Determination of the perihelion rotation using the barycenter method

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Abstract

Knowing the barycenter of the solar system and how far it is from the sun (up to twice the solar radius away), one can define fairly accurately how far the common barycenter of all planets (without the sun) is away, since the cumulative mass of the planets is well-known. From this, the orbital disturbance of planets close to the sun and also far from the sun can be reduced to a two-body problem. A very good approximation for the calculation of an orbital disturbance of a planet close to the sun using the barycenter method is ϕ = 2πba^2(1-e) per orbit, with b = 4*10^-28 [rad/m^2], a is the semimajor axis and e is the eccentricity of the planet's orbit. The results are Mercury 574.14 arc seconds, Earth 1161.74", Saturn 2040.15" and Mars 1598.78" per century. Thus, the orbital perturbations using a planetary barycenter are fairly consistent with the measured data. Determining the difference to Newton's theory using general relativity calculations makes no sense as long as the theoretical numbers have not been determined with sufficient accuracy and as long as there are no differences between the measurement data and the theory when using the barycentric method. A computer simulation could help to exactly determine the perihelion rotation of the planets. The angle between the force effect and the orbital direction and the respective distance of the planet to the time-dependent variable planetary barycenter as well as the mean values from this would have to be determined frequently enough.

A possible reason for deviations from the $1/r^2$ central force law is the presence of other bodies that exert additional gravitational forces on the celestial body under consideration. In the case of the planetary orbits, the influence of the respective other planets is the main cause of the perihelion rotations (1-10).

Another cause can be deviations of the central body from the spherical shape. While an extended body of exactly spherical symmetry would produce the same strictly inverse-square gravitational field as a point-like body of the same mass, irregular mass distributions or the equatorial bulge of flattened central bodies lead to deviations from the inverse-square force law and thus to orbital disturbances. The equatorial bulge of the earth causes (among other orbital disturbances) perigee rotations in artificial earth satellites. The flattening of the sun causes perihelion rotations of the planetary orbits, which, however, are much smaller than the rotations caused by the planets themselves due to the insignificance of the flattening and the large distance between the planets.

The curvature of spacetime, an effect of general relativity, is thought to cause a deviation from Newton's equations of motion. This could create a contribution to the perihelion rotation, which is called Schwarzschild precession.

Knowing the barycenter of the solar system and how far it is from the sun (up to twice the solar radius away), one can define fairly accurately the distance P to the common barycenter of all planets (without the sun), since the cumulative mass of the planets is well-known.

$$a_B M = (P - a_B)M' \quad [1]$$

(a_B is the barycenter distance of the solar system, M is the mass of the sun, P is the distance to the barycenter of the planets and M' is the cumulative mass of the planets). The effect of nearby planets is unclear, since the effect is canceled out when the influencing planet is passed by the planet and is on average just as often in front of the planet as behind the planet. In contrast, the barycenter does not rotate on an elliptical orbit, but is sometimes in front of and behind Jupiter and can theoretically also move backwards, which emphasizes the influence of the barycenter as the sole factor.

Using the construct of a barycenter, the orbital disturbance of planets can be reduced to a two-body problem. In order to calculate the perihelion rotation, Newtonian calculations were used.

$$\varphi = \frac{2\pi a sin\alpha \cdot f \cdot 4{,}8481 \cdot 10^{-6}}{\left(\frac{P}{a}\right)^2 \left(\frac{M}{M'}\right)\left(\frac{\pi a_M^2(1-e_M)^2}{4\pi a_M^2}\right)\left(\frac{4\pi a^2}{\pi a^2(1-e)^2}\right)\left(\frac{a(1-e)}{a_M(1-e_M)}\right)\left(\frac{10^7 m}{43.1''}\right)} = 2\pi b f a^2(1-e)$$

$$b = 4.049 * 10^{-28}\,\frac{rad}{m^2} \quad [2]$$

(M’ is the cumulative mass of all planets, P is the distance to the common barycenter, M is the mass of the sun, a_M is the semiaxis of the Mercury used for calibration, a is the semiaxis of the planet, e is the eccentricity, e_M is the eccentricity of Mercury, 10^7 m corresponds to 43.1”, $4{,}8481 * 10^{-6}$ rad corresponds to one arc second, α is the angle between the force direction of the barycenter and the movement direction of the planet and f is a factor from the flattening of the planet and the reduced circumference of the orbit due to the eccentricity compared to 2πa). The formula is built out of the concept, that the barycenter influences the perihelion of a planet in such a way, that, in the time of a total orbiting the perihelion moves proportional to the lower mass and greater distance of the barycenter, while still considering the ratio between the circle area circumscribed by the aphelion (2a) and the area enclosed by that of the perihelion distance. Also of importance is the fact, that the different perihelion distances muss be taken into account.

The results are Mercury 574.14 arc seconds, Earth 1161.74", Saturn 2040.15" and Mars 1598.78" per century. In case of Mars, which is in the next neighborhood of Jupiter, the influence of Jupiter is sometimes (0.46) larger than that of all planets together, expressed by the planet’s barycenter, so that here the correction formula must be employed [3]. For the planet Saturn a correction factor [4] must be used, since the mass of Saturn must be subtracted from the mass of the barycenter and its position is therefore $1159.3 \cdot 10^9$ m distant to the sun.

$$\varphi = 2\pi bfa^2(1-e)\cdot 0.64\left(\frac{P}{J}\right)^2\left(\frac{M_J}{M'}\right) + 0.36\cdot 2\pi bfa^2(1-e) \quad [3]$$

(M_J is the mass of Jupiter, M' is the mass of the barycenter, J is the distance to Jupiter).

$$\varphi = \frac{M' - M_S}{M'}\cdot\left(\frac{P_S}{a_S}\right)^2\cdot 2\pi bfa^2(1-e) \quad [4]$$

(M_S is the mass of Saturn, P_S is the barycenter without Saturn, a_S is the semiaxis of Saturn).

	a (10^9 m)	e	f_C	f_F	φ (“)
Mercure	57.909	0.2056	1.0106	1.0	574.14
Earth	149.598	0.00167	1.00133	1.0033641	1161.74
Mars	227.99	0.00592	1.00178	1.00592	1598.78
Saturn	1433.4	0.1086	1.002796	1.1086	2040.15

$$f = \frac{1.009 f_F}{f_C} \quad [5]$$

Crantor, the largest of three asteroids with a diameter of 70 kilometers, has been suspected by astronomers for a few years to be in resonance with Uranus. This means that the orbital times of the two are related to each other in such a way that they regularly influence each other through their gravity. In the case of Crantor and Uranus, the ratio of these orbital times is almost exactly 1:1, so they take exactly the same amount of time to orbit the sun once. Carlos de la Fuente Marcos and Raúl de la Fuente Marcos from the Universidad Complutense de Madrid have now been able to confirm this with the help of a computer simulation. Furthermore, the perihelion rotation of Venus ist very small. On the one hand, the effect is very small because of the almost circular sun path of Venus. And then the "8:13 resonance" of the Earth-Venus system probably comes into play: in 8 orbits of the earth there are (almost exactly) 13 orbits of Venus. It is assumed that this is responsible for the fact that the earth slows down or stabilizes the Venus perihelion rotation instead of kicking it. In addition, Neptune with Kuiper belt objects and Jupiter with his Moons are also involved in orbit resonances. Therefore, these four planets have smaller perihelion rotations than calculated by the Newtonian theory. The elliptical shape of the planetary orbits was first described empirically in 1609 using Kepler's laws. The physical justification followed in the middle of the 17th century with Isaac Newton's celestial mechanics. With his universal law of force, which also describes gravitation, it had become possible to examine the orbital disturbances that the planets mutually cause in more detail. In particular, the observed apse rotations of the planets and the moon could be explained almost entirely by Newton's theory.

In the mid-19th century, however, Urbain Le Verrier used observations of Mercury's transits for a particularly accurate survey of Mercury's orbit and, using the improved data, found that Mercury's perihelion rotation was slightly stronger than expected. According to the celestial mechanical calculations, it should be about 530” (arc seconds) per century, with about 280” due to the influence of Venus, about 150” to perturbations by Jupiter and about 100” to the remaining planets (11). However, the observed perihelion rotation (modern value: 571.91”/century) (12) was significantly larger; the modern value for the discrepancy is 43.11”.

Le Verrier, who had already successfully discovered Neptune by investigating unexplained parts in the orbital disturbances of Uranus, suspected the cause of the discrepancy in Mercury to be a disturbance by a previously unknown planet on an orbit within Mercury's orbit. This planet was given the name Vulcan, but could not be discovered despite an extensive search - including during several solar eclipses. Likewise, no asteroid belt close to the sun responsible for the disturbances could be detected. Others suspected the dust belt responsible for the zodiacal light or saw at least part of the cause in a flattened shape of the sun due to its rotation (see also below), but were ultimately unsuccessful with their attempts at an explanation (13).

Further attempts at an explanation cast doubt on the validity of Newton's law of force. For example, Levy (1890) and especially Paul Gerber (1898) succeeded in deriving the excess on the basis of electrodynamic force laws, provided that the propagation speed of gravitation is equal to the speed of light. Gerber's formula for the deviation from perihelion was already formally identical to that later formulated by Einstein. However, the underlying force laws were wrong and theories of this kind had to be discarded (14,15).

Only Albert Einstein's General Theory of Relativity (GRT), which describes gravitation as the curvature of space-time, the structure of which is also influenced by the celestial bodies, was able to explain the apparent excess (16). This achievement is considered one of the mainstays of general relativity and its first major confirmation. The relativistically calculated fraction of 42.98″ (17) agrees very well with the calculated excess of 43.11″. The reason for the relativistic effect apparently lies in the slight deviation of the relativistic gravitational field from the strictly inverse-square behavior.

The agreement between the observation and the relativistic calculation was thought to be less good if a significant part of the observed excess was due to rotational flattening of the sun and the remaining part to be explained was therefore significantly smaller than calculated according to ART. Attempts to measure the extremely small oblateness of the sun have yielded conflicting results over a long period of time, so that it has always been a bit doubtful how well the relativistic prediction actually agrees with the observation. However, helio-seismological studies have now reliably determined the quadrupole moment of the sun to be $(2.18 \pm 0.06) \cdot 10-7$; this quadrupole moment contributes only a few hundredths of an arc second to the perihelion spin and is therefore negligible (19). Another way to determine uses the fact that the relativistic and the conditional part of the total perihelion rotation decrease at different rates with increasing distance from the sun and can thus be separated from each other by comparing the total rotations of different planets. Such an investigation (20) delivered a result of = $(1.9 \pm 0.3)\cdot10^{-7}$ that is close to that of helioseismology.

Conclusions

In conclusion, the orbital perturbations using a planetary barycenter are fairly consistent with the measured data. Determining the difference to Newton's theory using general relativity calculations makes no sense as long as the theoretical numbers have not been determined with sufficient accuracy and as long as there are no differences between the measurement data and the theory when using the barycentric method. A computer simulation could help to exactly determine the perihelion rotation of the planets. The angle between the force effect and the orbital direction and the respective distance of the planet to the time-dependent variable planetary barycenter as well as the mean values from this would have to be determined frequently enough.

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