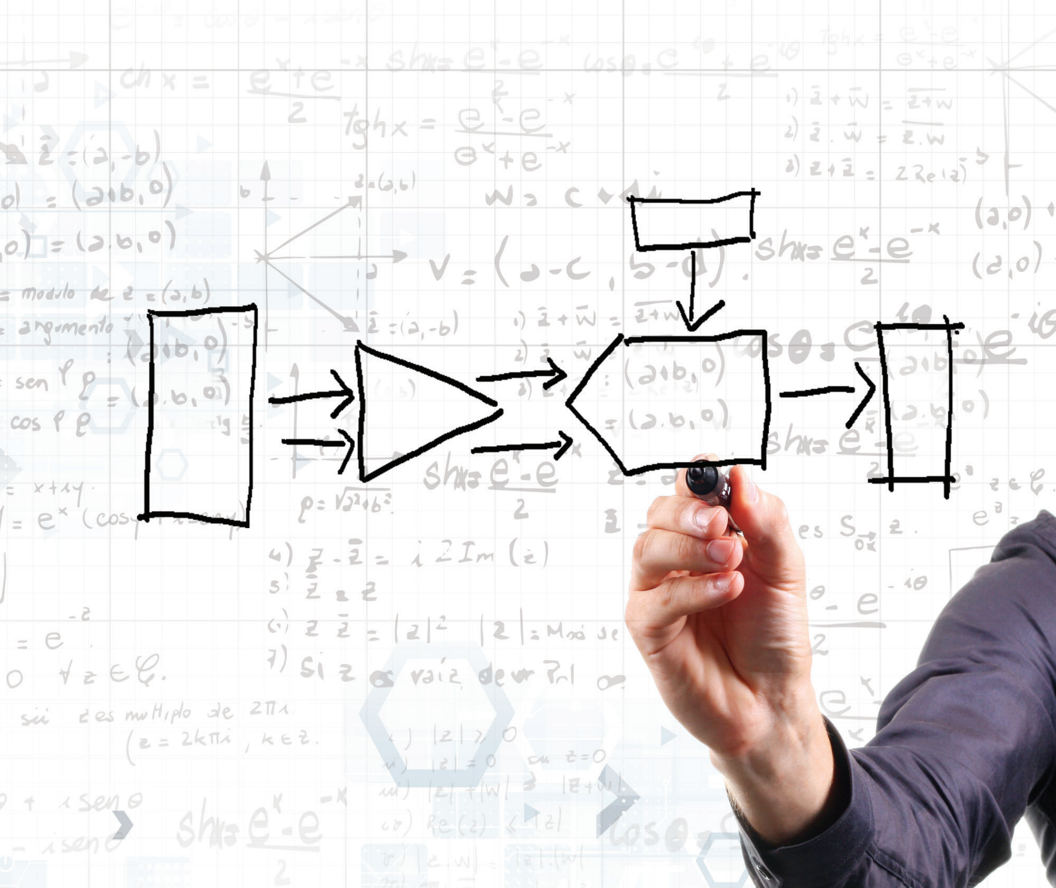
Analog Engineer's

Pocket Reference

Art Kay and Tim Green, Editors



Texas Instruments

Analog Engineer's Pocket Reference

Third Edition

Edited by:

Art Kay and Tim Green

Special thanks for technical contribution and review:

Kevin Duke

Rafael Ordonez

John Caldwell

Collin Wells

Ian Williams

Thomas Kuehl

Message from the editors:

This pocket reference is intended as a valuable quick guide for often used board- and system-level design formulae. This collection of formulae is based on a combined 50 years of analog board- and system-level expertise. Much of the material herein was referred to over the years via a folder stuffed full of printouts. Those worn pages have been organized and the information is now available via this guide in a bound and hard-to-lose format!

Here is a brief overview of the key areas included:

- Key constants and conversions
- Discrete components
- AC and DC analog equations
- Op amp basic configurations
- OP amp bandwidth and stability
- Overview of sensors
- PCB trace R, L, C
- Wire L, R, C
- Binary, hex and decimal formats
- A/D and D/A conversions

We hope you find this collection of formulae as useful as we have. Please send any comments and/or ideas you have for the next edition of the *Analog Engineer's Pocket Reference* to **artkay_timgreen@list.ti.com**

Additional resources:

- Search for complete board-and-system level circuits in the TI Designs – Precision reference design library (**www.ti.com/precisiondesigns**).
- Read how-to blogs from TI precision analog experts at the Precision Hub (**www.ti.com/thehub**).
- Find solutions, get help, share knowledge and solve problems with fellow engineers and TI experts in the TI E2E™ Community (**www.ti.com/e2e**).

Conversions 7
Physical constants 8
Standard decimal prefixes 9
Metric conversions 9
Temperature conversions 10
Error conversions (ppm and percentage) 10

Discrete components 11
Resistor color code 12
Standard resistor values 13
Practical capacitor model and specifications 14
Practical capacitors vs. frequency 15
Capacitor type overview 16
Standard capacitance values 17
Capacitance marking and tolerance 17
Diodes and LEDs 18

Analog 19
Capacitor equations (series, parallel, charge, energy) 20
Inductor equations (series, parallel, energy) 21
Capacitor charge and discharge 22
RMS and mean voltage definition 24
RMS and mean voltage examples 25
Logarithmic mathematical definitions 27
dB definitions 28
Pole and zero definitions and examples 30

Amplifier 35
Basic op amp configurations 36
Op amp bandwidth 41
Full power bandwidth 42
Small signal step response 43
Noise equations 44
Stability equations 48
Stability open loop SPICE analysis 50

PCB and wire 55
PCB conductor spacing 56
Self-heating of PCB traces on inside layer 57
PCB trace resistance for 1oz and 2oz Cu 58
Package types and dimensions 60
PCB parallel plate capacitance 61
PCB via capacitance and inductance 62
Common coaxial cable specifications 64
Coaxial cable equations 65
Resistance per length for different wire types (AWG) 66
Maximum current for wire types 67

Sensor 69
Temperature sensor overview 70
Thermistor 71
Resistive temperature detector (RTD) 72
Diode temperature characteristics 74
Thermocouple (J and K) 76

A/D conversion 81
Binary/hex conversions 82
A/D and D/A transfer function 86
Quantization error 88
Signal-to-noise ratio (SNR) 89
Signal-to-noise and distortion (SINAD) 90
Total harmonic distortion (THD) 89
Effective number of bits (ENOB) 92
Noise free resolution and effective resolution 93

Conversions

- Standard decimal prefixes
- Metric conversions
- Temperature scale conversions
- Error conversions (ppm and percentage)

Conversions

Table 1: Physical constants

Constant	Symbol	Value	Units
Speed of light in a vacuum	c	3.00×10^{8}	m/s
Permittivity of vacuum	ε_o	8.85×10^{-12}	F/m
Permeability of free space	μ_o	1.26×10^{-6}	H/m
Plank's constant	h	6.63×10^{-34}	Js
Boltzmann's constant	k	1.38×10^{-23}	J/K
Faraday's constant	F	9.65×10^{4}	C/mol
Avogadro's constant	N_A	6.02×10^{23}	/mol
Unified atomic mass unit	m_u	1.66×10^{-27}	kg
Electronic charge	q	1.60×10^{-19}	C
Rest mass of electron	m_e	9.11×10^{-31}	kg
Mass of proton	m_p	1.67×10^{-27}	kg
Gravitational constant	G	6.67×10^{-11}	Nm^2/kg
Standard gravity	g_n	9.81	m/s^2
Ice point	T_{ice}	273.15	K
Maximum density of water	ρ	1.00×10^{3}	kg/m^3
Density of mercury	ρ_{Hg}	1.36×10^{4}	kg/m^3
Gas constant	R	8.31	J/(K•mol)
Speed of sound in air (at 273 K)	c_{air}	3.31×10^{2}	m/s

Table 2: Standard decimal prefixes

Multiplier	Prefix	Abbreviation
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Table 3: English to metric conversions

Unit	Symbol	Equivalent	Unit	Symbol
inches	in	25.4 mm/in	millimeter	mm
mil	mil	0.0254 mm/mil	millimeter	mm
feet	ft	0.3048 m/ft	meters	m
yards	yd	0.9144 m/yd	meters	m
miles	mi	1.6093 km/mi	kilometers	km
circular mil	cir mil	5.067×10^{-4} mm^2/cir mil	square millimeters	mm^2
square yards	yd^2	0.8361 m^2	square meters	m^2
pints	pt	0.5682 L/pt	liters	L
ounces	oz	28.35 g/oz	grams	g
pounds	lb	0.4536 kg/lb	kilograms	kg
calories	cal	4.184 J/cal	joules	J
horsepower	hp	745.7 W/hp	watts	W

Table 4: Metric to English conversions

Unit	Symbol	Conversion	Unit	Symbol
millimeter	mm	0.0394 in/mm	inch	in
millimeter	mm	39.4 mil/mm	mil	mil
meters	m	3.2808 ft/m	feet	ft
meters	m	1.0936 yd/m	yard	yd
kilometers	km	0.6214 mi/km	miles	mi
square millimeters	mm^2	1974 cir mil/mm^2	circular mil	cir mil
square meters	m^2	1.1960 yd^2/ m^2	square yards	yd^2
liters	L	1.7600 pt/L	pints	pt
grams	g	0.0353 oz/g	ounces	oz
kilograms	kg	2.2046 lb/kg	pounds	lb
joules	J	0.239 cal/J	calories	cal
watts	W	1.341×10^{-3} hp/W	horsepower	hp

Example

Convert 10 mm to mil.

Answer

$$10\ \text{mm} \times 39.4\frac{\text{mil}}{\text{mm}} = 384\ \text{mil}$$

Table 5: Temperature conversions

$°C = \frac{5}{9}(°F - 32)$	Fahrenheit to Celsius
$°F = \frac{9}{5}(°C) + 32$	Celsius to Fahrenheit
$K = °C + 273.15$	Celsius to Kelvin
$°C = K - 273.15$	Kelvin to Celsius

Table 6: Error conversions

$\text{Error}(\%) = \frac{\text{Measured} - \text{Ideal}}{\text{Ideal}} \times 100$	Error in measured value
$\text{Error}(\%\ \text{FSR}) = \frac{\text{Measured} - \text{Ideal}}{\text{Full-scale range}} \times 100$	Error in percent of full-scale range
$\% = \frac{\text{ppm}}{10^6} \times 100$	Part per million to percent
$\text{m}\% = \frac{\text{ppm}}{10^6} \times 100 \times 1000$	Part per million to milli-percent
$\text{ppm} = \% \times 10^4$	Percent to part per million
$\text{ppm} = \text{m}\% \times 10$	Milli-percent to part per million

Example

Compute the error for a measured value of 0.12V when the ideal value is 0.1V and the range is 5V.

Answer

$$\text{Error}(\%) = \frac{0.12\text{V} - 0.1\text{V}}{0.1\text{V}} \times 100 = 20\%$$ Error in measured value

$$\text{Error}(\%\ \text{FSR}) = \frac{0.12 - 0.1\text{V}}{5\text{V}} \times 100 = 0.4\%$$ Percent FSR

Example

Convert 10 ppm to percent and milli-percent.

Answer

$$\frac{10\ \text{ppm}}{10^6} \times 100 = 0.001\%$$ Part per million to percent

$$\frac{10\ \text{ppm}}{10^6} \times 100 \times 1000 = 1\ \text{m}\%$$ Part per million to milli-percent

Discrete components

- Resistor color code
- Standard resistor values
- Capacitance specifications
- Capacitance type overview
- Standard capacitance values
- Capacitance marking and tolerance

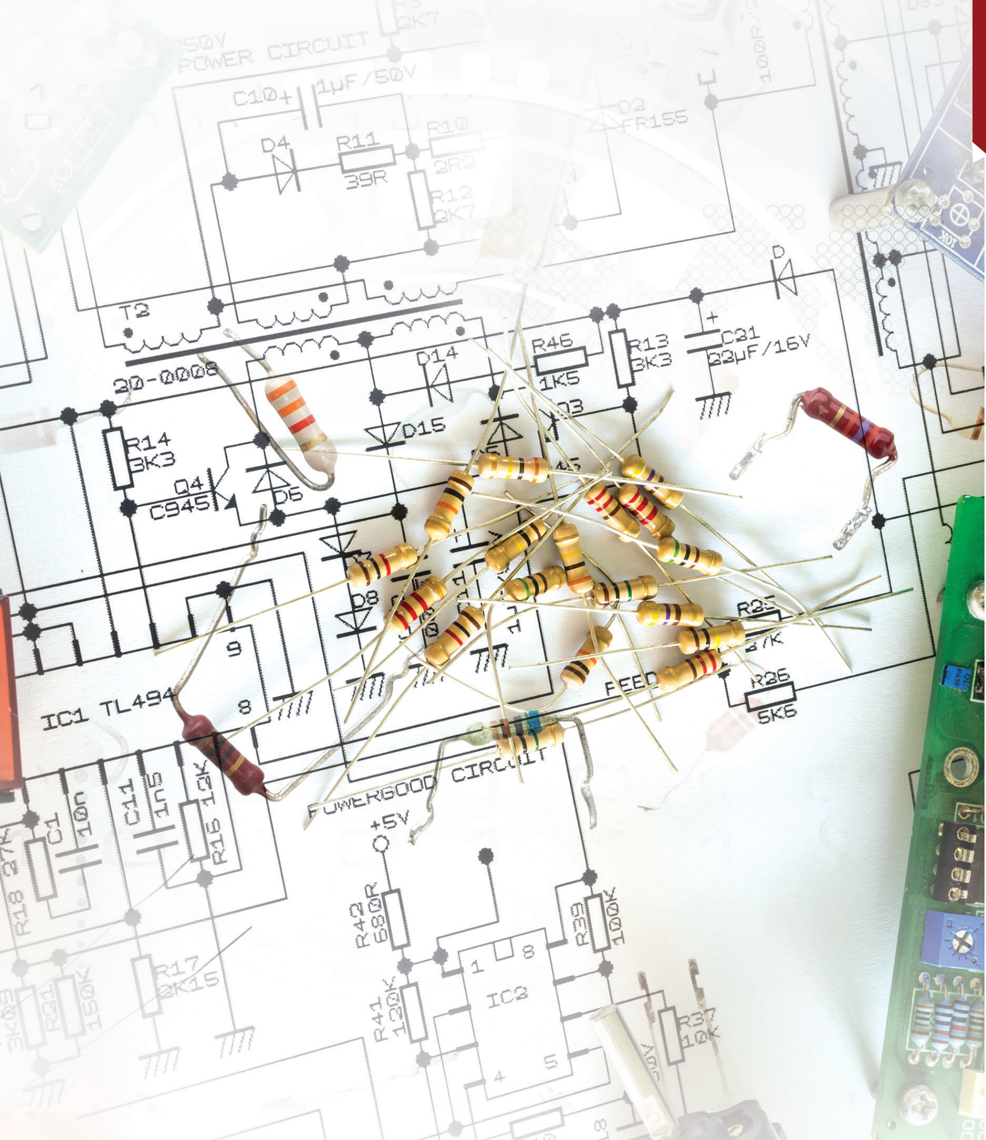


Table 7: Resistor color code

Color	Digit	Additional Zeros	Tolerance
None	-na-	-na-	20%
Silver	-na-	-2	10%
Gold	-na-	-1	5%
Black	0	0	
Brown	1	1	
Red	2	2	2%
Orange	3	3	
Yellow	4	4	
Green	5	5	
Blue	6	6	
Violet	7	7	
Grey	8		
White	9		

Example

Yellow, violet, orange and silver indicate 4, 7, and 3 zeros. or a 47 kΩ, 10% resistor.

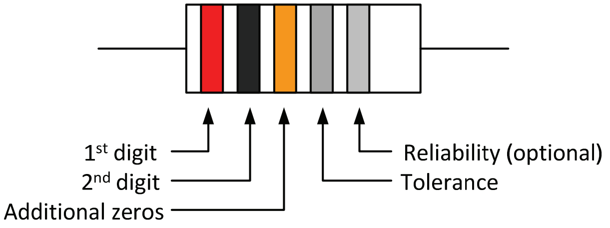


Figure 1: Resistor color code

Table 8: Standard resistor values

Standard resistance values for the 10 to 100 decade

0.1% 0.25% 0.5%	1%	2% 5% 10%	0.1% 0.25% 0.5%	1%	2% 5% 10%	0.1% 0.25% 0.5%	1%	2% 5% 10%	0.1% 0.25% 0.5%	1%	2% 5% 10%	0.1% 0.25% 0.5%	1%	2% 5% 10%	0.1% 0.25% 0.5%	1%	2% 5% 10%
10.0	10.0	10	14.7	14.7		21.5	21.5		31.6	31.6		46.4	46.4		68.1	68.1	68
10.1			14.9			21.8			32.0			47.0		47	69.0		
10.2	10.2		15.0	15.0	15	22.1	22.1	22	32.4	32.4		47.5	47.5		69.8	69.8	
10.4			15.2			22.3			32.8			48.1			70.6		
10.5	10.5		15.4	15.4		22.6	22.6		33.2	33.2	33	48.7	48.7		71.5	71.5	
10.6			15.6			22.9			33.6			49.3			72.3		
10.7	10.7		15.8	15.8		23.2	23.2		34.0	34.0		49.9	49.9		73.2	73.2	
10.9			16.0		16	23.4			34.4			50.5			74.1		
11.0	11.0	11	16.2	16.2		23.7	23.7		34.8	34.8		51.1	51.1	51	75.0	75.0	75
11.1			16.4			24.0		24	35.2			51.7			75.9		
11.3	11.3		16.5	16.5		24.3	24.3		35.7	35.7		52.3	52.3		76.8	76.8	
11.4			16.7			24.6			36.1		36	53.0			77.7		
11.5	11.5		16.9	16.9		24.9	24.9		36.5	36.5		53.6	53.6		78.7	78.7	
11.7			17.2			25.2			37.0			54.2			79.6		
11.8	11.8		17.4	17.4		25.5	25.5		37.4	37.4		54.9	54.9		80.6	80.6	
12.0		12	17.6			25.8			37.9			55.6			81.6		
12.1	12.1		17.8	17.8		26.1	26.1		38.3	38.3		56.2	56.2	56	82.5	82.5	82
12.3			18.0		18	26.4			38.8			56.9			83.5		
12.4	12.4		18.2	18.2		26.7	26.7		39.2	39.2	39	57.6	57.6		84.5	84.5	
12.6			18.4			27.1		27	39.7			58.3			85.6		
12.7	12.7		18.7	18.7		27.4	27.4		40.2	40.2		59.0	59.0		86.6	86.6	
12.9			18.9			27.7			40.7			59.7			87.6		
13.0	13.0	13	19.1	19.1		28.0	28.0		41.2	41.2		60.4	60.4		88.7	88.7	
13.2			19.3			28.4			41.7			61.2			89.8		
13.3	13.3		19.6	19.6		28.7	28.7		42.2	42.2		61.9	61.9	62	90.9	90.9	91
13.5			19.8			29.1			42.7			62.6			92.0		
13.7	13.7		20.0	20.0	20	29.4	29.4		43.2	43.2	43	63.4	63.4		93.1	93.1	
13.8			20.3			29.8			43.7			64.2			94.2		
14.0	14.0		20.5	20.5		30.1	30.1	30	44.2	44.2		64.9	64.9		95.3	95.3	
14.2			20.8			30.5			44.8			65.7			96.5		
14.3	14.3		21.0	21.0		30.9	30.9		45.3	45.3		66.5	66.5		97.6	97.6	
14.5			21.3			31.2			45.9			67.3			98.8		

Practical capacitor model and specifications

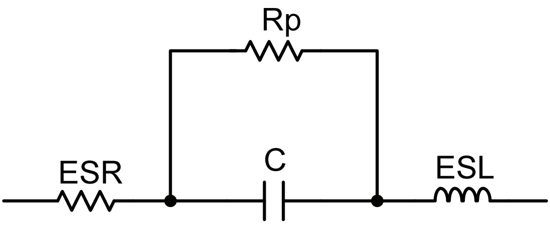


Figure 2: Model of a practical capacitor.

Table 9: Capacitor specifications

Parameter	Description
C	*The nominal value of the capacitance*. Table 11 lists standard capacitance values.
ESR	*Equivalent series resistance.* Ideally this is zero. Ceramic capacitors have the best ESR (typically in milliohms). Tantalum Electrolytic have ESR in the hundreds of milliohms and Aluminum Electrolytic have ESR in the ohms.
ESL	*Equivalent series inductance*. Ideally this is zero. ESL ranges from 100 pH to 10 nH.
Rp	*Rp is a parallel leakage resistance (or insulation resistance).* Ideally this is infinite. This can range from tens of megaohms for some electrolytic capacitors to tens of gigohms for ceramic.
Voltage rating	*The maximum voltage that can be applied to the capacitor.* Exceeding this rating damages the capacitor.
Voltage coefficient	*The change in capacitance with applied voltage in ppm/V*. A high-voltage coefficient can introduce distortion. C0G capacitors have the lowest coefficient. The voltage coefficient is most important in applications that use capacitors in signal processing such as filtering.
Temperature coefficient	*The change in capacitance with across temperature in ppm/℃.* Ideally, the temperature coefficient is zero. The maximum specified drift generally ranges from 10 to 100ppm/°C depending on the resistor type.

Practical capacitors vs. frequency

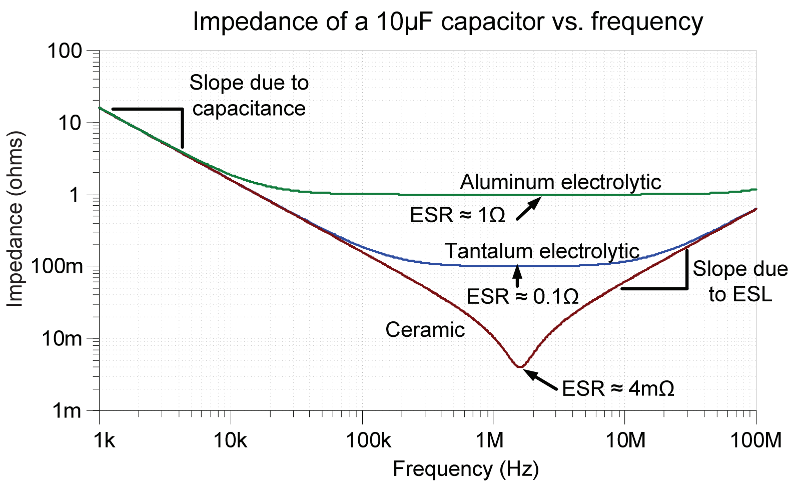


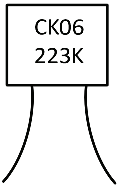
Figure 3: Effect of ESR and ESL on capacitor frequency response

Table 10: Capacitor type overview

Capacitor type	Description
C0G/NP0 (Type 1 ceramic)	*Use in signal path, filtering, low distortion, audio, and precision* Limited capacitance range: 0.1 pF to 0.47 µF Lowest temperature coefficient: ±30 ppm/℃ Low-voltage coefficient Minimal piezoelectric effect Good tolerance: ±1% to ±10% Temperature range: –55℃ to 125℃ (150℃ and higher) Voltage range may be limited for larger capacitance values
X7R (Type 2 ceramic)	*Use for decoupling and other applications where accuracy and low distortion are not required* X7R is an example of a type 2 ceramic capacitor See **EIA capacitor tolerance table** for details on other types Capacitance range: 10 pF to 47 µF Temperature coefficient: ±833 ppm/℃ (±15% across temp range) Substantial voltage coefficient Tolerance: ±5% to –20%/+80% Temperature range: –55℃ to 125℃ Voltage range may be limited for larger capacitance values
Y5V (Type 2 ceramic)	*Use for decoupling and other applications where accuracy and low distortion are not required* Y5V is an example of a type 2 ceramic capacitor See **EIA capacitor tolerance table** for details on other types Temperature coefficient: –20%/+80% across temp range Temperature range: –30℃ to 85℃ Other characteristics are similar to X7R and other type 2 ceramic
Aluminum oxide electrolytic	*Use for bulk decoupling and other applications where large capacitance is required* Note that electrolytic capacitors are polarized and will be damaged, if a reverse polarity connection is made Capacitance range: 1 µF to 68,000 µF Temperature coefficient: ±30 ppm/℃ Substantial voltage coefficient Tolerance: ±20% Temperature range: –55℃ to 125℃ (150℃ and higher) Higher ESR than other types
Tantalum electrolytic	Capacitance range: 1 µF to 150 µF Similar to aluminum oxide but smaller size
Polypropylene film	Capacitance range: 100 pF to 10 µF Very low voltage coefficient (low distortion) Higher cost than other types Larger size per capacitance than other types Temperature coefficient: 2% across temp range Temperature range: –55℃ to 100℃

Table 11: Standard capacitance table

Standard capacitance table											
1	1.1	1.2	1.3	1.5	1.6	1.8	2	2.2	2.4	2.7	3
3.3	3.6	3.9	4.3	4.7	5.1	5.6	6.2	6.8	7.5	8.2	9.1



Example

Translate the capacitor marking

2 2 3 K → "K" = ±10%

22 000 pF

= 22nF = 0.022µF

Figure 4: Capacitor marking code

Table 12: Ceramic capacitor tolerance markings

Code	Tolerance		Code	Tolerance
B	± 0.1 pF		J	± 5%
C	± 0.25 pF		K	± 10%
D	± 0.5 pF		M	± 20%
F	± 1%		Z	+ 80%, –20%
G	± 2%			

Table 13: EIA capacitor tolerance markings (Type 2 capacitors)

First letter symbol	Low temp. limit		Second number symbol	High temp. limit		Second letter symbol	Max. capacitance change over temperature rating
Z	+10°C		2	+45°C		A	±1.0%
Y	–30°C		4	+65°C		B	±1.5%
X	–55°C		5	+85°C		C	±2.2%
			6	+105°C		D	±3.3%
			7	+125°C		E	±4.7%
						F	±7.5%
						P	±10.0%
						R	±15.0%
						S	±22.0%
						T	±22% ~ 33%
						U	±22% ~ 56%
						V	±22% ~ 82%

Example

X7R: –55°C to +125°C, ±15.0%

Diodes and LEDs

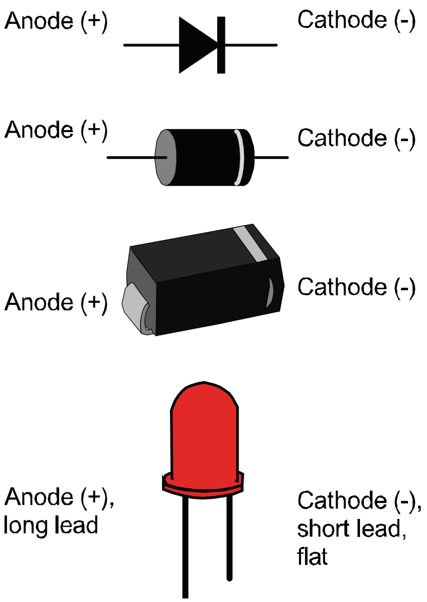


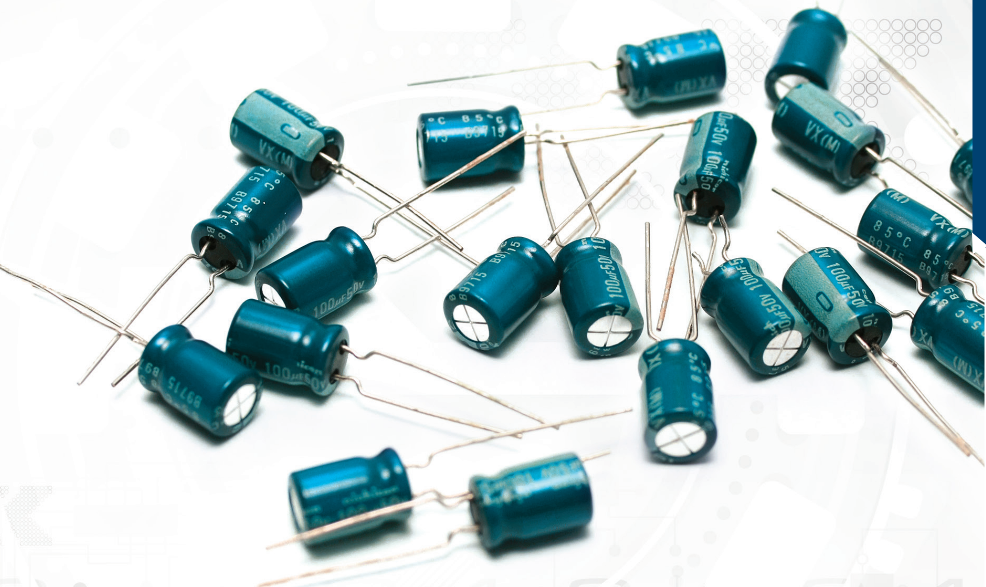
Figure 5: Diode and LED pin names

Color	Voltage
Infrared	1.4
Red	1.7 to 1.9
Orange / yellow	2
Green	2.1
Blue / white	3.4

Figure 6: LED forward voltage drop by color

Analog

- Capacitor equations (series, parallel, charge, energy)
- Inductor equations (series, parallel, energy)
- Capacitor charge and discharge
- RMS and mean voltage definition
- RMS for common signals
- Logarithm laws
- dB definitions
- Pole and zero definition with examples



Capacitor equations

$$C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}}$$ (1) Series capacitors

$$C_t = \frac{C_1 C_2}{C_1 + C_2}$$ (2) Two series capacitors

$$C_t = C_1 + C_2 + \cdots + C_N$$ (3) Parallel capacitors

Where
C_t = equivalent total capacitance
C_1, C_2, C_3…C_N = component capacitors

$$Q = CV$$ (4) Charge storage

$$Q = It$$ (5) Charge defined

Where
Q = charge in coulombs (C)
C = capacitance in farads (F)
I = current in amps (A)
t = time in seconds (s)

$$i = C\frac{dv}{dt}$$ (6) Instantaneous current through a capacitor

Where
i = instantaneous current through the capacitor
C = capacitance in farads (F)
$\frac{dv}{dt}$ = the instantaneous rate of voltage change

$$E = \frac{1}{2}CV^2$$ (7) Energy stored in a capacitor

Where
E = energy stored in an capacitor in Joules (J)
V = voltage in volts
C = capacitance in farads (F)

Inductor equations

$$L_t = L_1 + L_2 + \cdots + L_N$$ (8) Series inductors

$$L_t = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}}$$ (9) Parallel inductors

$$L_t = \frac{L_1 L_2}{L_1 + L_2}$$ (10) Two parallel inductors

Where
L_t = equivalent total inductance
L_1, L_2, $L_3 \ldots L_N$ = component inductance

$$v = L\frac{di}{dt}$$ (11) Instantaneous voltage across an inductor

Where
v = instantaneous voltage across the inductor
L = inductance in Henries (H)
$\frac{d}{dt}$ = the instantaneous rate of voltage change

$$E = \frac{1}{2}LI^2$$ (12) Energy stored in an Inductor

Where
E = energy stored in an inductor in Joules (J)
I = current in amps
L = inductance in Henries (H)

Equation for charging a capacitor

$$V_C = V_S\left[1 - e^{\left(\frac{-t}{\tau}\right)}\right] \qquad (13) \quad \text{General relationship}$$

Where
V_C = voltage across the capacitor at any instant in time (t)
V_S = the source voltage charging the RC circuit
t = time in seconds
τ = RC, the time constant for charging and discharging capacitors

Graphing equation 13 produces the capacitor charging curve below. Note that the capacitor is 99.3% charged at five time constants. It is common practice to consider this *fully charged*.

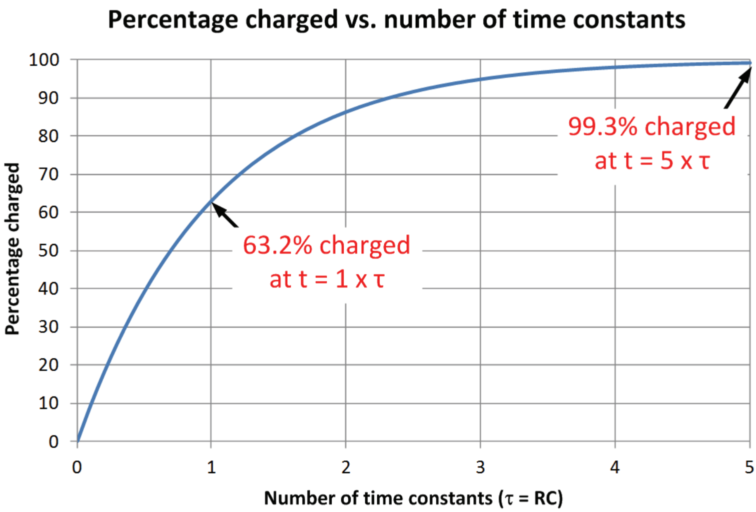


Figure 7: RC charge curve

Equation for discharging a capacitor

$$V_C = V_i\left[e^{\left(\frac{-t}{\tau}\right)}\right] \quad (14)$$ General relationship

Where
V_C = voltage across the capacitor at any instant in time (t)
V_I = the initial voltage of the capacitor at t=0s
t = time in seconds
τ = RC, the time constant for charging and discharging capacitors

Graphing equation 14 produces the capacitor discharge curve below. Note that the capacitor is 0.7% charged at five time constants. It is common practice to consider this *fully discharged*.

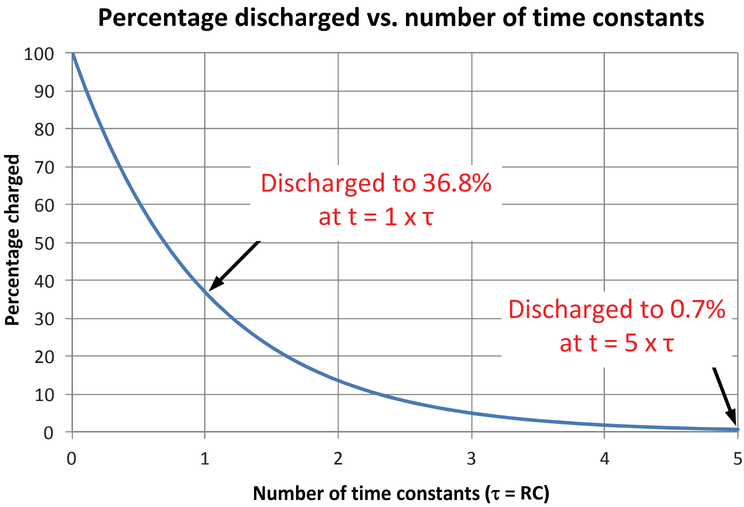


Figure 8: RC discharge curve

RMS voltage

$$V_{RMS} = \sqrt{\frac{1}{(T_2 - T_1)} \int_{T1}^{T2} [V(t)]^2 dt}$$

(15) General relationship

Where
V(t) = continuous function of time
t = time in seconds
$T_1 \leq t \leq T_2$ = the time interval that the function is defined over

Mean voltage

$$V_{MEAN} = \frac{1}{(T_2 - T_1)} \int_{T1}^{T2} V(t) dt$$

(16) General relationship

Where
V(t) = continuous function of time
t = time in seconds
$T_1 \leq t \leq T_2$ = the time interval that the function is defined over

$$V_{RMS} = \frac{V_{PEAK}}{\sqrt{2}}$$

(17) RMS for full wave rectified sine wave

$$V_{MEAN} = \frac{2 \times V_{PEAK}}{\pi}$$

(18) Mean for full wave rectified sine wave

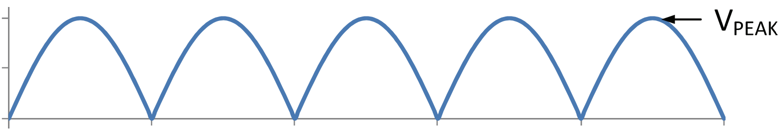


Figure 9: Full wave rectified sine wave

RMS voltage and mean voltage

$$V_{RMS} = V_{PEAK}\sqrt{\frac{T}{2\tau}} \quad (19)$$

RMS for a half-wave rectified sine wave

$$V_{MEAN} = \frac{2 \times V_{PEAK}}{\pi}\left(\frac{T}{\tau}\right) \quad (20)$$

Mean for a half-wave rectified sine wave

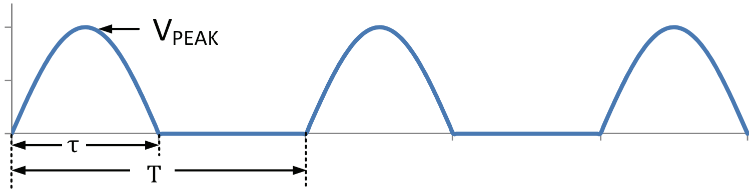


Figure 10: Half-wave rectified sine wave

$$V_{RMS} = V_{PEAK}\sqrt{\frac{\tau}{T}} \quad (21)$$

RMS for a square wave

$$V_{MEAN} = V_{PEAK}\frac{\tau}{T} \quad (22)$$

Mean for a square wave

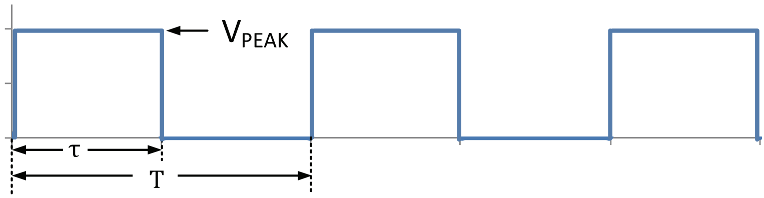


Figure 11: Square wave

RMS voltage and mean voltage

$$V_{RMS} = \frac{\tau}{T}\sqrt{\frac{V_a{}^2 + V_aV_b + V_b{}^2}{3}}$$ (23) RMS for a trapezoid

$$V_{MEAN} = \frac{\tau}{2T}(V_a + V_b)$$ (24) Mean for a trapezoid

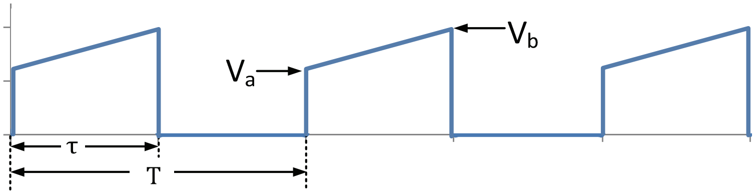


Figure 12: Trapezoidal wave

$$V_{RMS} = V_{PEAK}\sqrt{\frac{\tau}{3T}}$$ (25) RMS for a triangle wave

$$V_{MEAN} = \frac{\tau}{2T}V_{PEAK}$$ (26) Mean for a triangle wave

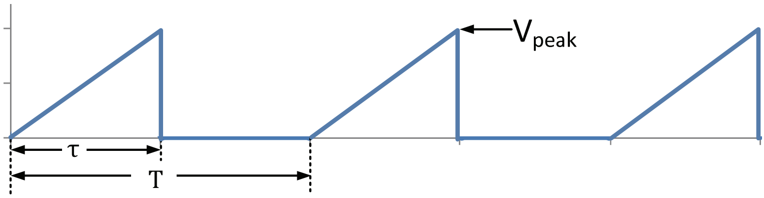


Figure 13: Triangle wave

Logarithmic mathematical definitions

$\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$	(27)	Log of dividend
$\log(AB) = \log(A) + \log(B)$	(28)	Log of product
$\log(A^x) = x\log(A)$	(29)	Log of exponent
$\log_b(X) = \frac{\log_a(X)}{\log_a(b)}$	(30)	Changing the base of log function
$\log_2(X) = \frac{\log_{10}(X)}{\log_{10}(2)}$	(31)	Example changing to log base 2
$\ln(X) = \log_e(X)$	(32)	Natural log is log base e
$e = 2.718282$	(33)	Exponential function to 6 digits.

Alternative notations

$\exp(x) = e^x$	(34)	Different notation for exponential function
$3.54E-2 = 3.54 \times 10^{-2}$	(35)	Different notation for scientific notation, sometimes confused with exponential function

dB definitions

Bode plot basics

The frequency response for the magnitude or gain plot is the change in voltage gain as frequency changes. This change is specified on a Bode plot, a plot of frequency versus voltage gain in dB (decibels). Bode plots are usually plotted as semi-log plots with frequency on the x-axis, log scale, and gain on the y-axis, linear scale. The other half of the frequency response is the phase shift versus frequency and is plotted as frequency versus degrees phase shift. Phase plots are usually plotted as semi-log plots with frequency on the x-axis, log scale, and phase shift on the y-axis, linear scale.

Definitions

$$\text{Voltage gain (dB)} = 20\log\left(\frac{V_{OUT}}{V_{IN}}\right)$$ (36) Voltage gain in decibels

$$\text{Power gain (dB)} = 10\log\left(\frac{P_{OUT}}{P_{IN}}\right)$$ (37) Power gain in decibels

$$\text{Power gain (dBm)} = 10\log\left(\frac{P_{OUT}}{1\text{ mW}}\right)$$ (38) Power gain in decibel milliwatt

Table 14: Examples of common gain values and dB equivalent

A (V/V)	A (dB)
0.001	–60
0.01	–40
0.1	–20
1	0
10	20
100	40
1,000	60
10,000	80
100,000	100
1,000,000	120
10,000,000	140

Roll-off rate is the decrease in gain with frequency

Decade is a tenfold increase or decrease in frequency.(from 10 Hz to 100 Hz is one decade)

Octave is the doubling or halving of frequency (from 10 Hz to 20 Hz is one octave)

Figure 14 illustrates a method to graphically determine values on a logarithmic axis that are not directly on an axis grid line.

1. Given L = 1 cm; D = 2cm, measured with a ruler.
2. $L/D = \log_{10}(f_P)$
3. $f_P = 10^{(L/D)} = 10^{(1CM/2CM)} = 3.16$
4. Adjust for the decade range (for example, 31.6 Hz)

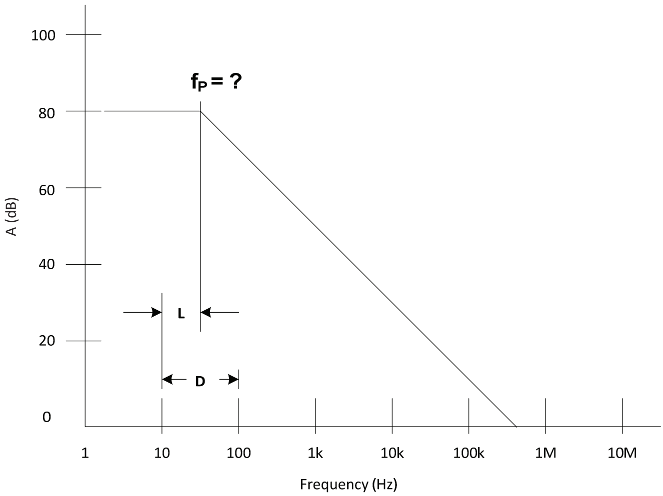


Figure 14: Finding values on logarithmic axis not directly on a grid line

Bode plots: Poles

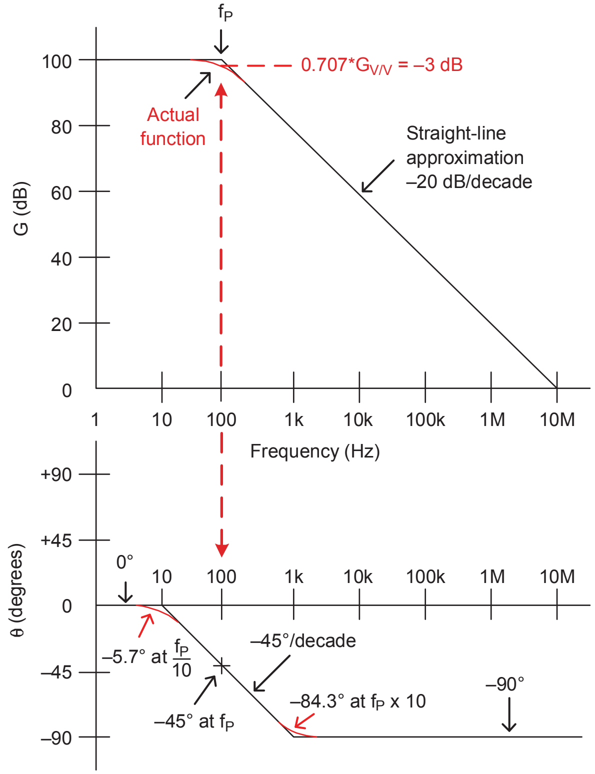


Figure 15: Pole gain and phase

Pole Location = f_P (cutoff freq)
Magnitude ($f < f_P$) = Gdc (for example, 100 dB)
Magnitude ($f = f_P$) = –3 dB
Magnitude ($f > f_P$) = –20 dB/decade
Phase ($f = f_P$) = –45°
Phase ($0.1\ f_P < f < 10\ f_P$) = –45°/decade
Phase ($f > 10\ f_P$) = –90°
Phase ($f < 0.1\ f_P$) = 0°

Pole (equations)

$$G_V = \frac{V_{OUT}}{V_{IN}} = \frac{G_{DC}}{i\left(\frac{f}{f_P}\right) + 1}$$ (39) As a complex number

$$G_V = \frac{V_{OUT}}{V_{IN}} = \frac{G_{DC}}{\sqrt{\left(\frac{f}{f_P}\right)^2 + 1}}$$ (40) Magnitude

$$\theta = -\tan^{-1}\left(\frac{f}{f_P}\right)$$ (41) Phase shift

$$G_{dB} = 20\ \mathrm{Log}(G_V)$$ (42) Magnitude in dB

Where
G_v = voltage gain in V/V
G_{dB} = voltage gain in decibels
G_{dc} = the dc or low frequency voltage gain
f = frequency in Hz
f_P = frequency at which the pole occurs
θ = phase shift of the signal from input to output

Bode plots (zeros)

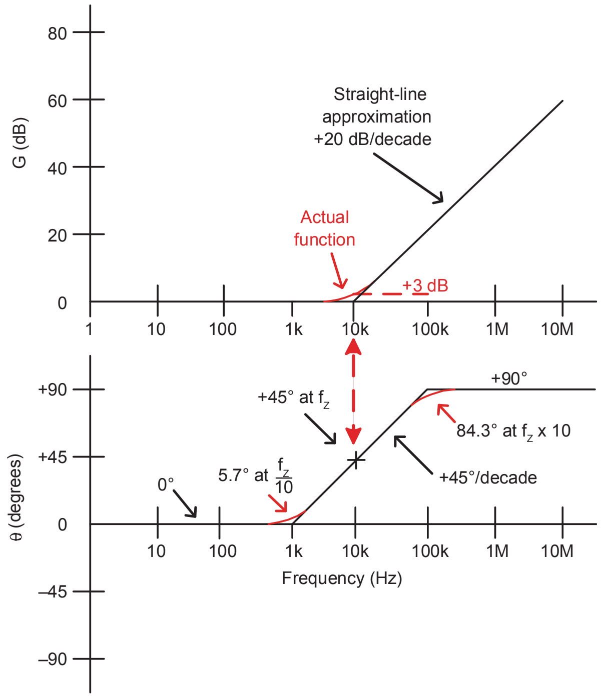


Figure 16: Zero gain and phase

Zero location = f_Z
Magnitude ($f < f_Z$) = 0 dB
Magnitude ($f = f_Z$) = +3 dB
Magnitude ($f > f_Z$) = +20 dB/decade
Phase ($f = f_Z$) = +45°
Phase ($0.1\ f_Z < f < 10\ f_Z$) = +45°/decade
Phase ($f > 10\ f_Z$) = +90°
Phase ($f < 0.1\ f_Z$) = 0°

Zero (equations)

$$G_V = \frac{V_{OUT}}{V_{IN}} = G_{DC}\left[i\left(\frac{f}{f_Z}\right) + 1\right]$$ (43) As a complex number

$$G_V = \frac{V_{OUT}}{V_{IN}} = G_{DC}\sqrt{\left(\frac{f}{f_Z}\right)^2 + 1}$$ (44) Magnitude

$$\theta = \tan^{-1}\left(\frac{f}{f_Z}\right)$$ (45) Phase shift

$$G_{dB} = 20\ \mathrm{Log}(G_V)$$ (46) Magnitude in dB

Where
G_V = voltage gain in V/V
G_{dB} = voltage gain in decibels
G_{DC} = the dc or low frequency voltage gain
f = frequency in Hz
f_Z = frequency at which the zero occurs
θ = phase shift of the signal from input to output

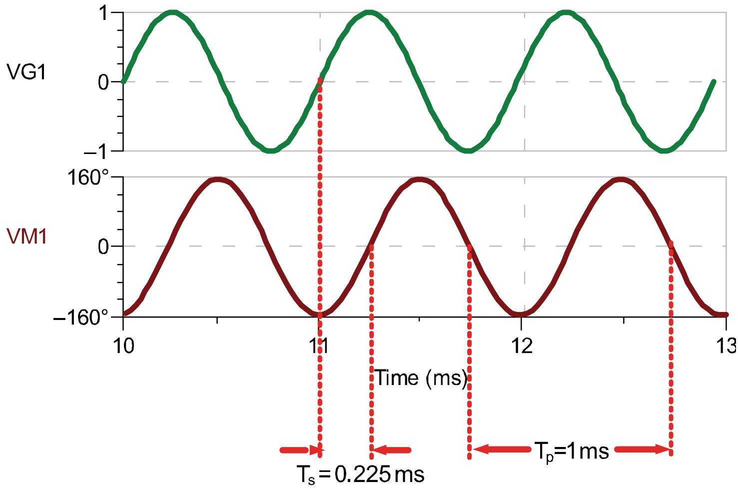


Figure 17: Time to phase shift

$$\Theta = \frac{T_S}{T_P} * 360° \qquad (47)$$

Phase shift from time

Where
T_S = time shift from input to output signal
T_P = period of signal
θ = phase shift of the signal from input to output

Example

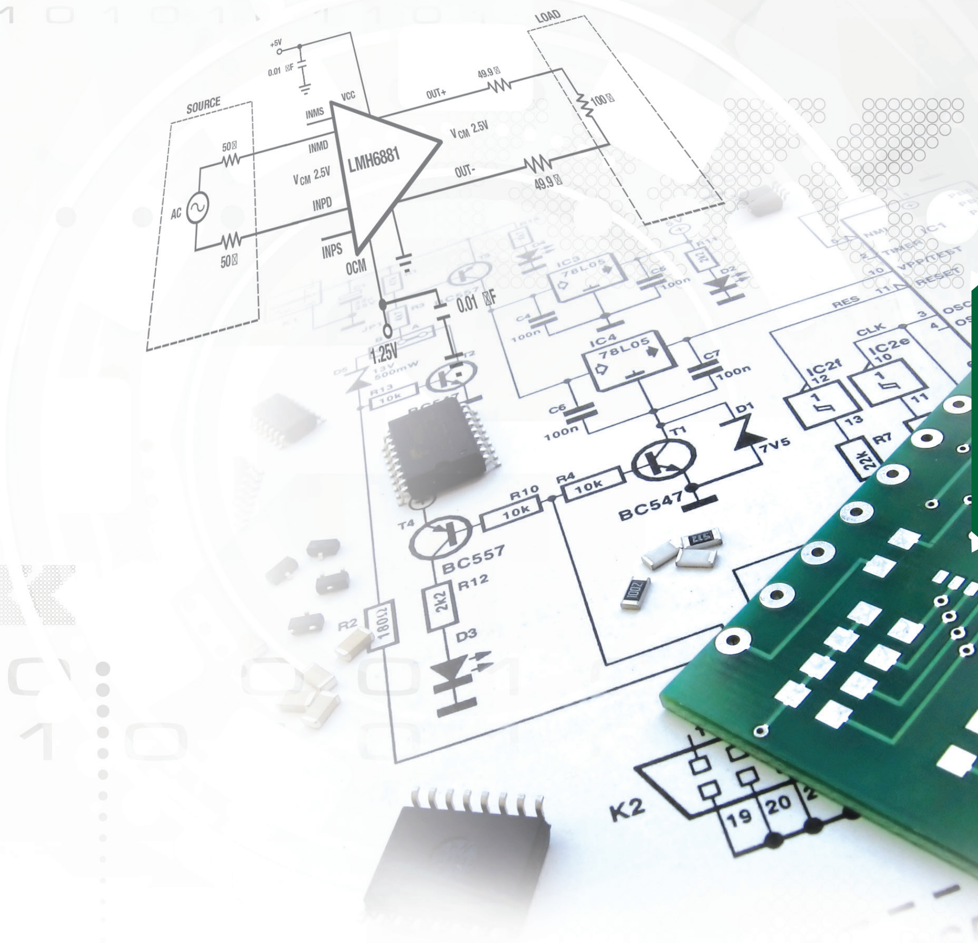
Calculate the phase shift in degrees for Figure 17.

Answer

$$\theta = \frac{T_S}{T_P} * 360° = \left(\frac{0.225\text{ ms}}{1\text{ ms}}\right) * 360° = 81°$$

Amplifier

- Basic op amp configurations
- Op amp bandwidth
- Full power bandwidth
- Small signal step response
- Noise equations
- Stability equations
- Stability open loop SPICE analysis



Basic op amp configurations

$G_{CL} = 1$ (48) Gain for buffer configuration

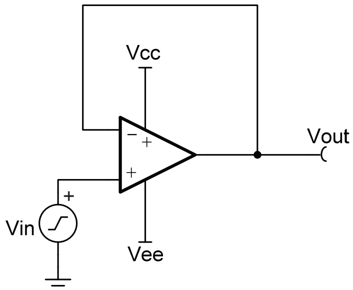


Figure 18: Buffer configuration

$$G_{CL} = \frac{R_f}{R_1} + 1$$ (49) Gain for non-inverting configuration

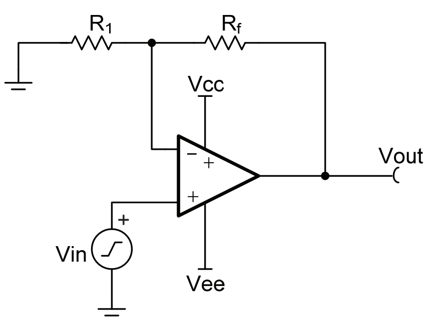


Figure 19: Non-inverting configuration

Basic op amp configurations (cont.)

$$G_{CL} = -\frac{R_f}{R_1} \quad (50)$$

Gain for inverting configuration

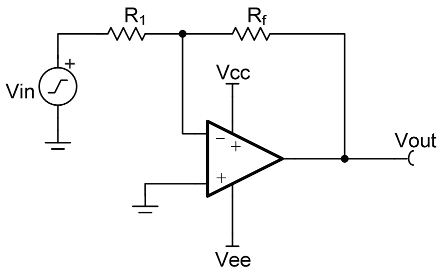


Figure 20: Inverting configuration

$$V_{OUT} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots + \frac{V_N}{R_N}\right) \quad (51)$$

Transfer function for inverting summing amplifier

$$V_{OUT} = -\frac{R_f}{R_1}(V_1 + V_2 + \cdots + V_N) \quad (52)$$

Transfer function for inverting summing amplifier, assuming $R_1 = R_2 = \ldots = R_N$

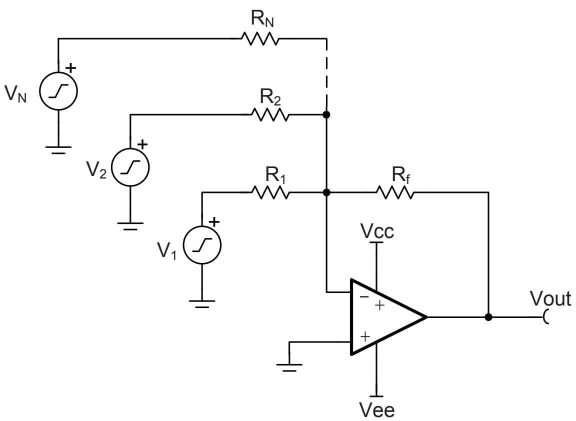


Figure 21: Inverting summing configuration

Basic op amp configurations (cont.)

$$V_{OUT} = \left(\frac{R_f}{R_{in}} + 1\right)\left[\frac{V_1}{N} + \frac{V_2}{N} + \cdots + \frac{V_N}{N}\right] \quad (53)$$

Transfer function for non-inverting summing amplifier for equal input resistors

Where
$R_1 = R_2 = \ldots = R_N$
N = number of input resistors

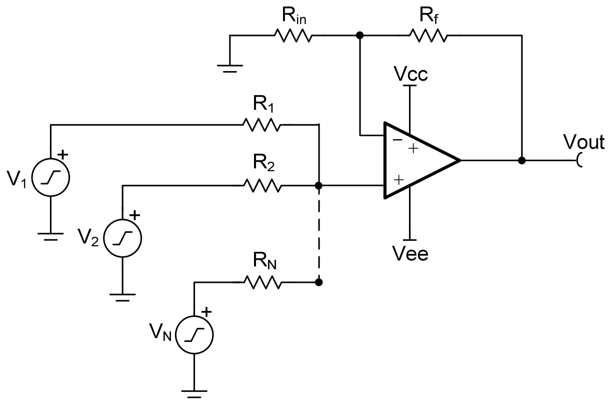


Figure 22: Non-inverting summing configuration

Simple non-inverting amp with C_f filter

$$G_{LF} = \frac{R_f}{R_1} + 1 \quad (54)$$

Gain for non-inverting configuration for f < f_c

$$G_{HF} = 1 \quad (55)$$

Gain for non-inverting configuration for f >> f_c

$$f_C = \frac{1}{2\pi R_f C_f} \quad (56)$$

Cut off frequency for non-inverting configuration

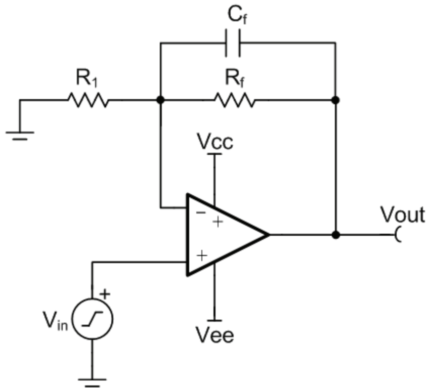


Figure 23: Non-inverting amplifier with C_f filter

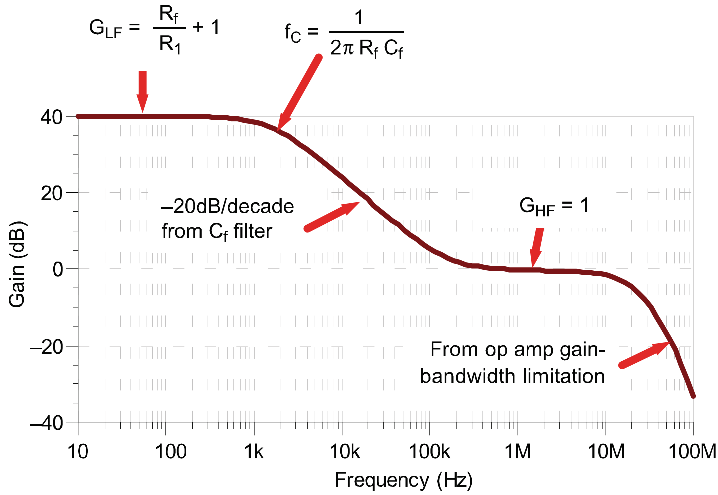


Figure 24: Frequency response for non-inverting op amp with C_f filter

Simple inverting amp with C_f filter

$G_{LF} = -\frac{R_f}{R_1}$	(57)	Gain for inverting configuration for $f < f_C$
$G_{HF} = 1$	(58)	Gain for inverting configuration for $f >> f_C$
$f_C = \frac{1}{2\pi\, R_f\, C_f}$	(59)	Cutoff frequency for inverting configuration

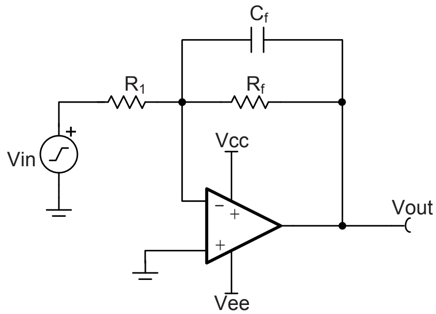


Figure 25: Inverting amplifier with C_f filter

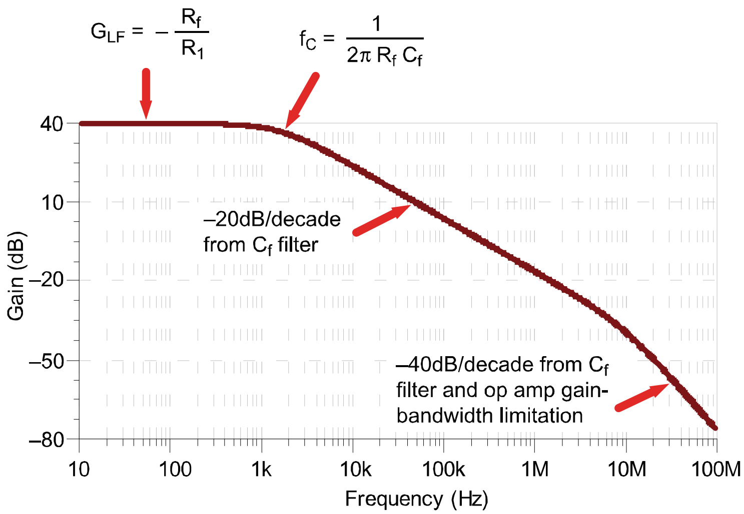


Figure 26: Frequency response for inverting op amp with C_f filter

Op amp bandwidth

$$\text{GBW} = \text{Gain x BW} \qquad (60)$$ Gain bandwidth product defined

Where
GBW = gain bandwidth product, listed in op amp data sheet specification table
Gain = closed loop gain, set by op amp gain configuration
BW = the bandwidth limitation of the amplifier

Example

Determine bandwidth using equation 60

$\text{Gain} = 100$ (from amplifier configuration)

$\text{GBW} = 22\text{MHz}$ (from data sheet)

$$\text{BW} = \frac{\text{GBW}}{\text{Gain}} = \frac{22\text{MHz}}{100} = 220\text{kHz}$$

Note that the same result can be graphically determined using the A_{OL} curve as shown below.

Open-loop gain and phase vs. frequency

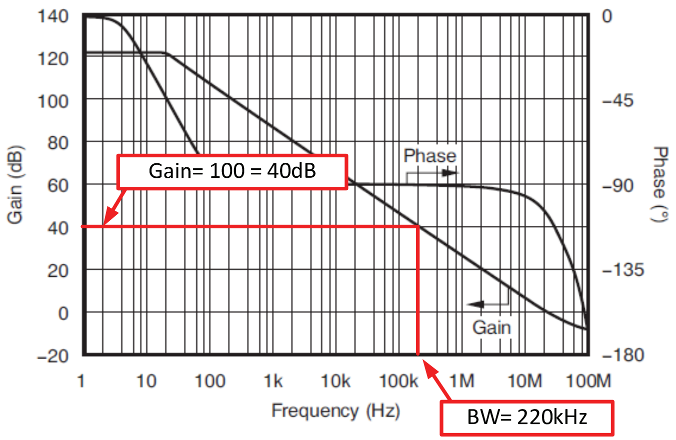


Figure 27: Using A_{OL} to find closed-loop bandwidth

Full power bandwidth

$$V_P = \frac{SR}{2\pi f} \quad (61)$$

Maximum output without slew-rate induced distortion

Where
V_P = maximum peak output voltage before slew induced distortion occurs
SR = slew rate
f = frequency of applied signal

Maximum output voltage vs. frequency

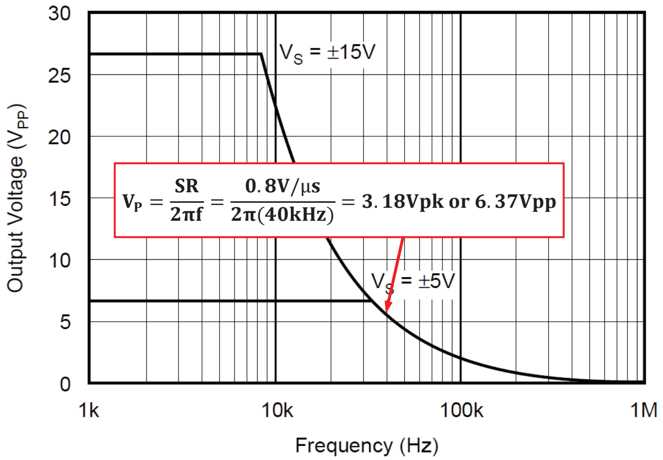


Figure 28: Maximum output without slew-rate induced distortion

Notice that the above figure is graphed using equation 61 for the OPA188. The example calculation shows the peak voltage for the OPA277 at 40kHz. This can be determined graphically or with the equation.

Example

$$V_P = \frac{SR}{2\pi f} = \frac{0.8V/\mu s}{2\pi(40kHz)} = 3.18Vpk \text{ or } 6.37Vpp$$

Small signal step response

$$\tau_R = \frac{0.35}{f_C} \qquad (62)$$

Rise time for a small signal step

Where

τ_R = the rise time of a small signal step response

f_C = the closed-loop bandwidth of the op amp circuit

Small signal step response waveform

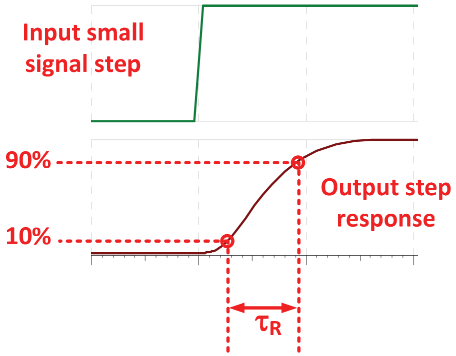


Figure 29: Small signal step response

Op amp noise model

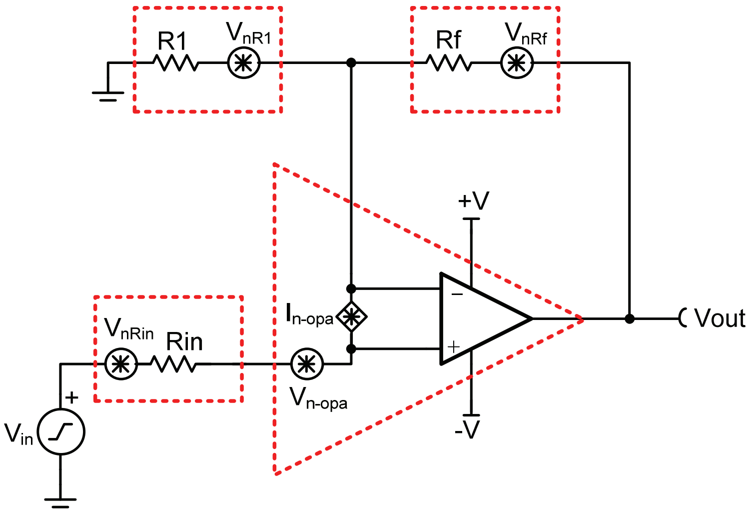


Figure 30: Op amp noise model

Op amp intrinsic noise includes:

- Noise caused by op amp (current noise + voltage noise)
- Resistor noise

Noise bandwidth calculation

$BW_N = K_N f_C$ (63) Noise bandwidth

Where
BW_N = noise bandwidth of the system
K_N = the brick wall correction factor for different filter order
f_C = –3 dB bandwidth of the system

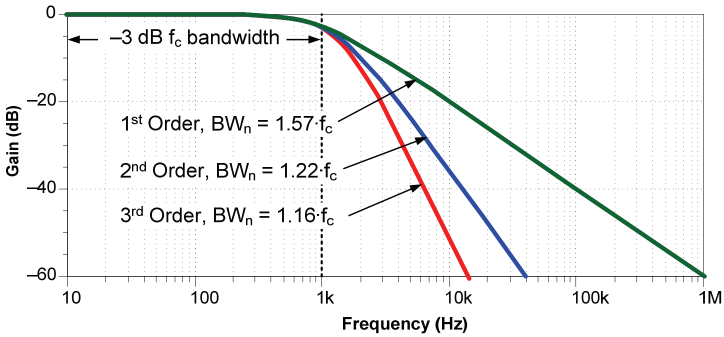


Figure 31: Op amp bandwidth for three different filters orders

Table 15: Brick wall correction factors for noise bandwidth

Number of poles	K_N brick wall correction factor
1	1.57
2	1.22
3	1.13
4	1.12

Broadband total noise calculation

$E_N = e_{BB}\sqrt{BW_N}$ (64) Total rms noise from broadband

Where
E_N = total rms noise from broadband noise
e_{BB} = broadband noise spectral density (nV/rtHz)
BW_N = noise bandwidth (Hz)

1/f total noise calculation

$$E_{N\_NORMAL} = e_{BF}\sqrt{f_O} \quad (65)$$

Normalized 1/f noise at 1 Hz

Where
E_{N_NORMAL} = 1/f noise normalized to 1 Hz
e_{BF} = noise spectral density measured in the 1/f region
f_O = the frequency that the 1/f noise e_{BF} is measured at

$$E_{N\_FLICKER} = E_{N\_NORMAL}\sqrt{\ln\left(\frac{f_H}{f_L}\right)} \quad (66)$$

1/f total noise calculation

Where
$E_{N\_FLICKER}$ = total rms noise from flicker
E_{N_NORMAL} = 1/f noise normalized to 1 Hz
f_H = upper cutoff frequency or noise bandwidth
f_L = lower cutoff frequency, normally set to 0.1 Hz

Table 16: Peak-to-peak conversion

Number of standard deviations	Percent chance reading is in range
2σ (same as ±1σ)	68.3%
3σ (same as ±1.5σ)	86.6%
4σ (same as ±2σ)	95.4%
5σ (same as ±2.5σ)	98.8%
6σ (same as ±3σ)	99.7%
6.6σ (same as ±3.3σ)	99.9%

Thermal noise calculation

$$E_{N\_R} = \sqrt{4\,kTR\Delta f} \qquad (67)$$

Total rms thermal noise

Where

E_{N_R} = total rms noise from resistance, also called thermal noise

k = Boltzmann's constant 1.38×10^{-23} J/K

T = temperature in Kelvin

Δf = noise bandwidth in Hz

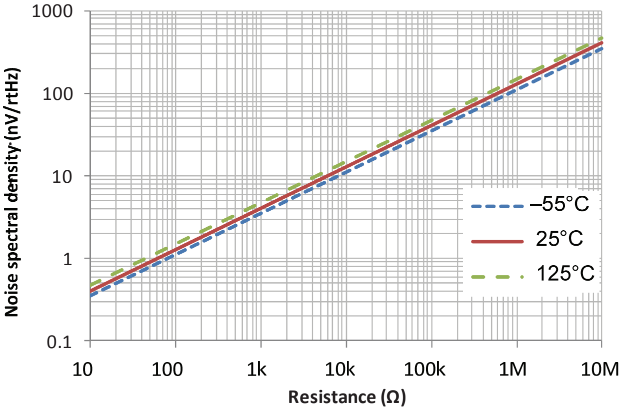


Figure 32: Noise spectral density vs. resistance

Ac response versus frequency

Figure 33 illustrates a bode plot with four different examples of ac peaking.

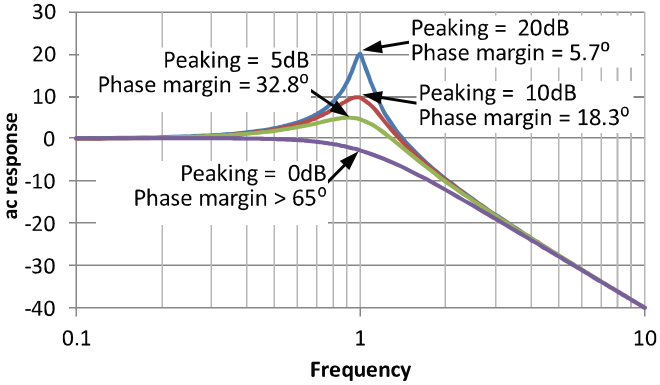


Figure 33: Stability – ac peaking relationship example

Phase margin versus ac peaking

This graph illustrates the phase margin for any given level of ac peaking. Note that 45° of phase margin or greater is required for stable operation.

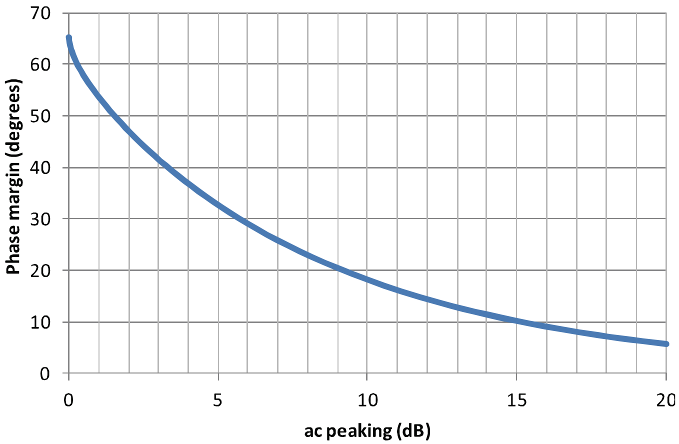


Figure 34: Stability – phase margin vs. peaking for a two-pole system

Transient overshoot

Figure 35 illustrates a transient response with two different examples of percentage overshoot.

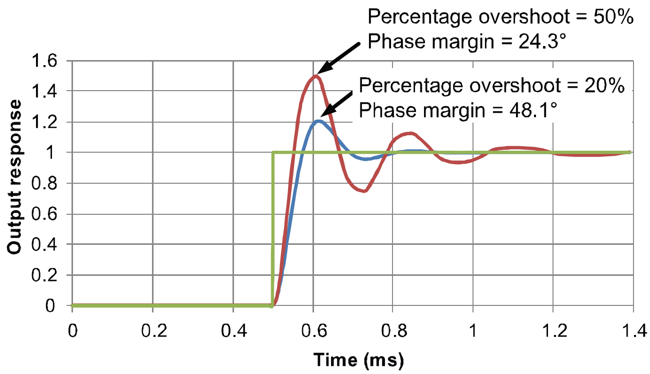


Figure 35: Stability – transient overshoot example

Phase margin versus percentage overshoot

This graph illustrates the phase margin for any given level of transient overshoot. Note that 45° of phase margin or greater is required for stable operation.

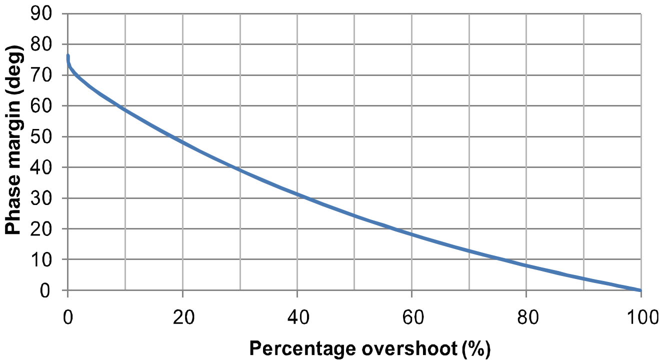


Figure 36: Stability – phase margin vs. percentage overshoot

Note: The curves assume a two-pole system.

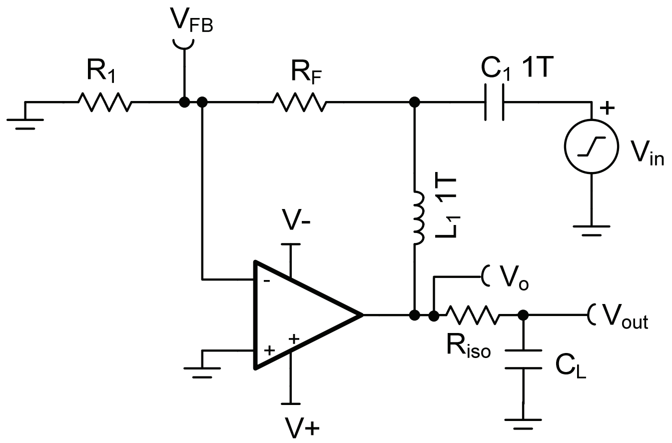


Figure 37: Common spice test circuit used for stability

$A_{OL\_LOADED} = \frac{V_O}{V_{FB}}$	(68)	Loaded open-loop gain
$\beta = V_{FB}$	(69)	Feedback factor
$\frac{1}{\beta} = \frac{1}{V_{FB}}$	(70)	Closed-loop noise gain
$A_{OL\_LOADED} \times \beta = V_O$	(71)	Loop gain

Where
V_O = the voltage at the output of the op amp.
V_{OUT} = the voltage output delivered to the load, which may be important to the application but is not considered in stability analysis.
V_{FB} = feedback voltage
R_F , R_1, R_{ISO} and C_L = the op amp feedback network and load. Other op amp topologies will have different feedback networks; however, the test circuit will be the same for most cases. Figure 38 shows the exception to the rule (multiple feedback).
C_1 and L_1 are components that facilitate SPICE analysis. They are large (1TF, 1TH) to make the circuit closed-loop for dc, but open loop for ac frequencies. SPICE requires closed-loop operation at dc for convergence.

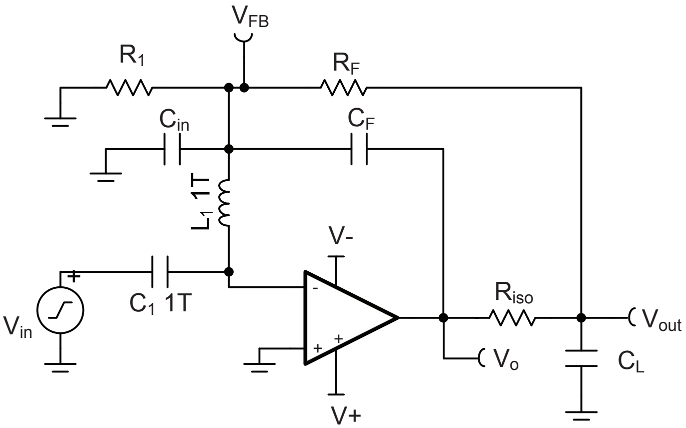


Figure 38: Alternative (multiple feedback) SPICE test circuit used for stability

$A_{OL\_LOADED} = V_O$	(72)	Loaded open loop gain
$\beta = \frac{V_{FB}}{V_O}$	(73)	Feedback factor
$\frac{1}{\beta} = \frac{V_O}{V_{FB}}$	(74)	Closed-loop noise gain
$A_{OL\_LOADED} \times \beta = V_{FB}$	(75)	Loop gain

Where
V_O = the voltage at the output of the op amp.
V_{OUT} = the voltage output delivered to the load. This may be important to the application but is not considered in stability analysis.
V_{FB} = feedback voltage
R_F, R_1, R_{ISO} and C_F = the op amp feedback network. Because there are two paths for feedback, the loop is broken at the input.
C_1 and L_1 are components that facilitate SPICE analysis. They are large (1TF, 1TH) to make the circuit closed loop for dc, but open loop for ac frequencies. SPICE requires closed-loop operation at dc for convergence.
C_{IN} = the equivalent input capacitance taken from the op amp datasheet. This capacitance normally does not need to be added because the model includes it. However, when using this simulation method the capacitance is isolated by the 1TH inductor.

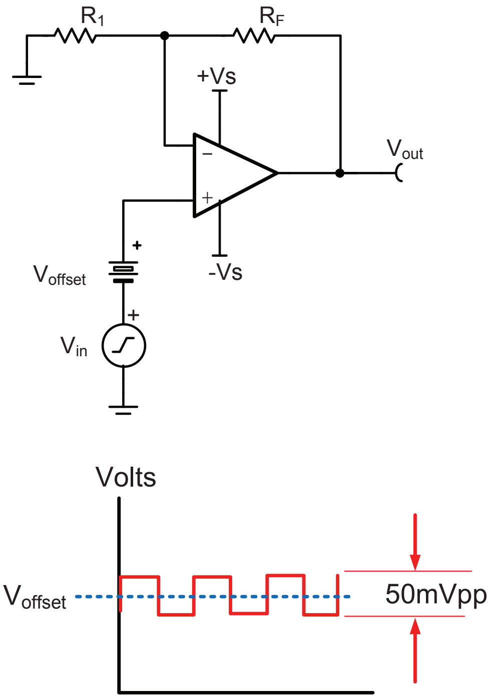


Figure 39: Transient real world stability test

Test tips

- Choose test frequency << f_{cl}
- Small signal (Vpp ≤ 50 mV) ac output square wave (for example, 1 kHz)
- Adjust V_{in} amplitude to yield output ≤ 50 mVpp
- Worst cases is usually when V_{offset} = 0 (Largest Ro, for I_{out} = 0A).
- User V_{offset} as desired to check all output operating points for stability
- Set scope = ac couple and expand vertical scope scale to look for amount of overshoot, undershoot, and ringing on V_{out}
- Use 1x attenuation scope probe on V_{out} for best resolution

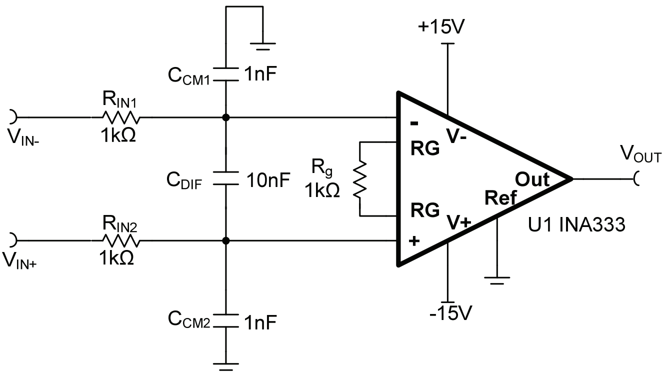


Figure 40: Input filter for instrumentation amplifier

$\text{Select } C_{DIF} \geq 10C_{CM1}$	(76)	Differential filter is sized 10 times the common-mode filter
$R_{IN1} = R_{IN2}$	(77)	Input resistors must be equal
$C_{CM1} = C_{CM2}$	(78)	Common-mode capacitors must be equal
$f_{CM} = \frac{1}{2\pi R_{IN1} C_{CM1}}$	(79)	Differential filter cutoff
$f_{DIF} = \frac{1}{2\pi(2R_{IN1})(C_{DIF} + \frac{1}{2}C_{CM1})}$	(80)	Common-mode filter cutoff

Where
f_{DIF} = differential cutoff frequency
f_{CM} = common-mode cutoff frequency
R_{IN} = input resistance
C_{CM} = common-mode filter capacitance
C_{DIF} = differential filter capacitance

Note: Selecting $C_{DIF} \geq 10\ C_{CM}$ sets the differential mode cutoff frequency 10 times lower than the common-mode cutoff frequency. This prevents common-mode noise from being converted into differential noise due to component tolerances.

Notes

PCB and wire

- PCB trace resistance for 1oz and 2oz Cu
- Conductor spacing in a PCB for safe operation
- Current carrying capacity of copper conductors
- Package types and dimensions
- PCB trace capacitance and inductance
- PCB via capacitance and inductance
- Common coaxial cable specifications
- Coaxial cable equations
- Resistance per length for wire types
- Maximum current for wire types

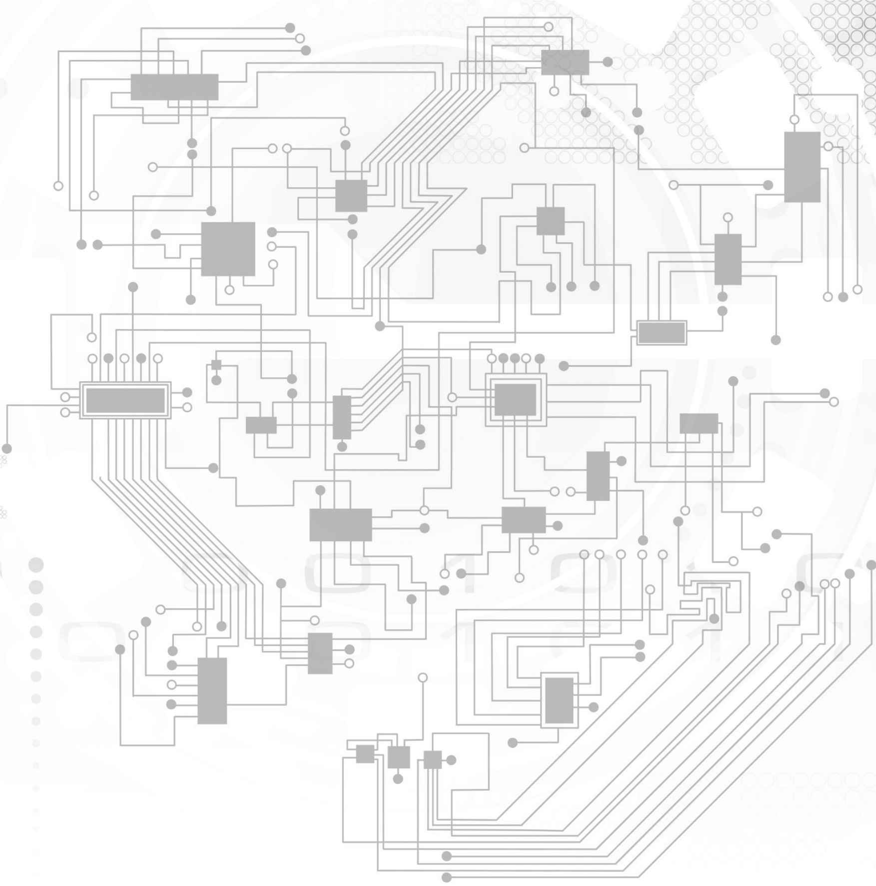
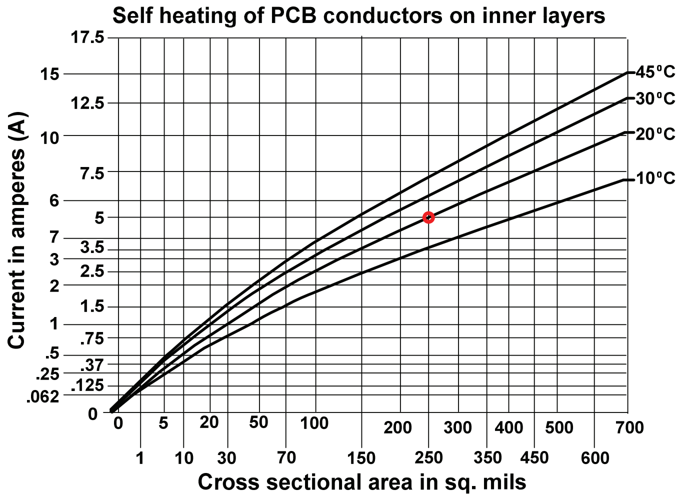


Table 17: Printed circuit board conductor spacing

Voltage between conductors (dc or ac peaks)	Minimum spacing						
	Bare board				Assembly		
	B1	B2	B3	B4	A5	A6	A7
0-15	0.05 mm [0.00197 in]	0.1 mm [0.0039 in]	0.1 mm [0.0039 in]	0.05 mm [0.00197 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]
16-30	0.05 mm [0.00197 in]	0.1 mm [0.0039 in]	0.1 mm [0.0039 in]	0.05 mm [0.00197 in]	0.13 mm [0.00512 in]	0.25 mm [0.00984 in]	0.13 mm [0.00512 in]
31-50	0.1 mm [0.0039 in]	0.6 mm [0.024 in]	0.6 mm [0.024 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]	0.4 mm [0.016 in]	0.13 mm [0.00512 in]
51-100	0.1 mm [0.0039 in]	0.6 mm [0.024 in]	1.5 mm [0.0591 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]	0.5 mm [0.020 in]	0.13 mm [0.00512 in]
101-150	0.2 mm [0.0079 in]	0.6 mm [0.024 in]	3.2 mm [0.126 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.4 mm [0.016 in]
151-170	0.2 mm [0.0079 in]	1.25 mm [0.0492 in]	3.2 mm [0.126 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.4 mm [0.016 in]
171-250	0.2 mm [0.0079 in]	1.25 mm [0.0492 in]	6.4 mm [0.252 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.4 mm [0.016 in]
251-300	0.2 mm [0.0079 in]	1.25 mm [0.0492 in]	12.5 mm [0.492 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.8 mm [0.031 in]
301-500	0.25 mm [0.00984 in]	2.5 mm [0.0984 in]	12.5 mm [0.492 in]	0.8 mm [0.031 in]	0.8 mm [0.031 in]	1.5 mm [0.0591 in]	0.8 mm [0.031 in]

B1 Internal conductors
B2 External conductors uncoated sea level to 3050 m
B3 External conductors uncoated above 3050 m
B4 External conductors coated with permanent polymer coating (any elevation)
A5 External conductors with conformal coating over assembly (any elevation)
A6 External component lead/termination, uncoated, sea level to 3050 m
A7 External component lead termination, with conformal coating (any elevation)

Extracted with permission from IPC-2221B, Table 6-1.
For additional information, the entire specification can be downloaded at www.ipc.org



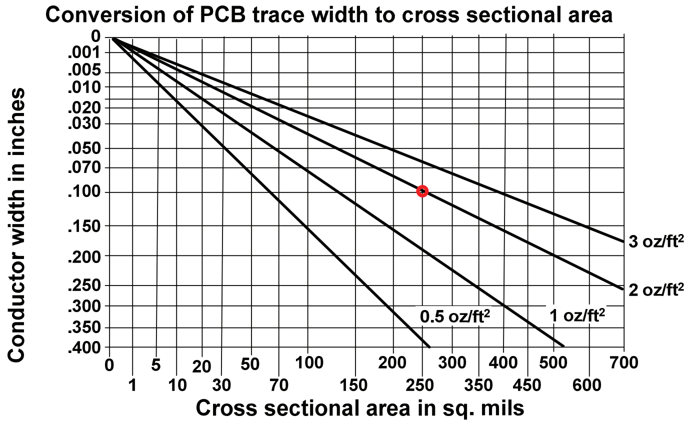


Figure 41: Self heating of PCB traces on inside layer

Example

Find the current that will cause a 20℃ temperature rise in a PCB trace that is 0.1 inch wide and uses 2 oz/ft^2 copper. (Assume traces on outside of PCB.)

Answer

First translate 0.1 inch to 250 sq. mils. using bottom chart. Next find the current associated with 10℃ and 250 sq. mils. using top chart (Answer = 5A).

Extracted with permission from IPC-2152, Figure 5-1.
For additional information the entire specification can be downloaded at www.ipc.org

PCB trace resistance for 1 oz Cu

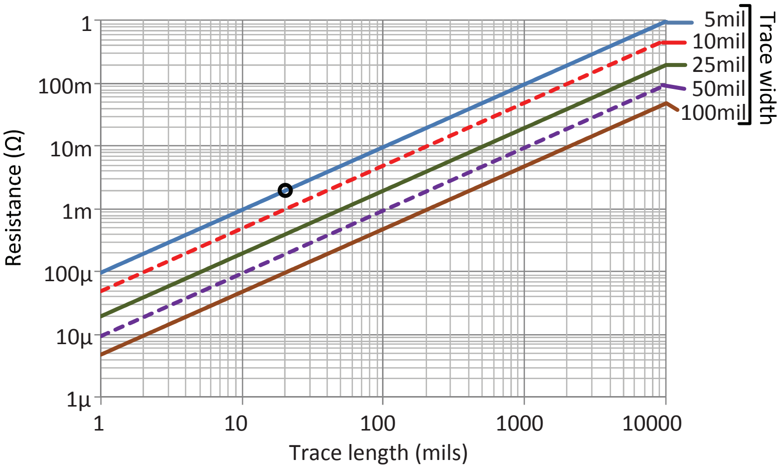


Figure 42: PCB trace resistance vs. length and width for 1 oz-Cu, 25°C

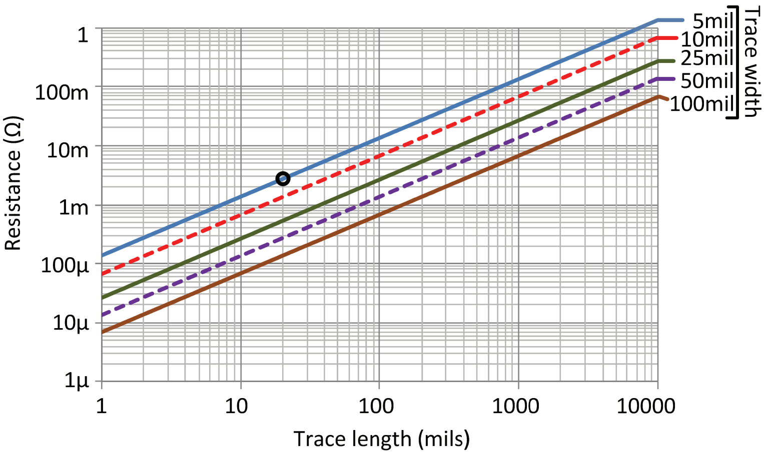


Figure 43: PCB trace resistance vs. length and width for 1 oz-Cu, 125°C

Example

What is the resistance of a 20 mil long, 5 mil wide trace for a 1 oz-Cu thickness at 25°C and 125°C?

Answer

R25C = 2 mΩ, R125C = 3 mΩ. The points are circled on the curves.

PCB trace resistance for 2 oz Cu

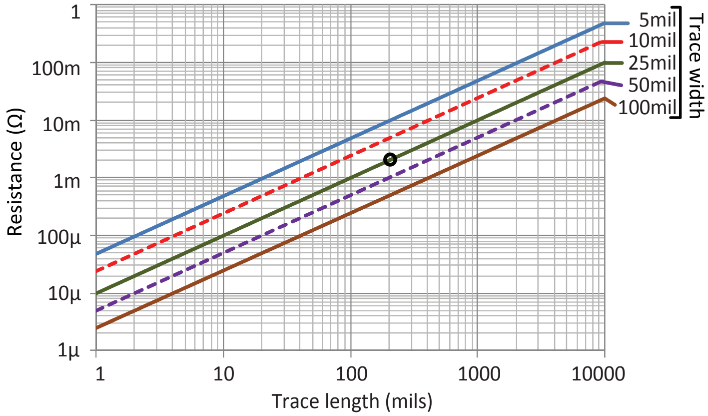


Figure 44: PCB trace resistance vs. length and width for 2 oz-Cu, 25°C

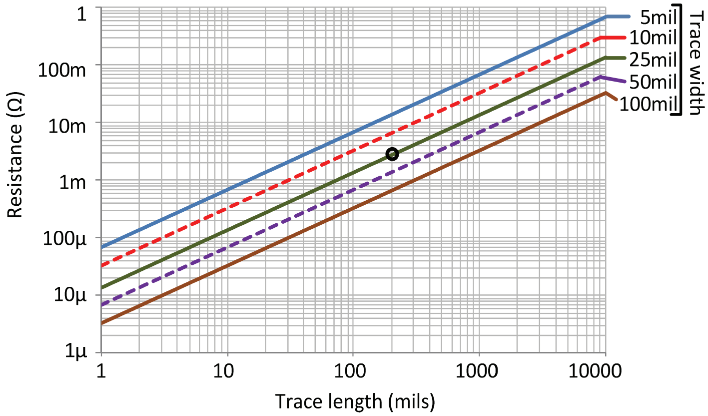


Figure 45: PCB trace resistance vs. length and width for 2 oz-Cu, 125°C

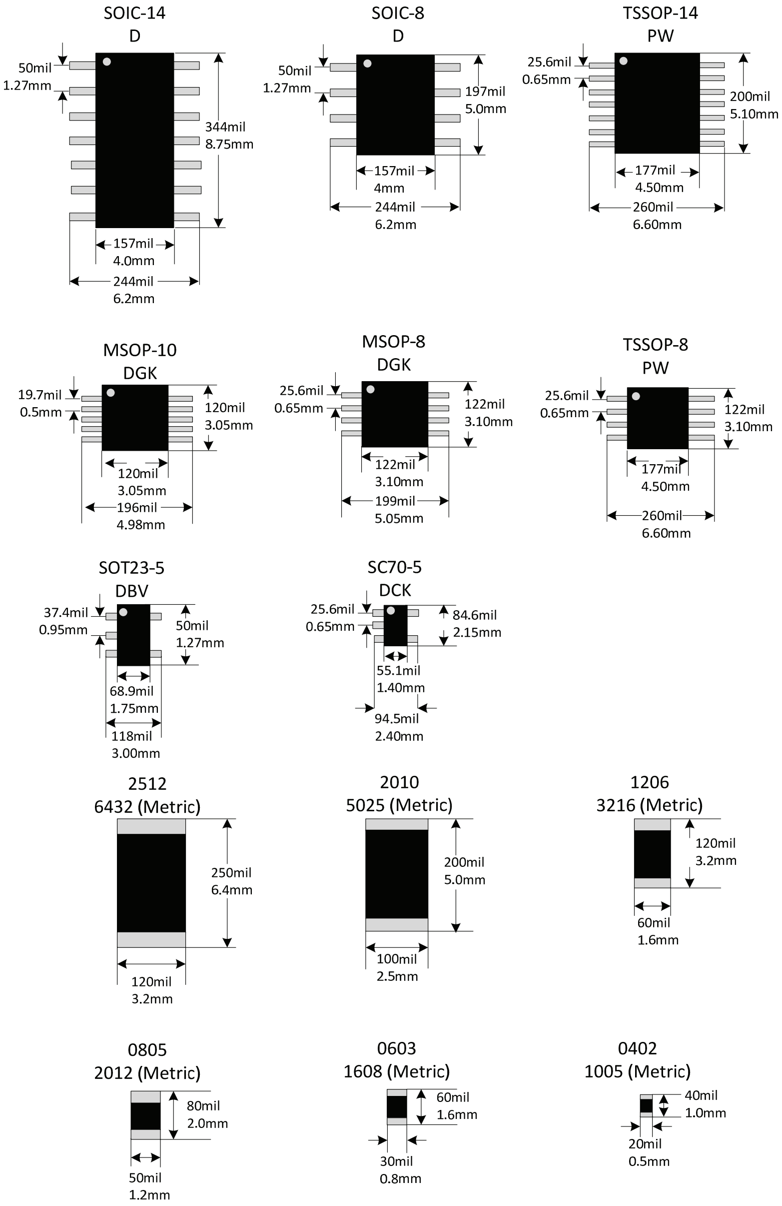
Example

What is the resistance of a 200 mil long, 25 mil wide trace for a 2 oz-Cu thickness at 25°C and 125°C?

Answer

R25C = 2 mΩ, R125C = 3 mΩ. The points are circled on the curves.

Common package type and dimensions



PCB parallel plate capacitance

$$C(\text{pF}) = \frac{2.249 * 10^{-4} * \varepsilon_r * l * w}{h} \qquad (81)$$

Capacitance for parallel copper planes

Where

ε_r = PCB dielectric constant ($\varepsilon_r \approx 4.2$ for FR-4)
l = common length of copper planes (mils)
w = common width of copper planes (mils)
h = separation between copper planes (mils)

Example

ε_r = 4.2
l = 400 mils
w = 400 mils
h = 63 mils

Answer

C = 2.4 pF

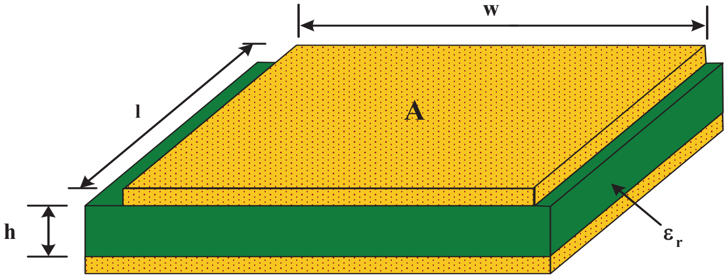


Figure 46: PCB parallel plate capacitance

PCB via capacitance and inductance

$$L(nH) \approx \frac{h}{197}\left[1 + \ln\left(\frac{4 * h}{d}\right)\right] \quad (82)$$ Inductance for via

$$C(pF) \approx \frac{1.41 * 10^{-3} * \varepsilon_r * h * d_1}{d_2 - d_1} \quad (83)$$ Capacitance for via

Where
h = separation between planes (mils)
d = diameter of via hole (mils)
ε_r = PCB dielectric constant ($\varepsilon_r \approx 4.2$ for FR-4)
d_1 = diameter of the pad surrounding the via (mils)
d_2 = diameter of the clearance hole in the plane (mils)

Example

h = 63 mils
d = 16 mils
ε_r = 4.2
d_1 = 32 mils
d_2 = 63 mils

Answer

L = 1.20 nH
C = 0.39 pF

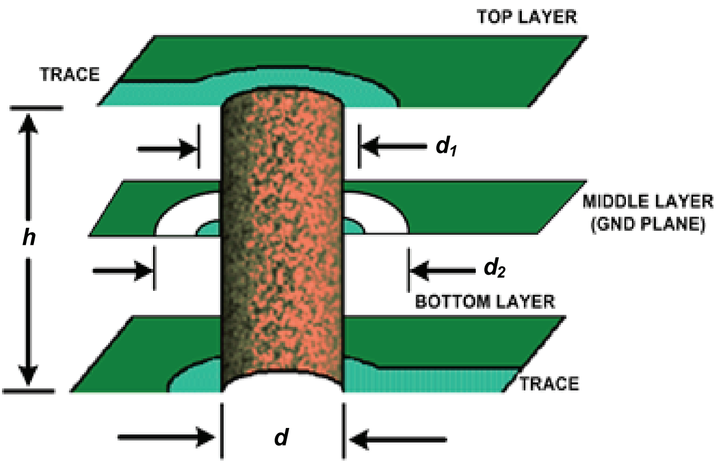


Figure 47: Inductance and capacitance of via

Adjacent copper trace capacitance

$$C\,(\mathrm{pF}) \approx \frac{2.249 * 10^{-4} * l * t}{d} \qquad (58) \qquad \text{Same layer}$$

$$C\,(\mathrm{pF}) \approx \frac{2.249 * 10^{-4} * \varepsilon_r * l * w}{h} \qquad (59) \qquad \text{Different layers}$$

Where
l = length of copper trace (mils)
t = thickness of copper trace (mils)
copper thickness (mils) = 1.37 * (number of ounces)
ex: 1 oz. copper thickness = 1.37 mils
ex: ½ oz. copper thickness = 0.685 mils
d = distance between traces (mils)
ε_r = PCB dielectric constant ($\varepsilon_r \approx 4.2$ for FR-4)
w = width of copper trace (mils)
h = separation between planes (mils)

Example

l = 100 mils
t = 1.37 mils (1 oz. copper)
d = 10 mils
ε_r = 4.2
w = 25 mils
h = 63 mils

Answer

C (same layer) = 0.003 pF
C (different layers) = 0.037 pF

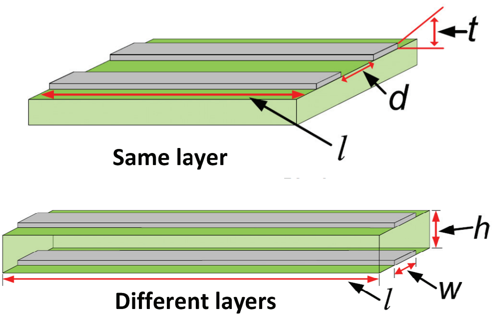


Figure 48: Capacitance for adjacent copper traces

Table 18: Coaxial cable information

Type	Z_O	Capacitance / length (pF/feet)	Outside diameter (inches)	dB attenuation /100 ft at 750 MHz	Dielectric type	Application
RG-58	53.5 Ω	28.8	0.195	13.1	PE	Test equipment and RF power to a few hundred watts, and a couple hundred MHz
RG-8	52 Ω	29.6	0.405	5.96	PE	RF power to a few kW, up to several hundred MHz
RG-214/U	50 Ω	30.8	0.425	6.7	PE	
9914	50 Ω	26.0	0.405	4.0	PE	
RG-6	75 Ω	20	0.270	5.6	PF	Video and CATV applications. RF to a few hundred watts, up to a few hundred MHz, sometimes to higher frequencies if losses can be tolerated
RG-59/U	73 Ω	29	0.242	9.7	PE	
RG-11/U	75 Ω	17	0.412	3.65	PE	RF power to a few kW, up to several hundred MHz
RG-62/U	93 Ω	13.5	0.242	7.1	ASP	Used in some test equipment and 100 Ω video applications
RG-174	50 Ω	31	0.100	23.5	PE	Miniature coax used primarily for test equipment interconnection. Usually short runs due to higher loss.
RG-178/U	50 Ω	29	0.071	42.7	ST	

Coaxial cable equations

$$\frac{C}{l} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{d}\right)} \quad (84)$$ Capacitance per length

$$\frac{L}{l} = \frac{\mu}{2\pi}\ln\left(\frac{D}{d}\right) \quad (85)$$ Inductance per length

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}} \quad (86)$$ Characteristic impedance

Where
L = inductance in henries (H)
C = capacitance in farads (F)
Z = impedance in ohms (Ω)
d = diameter of inner conductor
D = inside diameter of shield, or diameter of dielectric insulator
ε = dielectric constant of insulator ($\varepsilon = \varepsilon_r \varepsilon_o$)
μ = magnetic permeability ($\mu = \mu_r \mu_o$)
l = length of the cable

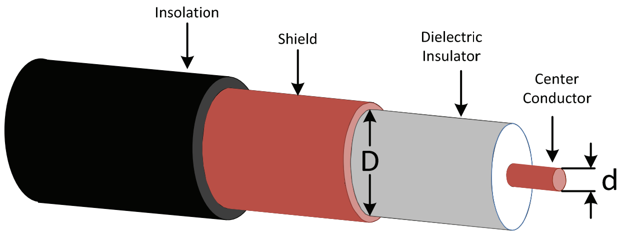


Figure 49: Coaxial cable cutaway

Table 19: Resistance per length for different wire types (AWG)

AWG	Stds	Outside diameter		Area		dc resistance	
		in	mm	circular mils	mm^2	Ω / 1000 ft	Ω / km
36	Solid	0.005	0.127	25	0.013	445	1460
36	7/44	0.006	0.152	28	0.014	371	1271
34	Solid	0.0063	0.160	39.7	0.020	280	918
34	7/42	0.0075	0.192	43.8	0.022	237	777
32	Solid	0.008	0.203	67.3	0.032	174	571
32	7/40	0.008	0.203	67.3	0.034	164	538
30	Solid	0.010	0.254	100	0.051	113	365
30	7/38	0.012	0.305	112	0.057	103	339
28	Solid	0.013	0.330	159	0.080	70.8	232
28	7/36	0.015	0.381	175	0.090	64.9	213
26	Solid	0.016	0.409	256	0.128	43.6	143
26	10/36	0.021	0.533	250	0.128	41.5	137
24	Solid	0.020	0.511	404	0.205	27.3	89.4
24	7/32	0.024	0.610	448	0.229	23.3	76.4
22	Solid	0.025	0.643	640	0.324	16.8	55.3
22	7/30	0.030	0.762	700	0.357	14.7	48.4
20	Solid	0.032	0.813	1020	0.519	10.5	34.6
20	7/28	0.038	0.965	1111	0.562	10.3	33.8
18	Solid	0.040	1.020	1620	0.823	6.6	21.8
18	7/26	0.048	1.219	1770	0.902	5.9	19.2
16	Solid	0.051	1.290	2580	1.310	4.2	13.7
16	7/24	0.060	1.524	2828	1.442	3.7	12.0
14	Solid	0.064	1.630	4110	2.080	2.6	8.6
14	7/22	0.073	1.854	4480	2.285	2.3	7.6

Table 20: Maximum current vs. AWG

Wire gauge	Polyethylene Neoprene Polyvinylchloride (semi-ridged) at 80°C	Polypropylene Polyethylene (high density) at 90°C	Polyvinylchloride Nylon at 105°C	Kynar Polyethylene Thermoplastic at 125°C	Kapton Teflon Silicon at 200°C
AWG	I_{max} (A)	I_{max} (A)	I_{max} (A)	I_{max} (A)	I_{max} (A)
30	2	3	3	3	4
28	3	4	4	5	6
26	4	5	5	6	7
24	6	7	7	8	10
22	8	9	10	11	13
20	10	12	13	14	17
18	15	17	18	20	24
16	19	22	24	26	32
14	27	30	33	40	45
12	36	40	45	50	55
10	47	55	58	70	75
Note: Wire is in free air at 25°C					

Example

What is the maximum current that can be applied to a 30 gauge Teflon wire in a room temperature environment? What will the self-heating be?

Answer

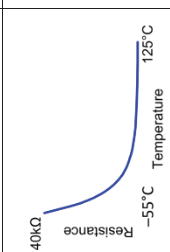
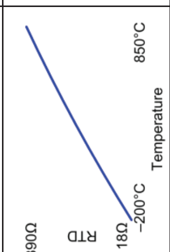
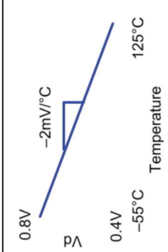
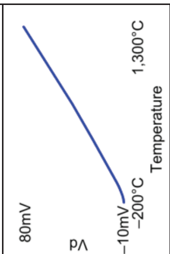
I_{max} = 4A
Wire temperature = 200°C

Notes

Sensor

- Thermistor
- Resistive temperature detector (RTD)
- Diode temperature characteristics
- Thermocouple (J and K)

Table 21: Temperature sensor overview

	Thermistor	RTD	Diode	Thermocouple
				
Temp range	–55°C < T < 150°C	–200°C < T < 850°C	–55°C < T < 150°C	–250°C < T < 1800°C
Cost	Low	High	Low	Low
Accuracy	Good accuracy at one temperature. Less accurate over full range.	Excellent accuracy	Poor accuracy without calibration.	Good accuracy with polynomial correction.
Linearity	Very nonlinear. Follows reciprocal of logarithmic function.	Fairly linear Nonlinearity < 4.5% of full scale. Relatively simple quadratic function.	Fairly linear Slope ≈ -2mV/C Slope varies according to current excitation, diode type, and diode processing.	Fairly linear Nonlinearity < 10% of full scale Complex 10th order polynomial
Construction	Less rugged	Depends on Type (can be rugged)	Rugged	Most rugged
Output range	Typically 10s to 100s of kΩ full scale. Very wide variation in resistance.	18 to 390 Ω for PT100 180 to 3.9 kΩ for PT1000	0.4 to 0.8V	10s of millivolts
Applications	General purpose	Scientific and industrial	Low cost temperature monitor Low cost linear response	Industrial temperature measurement
General	Requires excitation	Requires excitation	Requires excitation	Self-powered Requires cold junction comp

Thermistor: Resistance to temperature, Steinhart-Hart equation

$$\frac{1}{T} = a + b\ln(R) + c\left(\ln(R)\right)^3 \quad (87)$$

Convert resistance to temperature for a thermistor

Where
T = temperature in Kelvin
a, b, c = Steinhart-Hart equation constants
R = resistance in ohms

Thermistor: Temperature to resistance, Steinhart-Hart equation

$$R = \exp\left(x - \frac{y}{2}\right)^{\frac{1}{3}} - \exp\left(x - \frac{y}{2}\right)^{\frac{1}{3}} \quad (88)$$

Convert temperature to resistance for a thermistor

$$y = \frac{a - \frac{1}{T}}{c} \quad (89)$$

Factor used in Equation 88

$$x = \sqrt{\left(\frac{b}{3\,c}\right)^3 + \frac{y^2}{4}} \quad (90)$$

Factor used in Equation 88

Where
R = resistance in ohms
T = temperature in Kelvin
a, b, c = Steinhart-Hart equation constants
x, y = Steinhart-Hart factors used in temperature to resistance equation

RTD equation temperature to resistance

$$R_{rtd} = R_o[1 + A_oT + B_oT^2 + C_o(T - 100)T^3] \quad (91)$$ RTD resistance for T<0°C

$$R_{rtd} = R_o[1 + A_oT + B_oT^2] \quad (92)$$ RTD resistance for T>0°C

Where
R_{rtd} = resistance of RTD over temperature range of (–200°C < T < 850°C)
R_o = 100 Ω for PT-100, 1000 Ω for PT-1000
A_O, B_O, C_O = Callendar-Van Dusen coefficients
T = temperature in degrees Celsius (℃)

RTD equation resistance to temperature (T>0°C)

$$T = \frac{-A_0 + \sqrt{{A_0}^2 + 4B_0\left(1 - \frac{R_{RTD}}{R_O}\right)}}{2B_0} \quad (93)$$ RTD resistance for T>0°C

Where
R_{RTD} = resistance of RTD over temperature range of (–200°C < T < 850°C)
R_o = 100 Ω
A_O, B_O, C_O = Callendar-Van Dusen coefficients
T = temperature in degrees Celsius (℃)

Table 22: Callendar-Van Dusen coefficients for different RTD standards

	IEC-751 DIN 43760 BS 1904 ASTM-E1137 EN-60751	JISC 1604	US Industrial Standard D-100 American	US Industrial Standard American	ITS-90
A0	+3.9083E-3	+3.9739E-3	+3.9787E-3	+3.9692E-3	+3.9888E-3
B0	–5.775E-7	–5.870E-7	–5.8686E-7	–5.8495E-7	–5.915E-7
C0	–4.183E-12	–4.4E-12	–4.167E-12	–4.233E-12	–3.85E-12

Example

What is the temperature given an ITS-90 PT100 resistance of 120 Ω?

Answer

$$T = \frac{-(3.9888 \cdot 10^{-3}) + \sqrt{(3.9888 \cdot 10^{-3})^2 + 4(-5.915 \cdot 10^{-7})\left(1 - \frac{120}{100}\right)}}{2(-5.915 \cdot 10^{-7})} = 50.5℃$$

RTD equation resistance to temperature (T<0°C)

$$T = \sum_{i=0}^{n} \alpha_i \, (R_{rtd})^i \qquad (94)$$

RTD resistance for T<0°C

Where
T = temperature in degrees Celsius (°C)
R_{RTD} = resistance of RTD over temperature range of (T<0°C)
α_I = polynomial coefficients for converting RTD resistance to temperature for T<0°C

Table 23: Coefficients for 5th order RTD resistance to temperature

	IEC-751 DIN 43760 BS 1904 ASTM-E1137 EN-60751	JISC 1604	US Industrial Standard D-100 American	US Industrial Standard American	ITS-90
α0	–2.4202E+02	–2.3820E+02	–2.3818E+02	–2.3864E+02	–2.3791E+02
α1	2.2228E+00	2.1898E+00	2.1956E+00	2.1973E+00	2.2011E+00
α2	2.5857E-03	2.5226E-03	2.4413E-03	2.4802E-03	2.3223E-03
α3	–4.8266E-06	–4.7825E-06	–4.7517E-06	–4.7791E-06	–4.6280E-06
α4	–2.8152E-08	–2.7009E-08	–2.3831E-08	–2.5157E-08	–1.9702E-08
α5	1.5224E-10	1.4719E-10	1.3492E-10	1.4020E-10	1.1831E-10

Example

Find the temperature given an ITS-90 PT100 resistance of 60 Ω.

Answer

$$T = (-2.3791E+02) * (60)^0 + (2.2011E+00) * (60)^1 + (2.3223E-03) * (60)^2 + \cdots + (2.3223E-03) * (60)^5 = -98.6°C$$

Diode equation vs. temperature

$$V_D = \frac{nkT}{q}\ln\left(\frac{I}{I_S}+1\right) \approx \frac{nkT}{q}\ln\left(\frac{I}{I_S}\right)$$ (95) Diode voltage

Where
V_D = diode voltage vs. temperature and current
n = diode ideality factor (ranges from 1 to 2)
k = 1.38 x 10^{-23} J/K, Boltzmann's constant
T = temperature in Kelvin
q = 1.60 x 10^{-19} C, charge of an electron
I = forward diode current in amps
I_S = saturation current

$$I_S = \alpha T^{(3/n)}\exp\left(-\frac{qV_G}{nkT}\right)$$ (96) Saturation current

Where
I_S = saturation current
α = constant related to the cross sectional area of the junction
V_G = diode voltage vs. temperature and current
n = diode ideality factor (ranges from 1 to 2)
k = 1.38 x 10^{-23} J/K, Boltzmann's constant
T = temperature in Kelvin
q = 1.60 x 10^{-19} C, charge of an electron

Diode voltage versus temperature

Figure 50 shows an example of the temperature drift for a diode. Depending on the characteristics of the diode and the forward current the slope and offset of this curve will change. However, typical diode drift is about –2mV/°C. A forward drop of about 0.6V is typical for room temperature.

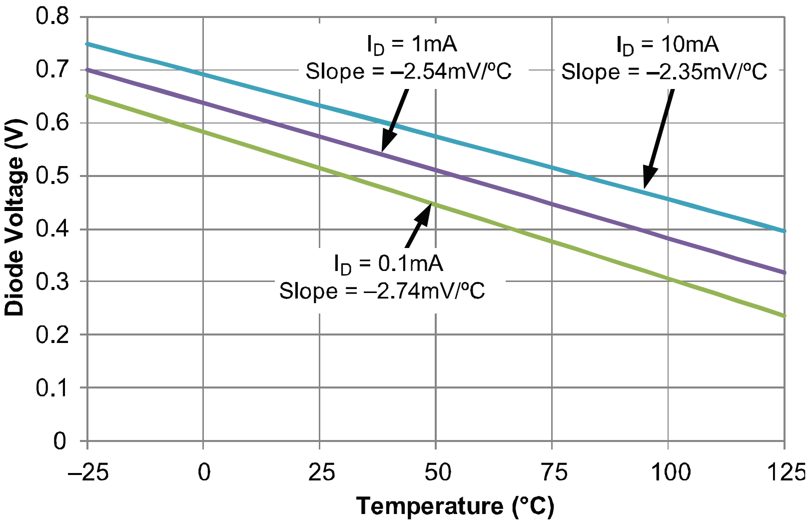


Figure 50: Diode voltage drop vs. temperature

Type J thermocouples translating temperature to voltage (ITS-90 standard)

$$V_t = \sum_{i=0}^{n} c_i \, (T)^i \quad (97)$$

Thermoelectric voltage

Where
V_T = thermoelectric voltage
T = temperature in degrees Celsius
c_i = translation coefficients

Table 24: Type J thermocouple temperature to voltage coefficients

Type J thermocouple temperature to voltage		
	–219℃ to 760℃	**760℃ to 1,200℃**
c0	0.0000000000E+00	2.9645625681E+05
c1	5.0381187815E+01	–1.4976127786E+03
c2	3.0475836930E-02	3.1787103924E+00
c3	–8.5681065720E-05	–3.1847686701E-03
c4	1.3228195295E-07	1.5720819004E-06
c5	–1.7052958337E-10	–3.0691369056E-10
c6	2.0948090697E-13	- -
c7	–1.2538395336E-16	- -
c8	1.5631725697E-20	- -

Type J thermocouples translating voltage to temperature (ITS-90 standard)

$$T = \sum_{i=0}^{n} c_i \, (V_t)^i \qquad (98) \quad \text{Temperature}$$

Table 25: Type J thermocouple voltage to temperature coefficients

Type J thermocouple voltage to temperature			
	–219°C to 0°C	**0°C to 760°C**	**760°C to 1,200°C**
c0	0.000000000E+00	0.000000000E+00	–3.113581870E+03
c1	1.952826800E-02	1.978425000E-02	3.005436840E-01
c2	–1.228618500E-06	–2.001204000E-07	–9.947732300E-06
c3	–1.075217800E-09	1.036969000E-11	1.702766300E-10
c4	–5.908693300E-13	–2.549687000E-16	–1.430334680E-15
c5	–1.725671300E-16	3.585153000E-21	4.738860840E-21
c6	–2.813151300E-20	–5.344285000E-26	- -
c7	–2.396337000E-24	5.099890000E-31	- -
c8	–8.382332100E-29	- -	- -

Type K thermocouples translating temperature to voltage (ITS-90 standard)

$$V_T = \sum_{i=0}^{n} c_i\,(T)^i$$ (99) Thermoelectric voltage for T<0℃

$$V_t = \left[\sum_{i=0}^{n} c_i\,(T)^i\right] + \alpha_0 e^{[\alpha_1(T-126.9686)]^2}$$ (100) Thermoelectric voltage forT>0℃

Where
V_T = thermoelectric voltage
T = temperature in degrees Celsius
c_i = translation coefficients
α_0, α_1 = translation coefficients

Table 26: Type K thermocouple temperature to voltage coefficients

	–219°C to 760°C	**760°C to 1,200°C**
c_0	0.0000000000E+00	–1.7600413686E+01
c_1	3.9450128025E+01	3.8921204975E+01
c_2	2.3622373598E-02	1.8558770032E-02
c_3	–3.2858906784E-04	–9.9457592874E-05
c_4	–4.9904828777E-06	3.1840945719E-07
c_5	–6.7509059173E-08	–5.6072844889E-10
c_6	–5.7410327428E-10	5.6075059059E-13
c_7	–3.1088872894E-12	–3.2020720003E-16
c_8	–1.0451609365E-14	9.7151147152E-20
c_9	–1.9889266878E-17	–1.2104721275E-23
c_{10}	–1.6322697486E-20	- -
α_0	- -	1.1859760000E+02
α_1	- -	–1.1834320000E-04

Type K thermocouples translating voltage to temperature (ITS-90 standard)

$$T = \sum_{i=0}^{n} c_i (V_t)^i \qquad (101)$$ Temperature

Table 27: Type K thermocouple voltage to temperature coefficients

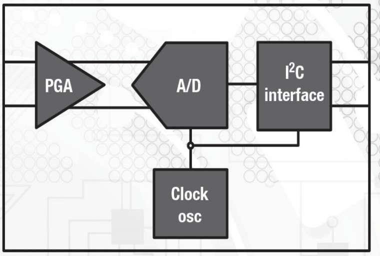
	–219°C to 0°C	0°C to 760°C	760°C to 1,200°C
c0	0.0000000E+00	0.0000000E+00	–1.3180580E+02
c1	2.5173462E-02	2.5083550E-02	4.8302220E-02
c2	–1.1662878E-06	7.8601060E-08	–1.6460310E-06
c3	–1.0833638E-09	–2.5031310E-10	5.4647310E-11
c4	–8.9773540E-13	8.3152700E-14	–9.6507150E-16
c5	–3.7342377E-16	–1.2280340E-17	8.8021930E-21
c6	–8.6632643E-20	9.8040360E-22	–3.1108100E-26
c7	–1.0450598E-23	–4.4130300E-26	- -
c8	–5.1920577E-28	1.0577340E-30	- -
c9	- -	–1.0527550E-35	- -

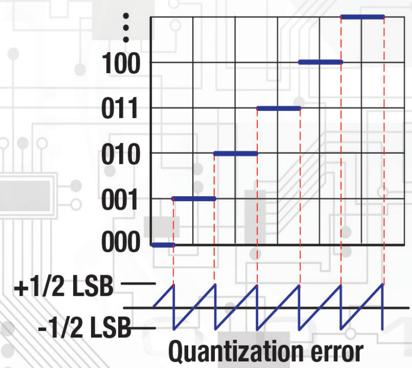
Table 28: Seebeck coefficients for different material

Material	Seebeck coefficient	Material	Seebeck coefficient	Material	Seebeck coefficient
Aluminum	3.5	Gold	6.5	Rhodium	6.0
Antimony	47	Iron	19	Selenium	900
Bismuth	–72	Lead	4.0	Silicon	440
Cadmium	7.5	Mercury	0.6	Silver	6.5
Carbon	3.0	Nichrome	25	Sodium	–2.0
Constantan	–35	Nickel	–15	Tantalum	4.5
Copper	6.5	Platinum	0	Tellurium	500
Germanium	300	Potassium	–9.0	Tungsten	7.5

Note: Units are µV/°C. All data at temperature of 0°C

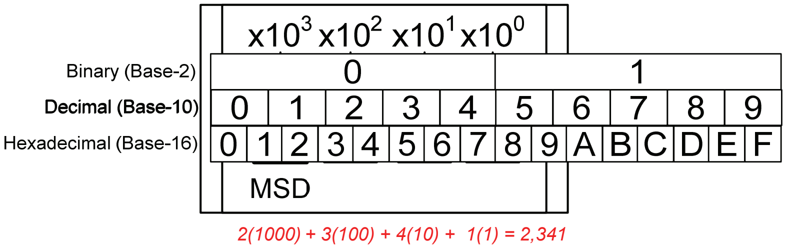
- Binary/hex conversions
- A/D and D/A transfer function
- Quantization error
- Signal-to-noise ratio (SNR)
- Signal-to-noise and distortion (SINAD)
- Total harmonic distortion (THD)
- Effective number of bits (ENOB)
- Noise-free resolution and effective resolution





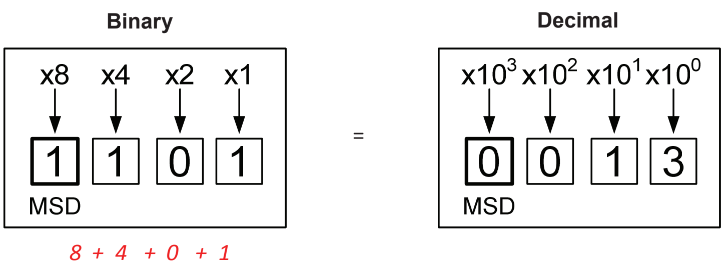
A/D conversion

Numbering systems: Binary, decimal, and hexadecimal

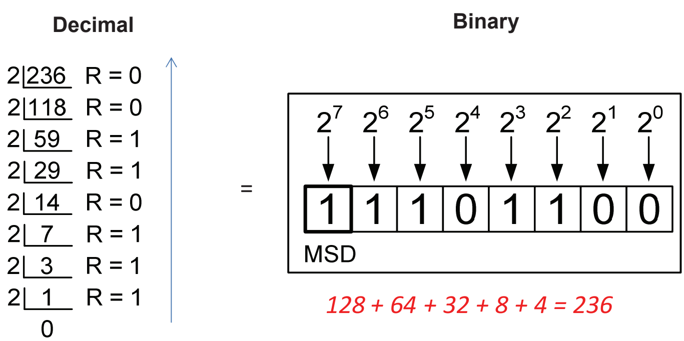


MSD = Most significant digit

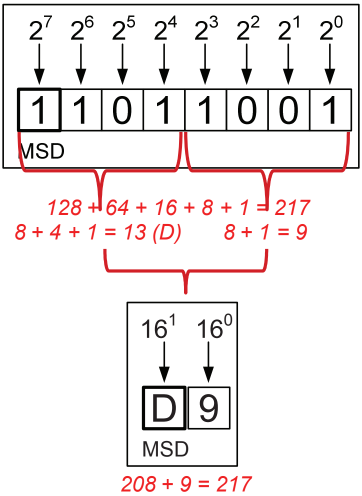
Example conversion: Binary to decimal



Example conversion: Decimal to binary



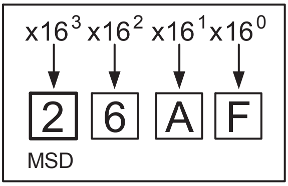
Example conversion: Binary to hexadecimal



Example Conversion: Hexadecimal to binary

Decimal (Base-10)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal (Base-16)	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Hexadecimal



=

Decimal

16 | 9903 R = 15 (F)
16 | 618 R = 10 (A)
16 | 38 R = 6 (6)
16 | 38 R = 2 (2)

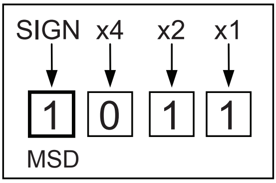
2(4096) + 6(256) + 10(16) + 16(1) = 9903

Table 29: Different data formats

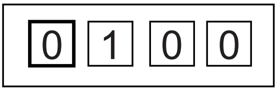
Code	Straight binary	Offset binary	2’s complement
Binary	**Decimal value**	**Decimal value**	**Decimal value**
11111111	255	127	–1
11000000	192	64	–64
10000000	128	0	–128
01111111	127	–1	127
01000000	64	–64	64
00000000	0	–128	0

Converting two’s complement to decimal: Negative number example

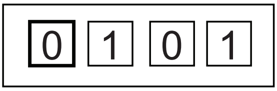
Step 1: Check sign bit
This case is negative



Step 2: Invert all bits



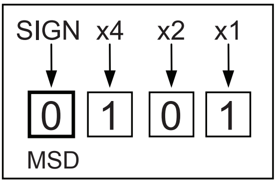
Step 3: Add 1



Final result **–(4+1) = –5**

Converting two’s complement to decimal: Positive number example

Just add bit weights



Final result **4+1 = 5**

Table 30: LSB voltage vs. resolution and reference voltage

		Reference voltage			
		1.024	**1.25**	**2.048**	**2.5**
Resolution	**8**	4 mV	4.88 mV	8 mV	9.76 mV
	10	1 mV	1.22 mV	2 mV	2.44 mV
	12	250 µV	305 µV	500 µV	610 µV
	14	52.5 µV	76.3 µV	125 µV	152.5 µV
	16	15.6 µV	19.1 µV	31.2 µV	38.14 µV
	18	3.91 µV	4.77 µV	7.81 µV	9.53 µV
	20	0.98 µV	1.19 µV	1.95 µV	2.384 µV
	22	244 nV	299 nV	488 nV	596 nV
	24	61 nV	74.5 nV	122 nV	149 nV

Table 31: LSB voltage vs. resolution and reference voltage

		Reference voltage			
		3	**3.3**	**4.096**	**5**
Resolution	**8**	11.7 mV	12.9 mV	16 mV	19.5 mV
	10	2.93 mV	3.222 mV	4 mV	4.882 mV
	12	732 µV	806 µV	1 mV	1.221 mV
	14	183 µV	201 µV	250 µV	305 µV
	16	45.77 µV	50.35 µV	62.5 µV	76.29 µV
	18	11.44 µV	12.58 µV	15.6 µV	19.07 µV
	20	2.861 µV	3.147 µV	3.91 µV	4.768 µV
	22	715 nV	787 nV	976 nV	1.192 µV
	24	179 nV	196 nV	244 nV	298 nV

DAC definitions

Resolution = n	The number of bits used to quantify the output
Codes = 2^n	The number of input code combinations
Reference voltage = V_{REF}	Sets the LSB voltage or current size and converter range
LSB = $V_{REF} / 2^n$	The output voltage or current step size of each code
Full-scale code = $2^n - 1$	The largest code that can be written
Full-scale voltage = $V_{REF} - 1LSB$	Full-scale output voltage of the DAC
Transfer function = $V_{REF} \times (code/ 2^n)$	Relationship between input code and output voltage or current

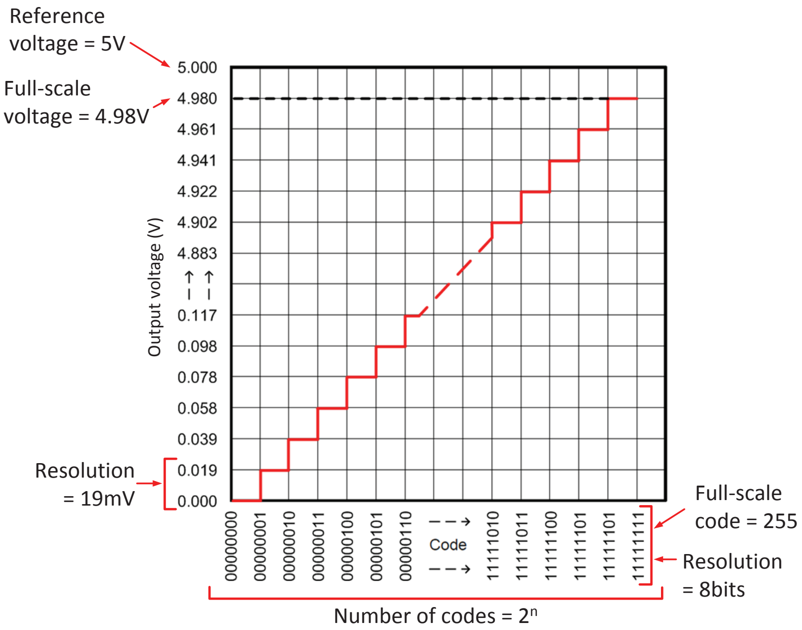


Figure 51: DAC transfer function

ADC definitions

Resolution = n	The number of bits used to quantify the output
Codes = 2^n	The number of input code combinations
Reference voltage = V_{REF}	Sets the LSB voltage or current size and converter range
LSB = $V_{REF} / (2^n - 1)$	The voltage step size of each code. Note that some topologies may use 2^n as opposed to $2^n - 1$ in the denominator.
Full-scale code = $2^n - 1$	The largest code that can be written.
Full-scale voltage = V_{REF}	Full-scale output voltage of the DAC. Note that the full-scale voltage will differ if the alternative definition for resolution is used.
Transfer function = V_{REF} x (code/ 2^n)	Relationship between input code and output voltage or current

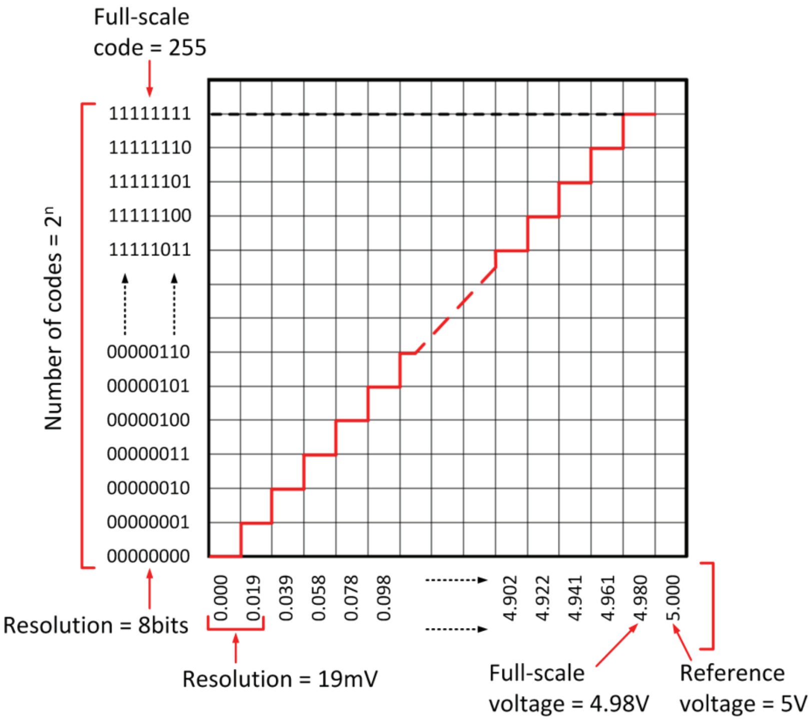


Figure 52: ADC transfer function

Quantization error of ADC

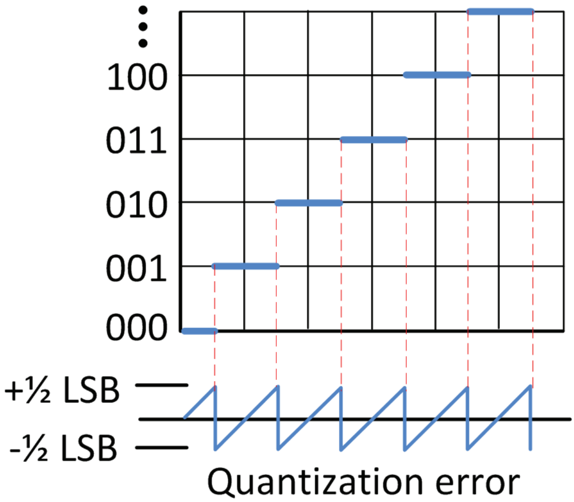


Figure 53: Quantization error of an A/D converter

Quantization error

The error introduced as a result of the quantization process. The amount of this error is a function of the resolution of the converter. The quantization error of an A/D converter is ½ LSB. The quantization error signal the difference between the actual voltage applied and the ADC output (Figure 53). The rms of the quantization signal is $1\text{LSB}/\sqrt{12}$

Signal-to-noise ratio (SNR) from quantization noise only

$$\text{MaxRMSSignal} = \frac{\text{FSR}/2}{\sqrt{2}} = \frac{1\text{LSB} \times 2^{\text{N}-1}}{\sqrt{2}} \quad (102)$$

$$\text{RMSNoise} = \frac{1\text{LSB}}{\sqrt{12}} \text{ from quantization only} \quad (103)$$

$$\text{SNR} = \frac{\text{MaxRMSSignal}}{\text{RMSNoise}} = \frac{1\text{LSB} \times 2^{\text{N}-1}/\sqrt{2}}{1\text{LSB}/\sqrt{12}} = 2^{\text{N}-1}\sqrt{6} \quad (104)$$

$$\text{SNR(dB)} = 20\log(\text{SNR}) = [20\log(2)]\text{N} + 20\log\left(\frac{\sqrt{6}}{2}\right) \quad (105)$$

$$\text{SNR(dB)} \approx 6.02\text{N} + 1.76 \quad (106)$$

Where
FSR = full-scale range of the A/D converter
1LSB = the voltage of 1LSB, $V_{REF}/2^n$
N = the resolution of the A/D converter
MaxRMSSignal = the rms equivalent of the ADC's full-scale input
RMSNoise = the rms noise from quantization
SNR = the ratio of rms signal to rms noise

Example

What is the SNR for an 8-bit A/D converter with 5V reference, assuming only quantization noise?

Answer

$$\text{SNR} = 2^{\text{N}-1}\sqrt{6} = 2^{8-1}\sqrt{6} = 314$$

$$\text{SNR(dB)} = 20\log(314) = 49.9\text{ dB}$$

$$\text{SNR(dB)} = 6.02(8) + 1.76 = 49.9\text{ dB}$$

Total harmonic distortion (Vrms)

$$\mathrm{THD} = \left(\frac{\mathrm{RMSDistortion}}{\mathrm{MaxRMSSignal}}\right) = \frac{\sqrt{{V_2}^2 + {V_3}^2 + {V_4}^2 + \cdots + {V_n}^2}}{V_1} \tag{107}$$

$$\mathrm{THD(dB)} = 20\log\left(\frac{\mathrm{RMSDistortion}}{\mathrm{MaxRMSSignal}}\right) \tag{108}$$

Where
THD = total harmonic distortion, the ratio of the rms distortion to the rms signal
RMSDistortion = the rms sum of all harmonic components
MaxRMSSignal = the rms value of the input signal
V_1 = the fundamental, generally the input signal
V_2, V_3, V_4, …V_n = harmonics of the fundamental

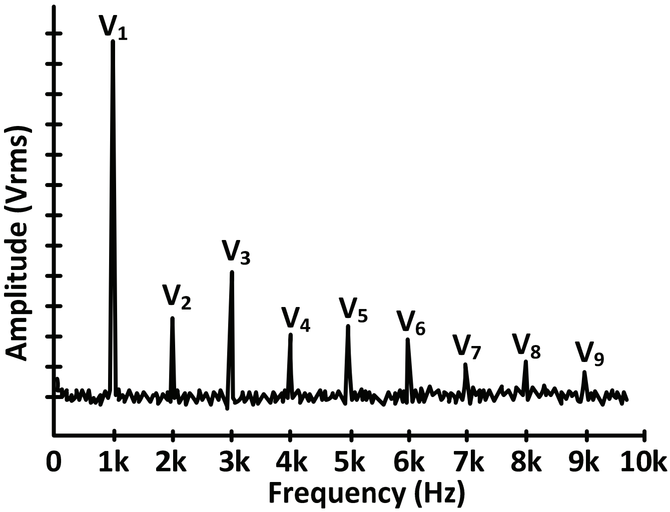


Figure 54: Fundamental and harmonics in Vrms

Total harmonic distortion (dBc)

$$\mathrm{THD} = 10\log\left[10^{\left(\frac{D_2}{10}\right)} + 10^{\left(\frac{D_3}{10}\right)} + 10^{\left(\frac{D_4}{10}\right)} + \cdots + 10^{\left(\frac{D_n}{10}\right)}\right] \quad (109)$$

Where
THD = total harmonic distortion. The ratio of the rms distortion to the rms signal
D_1 = the fundamental, generally the input signal. This is normalized to 0 dBc
D_2, D_3, D_4, …D_n = harmonics of the fundamental measured relative to the fundamental

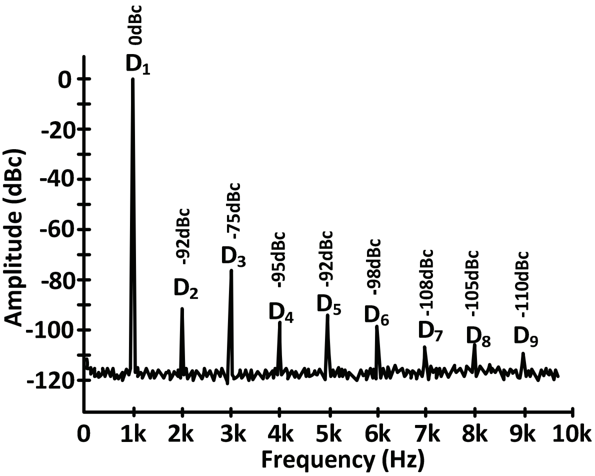


Figure 55: Fundamental and harmonics in dBc

Example

Determine THD for the example above.

Answer

$$\mathrm{THD} = 10\log\left[10^{\left(\frac{-92}{10}\right)} + 10^{\left(\frac{-75}{10}\right)} + 10^{\left(\frac{-95}{10}\right)} + \cdots + 10^{\left(\frac{-110}{10}\right)}\right]$$

$$\mathrm{THD} = -74.76\ \mathrm{dB}$$

Ac signals

Signal-to-noise and distortion (SINAD) and effective number of bits (ENOB)

$$\mathrm{SINAD(dB)} = 20\log\left(\frac{\mathrm{MaxRMSSignal}}{\sqrt{\mathrm{RMSNoise}^2 + \mathrm{RMSDistortion}^2}}\right) \quad (110)$$

$$\mathrm{SINAD(dB)} = -20\log\left(\sqrt{10^{\left(\frac{-\mathrm{SNR(dB)}}{10}\right)} + 10^{\left(\frac{\mathrm{THD(dB)}}{10}\right)}}\right) \quad (111)$$

$$\mathrm{ENOB} = \frac{\mathrm{SINAD(dB)} - 1.76\mathrm{dB}}{6.02} \quad (112)$$

Where
MaxRMSSignal = the rms equivalent of the ADC's full-scale input
RMSNoise = the rms noise integrated across the A/D converters
RMSDistortion = the rms sum of all harmonic components
SINAD = the ratio of the full-scale signal-to-noise ratio and distortion
THD = total harmonic distortion. The ratio of the rms distortion to the rms signal.
SNR = the ratio of rms signal to rms noise

Example

Calculate the SNR, THD, SINAD and ENOB given the following information:
MaxRMSSignal = 1.76 Vrms
RMSDistortion = 50 μVrms
RMSNoise = 100 μVrms

Answer

$$\mathrm{SNR(dB)} = 20\log\left(\frac{1.76\ \mathrm{Vrms}}{100\ \mu\mathrm{Vrms}}\right) = 84.9\ \mathrm{dB}$$

$$\mathrm{THD(dB)} = 20\log\left(\frac{50\ \mu\mathrm{Vrms}}{1.76\ \mathrm{Vrms}}\right) = -\,90.9\ \mathrm{dB}$$

$$\mathrm{SINAD(dB)} = 20\log\left(\frac{1.76\mathrm{V\ rms}}{\sqrt{(100\ \mu\mathrm{Vrms})^2 + (50\ \mu\mathrm{Vrms})^2}}\right) = 83.9\ \mathrm{dB}$$

$$\mathrm{SINAD(dB)} = -20\log\left(\sqrt{10^{\left(\frac{-83.9\ \mathrm{dB}}{10}\right)} + 10^{\left(\frac{-90.9\ \mathrm{dB}}{10}\right)}}\right) = 83.9\ \mathrm{dB}$$

$$\mathrm{ENOB} = \frac{83.9\mathrm{dB} - 1.76\mathrm{dB}}{6.02} = 13.65$$

Dc signals

Noise free resolution and effective resolution

$$\text{NoiseFreeResolution} = \log_2\left(\frac{2^{\mathrm{N}}}{\text{PeaktoPeakNoiseinLSB}}\right) \qquad (113)$$

$$\text{EffectiveResolution} = \log_2\left(\frac{2^{\mathrm{N}}}{\text{rmsNoiseinLSB}}\right) \qquad (114)$$

$$\text{PeaktoPeakNoiseinLSB} \approx 6.6 \times \text{rmsNoiseinLSB} \qquad (115)$$

$$\text{EffectiveResolution} \approx \text{NoiseFreeResolution} + 2.7 \qquad (116)$$

Note: The maximum *effective resolution* is never greater than the ADC resolution. For example, a 24-bit converter cannot have an effective resolution greater than 24 bits.

Example

What is the noise-free resolution and effective resolution for a 24-bit converter assuming the peak-to-peak noise is 7 LSBs?

Answer

$$\text{NoiseFreeResolution} = \log_2\left(\frac{2^{24}}{7}\right) = 21.2$$

$$\text{EffectiveResolution} = \log_2\left(\frac{2^{24}}{\frac{7}{6.6}}\right) = 23.9$$

$$\text{EffectiveResolution} = 21.2 + 2.7 = 23.9$$

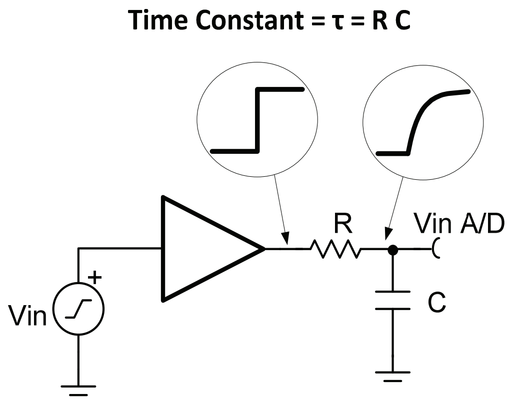


Figure 56: Settling time for RC circuit-related to A/D converters

Table 32: Conversion accuracy achieved after a specified time

Settling time in time constants (N_{TC})	Accuracy in bits	Settling time in time constants (N_{TC})	Accuracy in bits
1	1.44	10	14.43
2	2.89	11	15.87
3	4.33	12	17.31
4	5.77	13	18.76
5	7.21	14	20.20
6	8.66	15	21.64
7	10.10	16	23.08
8	11.54	17	24.53
9	12.98	18	25.97

$$N = \log_2\left(e^{-N_{TC}}\right) \tag{117}$$

Where
N = the number of bits of accuracy the RC circuit has settled to after N_{TC} number of time constants.
N_{TC} = the number of RC time constants

Table 33: Time required to settle to a specified conversion accuracy

Accuracy in bits (N)	Settling time in time constants (N_{TC})	Accuracy in bits (N)	Settling time in time constants (N_{TC})
8	5.55	17	11.78
9	6.24	18	12.48
10	6.93	19	13.17
11	7.62	20	13.86
12	8.32	21	14.56
13	9.01	22	15.25
14	9.70	23	15.94
15	10.40	24	16.64
16	11.09	25	17.33

$$N_{TC} = \ln(2^N) \quad (118)$$

Where
N_{TC} = the number of time constants required to achieve N bits of settling
N = the number of bits of accuracy

Notes

IMPORTANT NOTICE

Texas Instruments Incorporated and its subsidiaries (TI) reserve the right to make corrections, enhancements, improvements and other changes to its semiconductor products and services per JESD46, latest issue, and to discontinue any product or service per JESD48, latest issue. Buyers should obtain the latest relevant information before placing orders and should verify that such information is current and complete. All semiconductor products (also referred to herein as “components”) are sold subject to TI’s terms and conditions of sale supplied at the time of order acknowledgment.

TI warrants performance of its components to the specifications applicable at the time of sale, in accordance with the warranty in TI’s terms and conditions of sale of semiconductor products. Testing and other quality control techniques are used to the extent TI deems necessary to support this warranty. Except where mandated by applicable law, testing of all parameters of each component is not necessarily performed.

TI assumes no liability for applications assistance or the design of Buyers’ products. Buyers are responsible for their products and applications using TI components. To minimize the risks associated with Buyers’ products and applications, Buyers should provide adequate design and operating safeguards.

TI does not warrant or represent that any license, either express or implied, is granted under any patent right, copyright, mask work right, or other intellectual property right relating to any combination, machine, or process in which TI components or services are used. Information published by TI regarding third-party products or services does not constitute a license to use such products or services or a warranty or endorsement thereof. Use of such information may require a license from a third party under the patents or other intellectual property of the third party, or a license from TI under the patents or other intellectual property of TI.

Reproduction of significant portions of TI information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations, and notices. TI is not responsible or liable for such altered documentation. Information of third parties may be subject to additional restrictions.

Resale of TI components or services with statements different from or beyond the parameters stated by TI for that component or service voids all express and any implied warranties for the associated TI component or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

Buyer acknowledges and agrees that it is solely responsible for compliance with all legal, regulatory and safety-related requirements concerning its products, and any use of TI components in its applications, notwithstanding any applications-related information or support that may be provided by TI. Buyer represents and agrees that it has all the necessary expertise to create and implement safeguards which anticipate dangerous consequences of failures, monitor failures and their consequences, lessen the likelihood of failures that might cause harm and take appropriate remedial actions. Buyer will fully indemnify TI and its representatives against any damages arising out of the use of any TI components in safety-critical applications.

In some cases, TI components may be promoted specifically to facilitate safety-related applications. With such components, TI’s goal is to help enable customers to design and create their own end-product solutions that meet applicable functional safety standards and requirements. Nonetheless, such components are subject to these terms.

No TI components are authorized for use in FDA Class III (or similar life-critical medical equipment) unless authorized officers of the parties have executed a special agreement specifically governing such use.

Only those TI components which TI has specifically designated as military grade or “enhanced plastic” are designed and intended for use in military/aerospace applications or environments. Buyer acknowledges and agrees that any military or aerospace use of TI components which have ***not*** been so designated is solely at the Buyer’s risk, and that Buyer is solely responsible for compliance with all legal and regulatory requirements in connection with such use.

TI has specifically designated certain components as meeting ISO/TS16949 requirements, mainly for automotive use. In any case of use of non-designated products, TI will not be responsible for any failure to meet ISO/TS16949.

Products

Audio	www.ti.com/audio
Amplifiers	amplifier.ti.com
Data Converters	dataconverter.ti.com
DLP® Products	www.dlp.com
DSP	dsp.ti.com
Clocks and Timers	www.ti.com/clocks
Interface	interface.ti.com
Logic	logic.ti.com
Power Management	power.ti.com
Microcontrollers	microcontroller.ti.com
RFID	www.ti-rfid.com
OMAP™ Applications Processors	www.ti.com/omap
Wireless Connectivity	www.ti.com/wirelessconnectivity

Applications

Automotive and Transportation	www.ti.com/automotive
Communications and Telecom	www.ti.com/communications
Computers and Peripherals	www.ti.com/computers
Consumer Electronics	www.ti.com/consumer-apps
Energy and Lighting	www.ti.com/energy
Industrial	www.ti.com/industrial
Medical	www.ti.com/medical
Security	www.ti.com/security
Space, Avionics and Defense	www.ti.com/space-avionics-defense
Video and Imaging	www.ti.com/video

TI E2E™ Community   e2e.ti.com

SSZZ022H

Mailing Address: Texas Instruments, Post Office Box 655303, Dallas, Texas 75265

SLYW038A