases (internal feeding regime). The Rabbit reaches its maximum size at the point where the regime changes.

And we think that last piece of information is of particular importance. If we want to stop the Great Rabbit, the first thing we need to do is stop it from multiplying. According to this model, a higher Witch's number means an earlier change in regimes: thus, limiting the area the rabbits can travel (lowering A_R), making the rabbits consume more energy (increasing E_C) or limiting the amount of potential prey in areas nearby the Great Rabbit (lowering ρ_{E0}) could have a significant impact in limiting its growth. Other measures could be thought out using this model as a starting point.

This model is not without its limitations, however. First and foremost, some of its base assumptions (particularly, the rabbit's need to eat) have yet to be confirmed experimentally. Also, it does not account for changes in the rabbits environment (as would be the rabbits moving from one ecosystem to another). With more field data it should be able to improve this model and make it more reliable.

6 Acknowledgements

We acknowledge the Kingdom of Lugunica for financially supporting this project. We would also like to acknowledge all the efforts made by previous researchers to gather solid data in regards to this and all other Great Mabeasts, and extend our deepest condolences to the families and friends of people who lost their lives due to these calamities. It is our hope that this work and other related efforts pave the path towards a brighter future.

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