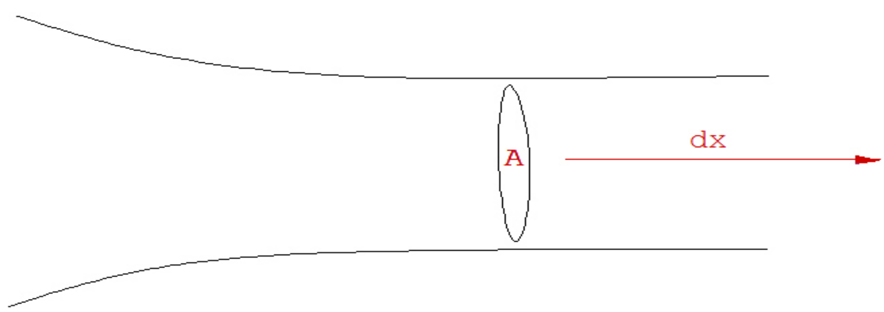
Axial Fan Design

fig.1



In fig.1, a mass of air is pushed a distance, Δx through an axial fan of area, A. This mass of air is the product of the air density, axial fan area and the distance travelled by the air.

$$\Delta m = \rho A \Delta x$$

Mass flux equals this mass of air divided by the time taken to travel the distance.

$$\frac{\Delta m}{\Delta t} = \rho A \frac{\Delta x}{\Delta t}$$

$\Delta x/\Delta t$ is the magnitude of the axial air velocity vector,

$$\frac{\Delta m}{\Delta t} = \rho A |\vec{v}|$$

According to Newton's 3rd law, the thrust, or the force exerted by the air on the fan is equal to the force exerted by the fan on the air. According to Newton's 2nd law the force exerted by the fan on the air is equal to the rate of change of the momentum of the air.

$$F = \frac{\Delta\left(m|\vec{v}|\right)}{\Delta t}$$

$$F = |\vec{v}| \frac{\Delta m}{\Delta t}$$

$$F = \rho A |\vec{v}|^2$$

Therefore the magnitude of the axial velocity vector is,

$$|\vec{v}|=\sqrt{\frac{F}{\rho A}}$$

If there are many blades, the blade geometry will be imposed onto the flow.

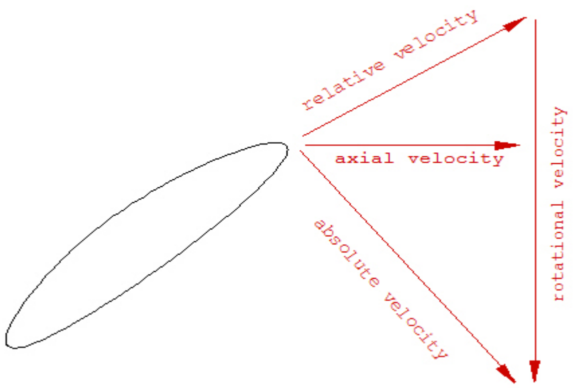


fig.4

The rotational velocity vector will be defined as $\vec{u}$,the angle between the axial and relative velocity vectors defined as ψ and the angle between the axial and absolute velocity vectors defined as φ.

The trigonometric relationship between $\vec{u}$, $\vec{v}$, ψ and φ is;

$$|\vec{u}|=|\vec{v}|\tan(\psi)+|\vec{v}|\tan(\varphi)$$

If the airfoil is uncambered, the angle between the relative velocity and rotational velocity vectors is the same as the angle between the camber line and the rotational velocity vector. This is the blade angle, θ.

The axial velocity vector is perpendicular to the rotational velocity vector and the sum of the internal angles in a triangle is 180°.

$$\theta+\psi+90=180$$

$$\psi=90-\theta$$

The incoming air flow is being deflected perpendicular to the airfoil camber line. Consequently,the angle between the absolute and relative velocity vectors is 90°

$$(90-\theta)+\varphi=90$$

$$\varphi=\theta$$

The magnitude of $\vec{u}$ is $r\omega$.

$$r\omega=(\tan(90-\theta)+\tan(\theta))\sqrt{\frac{F}{\rho A}}$$

$$\frac{r^2\omega^2}{(\tan(90-\theta)+\tan(\theta))^2}=\frac{F}{\rho A}$$

$$4F=\rho A r^2\omega^2\sin^2(2\theta)$$

Is the relationship between fan thrust and angular velocity.

Multiplying the fan thrust by the magnitude of $\vec{v}$ will give the power required.

$$P=F|\vec{v}|$$

$$P=F\sqrt{\frac{F}{\rho A}}$$

$$F\sqrt{F}=P\sqrt{\rho A}$$

$$F^3=P^2\rho A$$

Is the relationship between fan thrust and power.